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IEEE Journal of Quantum Electronics

DOI:
10.1109/JQE.2016.2609406

Published: 01/11/2016

Peer reviewed version

Dynamics and Stability of Mutually Coupled Nano-Lasers

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***Abstract*—The dynamics of mutually coupled nano-lasers has been explored theoretically. Calculations have been performed using rate equations which include the Purcell cavity-enhanced spontaneous emission factor *F* and the spontaneous emission coupling factor *β*. In the analysis, the influence of *F* and *β* has been evaluated for varying optical injection strength and distance between the two lasers. It is observed that, in general, for increased bias current the system can maintain stable output for a larger mutual coupling strength. It is also found that for short inter-laser distances and larger *F* the stability of mutually coupled nano-lasers is enhanced.**

***Index Terms*—Mutually-coupled semiconductor laser, nano-lasers, enhanced spontaneous emission**

I. Introduction

THE properties of mutually coupled lasers have been a topic of interest for many decades [1]. Activity on mutually coupled semiconductor lasers also has a significant pedigree [2], [3] with much effort having been given to identifying regimes of synchronization and instabilities [4]-[6]. In such work a variety of semiconductor lasers have been deployed with Vertical Cavity Surface Emitting Lasers (VCSELs) offering particularly rich dynamical scenarios [7]. Optical injection is well-known as a means for enhancing the modulation bandwidth of semiconductor lasers [8] and in recent work modulation bandwidth enhancement in mutually-coupled monolithically integrated laser diodes has been reported [9]. Semiconductor nano-lasers [10], [11] are of interest not least for their potential for incorporation in compact photonic integrated circuits. The motivation for the present paper is to determine the dynamical properties of mutually coupled nano-lasers with a particular view as to how such properties may assist with the practical utilisation of nanolasers.

Experimental investigation has been carried out on nanolaser structures such as, micro-post [12] nano-pillar and bowtie [13], [14], Fabry-Perot [15], nanowire [16], spaser-based [17], and nano-patch [18] lasers, where continuous wave lasing is observed by optical pumping [19] and electrical pumping [20].

Manuscript submitted July 4, 2016. The authors gratefully acknowledge financial support provided by the Sêr Cymru National Research Network in Advanced Engineering and the Materials, and from the International Science & Technology Cooperation Program of China (2014DFA50870).

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Such nano-lasers are anticipated to exhibit enhanced dynamical performance which may arise from a combination of physical factors including the Purcell spontaneous emission enhancement factor *F*, and enhanced spontaneous emission coupling expressed in the factor, *β*. In recent work, the impact of Purcell enhanced spontaneous emission on the modulation performance of nano-LEDs and nanolasers [21] have been examined. In complementary work on the dynamical performance of nano-lasers it was shown by means of a simple analysis that the direct-current modulation bandwidth of such lasers may suffer deleterious effects due to increased *F* and *β* [22]. A number of recent investigations of the dynamical performance of nanolasers have been undertaken. The behaviour of optically pumped nanolasers has been studied including the role of the spontaneous emission factor, *β*, in achieving single mode operation of nanolasers [23]. Ding et al. explored the dynamics of electrically pumped nanolasers where the effects of *F* and *β* on nanolaser performance were studied [24]. A more recent investigation of the effect of *F* and *β* shows that modulation bandwidth of up to 60GHz can be achieved for metal clad nano-lasers [25].Theoretical work has also been reported on the control of dynamical instability in such lasers [26].

Enhanced spontaneous emission, coupled with reduced laser threshold current, can lead to a reduction of the laser turn-on delay. Strong damping will give rise to a long tail in the switch-off dynamics of the laser and hence will compromise both analogue and digital direct current modulation of the laser [22]. In our recent work on the effect of external optical feedback in nano-lasers, it has been identified that strong damping of the relaxation oscillations due to high *F* and *β*, causes the chaos to occur at higher feedback fractions [27]. Similar conclusions have been drawn in explorations of phase conjugate optical feedback effects in nano-lasers [28]. Nano-lasers subject to external optical injection have also been predicted to exhibit more stable behaviour [28]. In this context it is apposite to investigate the dynamical behaviour of mutually coupled nan-lasers so as to ascertain whether this configuration may offer novel functionality – particularly in the context of photonic integrated circuits.

The paper is organized as follows. Firstly, the nano-laser dynamical model is introduced in section II, followed by the results in section III. Finally, in section IV, conclusions are drawn based on the results obtained.

II. Nano-laser Dynamics

A schematic diagram of mutually coupled nano-lasers is shown in Fig. 1.This system is modelled using modified forms

of rate equations which incorporate the Purcell enhanced spontaneous emission factor, F and spontaneous emission coupling factor, β have been included as introduced in [21].

Fig. 1 Schematic diagram of mutually coupled semiconductor nano-lasers.

Work by Gu. et al. [29] and Gerard et al. [30] has included detailed calculation of the spontaneous emission rate in nano-lasers. This work has shown that there is an interdependence between the spontaneous emission coupling factor and the Purcell enhancement factor. Such an approach has been adopted by [14] in the formulation of dynamical equations for nano-lasers. However, the precise relationship between these two factors is dependent upon the specific nano-laser structure under consideration. The aim of the present work is to explore how modifications of the spontaneous emission rate impact the performance of a generic nano-laser device subject to optical injection. In this context, and not with standing the work in [29], [30], the Purcell factor and the spontaneous emission factor are taken to be independent parameters. In this way it is possible to identify the trends in device performance consequent to changes in these two parameters. It is fully recognised, however, that in a practical context and due to the work of [29], [30], there will be constraints on the accessible values of these parameters and thus not all combinations of values of these parameters treated here will necessarily be available

It is underlined that the Purcell factor and the spontaneous emission coupling factor impact the spontaneous emission rate as shown in Eqs. (1) and (2) below. In contrast, the phase Eq. (3) is dependent on the laser gain and hence is not affected by the enhanced spontaneous emission. The gain compression does not appear in Eq. (3) because the gain saturation induced by spectral-hole burning is symmetrical around the emission frequency as in [31], [32].

$$\frac{dS_{\mathrm{I,II}}(t)}{dt}=\Gamma\left[\frac{F\beta N_{\mathrm{I,II}}(t)}{\tau_n}+\frac{g_n\left(N_{\mathrm{I,II}}(t)-N_0\right)}{1+\varepsilon S_{\mathrm{I,II}}(t)}S_{\mathrm{I,II}}(t)\right]-\frac{1}{\tau_p}S_{\mathrm{I,II}}(t)+2\frac{\kappa_{inj}}{\tau_{in}}\sqrt{S_{\mathrm{I,II}}(t)S_{\mathrm{II,I}}(t-\tau_{inj})}\cos(\theta_{\mathrm{I,II}}(t)) \quad (1)$$

$$\frac{dN_{\mathrm{I,II}}(t)}{dt}=\frac{I_{\mathrm{I,II}}}{eV_b}-\frac{N_{\mathrm{I,II}}(t)}{\tau_n}\left(F\beta+(1-\beta)\right)-\frac{g_n\left(N_{\mathrm{I,II}}(t)-N_0\right)}{1+\varepsilon S_{\mathrm{I,II}}(t)}S_{\mathrm{I,II}}(t) \quad (2)$$

$$\frac{d\phi_{\mathrm{I,II}}(t)}{dt}=\frac{\alpha}{2}\Gamma g_n\left(N_{\mathrm{I,II}}(t)-N_{th}\right)\pm\Delta\omega-\frac{\kappa_{inj}}{\tau_{in}}\sqrt{\frac{S_{\mathrm{II,I}}(t-\tau_{inj})}{S_{\mathrm{I,II}}(t)}}\sin(\theta_{\mathrm{I,II}}(t)) \quad (3)$$

$$\theta_{\mathrm{I,II}}(t)=\pm\Delta\omega t+\omega_{\mathrm{II,I}}\tau_{inj}+\phi_{\mathrm{I,II}}(t)-\phi_{\mathrm{II,I}}(t-\tau_{inj}) \quad (4)$$

In the rate equations the subscripts 'I' and 'II' represent laser I and laser II respectively.$S(t)$ is the photon density and $N(t)$ is the carrier density, $\emptyset(t)$ is the phase of laser, $\theta(\mathrm{t})$is the phase of injection laser. Γ is the confinement factor; τ_n and $\tau_p$are the radiative carrier lifetime and photon lifetime respectively. g_n is the differential gain that takes into account the effect of group velocity, N_o is the transparency carrier density, ϵ is the gain saturation factor and α is the linewidth enhancement factor. I is the dc bias current, V_a is the volume of the active region, e is the electron charge and N_{th} is the threshold carrier density. $\Delta\omega$ is the angle frequency detuning between laser I and laser II. $\tau_{inj}=D/c$ is the injection delay, where D is the distance beween laser I and II, c is the speed of light in free space. $\tau_{in}=2nL/c$ is the round-trip time in inner cavity of laser, where L is the cavity length and n is group refractive index. The mutual-coupled optical injection into the laser I and laser II is controlled by the injection fraction, κ_{inj}, which is relation to injection parameter, R_{inj}, and reflectivity of the laser, R. Here, κ_{inj} can be caculated by,

$$\kappa_{inj}=(1-R)\sqrt{R_{inj}/R} \quad (5)$$

The values of the nano-lasers device parameters used are those found in [27].

Attention is drawn to the fact that an increase of spontaneous emission via the Purcell factor F, or the spontaneous emission coupling factor β may lead to a reduction of the laser threshold current [22]. In the present analysis it has been found that for Purcell factors in the range $1 \le F \le 30$ and $\beta = 0.05$ or 0.1 used here, only a 2 % change in the threshold current occurs and has no impact on the general trends found via the following calculations. A threshold current of 1.1 mA is used here.

III. Results

The principal aim of the paper is to study the dynamical behaviour of mutually coupled nano-lasers giving attention to the role played by the Purcell spontaneous emission enhancement factor F and the spontaneous emission coupling factor β with different distances, D, between the lasers and for a range of laser bias currents I. That configuration offers a myriad of operating conditions which can be expected to give rise to many dynamical scenarios whose exhaustive exploration is anticipated to yield results which will be theoretically interesting and of practical relevance. As, to the best of our knowledge, this is the first attempt to detail the dynamics of mutually coupled electrically-pumped nano-lasers our focus is on highlighting novel features of the dynamics whilst utilizing only a relatively small part of the available parameter space. To this end, in the present calculations it is assumed that the nano-lasers are physically and operationally identical: specifically that they are driven with the same bias currents. That choice obviates exploration of interesting dynamical behaviours which can arise, e.g., due to frequency detuning between the lasers – as found for optical injected nano-lasers [33]. Such effects will merit future analysis. On the other hand, the adoption of a symmetric configuration has the advantage of clearly focussing effort on determining the dependence of the results on the parameters which distinguish nano-lasers. As is shown here, experimentally significant results emerge from the analysis of the defined symmetric configuration. Those results, in turn, motivate further exploration of the dynamics of mutually coupled nano-lasers.

The results presented here have been found using the rate Eqs. (1)–(4). As already indicated, the salient parameters for nano-lasers are the Purcell factor F and the spontaneous emission coupling factor, β. The bias current used to drive the lasers is an important operational parameter. For mutually coupled lasers the distance, D, between them is of significance. Variations in these parameters gives rise to the results presented here.

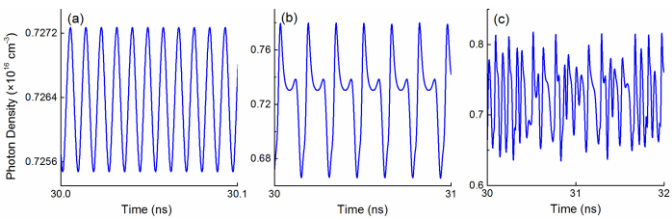
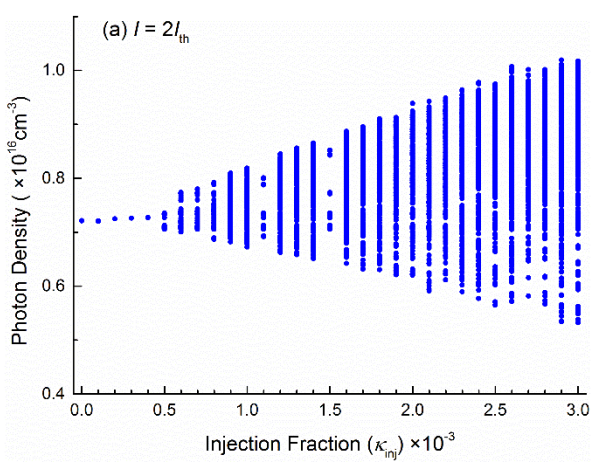


Fig. 2 Time series of photon densities for F = 14, β = 0.05, I=2I_{th}, D=1.5cm. (a) periodic signal at κ_{inj} = 0.4×10^{-3}. Unstable dynamics where the output is similar to period doubling at (b) κ_{inj} = 0.7×10^{-3}, and (c) at κ_{inj} = 1.0×10^{-3} where the output shows non-stationary period doubling.

So as to demonstrate explicitly the range of behaviors which arise in this configuration we show in Fig. 2, the changes in dynamics which are consequent to a change in optical coupling between the lasers for fixed bias currents and fixed distance between the nano-lasers. The results here are for the case that F = 14, β = 0.05. Figure 2(a)–(c) shows the dynamics of the nano-laser with increasing injection at zero detuning, including periodic output for κ_{inj} = 0.4×10^{-3} and period doubling at κ_{inj} = 0.7 × 10^{-3}. The laser output varies with a non-stationary period doubling at κ_{inj} = 1.0×10^{-3}.

A. Dynamics

In order to obtain a more generic representation of the dynamical behaviour of such systems extensive use is made of bifurcation analysis. Bifurcation diagrams are obtained for the photon density as a function of the optical coupling between the nano-lasers. Such representations have been offered in previous work [33] and have enabled conclusions to be drawn about the nature of the dynamics. Bifurcation diagrams are produced by using a long time series of the photon density S(t) as the coupling strength κ_{inj} is varied.



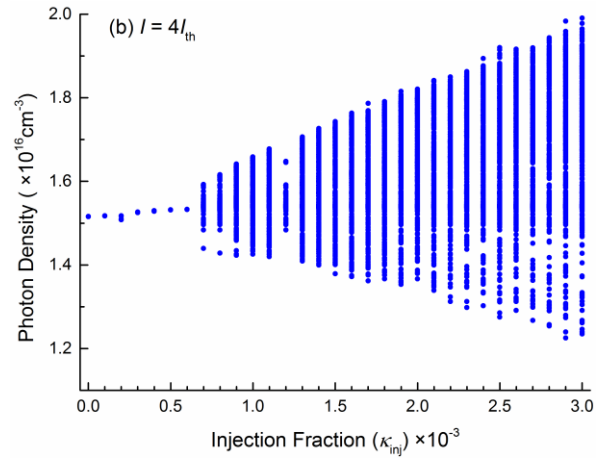


Fig. 3 Bifurcation diagram of photon density versus injection coupling at F=14, β=0.05, D=1.5cm at (a) I=2I_{th} and (b) I=4I_{th}.

Attention is given to the influence of the bias current I with simulation having been performed at I = 2I_{th} and I = 4I_{th} where the threshold current is I_{th} = 1.1 mA. Figure 3 shows that at higher bias current the system is more stable as exemplified by the increased optical coupling power required to initiate a bifurcation cascade which culminates in optical chaos. This tendency also can be found when β is increased to 0.1, as shown in Fig. 4, and for F=30 with D increasing to 3 cm shown in Fig. 5. For higher threshold, I=4I_{th}, the stable state is sustained for larger injection fractions.

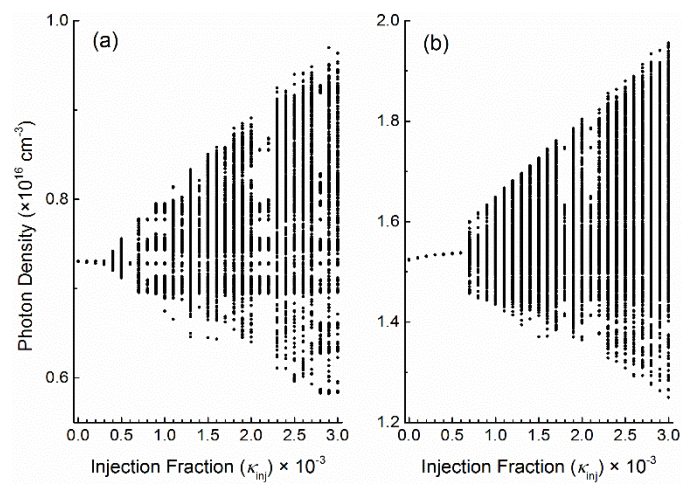
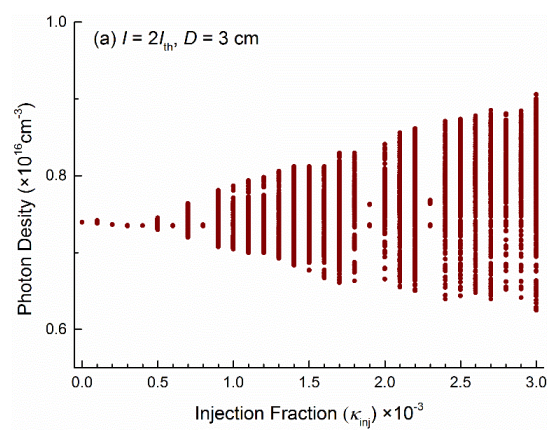


Fig. 4 Bifurcation diagram of photon density versus injection coupling at F=14, β=0.1, D=1.5cm at (a) I=2I_{th} and (b) I=4I_{th}.



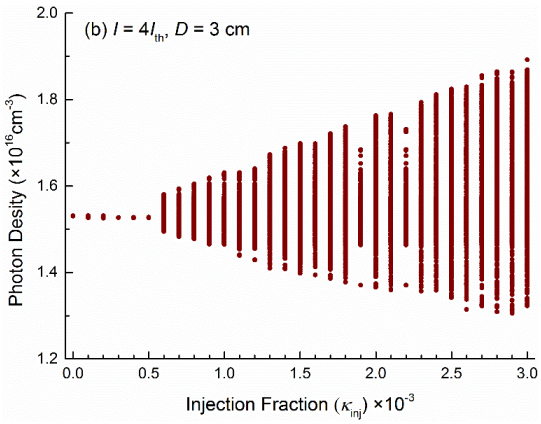


Fig. 5 Bifurcation diagram of photon density vs injection parameter at D=0.5 cm, F=30, β=0.1 and at (a) I=2I_{th}, (b) I=4I_{th}.

As the inter-laser distance is decreased to D= 0.5 cm, as shown in Fig. 6, the system exhibits more stable operation for F=30 and β=0.1. Conversely, at this distance but for the Purcell spontaneous emission enhancement value of F=14, shown in Fig. 7 the system become unstable when κ_{inj} is larger than 0.4 $\times 10^{-3}$. In contrast for the case of condition of D=1.5cm, the threshold for instability is κ_{inj} =0.7 $\times 10^{-3}$. It is, therefore, manifest that the present system of mutually-coupled nano-lasers provides a broad range of dynamical scenarios which will repay further analysis in order to identify optimal arrangements for practical applications.

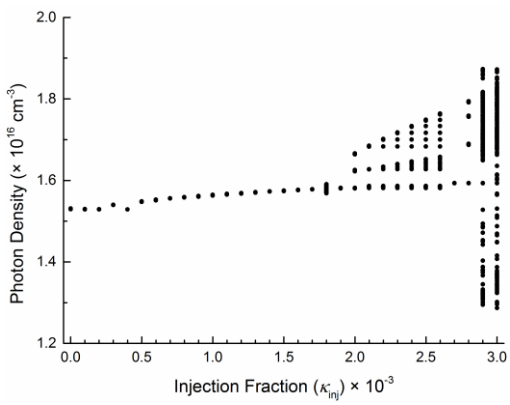


Fig. 6 Bifurcation diagram of photon density vs injection parameter at D=0.5 cm. F=30, β=0.1 and I=4I_{th}.

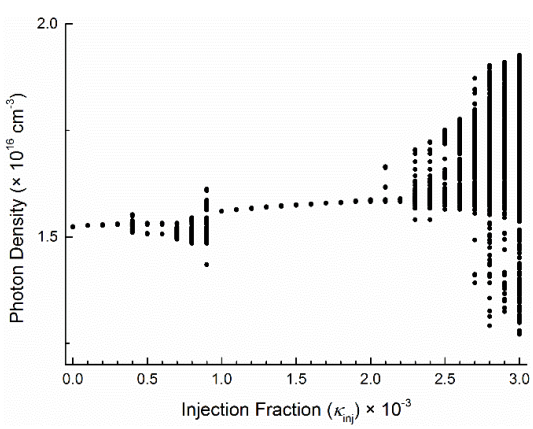


Fig. 7 Bifurcation diagram of photon density vs injection parameter at D=0.5 cm, F=14, β=0.1 and I=4I_{th}.

B. Stabilisation

The practically significant conclusion to be drawn from the results encapsulated in Figs. 3 to 5 is that use of higher bias currents enables the preservation of stable operation in this system. This is quantified by the strength of the mutual coupling required to de-stabilise the laser. This tendency also emerges at the larger β. The conclusion concerning the stabilisation is made transparent in Fig. 8 where the coupling power to destabilise the system is shown as a function of the laser bias current for two values of the Purcell factor. The clear trend over this range of bias currents is that by increasing the bias current the system remains stable for higher values of mutual coupling.

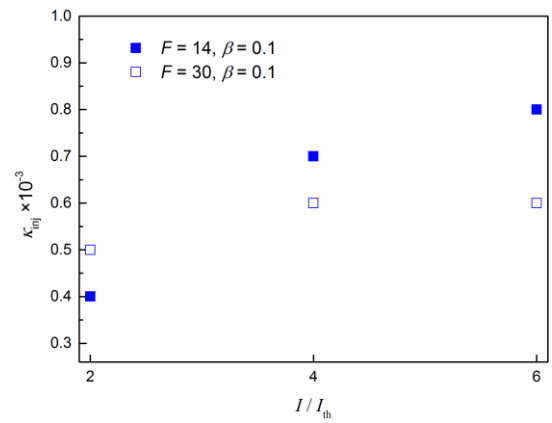


Fig. 8 Threshold for instability vs. bias current of mutual-coupled nano-lasers. Solid and open squares denote F=14 and F=30, respectively. Here, β=0.1 and D=1.5 cm.

It is expected that nano-lasers will possess rather low threshold currents and thus it is possible that they may be operated at very high currents above threshold. In that case it is valid to explore whether the trend exhibited in Fig. 8 persist at much higher bias currents. The line-width enhancement factor α is also expected to play a significant role in the determination of the stability properties of this system. The influences of bias current and the linewidth enhancement factor are explored in the results shown in Fig. 9.

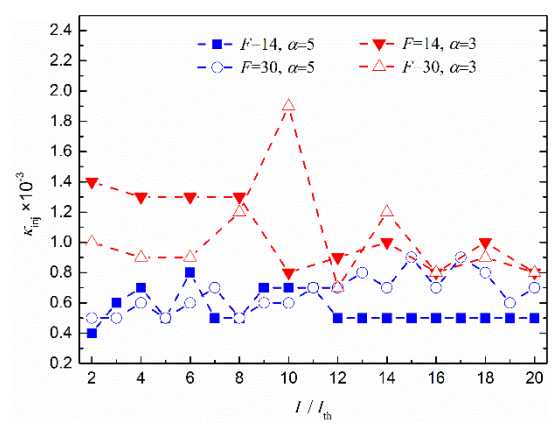


Fig. 9 Threshold for instability v. bias current of mutually-coupled nano-lasers for F = 14 (solid) and F = 30 (open) with β = 0.1 and D=1.5 cm.

Figure 9 confirms the general trend of Fig. 8 but simultaneous illustrates that the dependence is not monotonic. The impact of the linewidth enhancement factor is also apparent： for the lower value of the linewidth enhancement factor ($\alpha = 3$) a relatively high instability threshold is observed. However, if it becomes practical to access very high bias currents of order 10 times the threshold current then, particularly for the case where the linewidth enhancement factor is 5, the threshold for instability reaches a plateau and hence the system will be robust over a significant range of currents.

C. Distance between lasers

Apart from the driving bias current a feature of the system which can be easily changed is the distance between the lasers. It is of interest therefore to characterise the stability of the system as a function of the distance between the lasers. Such results are shown in Fig. 10. It is remarked that consideration is only given to relatively short distances – of order 1 cm – since such distances are expected to be of interest in possible applications of such lasers in photonic integration contexts.

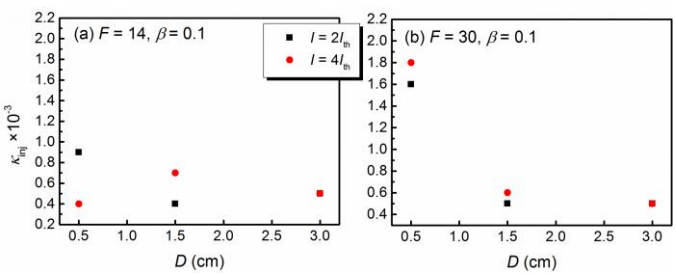


Fig. 10 Threshold for instability vs. distance between mutual-coupled nano-lasers. (a) F=14, β=0.1, (b) F=30, β=0.1. Black squares and red circles denote I=2I_{th} and I=4I_{th}, respectively.

The calculations summarised in Fig. 10 indicate that rather different behaviours arise depending on the accessed value of the Purcell factor. The higher value for the Purcell factor appears to increase the propensity for instability as the distance between the lasers is increased as seen in Fig. 10(b). For the lower value of the Purcell factor included in Fig. 10(a) there appears to be a limited range of distances over which stability is increased but eventually there is a tendency to reduce the instability threshold. It is noted that in Fig. 10(a) and 10(b) the threshold for instabilities is greater for larger values of the bias current – confirming the conclusions drawn from Figs. 8 and 9.

A more extensive analysis of the instability threshold versus distance is summarised in Fig. 11 for two values of the bias current. Here also we capture the influence of the linewidth enhancement factor. Although not conclusive, a trend seems to emerge for the larger value (α = 5) of the linewidth enhancement factor that for greater distances the threshold for instability is decreased. In the case of α = 3 a more involved dependence of the instability threshold emerges with, as expected, generally speaking higher instability thresholds than the case of the higher value of the linewidth enhancement factor. The further exploration of such dependences will be of particular value when, in future work, attention is given to mutual coupling of non-identical nano-lasers. In relation to possible future applications of nano-lasers in photonic integrated circuits, it is pointed that for the shortest distance considered in Fig. 11(b) namely D=0.5 cm, for α = 3, there is a markedly higher level of stability for larger values of F.

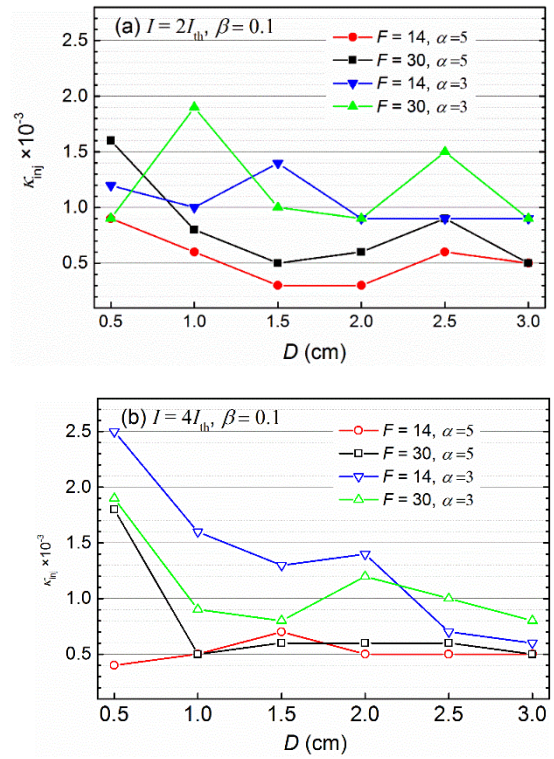


Fig. 11 Threshold for instability vs. distance between mutual-coupled nano-lasers for several values of F with β=0.1. (a) I=2I_{th} and (b) I=4I_{th}

IV. Conclusions

The first calculations have been undertaken of the dynamical properties of mutually-coupled identical nano-lasers. Attention has been given to the onset of instabilities in this system. It is shown that for higher bias currents there is a tendency for the system to be more robust to increased mutual coupling. It has been found that with very short distances between nano-lasers with high values of the Purcell factor stability is preserved for higher values of the mutual coupling. This augurs well for future applications. The present calculations provide a platform for further analysis of the dynamical properties of mutually-coupled nano-lasers where, in particular consideration may be given to physically non-identical lasers or lasers in different operating regimes. Such analysis would, in particular, incorporate effects arising due to detuning between non-identical lasers and would enable the detailed definition of the dynamical regimes accessed by this system - as has been previously performed in other configurations (see e.g. [34]). It is confidently expected that interesting dynamical behaviours will emerge from such further work.

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