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A conceptual e-learning system for teaching mathematics

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PRIFYSGOL
BANGOR
UNIVERSITY

School of Computer Science
College of Physical & Applied Sciences

**A conceptual e-learning system for
teaching mathematics**

Yasser Mohammed Alghurabi

Submitted in partial satisfaction of the requirements for the
Degree of Doctor of Philosophy
in Computer Science

Supervisor Dr. William J. Teahan

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“Tell me and I will forget. Show me and I will remember. Involve me and I will understand.”

— **Confucius**

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Statement of Originality

The work presented in this thesis/dissertation is entirely from the studies of the individual student, except where otherwise stated. Where derivations are presented and the origin of the work is either wholly or in part from other sources, then full reference is given to the original author. This work has not been presented previously for any degree, nor is it at present under consideration by any other degree awarding body.

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Abstract

Researchers have been attempting to implement effective e-learning methods that improve educational outcomes and consider their conformity with human psychology and current state of the technology. Hence, e-learning systems with a particular focus on cognitive-related aspects have emerged as a potential solution. This research study aimed on enhancing students' conceptualisations and mental perceptions of mathematical geometric concepts using an e-learning system that was developed based on key aspects of the Cognitive Theory of Gärdenfors' Conceptual Spaces (Gärdenfors, 2000), which utilises a combination of visual and audio.

The research achieved this through an effective agent-based model that was designed to help in teaching basic mathematical geometric in primary school. The e-learning systems adapts to the individual student's needs and pace to conceive the geometric concepts while maintaining the design objective of a flexible and proactive approach for the agents that is semi-autonomous.

This research study investigated the instruction of geometric concepts for primary school students based on three national curricula (UK, New Zealand and Saudi Arabia). Sets of questions were developed to study the students' understanding of selected concepts through a rigorous process of surveying primary school math teachers, determining the appropriate level of question difficulty and requesting the verification and appraisal of the type and format of the questions from the mathematical instructor community. Based on the generated sets of questions, a prototype of an e-learning system was developed and used in a pilot experiment that was conducted on students from the UK and Saudi Arabia to investigate whether students' misconceptions were consistent with Gärdenfors's Conceptual Spaces cognitive model. In addition to the variations in students' answers, the experiment revealed

consistent misconceptions based on the mistakes made on specific questions, which confirmed several aspects of the Conceptual Spaces cognitive model.

These results led to the implementation of the CABELS e-learning system, which was developed based on this theory, to enhance student conceptualisations of the previously determined common misconceptions in a way that assimilates their mental perceptions of the studied concepts. CABELS includes two parallel modules, which are the language-based and visual-based modules, and involves three stages: a pre-test, lessons explaining the concepts and a post-test.

The CABELS system was first used in experiments on primary school students in Saudi Arabia, which highly improved their understanding of lines and shape concepts. Furthermore, a statistical analysis of the experiment data showed that there was no noticeable effect of the teaching methods, groups, classrooms or genders on the students' scores nor any interdependence between these variables. Therefore, these results reinforce the effectiveness of CABELS for teaching basic geometric shape concepts to primary school pupils. The effectiveness of the CABELS system was also evaluated through post-session interviews with students to assess their satisfaction and experiences with CABELS. The results showed an overall satisfaction of students regarding the use of this system, which the students indicated was mainly due to its usability and usefulness.

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Chapter 1

Introduction and Motivation

1.1 Background of the Study

Technology is currently involved in all aspects of daily life and has positively affected people's lives around the world. For example, new technology can be used as a multi-dimensional tool and has created new opportunities in the field of education. Technology has also improved the ability of primary schools and high schools to provide services to villages and remote areas that previously had no access to modern education practices.

More importantly, technological developments have enabled the modelling of suitable learning environments that facilitate the learning process for students at schools or at care facilities. For instance, computers are increasingly employed in the learning and teaching processes of geometrical aspects at all levels of education through visual presentations (Atiyah, 2001). This evolution in geometry instruction has occurred due to its importance in the field of mathematics as well as in other science fields, such as engineering and computing.

The UK mathematician (Atiyah, 2001) stated that 'geometry is one of the two pillars of mathematics', and the other pillar is algebra. This statement emphasises that geometry is critical in learning mathematics, statistics and physics. Geometry is valuable when employed in spatial intuition or spatial perception because it enables the use of intuition.

Therefore, geometry should be taught using effective methods of instruction, such as an e-learning approach. Using appropriately designed software in

the teaching process may significantly influence positive changes in terms of learning content and in the roles of learners and teachers (Noss, 1998). Furthermore, using technology maximises student learning of different shape types, especially if geometric concepts are visualised.

In the last decade, several researchers in the field of e-learning have concluded that more effective methods are needed to improve education; however, a greater understanding of the design and cognitive issues related to e-learning is required (Weller, 2004; Zemsky and Massy, 2004). Currently, one of the most active areas of research in this area is agent-based e-learning in which the system acts autonomously and proactively in some manner on behalf of the users (Neji and Ben Ammar, 2007).

Accordingly, the e-learning system interface should be designed in such a way that it will be easy to use and easy to understand. As with agent-based modelling, human computer interaction (HCI) is a highly researched field in computer science. Therefore, a more effective HCI is a primary issue that should be taken into consideration when designing an e-learning system. For instance, with an effective HCI interface, students will be able to explore properties of congruence and similarities between geometric shapes. Hence, a primary purpose of the current study was to identify approaches that reinforce the usability of e-learning systems in education.

1.2 Motivation and Rationale of Conducting the Study

Over the last three decades, teaching and learning have been changing from traditional teaching methods to e-learning and blended learning methods (blended learning is defined as a mixture of e-learning techniques and face-to-face teaching and learning), which necessitates the development of new technologies that meet the requirements of designing e-learning modules. Therefore, the Ministry of Education in all countries should maintain up-to-date technology to advance the teaching and learning process in schools. In fact, there is a noticeable trend in which governments are increasingly supporting the adoption of e-learning. Hence, the general perception is that this issue will

impact learners. Furthermore, e-learning provides a solution that will improve the understanding of mathematics in general and geometrical shapes in particular.

More recently, several researchers who have participated in e-learning projects have concluded that e-learning offers more effective methods and an improved educational environment; however, a greater understanding of the design and cognitive issues related to an effective e-learning system is needed (Weller, 2004; Zemsky and Massy, 2004). Some of these issues include the ability of e-learning systems to proactively adapt to the individual user's needs and to present content in a way that is easy to understand.

The focus of this research is the investigation of problems that are related to students' understanding of geometry. In addition, attempts were made to enhance the means of helping students gain a better understanding of the concepts in a way that complements traditional in-class teaching. Furthermore, an approach is proposed that has the potential to guide the development of e-learning systems in countries that speak languages other than English. Therefore, the primary motivation of this study was to increase the effectiveness of teaching mathematics to primary school students beyond the allocated time in face-to-face, teacher-led classes, and more specifically, to help students improve their conceptualisations and mental perceptions of geometrical concepts.

1.3 Aim and Objectives

The aim of this research was to establish and evaluate an effective agent-based model that can help in teaching basic mathematics in primary school. This agent-based model follows a novel approach that focuses on the Cognitive Theory of Gärdenfors' Conceptual Spaces (Gärdenfors, 2000). This is mainly used to design an effective agent-based model. In this model, an agent is used to interpret pupils' answers and to provide feedback and further clarification to help them understand the correct answer.

Specifically, the aim was to achieve the following objectives:

- To conduct a comprehensive review of the most recent research to identify the best characteristics required to design an agent-based model suitable for primary school pupils with a particular focus on the learning of basic geometric shapes.
- To develop a novel e-learning model based on the Conceptual Spaces theory that helps in teaching basic geometric shape concepts to pupils in primary school classes.
- To validate the effectiveness of the developed software within the educational environment by comparing and assessing the students' achievement scores and by collecting feedback.

1.4 Research Questions

Specifically, this study attempted to answer this primary research question:

Is it possible to create a more effective e-learning system to teach basic geometrical shape concepts to primary school pupils?

To answer this question, five sub-questions were investigated:

- Sub-question 1: What type of interface for the e-learning system is more effective?
- Sub-question 2: Would a visual-based or textual-based system be more effective?
- Sub-question 3: Are language and culture relevant factors in the e-learning process?
- Sub-question 4: How can a sound cognitive model be integrated in the design of an effective e-learning system?
- Sub-question 5: Are there any significant differences between Saudi Arabia and United Kingdom's students in relation to the misconceptions of mathematical concepts?

1.5 Contributions

From the onset of undertaking this novel research, academic and practical areas of knowledge were drawn from the intersection of four different discipline areas: computer science, mathematics, psychology and education. The intersection between these various disciplines has enriched the work to be more effective and influencing many fields. Hence, the main contribution of this thesis has been in the development of a novel and effective educational software for creative and innovative learning, which promotes user requirements, adopts student needs, embraces new pedagogies, and involves blended learning mainly visual and language-based.

More specifically, our contributions consists of the following aspects:

- A novel effective type of e-learning system based on conceptual modelling.
- A developed e-learning system that has adopted visual and language modalities to acknowledge student needs.
- A new search unit technique which is related to the quality dimensions in conceptual space theory.
- A new map unit technique which is used to identify and recognise the similarities and differences of the quality dimensions of each student's concepts in their mental perception.
- A system that enhances the effective outcomes of teaching mathematics through the involvement of psychological theories, which has helped students to effectively conceptualise the mathematical geometric concepts.
- New experimental findings that indicate no significant differences between students in Saudi Arabia and the U.K. from different gender and while have different cultural language backgrounds.

1.6 Organisation of the Thesis

This study is organised into eight chapters. The first chapter provides an introduction to the study, its aim and objectives, a brief overview of related

work, the motivation for conducting this study, the contributions and the layout of the chapters.

The second chapter reviews the literature related to the main topics of the study. For instance, the second section in the chapter defines the e-learning approach, its importance and history and its aims and benefits for the education process. It also describes the types of e-learning and the techniques used, the adoption of e-learning in the Arab world, and more specifically in Saudi Arabia, and the current limitations of e-learning applications in the education sector. The third section discusses issues that are related to the implementation of an e-learning system, such as its requirements, effectiveness, evaluation and limitations. The fourth section presents a brief in learning development in children.

Chapter 3 reviews the related work to the main topics of the study. For instance, the second section presents the notion of learning. Section three the pedagogical methods for teaching mathematics. It also describes the types of pedagogical methods. Section four presents the teaching/learning geometric concepts in primary School. It also presents the teaching/learning objects in space, using computers to teach math in primary school, visual-based e-learning system, language-based e-learning system, game-based e-learning system and venn diagrams as teaching tool in mathematics. Section five presents a comparison of the e-learning systems for teaching mathematics. Section six in the chapter provides an overview of cognitive models and their types, such as symbolic models, Connectivist models, the Prototype theory and Conceptual Spaces. The seventh section presents a detailed introduction to the Theory of Concepts, the formation and learning of concepts, including the Behaviourist theory, and the Cognitive Development theory. The eighth section discusses the Conceptual Spaces Cognitive model, the quality dimensions and comparative concepts (structural spaces, inductive processes, etc.). The ninth section describes the agent-based models. it also presents the taxonomy of autonomous agents and design principles for autonomous agents. The final section in the chapter focuses on concepts in primary mathematics, including numbers, shapes, children's perceptions

of shapes, representing shapes, geometric properties of two and three dimensions, problem solving and concepts and misconceptions in primary school mathematics.

Chapter 4 begins with a discussion of the design and the prototype of an agent-based e-learning system. Section two presents the design considerations for an agent-based e-learning system that includes the selection of the development environment, the user interface and the user-agent interactions. Section three presents the methodology used to develop a set of questions that helped understand and determine the misconceptions of mathematical geometric line and shape concepts among primary school students. Section four describes an initial pilot experiment that was conducted to evaluate the prototype e-learning system. Section five discusses a second pilot experiment that was conducted using a paper-based test. Section six explains an additional experiment that was conducted using individual interviews. The purpose of these three pilot experiments was to understand the main sources for common misconceptions amongst students. Finally, section seven presents the details of a game that was developed as part of the e-learning system.

Chapter 5 presents a comparative study between students in Saudi Arabia and the United Kingdom. It discusses additional experiments conducted with the United Kingdom students and how they performed against earlier experiments that were conducted with Saudi Arabian students. The chapter consists of a description of the experiments, a statistical analysis of the collected data and the results that answered the study's hypotheses.

Chapter 6 presents the design and the methodology of the software application developed for the study. Section two describes the design of the e-learning system, including scope, design aim and objectives, rationale and design assumptions. Section three presents the agent-based design for CABELS. Section four presents the process flow for the application design and implementation. Section five describes application testing and integration. Section six presents the methodology used for the experiment of the e-

learning system design. Section seven discusses the population under study, and section eight provides the sample size for the study experiment. Section nine presents the details of the logistics involved in preparing and executing the main experiment. Finally, section ten summarises the chapter.

Chapter 7 presents a detailed analysis as well as discussions of the results obtained from the experiments performed in Chapter 6. This chapter discusses whether the e-learning system improved the conceptualisation of primary school students regarding geometric line and shape concepts as well as whether it was effective in enhancing student understanding.

Finally, Chapter 8 presents the summary, outcomes, conclusions and limitations of the study. In addition, it reviews the aims and objectives and presents future works of this research.

1.7 Publications

The following are the published (or accepted for publication) research papers that are relevant to this thesis:

1. Alghurabi, Y. & Teahan, W. J. 2014. A conceptual spaces agent-based model for e-learning of mathematics: teaching basic concepts concerning geometric shapes to school children. The 7th Saudi Arabia Students Conference UK, Edinburgh.
2. Alghurabi, Y. & Teahan, W. J. 2014. Towards conversational agent-based systems for e-learning of mathematics: teaching basic concepts concerning geometric shapes to school children. The International Conference on Competitiveness of learning: innovation and opening new markets, Poland, Bielsko-Biala.

Chapter 2

Background

2.1 Introduction

This chapter provides a background review of e-learning systems and assesses state-of-the-art methodologies to better understand its objectives, its importance and its practical applications. It also serves as a means to better understand and to identify any gaps in knowledge and to determine potential areas of e-learning and Conceptual Spaces (Oermann and Hays, 2015, p.48) to which the current research may contribute, and more particularly, its application in teaching mathematics to primary school students.

The chapter is structured as follows. First, an overview is provided of e-learning in general regarding the history of its evolution, various definitions, aims, benefits and techniques enabling its application in mathematics and its limitations followed by a summary of the literature reviewed to conclude the chapter.

2.2 E-learning

E-learning is a contemporary teaching method. It is an important and practical application of modern technological innovation and the development of computer technology. E-learning is a generic term used to describe a new, innovative means to deliver education or learning strategies in which the medium or instrument of teaching is computer technology (Johnson and Johnson, 2010, p.61). In practice, the e-learning method uses computer and internet technology for teaching, engaging and communicating with

students. The processes of delivering online education usually comprise an array of applications of computing tools, communication networks and related teaching techniques and strategies. The following section details the intricate evolution of e-learning and the technology and systems that enable e-learning.

2.2.1 History of E-learning

Remote communication systems were first developed in the late 19th century with the invention of the telegraph and the telephone. These early discoveries and technologies served as the basis and foundation of the invention of networks that led to the development of the Internet and computer communications; however, it was the evolution and combined effect of communication technology and the Internet and computer technologies that led to the development and introduction of e-learning systems toward the end of the 20th century (Harasim, 2006).

Since the 1960s, e-learning evolved through various methods, which has affected not only the education field but also the business, training and military fields (Fletcher and Rockway, 1986). Suppes (1964: 1966) stated that *'in the future it would be possible for all students to have access to the service of a personal tutor in the same way that ancient royals were once served by individual tutors, but this time the tutors would be in the form of a computer'*. Important events include the creation of packet-switched networks in 1969 and the email exchanges and computer conferencing in 1970, which were very important for the educational sector (Katy and Lyon, 1996).

Harasim (2006) made the following observation: "As these faculty introduced e-mail and computer conferencing to their academic curricula, they discovered expanded opportunities for student communication, interaction, and collaboration. Beyond what anyone anticipated, a sea change in education would emerge". According to Harasim (2006), these academics had to learn what email, computer conferencing and websites were and how to apply or utilise the available technology. This basic understanding of the

nature of computing, its functions and potentials was important in designing new processes and incorporating new technologies into pedagogy.

According to Waight, Willging and Wentling (2002), e-learning dates back to the early 1950s; however, the creation and market introduction of the Internet and computing technologies in the 1980s and 1990s led to the rapid expansion and development of e-learning, as the technological advances enabled schools and academics to communicate effectively with their students and to perform their teaching functions more efficiently. Researchers have identified at least four stages in the development of e-learning. These stages (Inoue, 2007) (Küren and Cellatoglu, 2008, p. 120) include: the pre-1983 instructor-led training period, the multimedia era from 1984 to 1993, the first wave of e-learning from 1994 to 1999 and the second wave of e-learning from 2000 to 2005.

The instructor-led training period took place before 1983 when computers were not yet widely available. This method required students to solely focus on academic studies and activities and to work together with their teachers and classmates. During this period, instructors used traditional tools, methods and processes and were significantly responsible for the students' learning process (Inoue, 2007).

The multimedia period that took place from 1984 to 1993 was widely characterised by the introduction and use of stand-alone computers and applications. During this period, traditional schools gradually developed into sophisticated learning environments, and educators focused on exploring ways in which available instructional resources could be utilised (Inoue, 2007). New computing products were integrated into learning techniques, methods and processes and led to the delivery of computer-based training (CBT) programmes. This new method reduced costs and triggered interest in further enhancing and developing the available resources.

The first wave of online education began between 1994 and 1999 when the World Wide Web, or simply 'the web', rapidly evolved and developed,

which enabled e-learning providers to begin exploring ways in which new technologies could improve the delivery of e-learning. It was during this era that a number of important technologies were introduced and made available in consumer markets. These include the Java programming language, which altered the face of multimedia learning, audio-video streaming, media players, HTML, web browsers and the pervasive use of electronic mail (Küren and Cellatoglu, 2008).

The second wave of e-learning, which took place from 2000 to 2005, was primarily motivated by the advent of next-generation communication technologies that were fully equipped and powerful enough to facilitate online programs and courses (Inoue, 2007, p. 297). Indeed, this was a period of fully developed web technology that not only allowed for the growth of e-learning but also enabled the expansion of e-commerce and e-governance (Chhabra and Kumar, 2011, p. 144).

During this period, there was a boom in the growth of e-learning. The growth of advanced technologies was the hallmark of the period between 2000 to 2005; however, there was no standardisation during the span of these five years (Harasim, 2006). This resulted in an almost exponential growth of e-learning in terms of the access to wireless networking, improved Power Point capabilities, educational games, e-learning repositories, etc. (Nicholson, 2007).

There was the Rapid Development Era between 2006 and 2010. During this period, some standardisation was observed in the tools of e-learning. The aim was to develop online tools and their related functions so that any person could learn more using e-learning tools (Nicholson, 2007). The subsequent technologies that were considered more advanced than preceding technologies included new benefits in which stakeholders of e-learning invested less, and learning could take place with no geographical boundaries (Harasim, 2006).

Furthermore, around the same period, the concept of online learning emerged, owing to the initiation of methodologies, such as podcasting, learning

analytics and the mobile Internet. In addition, there were other notable innovations, including educational gaming, digitisation of libraries, virtual worlds, cloud computing, mobile learning, e-books, etc. (Harasim, 2006).

During the period from 2006 to 2010, e-learning was an emerging industry; however, it was not completely embraced by academia, public education or places of employment. Around that time, various types of conferences were being held in North America regarding the benefits of e-learning and the reasons it should be embraced (Clark and Mayer, 2016).

At the same time, it must be conceded that there were some issues that accompanied the ever-unstoppable growth of e-learning. These issues included lack of bandwidth, compatibility of browsers and cross platform issues, which were among the most notable issues that required immediate resolutions.

Considerable educational research focused on the effect of technology, notwithstanding the incapacity of the researchers to investigate the broader and general applicability of the e-learning industry. There were several urgent issues that researchers addressed, such as the interaction between computers and humans, cognitive research on the use of the technology and computer-supported learning.

From 2011 onwards, e-learning has been further promoted by the availability of advanced tools that have simplified this type of learning. There is no doubt that courses offered through online learning are popular amongst academia (students, instructors, trainers and businesses). Currently, there are increasing presence of online lectures, virtual classrooms, instructor's advice and content coupled with the new online capacity for students to interact with lecturers (Wang, Wang and Shee, 2007).

Students can use these technologies to learn and gain expertise. Some technologies are considered to be the main factors in the development of e-learning since 2011. These technologies are tablet-based computing, cloud computing, learning through social platforms, MOOCs wearable technology

and advancements in learning analytics. Wang, Wang and Shee (2007) concluded that there is no denying that the future of e-learning is promising given the past history of the growth of advancing technologies.

E-learning has become a successful mode of learning and training across different sectors of the economy as well as in education. Though it was resisted for about 10 years, its potential benefits were accepted (Harasim, 2006). Currently, learners have a recognisable say in the matters related to e-learning in the sense that applications in areas such as communication, exploration, expression and networking are now in the hands of the learners due to the introduction of affordable technologies (Wang, Wang and Shee, 2007).

Following this overview of e-learning history, the definition of the term 'e-learning' is presented in the following section, which discusses the educational, technological and research scopes.

2.2.2 Definition of E-learning

The term 'e-learning', which is a short form of electronic learning, was first used and popularised in the mid-1990s to describe methods for online education (Friesen, 2009, p. 4). This term was not only used to describe a particular type of learning but also to describe instructional methods, teaching processes, course design, research and practices in education. Researchers and scholars have provided their own definitions of the term 'e-learning' since it attracted the interest of several fields within the academic sphere. Some of the other associated terms that were found in the e-learning literature include: e-learning research, e-learning theories, e-learning designs, e-learning technologies, e-learning methods and practices, e-learning philosophy and collaborative e-learning (Friesen, 2009, p. 5).

According to Reisman, Flores and Edge (2003), the e-learning philosophy was based on Andragogical and Pedagogical principles, which established e-learning theories. An Andragogical principle is a theory, a method, a technique or a set of assumptions that are used in adult learning (Davenport

and Davenport, 1985, p.152). According to this approach, the learning process focuses on learner's interests (Carlson, 1989). There are five assumptions in this principle: self-directed students; adult student learning is enriched by their wealth of experience in the educational setting; students are problem-centred when they are learning; and student motivation may be enhanced by internal factors.

There are a wide range of definitions of e-learning as a result of the various studies conducted on the subject. Thus, a common definition is needed to avoid confusion. Buzzetto-More (2007, p.28) observed that the definitions of the concept of e-learning are "partially exclusive and sometimes contradictory". Horton (2001) defined e-learning as "the use of information and computer technologies to create learning experiences". This definition focused on two distinct elements: use (or utility) and purpose. Association (2002), defined it as follows: "e-learning refers to the use of Internet technologies to deliver a broad array of solutions that enhance knowledge and performance", where the focus is on enablers of e-learning and its aims. Jarvis and Watts (2012, p.247) simply defined it as a type of 'learning that is supported via electronic means'.

There is a lack of a commonly accepted definition for other related terms, such as 'educational technology', 'blended learning', and 'technology-enhanced learning'. In addition, e-learning was initially used and considered as identical to distance learning. This is no longer the case with the all-encompassing nature of internet technology and the ease of access to the virtual world. This is why Holmes and Gardner (2006, p.14) simply defined it as 'online access to learning resources, anywhere and anytime'.

Other researchers cautioned that e-learning should not be confused with distance education or distance learning. Bates (2005, p.9) explained that e-learning covers a more extensive scope of processes and activities than distance learning, where students tend to possess quite diverse traits (age, profession, goals, etc.). The author also explicated that there are crucial pedagogical distinctions between classroom learning and distance learning

or even blended types of instruction. Bates and Poole (2003) further modified the definition of e-learning by presenting a model that suggested that the different types of e-learning methods can be regarded as a 'continuum' compared to the traditional definitions of e-learning that are characterised by the use of computers and the Internet. They suggested that e-learning should not be considered to simply involve a distance learning setup but that it can also be a face-to-face or a classroom setup that still allows interactions between teachers and students.

As a facet of modern learning, e-learning aims to fulfill several objectives from research, educational and organisational perspectives. These objectives are discussed in the following section.

2.2.3 Objectives of E-learning

E-learning is being widely adopted not only by educational institutions but also by industries and firms due to its various practical benefits. The value-adding role of e-learning makes it an attractive approach for universities, governments and organisations to achieve their learning objectives. As a result of its popularity and capacity to meet demands, schools that have maintained the traditional setup have been pressured to make significant changes and reforms and to begin providing e-learning solutions to their students (OECD, 2005). The research literature suggests that the objectives, or goals, of e-learning are purely utilitarian and endeavour to provide students with better opportunities (Nguyen and Preston, 2006, p.136).

From an educational perspective, and according to Garrison (2011), the e-learning approach aims to enhance and broaden the learning experience to take advantage of its learning or pedagogical processes and potentials (Garrison, 2011, p.4). He further stated "to realize the potential of e-learning as an open but cohesive system, it is essential that we rethink our pedagogy. Education is about ideas, not isolated bits of information". According to Garrison (2011), e-learning is not merely about technology but also about meeting the 'ideals of a higher educational experience'. Thus, its purpose

is to shape and enrich the skills and potentials of students to be productive members of society and to adapt to changing trends and developments.

From an organisational perspective, the primary objective of e-learning is to “underpin the enterprise’s search for quality improvement and competitiveness” (Bowles, 2004, p.137). Business organisations and industries may also adopt and implement e-learning to meet or target their corporate goals. From a consumer or student perspective, one of the primary objectives of e-learning is to offer students affordable and easy access to quality education and learning opportunities to take courses and partake in programs not provided by their schools (Talvitie-Siple, 2007, p.79). Another goal is to offer alternative education opportunities to people who are unable to attend or prefer not to attend a traditional school or university.

Since it has emerged in the educational field, e-learning has contributed several benefits, both from a stakeholder’s point-of-view and from a provider’s point-of-view. The various benefits are discussed in the following section.

2.2.4 Benefits of E-learning

Research has revealed substantial benefits and advantages of e-learning because online technology undergoes constant development and innovation. As technologies improve, online educators and schools also find ways to further enhance their e-learning systems and are compelled to improve their programs, processes, methods, systems and policies. This has many technological, social and economic benefits. The common benefits of e-learning include location and time flexibility, time and cost saving, freedom of choice and unlimited and open access and use of e-learning tools and materials (Vink, 2012, p.337).

Paola Bielli and Stefano Basaglia in Association (2002, p.876), also listed similar perceived and proven benefits of e-learning that include reduced learning costs, enhanced business responsiveness, improved content upgrading, learning customisation, improved learning accessibility, enhanced

community building, scalability and leveraged organisational investment on-line.

Apart from these benefits to stakeholders (students, employees, organisations and schools), e-learning can also significantly improve the quality of the education system as a whole (Lal and Aljondy, 2013)

1. by offering easy access to potential students who might be otherwise limited or restricted through physical impairment or economic reasons;
2. by contributing to the development and progress of theories and concepts of learning;
3. by offering effective instruments to students to use online technology;
4. by enhancing and supplementing the educational experience of students;
5. by contributing to the improvement and development in the way students learn.

In addition to providing different and often contradicting definitions of e-learning, scholars and researchers have also offered contrasting types of online education, which will be presented in the next section.

2.2.5 Types of E-learning

There is evidence that the types of e-learning developed are based on the technology that is available to institutions. According to Carliner (2002, p.2), there are two types of e-learning: formal and informal e-learning.

Formal e-learning is intentional and deliberate, as the purpose is to educate students through the adoption of e-learning courses, programmes and techniques and the application of new communication technologies. It is a basic alternative to traditional learning. It has the following goals: online education, online training and blended learning.

On the other hand, informal e-learning is delivered without the formalities of the first type of e-learning. This is usually provided by companies and organisations to their employees to gain additional knowledge, skills and

expertise. Its goals are as follows: knowledge management, electronic performance support and blended learning.

Bates and Poole (2003) suggested that there are three types of e-learning:

1. no e-learning: the same as the traditional learning setup that does not use computers, Internet or any new communication applications and materials;
2. blended or mixed e-learning: combines the tools, applications, benefits and practices of traditional learning and pure e-learning;
3. pure e-learning: a form of distance learning.

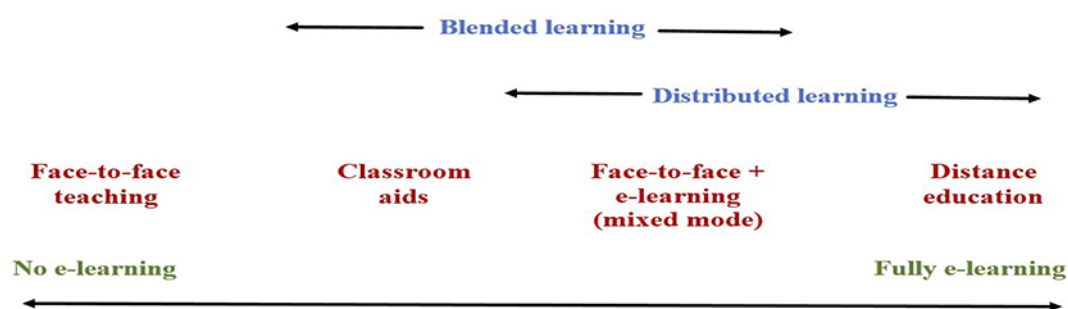


Figure 2.1: Different Forms of E-Learning based on Bates (2005, p.127) and Bates and Poole (2003)

Garrison (2011) argued that there can only be two types of e-learning: online and blended e-learning. Full e-learning can still be considered a type of distance education that is distinct from the classic distance learning via mail correspondences. Mixed learning, on the other hand, is the most widely used type of e-learning that still retains the traditional features of classroom education. It is the use of online applications, e-mails, online videos and other electronic teaching processes and tools that make a traditional instructional setup an e-learning combined experience.

The successful implementation of e-learning requires the exploitation of available technologies and tools as well as implementing specific techniques. These techniques are detailed in the following section.

2.2.6 Techniques Used in E-learning

Understanding the nature of e-learning, including the communication technologies and applications required, is not enough to make an online

educational program successful. There must also be consistent, well-integrated and comprehensive techniques to support e-learning programs, processes and practices. These techniques require the selection and strategic adoption of a specific implementation to ensure the successful outcome of a particular system and programme of learning.

The advent of mobile technologies and tablets also changed and improved certain e-learning processes, methods and activities, as students can now upload and send their assignments and worksheets via tablet and Bluetooth technology (Omatu et al., 2012, p.640). Smart phones also allow students to view and download their assignments, store and retrieve files, view lecture videos and communicate with instructors and fellow students. These developments and changes require the adoption of new techniques to make e-learning adaptive to the latest gadgets and technologies, trends, programmes and strategies.

The development of smart phones, which now function as mini-computers, led to the adoption of a new e-learning concept called 'm-learning', or mobile learning (Oliveira and Medina, 2008, p.274). To integrate m-learning into an e-learning system, there must be functional and effective e-learning techniques that allow for the efficient synergy of the two. The following are just some of the techniques used in e-learning (Oliveira and Medina, 2008):

1. Artificial Intelligence (AI) to better understand and determine methods to construct machines that execute tasks or activities normally carried out by individuals (Moreno et al., 2007).
2. Wireless technologies such as Bluetooth that can enhance the delivery of e-learning solutions to end-users.
3. E-learning design that accomplishes certain learning goals of the students according to the institution's framework of a particular learning situation (Camacho, R-Moreno and Obieta, 2007).
4. Virtual reality leveraging devices intended for 3D video games, such as Nintendo Wii (Chellali et al., 2008, p.153), to simulate scenarios that can greatly enhance learning experiences.

5. Designing web-based teaching, according to Horton (2001), with the goal of showing concepts to online students, verbally explaining them and requiring them to try and to repeat them after the instructor.
6. E-learning tools that are based on either starting with techniques offered by commercial tools or institution specific-techniques realised through the development of proprietary tools.

Now that e-learning concepts have been discussed from various perspectives, the following section will highlight the status of e-learning in Arab countries in general, and more specifically in Saudi Arabia, which was one of the countries investigated during the current study.

2.2.7 E-learning Use in Arab Countries

The growth and expansion of ICT globally led to the rise of e-commerce as well as e-learning in Arab countries. E-commerce in the Arab region is now estimated to be over \$100 billion, with e-learning also becoming a booming industry due to the reduced digital divide (Abdallah and Al-Badri, 2011, p.8). Many Arab governments, such as Egypt, Jordan, Kuwait and Saudi Arabia, have made tangible initiatives to integrate e-learning into their respective educational systems. Online learning solutions have also been established in other Arab nations to follow the global learning trends.

According to Ramady (2012, p.35), the rapid expansion of e-learning in the Gulf Cooperation Council (GCC) was primarily due to the Arab countries' massive financial resources and conscious willingness by the Arab leaders to finance and provide significant political support to education and information communications technology.

E-learning developments and expansion are mostly evident in Saudi Arabia and the United Arab Emirates (UAE), which are economically and politically stable. For example, the UAE government financially and politically supports online education along with its e-government initiatives to provide better education and welfare services to its citizens. UAE's massive investment in

e-learning has led to a rapid shift from the conventional classroom setting to the virtual, providing Emirati students with more educational opportunities (Ramady, 2012). This new education trend and initiative in at least six GCC nations was estimated at \$72 million in 2004. Ramady (2012) observed that Saudi Arabia and UAE have been massively investing in e-learning and technology-based education since 2004.

Ramady (2012, p.36) also stated that despite its spending on technology and education, the Saudi Arabian government is not yet ready to fully incorporate e-learning into the school system's curriculum because some Saudi Arabians fear that e-learning could lead to the desertion and abolition of traditional teaching methods and books. Despite the fact that e-learning is an essential component of King Abdullah's education program, many Saudi Arabian schools still do not have the capacity to provide e-learning to their students due to a lack of proper e-learning facilities and knowledge. To improve Saudi Arabian teachers' e-learning knowledge and skills, the Saudi Arabian government sponsored a 6-year e-learning training programme. Part of the ongoing development of the country's e-learning initiatives is the translation of more than 400 learning modules to the first Arabic e-learning curriculum in Office applications, Explorer and Microsoft Windows (Ramady, 2012).

To innovate and improve their educational systems, the Gulf nations decided to reassess their traditional educational techniques and to adopt comprehensive e-learning programmes. Arabic scholars have also emphasised the need to address educational deficiencies and to proactively improve schools' learning techniques and strategies. Apart from the need to upgrade learning strategies and to catch up with global educational trends and practices, Ramady (2012) stated that the following are some of the causative factors that influence e-learning adoption in the Arab world:

1. The baby boom of the past 20 years impacts the current and future demographics of the Arab nations. The Arab world's present and future generations have different educational demands.

2. There is a reduced dependency on oil. Current political scenarios and policies show that many oil-producing nations in the Gulf region have been investing in other industries, such as banking, manufacturing, tourism and services to reduce their dependence on the oil industry. This suggests that they need more skilled and talented professionals to support these sectors.

3. There are specific cultural dimensions. The Arab world's Islamic values and traditions have a significant impact on education. Ramady (2012) explained that Arabs consider jobs and the line of profession to be linked with the level of education as well as with family and social status.

Another strong causative factor is the need to maintain economic stability, as many Arab countries have been investing in manufacturing, services, tourism and other sectors. According to Elango, Gudep and Selvam (2008), the e-learning sector could serve as an effective and important alternative to traditional education in providing quality education to Arab students. The researchers also projected that among countries in the Gulf region, Saudi Arabia would dominate e-learning with its large and increasing student population. The UAE, on the other hand, is expected to lead in terms of business e-learning solutions.

As the current study focused on learning mathematic concepts in primary schools, a review of e-learning in the instruction of this discipline is presented in the following section.

2.2.8 E-learning for Teaching Mathematics

Several research studies have been conducted to determine the effectiveness of e-learning for teaching mathematics. In their study, Hu et al. (2005) found that e-learning will likely benefit assimilators, meaning students who acquire knowledge through reflective observation and abstract conceptualisation. These types of students usually learn through inductive reasoning rather than deductive reasoning, integration, concept-formation or theorisation. According to Kolb (1984), students typically excel in mathematics and basic sciences because of their ability to learn abstractly and to process

information reflectively. This means that e-learning could very well be suitable for mathematics students. Kolb's model of experiential learning involved learning of reflection on doing. It involved four stages in sequence: Concrete experience, reflective observation, abstract conceptualisation, and active experimentation.

In another study, Manochehr (2006, p.10) also found out that e-learning works best with assimilators and converges those who learn through observations and laboratories. This study suggested that if mathematics attracts assimilators and if e-learning is more effective with assimilators, it could also be concluded that through the process of deduction, online learning is suitable for teaching mathematics.

Only a few studies have focused on the pedagogical compatibility of e-learning with mathematics. Researchers have acknowledged that teaching mathematics to students is a quite challenging and time-consuming task (Elijah, 2012). Wetzel (2009) explained that this is mainly due to the application of ineffective teaching techniques used by math instructors due to improper training, lack of preparing for delivery of the content or simply a lack of the necessary skills and expertise. This then prompted a number of researchers to investigate whether e-learning is compatible with mathematics.

Because the most common tool used in e-learning is a computer, it follows that e-learning students must familiarise themselves with the technology, including other necessary applications and tools, and must undergo effective computer-assisted instruction (CAI) and computer-assisted learning (CAL). Elijah (2012) explained that CAI and CAL are not the same and that they entail different topics and subjects. To teach mathematics more effectively via an e-learning platform or model, he argued that instructors must acquire adequate training and utilise a proper integration of e-learning into the classroom.

Borba and Bartolini raised doubts about e-learning effectiveness when teaching mathematics, and they argued that e-learning may share the

same qualities with other technology-based learning models that have been practiced by teachers but have failed to generate positive results. Kidwell, Ackerberg-Hastings and Roberts (2008) cautioned against the use of the online learning paradigm and pointed out that it still requires additional improvements to ensure that its benefits and advantages are optimally exploited. Elijah (2012) stated that the main challenge or issue of using e-learning in mathematics is the lack of ICT knowledge and technical skills amongst mathematics teachers. In a recent study conducted in Saudi Arabia and the United Kingdom, it was found that school teachers who significantly lack computer and technical knowledge were unable to effectively utilise the e-learning packages and solutions (Hassana and Woodcock, 2010).

Thus, the availability of e-learning packages alone does not guarantee the success of an e-learning programme, as Hassana and Woodcock (2010) stated. They argued that it must be compatible with the planned e-learning math curriculum. Their study revealed that the use of commercially available e-learning solutions and tools must be properly designed and modified to suit the educational needs of e-learning programmes. Nonetheless, researchers have argued that teachers still play the most important role in providing quality education to their students (Elijah, 2012; Hassana and Woodcock, 2010).

Elijah (2012), explained that teachers can use e-learning systems to create a satisfying, memorable educational experience by: effectively adapting to the students' needs; motivating students; providing content that is curriculum and student-centred; and remaining versatile, reliable and flexible to achieve desired learning goals.

The implementation of e-learning systems involves several obstacles and difficulties that are related to specific issues, such as the effectiveness, evaluation and limitations of such systems. These issues are described in the following section.

2.3 Issues with the Implementation of E-learning Programmes

Despite the perceived and known effectiveness of e-learning, this new learning paradigm has issues, challenges, limitations and disadvantages. These matters affect all e-learning stakeholders, namely schools, teachers, administrators and students or end-users. There are several aspects that should be considered when implementing e-learning systems, which are presented in the following paragraphs.

2.3.1 What Is Involved in Acquiring an E-learning System?

Providing e-learning solutions to institutions that utilise this new means of instruction requires a cost that is dependent on the capability and goals envisioned for the programmes (Pralle, 2007, p.12):

1. E-learning requires a large amount of capital to implement and launch enabling technologies.
2. E-learning requires the acquisition of computers, Internet, new communication technologies, e-learning tools, applications, know-how, etc.
3. Schools must acquire skilled and effectively trained supporting know-how staff to operate the e-learning technology.
4. Institutions must design and adapt effective courses and programmes for e-learning programs.

Teachers must acquire proper e-learning and ICT training and knowledge to perform more effectively, including computer literacy and access to training and ICT knowledge. For students, online education requires a higher degree of attentiveness, responsibility and responsiveness. The effectiveness of e-learning systems is one of the success factors of these systems, which is described in the next section.

2.3.2 How Effective Is E-learning?

E-learning effectiveness is related to the following elements: e-learning programmes, teachers and e-learning practices (Sharma and Mishra, 2007, p.318). The success of an e-learning programme thus depends upon the efficacy and usefulness of these three elements. According to Latchem and Jung (2009), key success factors include designing effective courses, implementing efficient programmes and providing adequate training to teachers to improve their management and teaching skills.

The effectiveness of e-learning practices and programmes also depends on government policies and regulations for e-learning education. For e-learning programmes that are implemented or will be implemented by governments, Latchem and Jung (2009) observed that countries with e-learning policies in place tend to have more robust e-learning solutions and provisions.

As with any other implemented system, an e-learning system should be evaluated to assess its drawbacks and advantages; however, this evaluation is not always obvious, as it depends on factors that are discussed in the following paragraphs.

2.3.3 How Are E-learning Systems Evaluated?

Another challenge of e-learning programmes is the difficulty of assessing e-learning systems. Usually, this issue is due to the teachers' lack of knowledge of underlying technologies and proper and adequate evaluation skills. There are two aspects of an e-learning system that need to be evaluated: 1) the evaluation of the tool being used, and 2) evaluation of the effectiveness of the knowledge / exercises that the system delivers.

Assessing the pedagogical orientation of an e-learning system requires proper methodology and an assessor's relevant expertise (Buckley and Donert, 2004). In their study, Buckley and Donert assessed whether UNIGIS, an e-learning system offered to working students, was effective. Buckley and Donert used the following indicators to determine whether the system was: a) reflective,

b) adaptive, c) interactive and d) discursive. The results of their study showed that most criticisms focused on a lack of interaction between teachers and students.

E-learning systems might have some limitations if the necessary requirements are not fulfilled. These requirements include programme management, preparation, communication and technical knowledge, for example, which are discussed in the following section.

2.3.4 Limitations of E-learning Systems

The success of e-learning systems requires effective programme management. Since e-learning involves an orchestrated use of communication tools and the delivery of quality education to end-users, teachers must master new tools, techniques and technologies (Horton, 2001, p.43). Teachers must also dedicate adequate time preparing suitable discussions with students and engaging and motivating students so they can be active participants in virtual classes. Both the e-learning providers, particularly the teachers if they are working from different locations, and the students, must have high-quality communication links to the e-learning system to achieve the desired experience of the virtual classroom.

According to Horton (2001, p.35), mastering and acquiring the necessary e-learning techniques require the following skills that are rarely found among the practitioners: a) knowledge and training; b) soft skills and leadership skills; c) psychomotor skills; d) training of attitude; e) operator training and f) training in safe use of a computer. All of these skills entail the need for e-learning institutions to assess their global or local delivery, to guarantee consistency, to perform data collection and tracking and to access information.

2.4 Learning Development in Children

This section attempts to define who a child is, the various stages of childhood and their behavioural changes. It also includes their intellect and learning

curves giving a review and study on the various methods of learning and teaching that works best for them. For this purpose, it also delves into a few teaching and training methods in which children learn best and the varieties and strategies followed in each of them including examples and concepts.

Childhood in definition is a time to play, learn, grow and learn many aspects of life like love, confidence, family and many others. It is also a time when fear and violence should not be felt or found. Being safe from violence and abuse is crucial at this stage. Instead of simplifying the definition to merely a stage before adulthood this is the stage where the child's cognition and intellect is shaped and mentored making it the most significant part of its life (UNICEF, 2005).

The stages of childhood defined vary by different psychological experts. Especially, Kendra Cherry categorizes the stages of childhood as the development, cognitive, psychological, psychosexual and moral stages. Each stage has its own classifications to crystallize the development. In each of these stages, these classifications define each period and development of the child's intellect in a channelized manner for better understanding. Cherry (2016), Conversely, Erik Erikson who has the most comprehensive human development process among other psychologists has a more psychosocial approach to defining the stages as trust vs. mistrust, autonomy vs. shame and others as opposed to psychosexual stages of childhood development. McLeod (2013), these are two of the many different childhood stages that are explicated by various child psychologists.

Late childhood stage is a rather interesting one which can be defined in the ages of 6-12 years. This is the stage of pseud-maturity. This could be otherwise called as the latency period where this stage affects the child's social and personal adaptation. The development of the physical persona happens here. It is a creative age which is difficult and argumentative (KKHSOU, 2011).

The late childhood traits are multi-fold. There is physical development which happens rather slowly and consistently without any rapidity. Physically, there is improvement in more skills, strength and endurance and the child is usually restless which makes it the best time for engagement and keeping busy. There is a significant improvement in cognitive and intellectual ability with a willingness to learn. Socially, the child develops and extroversion occurs along with the ability to be creative and team-up. There is conceptual and emotional development. Homo-sexuality develops along with interests. Overall, this is the age where they are restless, ready to play, ready to team up and get extroversive and sexually develop as well. They will learn, develop hobbies, and exercise their abilities effectively. It is best at this age to have them busy and occupied so that they can find their interests. These are some of the traits that they develop in a glance (KKHSOU, 2011).

As mentioned earlier, the late childhood traits are prominent and diverse. A major part of this is intellectual as well. The level of improvement in the child's intellectual capabilities is very prominent. This is when the child is capable of utilizing memory power to recollect things. The child is more attentive and is able to think for itself and also imagine. This helps in solving problems when it arises. The knowledge acquired is not only in the receiving end for the child but becomes creative as this age progresses. The intellect of the child is very active and the IQ significantly develops and once this stage ends it gradually stabilizes. That being said, frustration goes before their successes as they find it quite hard to accept failures (Institute, 2011).

The ability to learn for the child significantly improves at this stage. The child is now mentally prepared to learn. The child increases the capacity to be attentive and acquire an interest in writing, reading and arithmetic actions. He learns to mimic and follow things and rules. The language development significantly increases and from responding in one-word answers, they progress to move complex grammatical structure towards their responses (Collin, 2010).

2.5 Summary

The main purpose of this chapter was to review background literature and previous studies pertaining to e-learning as well as to the use of technology. In the last two decades, many researchers have actively investigated different types of teaching methods in schools (face-to-face method, distance learning, e-learning, blended learning, etc.). Still, the use of technology is relatively new and has several potential applications in teaching mathematics in general and geometric shapes in particular. Based on the literature review, e-learning appears to be a favoured method of teaching as it expands the range of tools available for both the learner and the teacher in terms of time, location, repetition and communications amongst other things.

This chapter provides an overview of the historical background of e-learning and its conceptualisation, objectives, benefits and techniques. The chapter also addresses the issue of using e-learning in teaching. Although, studies showed that e-learning is an effective method of learning and teaching, it has several drawbacks and limitations, such as high costs, acquisition of computers and Internet, accessibility to the Internet and a lack of skilled teachers. The chapter also covers child development and stages of learning to help develop a matching methodology in the delivery of learning.

To fully understand new teaching methods such as e-learning, it is also necessary to review the theory behind cognitive models that contribute to child learning. This also helps in defining and modelling the dimensions of mathematical concepts. It also required a review of the Conceptual Spaces cognitive model, which plays an important role in improving teaching outcomes and learning experiences. This is discussed in the next chapter.

Chapter 3

Related Work

3.1 Introduction

This chapter aims to present the related work and the literature about the main areas of this study research that include pedagogical methods for teaching mathematics and learning methods. Initially, details of types of pedagogical methods are presented that were considered in building the e-learning system of this research. The chapter follows with the details of different teaching/learning methods that include the use of computers. The theory of the conceptual space and cognitive model are presented before detailing agent-based models. The chapter then presents concepts in primary school mathematics that were considered in this research. The chapter concludes with a summary of the reviewed literature.

3.2 Notion of Learning

Bloom (1956) offered a classification of learning as a set of objectives and mental abilities with levels of increasing complexity in order to facilitate communication between examiners. The original proposals consisted of six levels of learning: knowledge, understanding, application, analysis, synthesis and evaluation. Later on, Anderson and Krathwohl (2000) who studied under Bloom renamed the six levels as remember, understand, apply, analyse, evaluate, and create.

Bloom's argument about Knowledge is the ability of the student to observe and to recall and to recognize information previously presented. He further

explains that understanding represents the ability to grasp meaning, to compare, to contrast, to sort, to group, and to infer causes. In his view, application incorporates the use of methods or concepts in new situations. He also explained that analysis refers to finding patterns, and identifying components that enable the learner to differentiate and to classify structures. Similarly, he states that synthesizing is about the ability to generalise from existing knowledge and to predict consequences. His thought about evaluation is to compare and to discriminate between learned concepts.

Early approaches to teaching mathematics involved tables that had to be memorized and steps which had to be exactly replicated whether children understood or not. However, this approach made a significant shift when learning mathematics was made more interesting especially for primary school level children. The emphasis on getting it right to “making them understand” took a significant dominance in way teaching happened and it was enabled for children to talk about math in language rather than be bookish about it. There was an emphasis on articulation and memorization being only an advantage not a mandate. This was reinforced through real-life examples which made them relate and understand it better (Mumsnet, 2015).

This chapter discusses how mathematics is taught and understood by young children and how mathematical concepts should be communicated (or not) along the six stages of learning as defined in the Bloom taxonomy.

3.3 Pedagogical Methods for Teaching Mathematics

“What kind of mathematics education should be considered in the training of students?”, “What is the role of mathematics in the general education of people?”, “Is there a better methodology than another for the learning of mathematics?”, are just a few questions arising in the teaching of mathematics. These are some of the relevant questions from all teachers in this area before starting a course. The answers are not unique and it cannot be said that one way or another of teaching is better or worse than another.

Teachers have their ways of teaching; these could be either: epistemological or pedagogical or didactic.

Pedagogical methods refer to the elements used in a pedagogical scenario or referred to in a pedagogical strategy. An instructional design usually makes use of several kinds of pedagogical methods which are combined into some coherent storyboard (pedagogical scenario). It implements one or several strategies. There exist several kinds of attempts to create typologies. Some are easier to understand than others.

Khan (2001) list of methods and strategies focusses on 20 major "natural types" that he develops under the header "pedagogical" in his eight-components framework for e-learning: Presentation, Exhibits, Demonstration, Drill and Practice, Tutorials, Games, Story Telling, Simulations, Role-playing, Discussion, Interaction, modelling, Facilitation, Collaboration, Debate, Field Trips, Apprenticeship, Case Studies, Generative Development and Motivation.

3.3.1 Types of pedagogical methods

Vigotsky's contributions are fundamental in the area of education, especially his concept of a zone of proximal development that refers to "finding the distance between the child's actual level of development, determined by the ability to independently solve a problem, and the level of potential development, determined through the resolution of a problem under the guidance of an adult or in collaboration with another more capable partner".

Jean Piaget (1896-1980), a Swiss psychologist, founder of the School of Genetic Epistemology, is one of the most prestigious and relevant figures in twentieth-century psychology. He is one of the authors whose contributions have had more significance in Psycho-pedagogy. One of the most valuable contributions of Piaget's work is the eminently active and constructive feature he assigned to the developing subject, the subject he is learning. According to the previous image that prevailed before his studies, the different abilities arose and unfolded with the passage of time, almost automatically or pre-programmed, leaving the subject relegated to the role of passive spectator of

his learning or development. On the contrary, one of the basic pillars of the Piagetian theory is to consider and present the people who are learning as active constructors of their abilities and skills, which arise as a result of their interaction with the environment and their elementary need to understand the World around them and adapt to it.

David Ausubel believes that learning by discovery is effective if it fulfills some characteristics, these are:

- New knowledge is substantively incorporated into the student's cognitive structure.
- This is achieved thanks to a deliberate effort by the student to relate the new knowledge to his previous knowledge.

Thus, school learning could take place by reception or discovery, as a teaching strategy, and could achieve significant or rote and repetitive learning. According to meaningful learning, new knowledge is incorporated substantively into the student's cognitive structure. This is achieved when the student relates the new knowledge with the previously acquired, but it is also necessary for the student to be interested in learning what is being shown.

Other authors who contributed to the theory of constructivism are Alsup (2005), Suk (2005), David Pugalee (2001) and Romberg (1992). Alsup compared traditional and constructivist instruction in mathematics. His study shows that the second offers advantages. He examined the efficacy of constructivist instruction on anxiety toward mathematics, beliefs about efficiency, and perceptions about autonomy.

Suk (2005) investigated the efficacy of constructivist instruction in mathematics on academic performance, self-concept (beliefs each person has of self), learning strategies, and constructivist methodology preference. Their study concludes that constructivist teaching is more effective in terms of academic achievement and is not effective in terms of self-concept

improvement and that students have a preference for a constructivist environment.

Pugalee (2001) investigated the relationship between mathematics and meta-cognition. He validated that students' writing about their mathematical processes when solving problems shows evidence of cognitive goal behaviours. Student writings demonstrated the use of various cognitive target behaviours. The results promote the inclusion of process writing as an integral part of the mathematics curriculum.

Romberg (1992) illustrates the idea of doing mathematics in the arts faculties. In several European Schools, the Mathematics Schools belong to these faculties.

Polya (1945) warned that to understand a theory, one must know how it was discovered. Therefore, his teaching emphasized in the discovery process even more than simply developing appropriate exercises. To involve his students in solving problems, he generalized his method in the following four steps:

- Understand the problem.
- Set up a plan.
- Execute said plan.
- Look back and verify the result with real life.

Polya, also proposed a series of advice, for the teachers of Mathematics, to which it called. "The Ten Commandments for Teachers of Mathematics"; these are summarized in the following:

- Be interested in your subject.
- Know your subject.
- Read the faces of your students; See their expectations and difficulties; Put yourself in their place.
- To teach is not to transmit ideas to another, is to allow the other to discover them.

Table 3.1: Main pedagogical methods.

Lecture Method	Lectures are used to introduce new ideas, summarizing topics, relate between theory and practice, reemphasizing main points. Adaptable to many different settings (small or bigger groups). May be used to introduce a unit or a complete course.
Inductive-Deductive Method	Proceeds from specific to general and from concrete to abstract ideas. Have notion of the needs of the students, It is a development process. The students are taken from easy to difficult.
Heuristic Method	The teacher plays the role of resource person and a facilitator. The environment includes both freedom and structure with freedom having the upper hand. The content may very well be propositional truth in a general context.
Analytical-Synthetic Method	Breaking up the problem into simpler parts. Parts can be recombined to find solutions. The adopted procedure is to go from unknown to known.
Project Method	Appropriate for any level, but is often employed for senior levels of education. Using projects usually requires a lot of preparation by the teacher.
Brain Storming	A group creativity technique. Efforts are made to find a conclusion for a specific problem by gathering a list of ideas by its members.
Think-Pair-Share	A cooperative learning technique in which students think through questions using three distinct steps, encouraging individual participation. An excellent method for promoting critical thinking and articulate communication in the classroom.
Learn by Doing	A teaching method for active learning in scientific graduate education.

- Give your students not only information, promote mental attitudes and the habit of methodical work.
- Let them learn to guess.
- Check what you teach, so that you can learn to test what you have learned.
- Note that the exercises or problems of a subject will be useful in solving problems or future exercises: try to bring to the surface the general pattern that lies under the present concrete situation.
- Do not show all the secrecy at first: let your students guess beforehand; Let them find out for themselves as much as possible.

- Suggest them; do not make them swallow it by force.

The main pedagogical methods are shown in Table 3.1.

As the developed e-learning system that is discussed in chapter 3 involves game-based learning, language-based learning and visual-based learning, then these will be discussed in more detail in chapter 6.

3.4 Teaching/Learning Geometric Concepts in Primary School

Geometry gives a teacher more leverage to pick a manner in which they can teach primary school because most of it is imagery and deals with shapes and spaces. It greatly helps in problem-solving and is a form of a language. Albeit, it looks complicated and overloading for children, it is a great way to make it enjoyable for children with shapes and colours. This is achieved through making geometry relative to everyday life applications. Geometry teaching should enable the child to successfully translate that imagery to real world experiences rather than shapes. Use of multi-sensory activities where real materials are given to make them form shapes, getting them in groups, and displaying progress is an amazing and creative way of teaching geometry in a primary class. Not only does this make geometry relevant, it makes it fun and also understandable. At the end the methods should involve experiential where the children do an action to learn, informal where concepts are isolated and learned, and formal where a specific concept is labelled and studied to be understood (Cooper, 1986). For teaching children, a good place to start is to build the foundation right. An introduction to the basic shapes and teaching them about it is good way to begin. Visually relating this to shapes like circles including plates, cans and other things and helping them understand also goes a long way in getting the concept engraved in them (Sarama and Clements, 2006).

Teaching the basic shapes and concepts are best done through flash cards for primary school. This is the easiest medium in which this could be done. Utilizing choral repetition and pronunciation practice concepts, this could be

reiterated to their minds. Making them understand it and re-enact through symbols is a great way to reinforce understanding. Also, making it linguistic by asking them to name the shapes by forming a sentence also reiterates the learning done more effectively. Again, this should also be given to the students independently as an activity where they can make these shapes once again and combine both of the activities. This could be made more kinaesthetic as well which could be used as a quiz where they make the shapes as a team. This covers many aspects of learning such as visual, auditory and kinesthetic which makes it an effective tool to teach shapes to them (Arntsen, 2013).

Knowing basic shapes is very important for children as it will help them understand how they are formed and how to remember them in the future when they need to be used. One of the best ways to introduce them to shape concepts is by showing them a set of flash cards and walking them through the different kinds of shapes that are going to be discussed. Including basic shapes to complicated ones like circle, square, rectangle, star and diamond is a good place to start. Having them vocally say these shapes are important for them to linguistically relate the pictures. After this, the children can be made to make those shapes by their hand to physically get involved in the learning. This way, they will kinaesthetically learn the shape (Arntsen, 2013). This sums all of the learning styles including visual, auditory and kinaesthetic and makes the learning comprehensive for every child there (Constantinidou and Baker, 2002).

Like it is mentioned above, using different styles of learning help the late childhood learner understand concepts more effectively. In this case, the visual learning style is a powerful tool as the children are visual learners and they love exploring things. This could be achieved by introducing shapes in the form of cards or drawing them on a board. If a computer is used, walking them through a presentation of shapes of different kinds will be a great way to visually make them see what the shapes look like. Once they look at these shapes they will understand how to relate them to the words like a circle, square et al to reiterate their learning (Arntsen, 2013).

Children are not always visual learners and some will not learn through looking at pictures as this would be overwhelming for them (Constantinidou and Baker, 2002). They could be linguistic learners in the class as well and following a text-based learning methodology would work for them very well. Having the class go through the pictures and having them say the shapes and write them down is a great way of doing this. Once the children understand the pictures, texts could be written below those images denoting the name of the shapes and teach the children how to write them. The teacher can also include activities where the children make sentences (written) denoting what shapes they see around them. This will ensure that they understand what is being taught (Arntsen, 2013).

Games are an effective way to teach children any concept which could be complex faster. Games go a long way in reiterating a concept more strongly in their minds. There are so many ways in which games can be used to teach them shapes. There could be multiple activities where children could be asked to find shapes in a powder box and isolate them. The teacher can draw the shapes on a board and ask the children to name them and give points. This could also be an outdoor activity where children are allowed to move freely from a classroom environment. They could also be handed over with sticks to make shapes and a set of shapes could be assigned to a few groups or done individually. These are some of the many methods in which games could be introduced and concepts of shapes could be engraved in their minds much more efficiently than bookish learning (SG, 2013). This helps in kinaesthetic learning style where the late childhood audience gets their hands in involving their body and mind into it. This is fun and great for learning complex ideas (Constantinidou and Baker, 2002).

If two of these methods had to be juxtaposed to find a superior one, then the chances are there would be no conclusion. There are different types of learners in a class and it is very hard for teachers to identify the right learning style. The best way to make everyone learn a concept effectively is by using multiple learning styles. While visual will be great when pictures are shown to kids. Some will end up snoring through the sessions. This is

when these kids could be given books to read simultaneously as they see the visual content. So, the visual vs text should go hand in hand rather than independently as none of them is superior. We have different styles of learning and all learning types should be completed in a teaching session (Constantinidou and Baker, 2002). Explaining shapes is hard through text as there is no representation whether a round is actually round and how it looks. Visually, it could be shown but without telling them that it is a circle, they wouldn't know. They must go hand-in-hand to ensure that learning is complete (Project, 2013).

Geometry is best taught and understood when multiple level of learning styles and training methods are incorporated. Since children due to their lack of experience and age cannot formulate these ideas which are tough even for high schoolers at times, it is best for teachers to include various aspects of learning to get the best results for getting this done. Since at this late childhood stage, children are linguistically growing, it is best to utilize this to talk the language of geometry. They can formulate sentences and include geometric shapes in them that they identify in their surroundings. This helps to reiterate their understanding (Epstein, 2003). Some believe that starting with line concepts in geometry is a great way to start. The children should be go through the basic lines of geometry is great way to introduce them to this mathematical concept. Children can be given activities where they draw lines using strings and have arrows at the end denoting the direction. This way lines, segments, rays, horizontal and vertical lines can all be taught effectively. Children could be encouraged to try parallel lines, and then perpendicular lines and then asked to combine both of them like a wedding ceremony of a male line and a female line. This might induce a graphic picture and gets them understand it better. After this is done, they can start their activities independently where they are allowed to draw lines in sheets of paper and form a story and share it with their parents as well which includes these various line concepts (Mary, 2012).

The learning line concepts in primary school is no different from making them learn shapes. A good way to start is to talk about it. Making them frame

sentences where they speak about the shapes they see which is a good way to build their linguistics and make them learn more efficiently at it. Including words that denote shape is a very important part of it. This is how mostly educational institutions in the United States conduct their training method. The children should be shown complex pictures or imagery where they can identify shapes and lines. This makes them understand in a better way how shapes and lines are used to make that imagery which gives them a deeper level of knowledge of the same. This ends up in a better retention capacity for the children. Making it a habit by making them do activities which will have lines and shapes in them is a great way to reinforce this learning. Activities where they play games and get involved in forming lines is another great method as well (Project, 2013).

Projecting an art or an image with the use of computer is a better way to enhance the learning of shapes and lines in the class. To analyse if it is an effective method of teaching, for few minutes, students can be asked to identify the shapes and lines they saw and probably draw it as well. This visually enables them to find out these lines and shapes. This could be summarized with an activity where they are made to draw those shapes as a group or individually (Project, 2013).

Students can use language to understand the concepts being shown or taught to them. The children can be asked to write sentences in a piece of paper or a book about the lines they see and ask them to denote whether it was parallel or horizontal. Teaching them how to write a line makes them linguistically understand the concept (Project, 2013).

Games are by far the easiest way to make the children understand very complicated concepts rather faster. They are able to get involved much more eagerly and understand the concepts. It is blending fun by including the content slyly to ensure that learning is actively done. This could be achieved by making sure the games are organized and the children are encouraged to participate in it. This could be asking the class to form a line, ask them to intersect it and making them mark the beginning and end of it

after making them understand the concept. While having fun, these children are subconsciously memorizing the line concept and remembering it for a long time (Project, 2013).

Lines are one of the basic concepts in mathematics and it is a good place to start to teach the children. They will be able to understand visual concepts in this case as it is an easier form of a method. They would have seen straight lines in many places and once they are shown that this is a straight line, they will be able to grasp it. The problem occurs when multiple lines are shown and the beginning and end of it arrives (Project, 2013). This is where writing and textual learning helps and terms every line to give a better understanding. While visual learning surely has its part for better understanding, textual learning will also complete the learning (Constantinidou and Baker, 2002).

3.4.1 Teaching/Learning Objects in Space

Children in their late childhood have the ability to understand language better as their skills in linguistics improve. To teach geometric shapes, describing those objects by their shape is a great way of training them on the same. Phrases like, "I saw a square piece of cloth" and so on improves their understanding in shapes. Words like angle, flat, top, straight, curve, surface, solid, and point helps to be specific. Even art could be analysed as shapes in a group activity by these children to understand the lines and shapes in those art works. The concept of relativity helps here. This could be reinforced with provocative questions where they are forced to think in geometry spaces and shapes to answer like, "How a square be turned to a triangle on the board?" and so forth (Clements and Sarama, 2000). Structured-based learning activity could be followed by this where the children are asked to form lines and asked questions on who is starting and ending the line and comparison of shapes and sizes could be made by making them do an activity (Clements and Sarama, 2000).

3.4.2 Using Computers to Teach Math in Primary School

Concerning using computers to teach math in primary School, there are numerous institutions who have adopted computer technology to aid in teaching many subjects for children. This includes mathematics as well among others. The increase in computer technology has increased drastically. The performance of children in primary school learning maths through computers have increased greatly. Primary school students are more engaged and actively involved in the process of learning and during discussions get involved actively. Foster (2003), it is also understood that most of the incredible innovations came through mathematics and the proper execution of it and all of which was done using computers. This makes calculating by hand and paper obsolete. Not only is it obsolete but also tedious and time-consuming. Conrad Wolfram argues that mathematics should somehow be incorporated into computer language so that once the children have a basic idea, time could be saved by computerizing the concepts so that they could get answers faster and solve problems more efficiently. Wolfram (2010), it is safe to say that the use of computers for late childhood is a great tool as long as they are taught to understand the concepts. Once this is achieved they can use computers to aid them, making them more efficient.

The way children think could be altered with the introduction of computers. This alters their way of learning and how they communicate with others. Presenting very complex ideas in symbolism has increased the level of comprehension due to computer technology. The representations made in the computer is extensible making it more versatile to change. It provides better control for the students who use the medium to learn and is easier as well. Learning subjects like mathematics and science becomes far easier than it has to be when it is physical and becomes more interactive. This makes learning easier and enjoyable. Clements (1997), the use of mathematics in today's world including jobs and society in general has become very important over the past few years. This is why computers are a bridging mechanism in making education easier and efficient. This fundamental change in the way

math is taught was expected to revolutionize learning for children in the later childhood years.

3.4.3 Visual-Based E-Learning Systems

In later childhood training, it is important to get the children acquainted with new concepts in the shortest amount of time and in the most fun way possible. Visual-based learning in the case of e-learning is very effective where complex ideas are explained as concepts visually seen. For this, the content is made in a way to engaging graphic content that the late childhood can access. This also gives room for letting the children express themselves in a visual manner.

There are many benefits to this approach of teaching for later childhood learners. As mentioned previously, it simplifies complex concepts by imagery and reduces time spent on explaining things, allowing the brain to ruminate on the thought right away. Secondly, visual-based learning in the conceptual e-learning system gives the children a platform to re-create their understanding instead of having everything in their mind. This in turn contributes to the third benefit of the Visual-Based learning that helps boost the retention capabilities in children as well. If they can recreate, they are basically re-teaching themselves and that makes the concept stronger in their head and remember it later (Pappas, 2016). This helps children to keep their learned ideas in their memory for a longer time and improves their speed of retention and comprehension. This not only brings the concepts back but also adds emotional value to it making it a stronger bond and this keeps them motivated and learning. As long as this method is used correctly, it can benefit a huge audience (Gutierrez, 2014).

3.4.4 Language-Based E-Learning Systems

While visual-based e-learning involves a lot of graphic content and charts and pictures to illustrate complex ideas, there is no denying the fact that this is only a part of the equation as many of the late childhood audience is not going to learn from them. This is where the language-based learning comes into

play. Like visual, language-based e-learning uses language to reiterate ideas and complex concepts to make children understand. In the late childhood years, children are ready to understand linguistics better and this is used for more language-oriented children to learn the concepts through reading. While explanation of these concepts will help more of auditory learners, this language based system will help the children learn by themselves and master those concepts by reading them over a period of time (Roziewicz, 2015). The benefits of this is again multifold. This makes learning comprehensive and easier, repetitive content is available to access, new learning techniques can be introduced rather easily in the e-learning spectrum and it is accessible by the children at anytime (Sumner, 2013).

3.4.5 Game-Based E-Learning Systems

Game-Based Learning consists of the use of games as vehicles and tools to support learning, assimilation or knowledge assessment. It is an innovative methodology that offers both students and teachers a different and practical educational experience that can be applied to a subject or subject or integrate several subjects. If you choose digital educational games and the use of ICT, the game based learning is a complete approach that also works digital literacy. We explained the main advantages of this method of learning so that you are encouraged to try it in the next course.

Game-Based Learning is understood as the phenomenon that combines learning with different resources known as games, in particular referred to digital or computational in nature, with the Order to support and improve teaching, learning and / or evaluation. It is considered “an effective way to motivate the student and to engage the student in active learning experiences” (Charlier et al., 2012).

Advantages of game-based learning are:

- Motivate the student. One of the main advantages of the game-based learning is its ability to capture the attention of students, as it provides them with an environment that they like, amuse and find it very motivating.

The game makes the class more dynamic, awakens the interest previously and maintains it throughout the development, not only for the final victory but also for the ludic practice itself.

- It helps the student to reason and be autonomous. The game poses to the student situations in which he must reflect and make the right decisions, to solve failures and to recover of the defeats. With this method of learning, you will not only be assimilating concepts of the subject or subject in which the game is focused, but you will also be developing cognitive skills through critical thinking, reality analysis and problem-solving.
- It enables active learning. Game-based learning gives you the possibility to practice your knowledge in a practical way. By learning how to do the student experiences, practice test-error, establish relationships between previous and new knowledge and make decisions to improve.
- It gives the learner the control of their learning. By means of the game, the child or adolescent obtains instantaneous feedback regarding his or her knowledge about a subject or subject. This allows you to be aware of your degree of acquisition of what you have learned and helps you to find out what you should focus on and focus on.
- It provides useful information to the teacher. Besides the result and the surpassing or not of the game, also the choices that the student makes, the concrete problems that arise to him, the points in which he fails or in which he emphasizes they contribute much data to the teacher to detect strengths and weaknesses with respect to the subject or check the level of understanding of knowledge. In addition, it allows a much more profound approach to the student, in terms of their ability to reason, solve problems, make decisions or overcome failures.
- It empowers creativity and imagination. The game also implies freedom of improvisation and ability to imagine solutions to each challenge, which helps to open the student's mind and his perception of the world. This benefit is multiplied if it is the students themselves who design the game or modify it and improve it with a base already provided by the teacher,

a highly recommended practice to take a step further in the game-based learning.

- It fosters social skills. Game-based learning is perfect for collaborative learning. With this practice, the child interacts and works on emotional education, communication, dialogue and leadership, collaboration for a common goal, self-control or sportsmanship. This translates into a better climate in the classroom, cohesion among its members and the acquisition of values.
- It contributes to digital literacy. If you choose to use online games, video games or play applications you will not only be taking advantage of the game but you will also be adding the benefits of the application of ICT in the classroom. Students will gain knowledge on the central theme of the game and at the same time improve their handling of new technologies and practice the use of computer tools and digital devices in a safe environment and thought for learning.

3.4.6 Venn Diagrams as a Teaching Tool in Mathematics

Venn diagrams were first created by John Venn in 1891 to illustrate existing relationships between sets of objects. These diagrams are used today to teach elementary set theory. They are also used to illustrate simple set relationships, as well as to analyse collection of objects in different fields such as in probability, logic, statistics, linguistics and computer science.

A Venn diagram is a diagram that shows all possible logical relations between a finite collection of different sets. This diagram consists of multiple overlapping sets represented in form of closed curves, generally circles. Hence, the objects that are present inside a set S constitute its elements, whilst objects that are outside correspond to elements that are not part of this set.

The following figure is an example of Venn diagram. Considering the three following sets of numbers:

$A = \{2, 4, 6\}$;

$B = \{3, 6, 9\}$;

$C = \{0, 1\}$.

The Venn diagram shown in Figure 3.1 shows the overlapping that exist between the three sets.

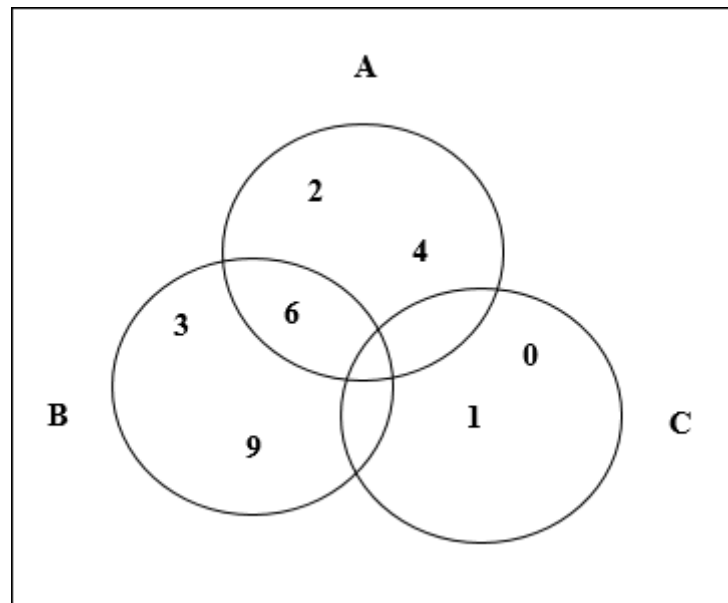


Figure 3.1: Example of Venn diagram

3.5 A comparison of e-learning systems for teaching mathematics

There are different e-learning systems that are available to the primary school students to learn mathematical geometry. Each has its strength and weaknesses in addition to its intended users and learning outcomes. Those systems that were marketed by for profit organisations appear to be reaching more students with online capability through government subsidies while the free online based e-learning system attract quite a dedicated contributors and parents who are looking for way to enhance the skills of their children.

A brief description of the selected e-learning systems are presented below and a summary table for high-level comparison purposes.

IXL Learning allows the students to find comprehensive, standard-aligned content for all grade levels. The developers of the IXL e-learning system had a certain goal that aimed at means to invigorate education environment using Web technology. The IXL's basic goal is to improve and help the learning experience of all stakeholders that include students, parents, and teachers. The developers of the IXL take the approach of making an educator's job easier, as well as save time by motivating the teachers and engaging students. The IXL e-learning system has no stated theoretical basis and it is claimed to be based on the realities and the current state of K-12 and higher education. For further details, refer to (<https://uk.ixl.com>).

Khan Academy claims to offer world-class education for anyone to learn what he or she wants. Khan Academy covers a massive number of topics, including K-12 math, science topics such as biology, chemistry and physics and humanities even with playlists in finance and history. Khan Academy, being a non-profit entity, sets out its mission to provide a free world-class education for anyone, anywhere. It is also strictly web-based set of tools that aim to personalise learning resource for all ages. Khan Academy offers practice exercises, instructional videos, and a personalised learning dashboard that empower learners to study at their pace in and outside of the classroom. Khan Academy provides free tools for parents and teachers that are intended to guide the learners from kindergarten to calculus using state-of-the-art, adaptive technology that identifies strengths and learning gaps. A critical point about the Khan Academy learning environment is the number of scattered tools that the learner should use or engage them. These include video lectures on YouTube, chatterbot on a web-based interface and finally a testing interface. For more information, refer to (<https://www.khanacademy.org/>).

E-learning for Kids is another initiative by a non-profit organization that aims to provide the children aged between 5 and 12 years a free learning via Internet to master the basics of reading, science, math and computers. The stated goal of the E-learning for Kids is to help address the challenges to the children and their parents that are related to the costs, class sizes and other

issues that often prevent children to access to quality online learning that can support and reinforce these essential skills. It claims that its math curriculum consists of 336 e-lessons in English based on the international baccalaureate standards and 25 Math e-lessons on specific topics. In certain conditions upon completing the required formalities, the e-learning system becomes available even offline through downloadable modules for local access. For further information refer to (<http://www.e-learningforkids.org>).

NRICH Project aims to enrich the mathematical experiences of all learners. NRICH states that they are a team of qualified teachers who are also practitioners in NRICH mathematical thinking. Based on their unique blend NRICH believes that it is ideally placed to offer advice and support to both learners and teachers of mathematics as well as preparing the carers. NRICH's stated goals include: enrich the experience of the mathematics curriculum for all learners, offer challenging and engaging activities, develop mathematical thinking and problem-solving skills, show rich mathematics in meaningful contexts, and maintain a partnership with the teachers, schools and other educational settings. It achieves these goals by providing the learners free and interesting mathematical games, problems and articles while encouraging them to share their solutions to NRICH mathematical problems and having mathematicians who can help them to solve problems in a safe online space where they can also meet others with similar interests. For more information refer to (<https://nrich.maths.org>).

Kids Math Games Online provides a wide range of free math games, interactive learning activities and fun educational resources that will engage students while they learn mathematics. It is unique in attempting to attract the students and retain their attention while they are using the resources it provides through the online. Particularly, Kids Math Games aims through its geometric games to offer the children a great range of free math activities and interactive practice problems that will help kids improve their math skills online. It focuses on a fun interaction for the children while improving the skills of the students by teaching shapes, grids, weights, building blocks, angles, measurements, reflection, rotation,

transformations and more. Kids Math Games also provides geometry facts, quizzes and videos as additional learning tools. For more information, refer to (<http://www.kidsmathgamesonline.com/geometry.html>).

The Comparisons of e-learning features for the selected systems are shown in Table 3.2.

Table 3.2: Comparisons of e-learning features for the selected systems.

E-learning Feature	E-learning Systems					
	Kids Math Games	IXL	Khan Academy	NRICH	e-learning for Kids	CABELS
Target year	1-13	1-13	1-13	1-13	1-6	9-12
Personalisation	no	yes	yes	yes	yes	yes
Learners' profile	yes	yes	yes	yes	yes	yes
User-agent	no	no	yes	no	no	yes
Tutor-agent	no	no	no	no	no	yes
Audio for the text	no	yes	yes	yes	yes	yes
Pre-test	no	no	no	no	no	yes
Post-test	yes	yes	yes	yes	yes	yes
Visual-based	yes	yes	yes	yes	yes	yes
Language-based	no	yes	yes	yes	yes	yes
Game-based	yes	no	no	no	no	yes
Map-based	yes	no	no	no	no	yes
Search-based	no	no	no	no	no	yes
Explanation	no	yes	yes	yes	yes	yes
Revision	no	no	no	no	no	yes
Sound-effect	yes	yes	yes	yes	yes	yes
Online-based	yes	yes	yes	yes	yes	no
Offline-based	no	no	no	no	yes	yes
Based on Theory	no	no	no	no	no	Yes

3.6 Cognitive Models

The purpose of this section is to examine how defining and modelling the quality dimensions of specific mathematical concepts in particular, basic

shapes, how using a Conceptual Spaces cognitive model will improve teaching outcomes and the learning experience of primary school students. Therefore, a review of cognitive models is presented in the next sections, including the Conceptual Spaces theory that was adopted in the current study.

3.6.1 Symbolic Models

The symbolic model describes cognition as a series of computations that is like a software programme. According to Lewis (1999), this model draws from Turing's computation, and the brain is described as a system that is "capable of manipulating and composing symbols and symbol structures". The strongest models attempt to replicate the algorithms utilised for human processing by using programming languages that are known as production systems. These programs attempt to precisely specify a particular level of abstraction by a set of independent production rules that may active anytime their conditions are satisfied.

Gärdenfors (2004b) suggested that this model was 'too coarse' to adequately represent concepts and cognitive processes, in that each 'symbol' is too broad to adequately explain some of the finer details related to quality dimensions. Aisbett and Gibbon (2001) explained that within symbolic theory, concepts are not actually modelled at all; they are just named. Symbolic representation infers the meanings of the symbols, which makes it difficult to represent in a formal logic system because the meanings can vary significantly from person to person and from system to system. Furthermore, one cannot determine the effects of change on a symbol, other than to suggest a new symbol is created when even the smallest characteristic (or perceptions of characteristics) of the concept or object changes.

3.6.2 Connectivist Models

Connectivism is a cognitive model that explains cognition as a by-product of activation patterns in the neural network of the brain. This model is represented by a manufactured creation that seeks to simulate the neural network of the brain, though the simulations are significantly simpler in that

the number of 'units' representing neurons is far lower than that which is found in corresponding regions of the brain. In a simulation, 'an active mental representation' is replicated via a 'pattern of activation' between the units. Learning occurs when the network adapts to a particular pattern of connection weights. In this model, memories are stored as patterns that can be activated, either partially or in full, by current stimuli (McClelland, 1999).

Gärdenfors (2004a) suggested that this model was 'too fine-grained' in that it excluded 'similarity', particularly similarities that were related to quality dimensions. Quality dimensions consist of the three ordinary spatial dimensions (height, width and depth) in addition to weight, temperature, brightness and pitch (Schiffman, 1990). The main function of the quality dimensions is to denote different types of qualities of objects. They are also used to judge similarities and differences between objects and to provide a stimuli ordering relation (Clark, 1992, p.114).

3.6.3 Prototype Theory

The classical prototype theory contends that concepts are defined by their similarities to a class of attributes. Concepts (objects) can be compared based on their degree of typicality, or the extent to which they are representative of the typical model of the 'idealized concept instance' (Dietz, 2013, p.7). The similarity between an object 'A' and a prototype for the concept 'B' can be defined as the sum of the weights of the attribute-values possessed by 'A' (Dietz, 2013, p.7).

This theory allows for extensional vagueness for borderline cases and even the movement of borderline cases into alternate prototype classifications as more information becomes available.

3.6.4 Conceptual Spaces Theory

The Conceptual Spaces theory is a cognitive model that was developed by Gärdenfors (2000). This theory extends the classical prototype theory and represents objects and abstract concepts with geometrical structures that

are described and defined by sets of quality dimensions and their associated values. Aisbett and Gibbon (2001, p.196) explained that this model employs symbols; however, the meanings of the symbols are not randomly specified but are defined as 'the underlying conceptual structure' that is formed due to judgments of similarity of the symbols of the quality dimensions of the concept.

Dietz (2013, p.141-142) summarised Gärdenfors' quality dimensions as features by which 'objects may be judged as more or less similar or different'. In this model, any point in a space is determined by vectors in the conceptual space that represent the defining dimensions. Dimensions are modelled geometrically or topologically, and mathematical similarities and differences are represented as a distance metric between each defined point (vector), where the similarity between two points is an inverse to their distance.

This theory is discussed in more detail in the next section and particularly in section 3.8, as it is central to the cognitive modelling approach that is discussed in the later chapters of this dissertation.

Now that cognitive models have been discussed, the Theory of Concepts is discussed in the following section.

3.7 Theory of Concepts

In cognitive psychology, concepts are the symbolic representations of human knowledge, perceptions and experience. They are 'mental representations of the world' (Hampton, 1999). According to Eschenbach et al. (1998), concepts are units of an internal structure in which humans represent and categorise their world. In relation to this aspect, Rice (2013) stated "concepts are constituents of thoughts and perceptual representations".

Gärdenfors' theory of Conceptual Spaces (Gärdenfors, 2000) describes concepts as regions or sets of regions in a mathematical sense and objects as entities that are linked to these concepts based on their quality dimensions. In the cases where quality dimensions regularly co-occur, the subspaces

that are formed are called quality domains. Gärdenfors divided the notion of concept into a simple concept in which few quality domains are related to it and complex concepts, which connect several quality domains.

3.7.1 Natural Concepts

In Gärdenfors' theory, he defined some concepts as primitive (colour, shape, texture, etc.) in the way that they are re-presentable by only one domain as a convex region within the conceptual space. These primitive concepts are also characteristics of other more complex concepts. Gärdenfors referred to these types of concepts as properties. Properties that are required when conducting cognitive tasks (e.g. inductive reasoning, communication, memory, etc.) are referred to as natural properties and are defined by three criteria: connectedness, star-shapedness and convexity (Dietz, 2013, p.144)

Aisbett and Gibbon (2001, p.211) referred to Gärdenfors' definition in their work as properties and objects. Basic (or natural) properties are the building blocks of representations and the variants within the domains of the natural properties that allow for the *"subtleties exhibited in different knowledge domains"*. Simple objects are singular points within a conceptual space. A natural property is considered a region in n-dimensional space that is convex in shape that defines a simple object. Furthermore, a complex object can be defined by its natural properties or quality categories.

It is also important to note that not all qualities that define a concept are equally important. Some characteristics (properties) may be more relevant than others and can be represented in a model with differing sizes.

3.7.2 Concept Formation and Learning

In this section, various conceptual theories are described, namely the Behaviourist theory, the Cognitive Development theory and the Conceptual Spaces theory. The agent-based modelling that was adopted for this study is also presented. In addition, quality dimensions and structural space are discussed. Furthermore, the mathematics topics that are taught at primary

schools (numbers, shapes, etc.) as well as the students' perceptions of mathematical concepts are described.

3.7.2.1 Behaviourist Theory

Behaviourists utilise learning models that seek to describe concept learning as trial and error with feedback processes. Hampton (1999) developed two learning models: rule-based and prototype-based. In the rule-based learning model, students form hypotheses of the concepts based on the feedback they receive during their learning trials. In the prototype-based model, students form representations of an average or prototypical stimulus that is dependent on previously learned exemplars. Both models have merits in describing fundamental learning processes, but behaviourists' dependence on structural aspects that rely solely on stimulus domains does not adequately explain cognitive or language development.

Anderson, Reder and Simon (1999) explained that mathematics learning within the behaviourist theory is based on factual and procedural rules; however, the models for learning in cognitive psychology work well with technology-based tools and have contributed to a significant increase in methodologies that facilitate learning mathematical concepts.

3.7.2.2 Cognitive Development Theory

Piaget's constructivist cognitive development approach is a developmental assimilation of concepts or schemas. This theory holds that conceptual change and the acquisition of schemas are a natural part of neurological maturation as a child develops and constructs increasingly complex concepts as a result of experiences and reflections. Although there is some support for the immature brain being unable to develop specific concepts until sufficient neurological development has occurred, the theory does not account for difficulties in learning concepts, particularly in adults (Hampton, 1999; Hearron and Hildebrand, 2010).

Concept learning may be inhibited if components of the concepts require a higher order thinking or processing that has not been yet been achieved. While behaviourists contend that all that is required is appropriate stimuli and feedback, Piaget's work shows that there are constraints in processing some stimuli before a developmental milestone has been met. In the pre-operational stage, which includes children under seven years old, it is usually impossible to process all the aspects of a problem concentration, and as such, it is impossible to demonstrate conservation when shapes of an object change (e.g. water is transferred into a taller, more slender container from the initial shorter, broader container).

Piaget also identified cognitive constraints in the primary age group. Children aged between seven and eleven are capable of mentally reversing an operation and are capable of focusing on more than one aspect of a problem. Piaget contended that this stage is a concrete operational period where the operations children conduct require a tangible object, image or event. Although there is significant evidence supporting that concept learning and formation are influenced and constrained by cognitive development maturation processes, (Dasen, 1994; Rogoff, 1990) have shown that Piaget's developmental stages are not entirely constrained by the age groups only and that a child's development is heavily influenced by culture and other elements of the environment.

Piaget's theory has given rise to a number of new Piagetian theories that seek to explain the processes and constraints that Piaget described. The next section explores Conceptual Spaces in more detail, as it is relevant to the work described in subsequent chapters.

3.8 Conceptual Spaces

The Conceptual Spaces model focuses on objects and abstract concepts that are represented by quality dimensions of features or properties that describe the concept (Gärdenfors, 2004a; Gärdenfors and Williams, 2001). A specific concept is defined as a region from the domains within an n -dimensional

geometric space. Each domain is composed of set of features, where the main domain function is to illustrate various qualities of objects or situations.

Gärdenfors (2011) contended that concept formation is a perceptual mechanism that is stimulated by sensory input at the sub-conceptual level and is processed at the conceptual level by invoking a prototype effect consisting of quality dimensions that are defined by sensory and non-sensory abstract categories. He argued that the mapping of the interpretations of the psychological and scientific dimensions of the quality in the human mind is not identical. This establishes limitations because human perceptions are less precise than external scientific measurements of perceived phenomena; however, Gärdenfors argued that within these constraints, learning does occur.

3.8.1 Quality Dimensions

Gärdenfors (2000) explained that judgments of similarity are central to cognitive processes and defined categories of dimensions for human perceptions and their structures. These dimensions are considered the building blocks of mental representations at the conceptual level, and he developed three categories for the dimensions. It is notable that these dimensional qualities of human perception are essential components for e-learning content and the presentation to students for teaching mathematics:

1. *Distances*, where similarity is designated or defined by the relative closeness of the cognitive representations of objects.
2. *Sensory dimensions*, where characteristics are received through the sensory systems, e.g. visual, auditory, thermal and kinaesthetic.
3. *Non-sensory dimensions*, where characteristics are judged in terms of properties of objects in accordance with specific characteristics, e.g. size, pitch and shades of a colour.

Gärdenfors (2011) suggested that some of the quality dimensions are biologically innate (e.g. colour, pitch, space, etc.), whilst others are learned (e.g. social roles, artefact functions, culturally dependent notions, new

scientific discoveries, etc.). He argued that the concept of a 'paradigm shift' is in fact a drastic change in existing Conceptual Spaces. He also suggested that learning new quality categories is simply a matter of introducing the correct phenomena or stimulus to bring about the paradigm shift. This appears to contradict Piaget's theory that some concepts cannot be conceived by a child until she/he reaches and attains sufficient neurological maturation (Gärdenfors and Warglien, 2012).

3.8.2 Structural Space

Within a conceptual space, Fiorini, Abel and Gärdenfors (2013, p.74) introduced a subspace called a structural space in which vectors encode structural information about the parts that compose and relate to the whole. From a cognitive perspective, the precise description of the structural aspect of the subspaces becomes too cumbersome and leads to 'dimensional explosion'. This complexity results from being unable to determine which part is directly related to the whole or the process of their selection, which is based solely on experience and perception. To enable broader representations, Fiorini developed the concept of a dimensional filter that is controlled by attention and context, which 'is a conceptual operation that projects a subset of the quality domains of a concept onto a smaller subset'.

3.8.3 Inductive Processes

According to Gärdenfors (2011), the process of induction is limited to natural properties, which are quality categories that are within a singular convex region. He argued that even scientific theorisation takes place solely at the conceptual level, wherein all the relevant dimensions used in explaining a phenomenon are individual natural properties. Identifying the appropriate dimensions that are required to explain a phenomenon is considered a major scientific activity. As such, mathematical reasoning may be negatively impacted if a student has an incomplete model of a specific concept in that he/she has not integrated all the relevant quality categories or natural properties.

3.8.4 Prototyping and Modelling

The Cognitive Spaces theory is derived from the prototype theory. Gärdenfors (2011) explained that the most prototypical examples of the concepts represented by the conceptual space are in the Cognitive Space centre. Dietz (2013) defined concepts as a set of ‘more than’ or ‘less than’ rules for ordering objects on a scale. Dietz also explained that categorical concepts are operational rules for classifying objects into subsets based on prototypical categories. Gärdenfors and Williams (2001) demonstrated that it is easier to learn categories with a central natural prototype than a prototype that is a more peripheral.

To model a concept, its relevant domains must be determined, and then the appropriate geometric structure of the identified domains must be chosen. For example, using the concept of an apple, the intuitive domains are likely colour, shape, texture and taste (see Table 3.3). Biological classification and nutritional values may be relevant if these domains have already been learned. The quality dimensions of the taste domain are sweetness, saltiness, sourness and bitterness. Some of the other domains are not as easy to express. For example, the biological classification of ‘fruit’ is a bit more complex and may require more specialised knowledge to define.

Table 3.3: An example of a concept: “Apple” (based on Gärdenfors.(2000))

Domain	Sample regions
Fruit	Values for skin, flesh and seed type.
Colour	Red-green-yellow.
Taste	Regions of the sweet and sour dimensions.
Shape	“Round” region of shape space.
Nutrition	Values for sugar, vitamin C, fibres etc.

Once the domains are determined, they must be mapped and evaluated according to their relationships. Mapping relationships of certain domains are relatively intuitive, while it is difficult to positively correlate others with their relationships with other domains. Gärdenfors (2011, p.4) provided criteria to map domains using a set of convex regions along with information about how the various domains’ regions can be correlated.

Gärdenfors (2000) mapped the quality dimensions of concepts using a Voronoi tessellation, wherein the centre of each cell in the tessellation is a prototype classification of a typical case of the dimension. Hence, the use of the cells simplifies the quality dimensions of the concept, and they are mapped to various points based on distance using a Euclidean metric of the observed characteristic from the prototypical norm in the centre. This simplification enabled the tessellation method of prototyping to represent discrete stimuli in a continuous space of stimulus. The resulting cognitive effect of this simplification was that the discretisation improved the learning process and provided a cognitively economical way of representing information about concepts and their classifications (Gärdenfors, 2000, p.89),

Douven et al. (2013) suggested that Gärdenfors' use of Voronoi diagrams may not have provided considerable empirical evidence because the mapping of the information in the human brain is not well understood, but it is a useful method for generating categorisations. They also suggested that well-arranged Voronoi orderings can adequately define comparative concepts based on the prototypicality and similarity of the distances of the points when mapping complex concepts.

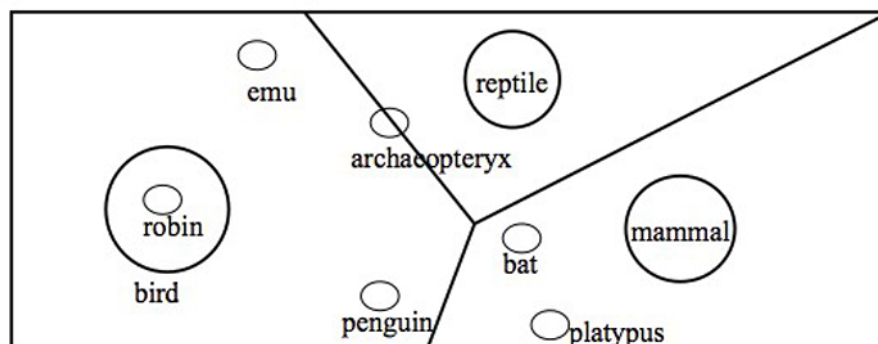


Figure 3.2: Prototype regions representing animal concepts based on Gärdenfors' Conceptual Spaces cognitive model; see diagram in Gärdenfors & Williams (2001)

Figure 3.2 provides an abstract representation of prototype regions in an animal space to illustrate how concepts such as birds, mammals and reptiles might be represented. The figure consists of a number of circle and oval shapes that represent regions corresponding to the concepts of animals. The bird region is larger than the emu, which encompasses the robin region because a robin is also a bird; however, a penguin is also a bird

regardless of whether the stimulus is the name or the image (Gärdenfors and Williams, 2001). The figure indicates that birds can be tessellated independently of reptiles and mammals, so that $\text{bird} = (\text{robin}) \cup (\text{penguin}) \cup (\text{emu}) \cup (c \cap c (\text{bird}) (\text{archaeopteryx}))$. Some concepts may also exist at the boundary between more generally related concepts. For example, a bat is positioned between the mammal and the bird concepts.

Conceptual Spaces have been discussed in the previous paragraphs, and mathematic concepts that are taught in primary schools, such as numbers and shapes, are the focus of the next section.

3.9 Agent-Based Modelling

This section briefly describes the main concepts of agent-based models (ABM). It starts with a simple introductory text to guide those who have not had contact prior with this modelling technique and predict that it may be useful for their research. For this reason, the document includes a brief definition of Agent-Based Models, their components, their potential usefulness, their advantages and some guidelines to define if this technique is appropriate for the problem to be solved.

An agent was defined in Wooldridge and Jennings (1994) according to two main usages, general and specific. The general usage of agent corresponds to a computer system that is characterized by autonomy, self-containing, reactivity as well pro-activity. The agent specific usage corresponds to a computer system that is implemented according to human concepts, e.g. beliefs, desires and intentions (Wooldridge and Jennings, 1994).

The meaning of autonomy is defined in Wooldridge and Jennings (1994) as “the assumption that, although we generally intend agents to act on our behalf, they nevertheless act without direct human or other intervention, and have some control over their internal state.”

Reactive agents can be defined as “capable of maintaining an ongoing interaction with the environment, and responding in a timely fashion to

changes that occur in it". Note that the term is now widely used to mean a system that includes no symbolic representation or reasoning: such an agent does not reflect on the long-term effects of its actions, and does not consider the coordination of activity with other agents. Thus, "a reactive agent will always respond in a timely fashion to external stimulus." (Wooldridge and Jennings, 1994). Note that there is no precise definition here of the type of response, the type of external stimulus and degree of response. For example, a system could be deemed to be a reactive agent even if the response is trivial.

Finally, a pro-active agent can be described as "capable of taking the initiative; not driven solely by events, but capable of generating goals and acting rationally to achieve them." (Wooldridge and Jennings, 1994).

A model is a simplified representation of reality. Thus, a model simplifies the different components and processes that are part of the system being studied. The process model helps to identify, select and sort available information in relation to the functioning of the study system. The model - the result of the modelling process - and the simulation is a useful tool to understand the functioning of a system and evaluate its response to changes in different components of internal or external system.

In agent-based models (ABMs) - also known as multi-agent systems, agent-based systems, etc. a system is modelled as a collection of autonomous decision-making entities called agents. Each agent evaluates their situation and decisions on the basis of a set of decision rules. Unlike analytical methods, which focus on modelling and characterizing the equilibrium of a system (top-down approach), the models based agents offer the possibility of generating (e.g. shaping) that balance. In this sense, the ABM have a "bottom-up" approach: this means that the modeller represents the behaviour and the interactions of the individual agents and local objects that compose the system and obtains result a pattern of behaviour at the system level.

An agent-based system is composed of a collection of agents, an environment through which agents interact and rules that define the relationships between agents and their environment and that determine the sequence of actions in the model (Parker et al., 2003). Agents are physical or virtual entities that make decisions autonomously. They can represent atoms, cells, animals, people, or organizations depending on their application. The environment is the virtual space in which agents (e.g. may represent a geographical space). The agents have their resources (e.g. land, capital), objectives (e.g. maximise income, decrease risk) and sensory capabilities, e.g. have information on attributes and status of other agents and the environment. Agents take decisions based on the rules and analytical functions prescribed by the modeller; the decisions are based on the information that the agent has available (information about himself, about other agents and the environment). Through their decisions, agents react and adapt to situations or conditions of the environment.

In Teahan (2010), there are several perspectives for the meaning of the term 'agent':

1. From the Artificial Intelligence perspective: the key idea is that an agent is embodied (e.g. situated) in an environment and makes its decisions. It perceives the environment through sensors and acts on the environment through actuators. For example, Intelligent Agents. Intelligent Systems. Robotics.
2. From the Distributed Computing perspective: the key idea is that an agent is an autonomous software process or thread. For example, 3-Tier model (using agents). Peer-to-peer networks. Parallel and Grid Computing.
3. From the Internet-based Computing perspective: the key idea is that the agent performs a task on behalf of a user. e.g. The agent acts as a proxy; the user cannot perform (or chooses not to perform) the task themselves. For example, Web spiders and crawlers. Web scrapers. Information Gathering, Filtering and Retrieval.
4. From the Simulation and Modelling perspective: the key idea is that an agent provides a model for simulating the actions and interactions of

autonomous individuals in a network. For example, Game Theory. Complex Systems. Multi-agent systems. Evolutionary Programming.

There are also several meanings for agent which causes confusion with the use of the term agent is that it is often related to the concept of “agency”, which itself can have multiple meanings. One meaning of the term agency is the capacity of an agent to act in a world – for humans, it is related to their ability to make their choices which will then affect the world that they live in. Another meaning of agency is authorization to act on another’s behalf – for example, a travel agency is authorized to act on behalf of its customers to find the most competitive travel options. The third meaning of agent relates to the meaning of agency often used in general English, such as used in the common phrases ‘insurance agent’, ‘modelling agent’, ‘advertising agent’, ‘secret agent’, and ‘sports agent’.

All of these similar, but slightly different, meanings spring from the underlying concept of an ‘agent’.

3.10 Concepts in Primary School Mathematics

This section explores how to teach mathematics concepts to primary school students. This process should begin with the goals and objectives of the educational institution, and then the cognitive structures that are already available to the students attending the institution should be assessed. This is followed by conducting a structural analysis of the instruction material in a manner that can be assimilated by these structures (Case and Okamoto, 1996, p.219).

Modelling the natural properties and quality dimensions of complex concepts may provide insight into developmental categories that may inhibit a student’s ability to conceive the concept. Learning activities can then focus on stimulating the development of the missing or incomplete natural property and its connections to other relevant dimensions in the new domain.

3.10.1 Numbers

Case (1993) determined that before the age of six, children tend to approach quantitative problems in a global fashion and address problems of a relative magnitude separately from problems of enumeration. He suggested that before the age of six, there are two separate cognitive structures used to address these problems; however, between the ages of five and seven, these two structures are integrated into a single structure. By evaluating Case's points and Gärdenfors' theory, it can be stated that the natural properties of the two age groups, five-six and six-seven year olds, had similar stimuli distances on the Voronoi diagrams; however, it is important to note that while the stimuli used to link the natural properties was the impetus for a new learning process, actual learning could not occur until cognitive development was sufficiently capable of facilitating the connection.

3.10.2 Shape

Shape is defined primarily as one of the visual attributes that include colour, shade and lighting, and it represents an unambiguous identification mainly due to its constancy (Pinna, 2011). Shapes are not usually considered a creation of the brain but rather appear vertically as part of the physical world. The essence of the shape is a major interest that is an integral part of mathematics from topology and mathematical analysis to trigonometry and geometry. Shapes are an important aspect of learning mathematics that are taught to students aged between seven to eleven years.

Ojose (2008) examined Piaget's developmental theory in relation to learning mathematical concepts and explained that Piaget identified the concrete operational stage, which includes the ages between seven and eleven, as the stage in which children master serialisation and classification. Both of these abilities are requisites for making cognitive connections within a conceptual space. Classification is necessary for constructing prototypes, and serialisation is related to making complex comparisons that evaluate something that has more than one dimension or natural property.

This illustrates the need to understand not only the quality dimensions and natural properties of a particular complex shape concept but also how the dimensions connect. Difficulty in mastering a concept may be attributable to a missing, albeit minor, dimension or skill.

Clements et al. (1999, p.192) noted that students in the United States struggle with learning basic geometry, primarily because the lessons require rote learning. They noted that this method has shown that these students 'do not recognise components, properties, and relationships between properties'. They recommended that lessons should be built upon the foundation of previously known concepts and discussed van Hiele's theory, which suggests that learning geometry is a guided progression through levels of thought. The development of the thinking process begins on an initial, Gestalt-like visual level and throughout increasingly various levels: analytic and descriptive, relational and abstract, formal deductive and mathematically rigorous (Clements et al., 1999, p.193).

3.10.2.1 Children's Perceptions of Shapes

Clements et al. (1999) suggested that initially, children do not recognise shapes based on previously observed templates but evaluate shapes based on 'recognition of combinations of essential features'. These are the natural properties and quality categories of each shape, which suggest that young children have cognitive access to these basic concepts, though they may not yet have integrated the knowledge. They explain that there is evidence to suggest that even very young children can recognise and match shapes even if they cannot identify (name) them.

In their research study examining the concepts of shape in young children (ages four to six years old), Clements et al. (1999) found that their subjects relied upon what they called unconscious schemas of properties to identify and classify specific shapes based upon features such as 'closed' and 'rounded'. They found that the youngest subjects were able to produce prototypes of shapes, particularly when exposed to exemplars (or non-

exemplars) of the shapes. They also noted that the children were able to recognise and identify features or properties of the shapes, although the language they used was not always clearly defined (e.g. 'pointy bits' = 'angles').

Clements et al. (1999) did notice that when the older children relied upon features of the shape in their efforts to classify, they made more classification errors, which suggests that while they have the ability to identify the features as belonging to a particular class of shapes, they have not yet learned some of the exclusion rules (e.g. Identifying a rhombus as a square because it has four sides and four points). They also noted that the children made more classification errors with shapes that had more variability (e.g. triangles and rectangles).

They advocated for a 'cognitively guided' curriculum that builds upon the schemas already developed in young children. Building models of natural properties and quality categories for each shape will allow the lessons to build upon natural knowledge based on these properties by first classifying (naming) these properties and then bringing them together to classify and identify the shapes.

Clements et al. (1999, p.208) advised teachers to encourage students to justify and describe whether or not a figure belongs to a specific shape category. Visual descriptions (e.g. prototype-based) should be expected and accepted; however, property-based responses should also be encouraged. These verbal descriptions may be spontaneous for shapes that have fewer but stronger prototypes (such as square, circle, etc.). Furthermore, shapes with additional possible prototypes (e.g. triangles) should be instructionally supported; however, in all cases, the traditional approach that is only based on the prototype with limited exemplars should be discarded.

In their work entitled 'A Guide to Effective Instruction in Mathematics: Geometry and Spatial Sense, Kindergarten to Grade 3' Education (2005, p.5-7), the authors identified 'three big ideas' to guide teaching and learning

geometry and spatial sense. The first of these ideas is an understanding of the 'properties of two-dimensional shapes and three-dimensional figures', followed by geometric relationships and then movement and location. This text provides teaching examples for guiding students in constructing models of two- and three-dimensional structures. The text emphasises that children initially describe their environment 'using vocabulary related to observable attributes' and that instruction in geometry should build upon this by focusing on the students' attention to specific features and properties of shapes. This process will assist students in the development of the 'ability to analyse and describe the geometric properties of shapes and figures' instead of spending learning time on memorising definitions.

3.10.2.2 Representing Shapes

Students learn how to classify images and how to explore the spatial distribution of a shape, which consequently leads to recognising the shape. Representation is a visual method that help students classify images and shapes (Zhang et al., 2010).

Lovett and Forbus (2010) posited that shapes are represented in two ways. Firstly, a shape is represented as a qualitative representation, which is a structure with a set of elements, such as edges and corners. Secondly, shapes are represented quantitatively in which new representations of a shape are compared to the qualitative representation already formed. This theory fits the Conceptual Spaces model in that new shapes are learned by integrating the quality categories into a concept of a particular shape. Once the new concept is formed cognitively, it can serve as a model (prototype) of a singular concept that can be used to compare other similar representations.

In turn, the prototype will be updated as the individual learns more and updates the original concept to integrate newly learned information (Moravčík, 2005). Lovett and Forbus (2010) tested their theory with a computational model of structural alignment called the Structure Mapping Engine (SME).

The SME is fed two shapes (structured representations), a base item and a target item, and the programme computes a structural evaluation score based upon how similar the two shapes map together. The primary bases for comparison are the shapes' edges and then the orientation of the corresponding edges. The programme can then identify whether the shapes are rotations or reflections of one another.

3.10.2.3 Quality Dimensions for Two-Dimensional Shapes and Three-Dimensional Figures

OECD (2005, p.7) developed a text to assist teaching efforts in primary geometry (children aged five to eight years). They explained that the properties of two- and three-dimensional shapes have 'properties that allow them to be identified, compared, sorted, and classified'. This is the primary purpose of modelling a concept in conceptual space, and to that end, the properties as outlined in this text are examined in the next section.

3.10.2.4 Geometric Properties of Two-Dimensional Shapes

Students in elementary grades are encouraged to learn about the geometry of shapes and figures with two and three dimensions. Two- and three-dimensional shapes are characterised by specific properties that should be understood by students. The properties consist of sides, angles and parallelism, for example. These properties help students identify, sort, compare, compose, classify and decompose geometric forms, which enables students to continue to learn about dimensional shapes.

Furthermore, properties enable students to understand that angles are considered measures of turn and that they are categorised by their degree of rotation. Students can also understand polygons and their attributes, which facilitates the exploration and investigation of concepts in measurement and geometry (Zealand, 2011).

Amongst the significant properties used in identifying and describing two-dimensional shapes, there are:

- a. Number of shape sides:** this is one of the main properties that students begin learning, which helps in the identification of various shapes (e.g. triangles, quadrilaterals, pentagons, hexagons).
- b. Number of shape vertices:** for young students, this is referred to as 'corners' for two-dimensional shapes. In later grades (e.g. 2 and 3), students learn that a vertex is the meeting point of two sides. They also become aware of the fact that a shape's vertices number is equal to its sides number.
- c. Length of shape sides:** students learn about the importance of this property for many two-dimensional shapes, and they learn that square sides and rhombus sides are of equal length, whilst in rectangles and parallelograms, the lengths of opposite sides are equal.
- d. Size of angles:** In primary grades, students should be able to understand basic ideas about angles and how they are formed. Students can also visually compare the angle sizes using relationships such as 'small', 'equal' or 'bigger'. In addition, primary school students learn about the concept of a right angle (or 90 degree angle) and refer to it using informal expressions, such as a 'square angle' or a 'square corner'.
- e. Parallel lines:** This is informally introduced to students at primary schools as 'two lines that run side by side in the same direction and remain the same distance apart'. Furthermore, and to help students identify and describe parallel lines in parallelograms, they are provided with examples of parallel lines that exist in the classroom (such as the opposite sides of a book cover, the top and bottom edges of a bulletin board, etc.) (OECD, 2005, pp.13-14).

3.10.2.5 Geometric Properties of Three-Dimensional Figures

- a. Number and shape of faces:** this is the best way to describe the properties of a polyhedron (which is a three-dimensional figure whose faces are polygons). For instance, a cube is composed of six square

faces, whilst a rectangular prism is composed of six rectangular faces (OECD, 2005, p.14).

b. Number of vertices or edges: these terms help students describe three-dimensional figures and their properties.

Hence, the use of these terms and expressions when teaching geometry to primary school students provides the language that is necessary to express ideas about geometry, rather than providing definitions to memorise (OECD, 2005, pp.16).

3.10.3 problem-solving

Mathematics not only involves mastering concepts but also applying those concepts to solve problems. For instance, Lesh and English (2005, p.195) suggested that modelling research should be conducted to develop models that express the context of conceptual tools that represent the concepts necessary for solving real-world mathematical problems. This too can be modelled within Conceptual Spaces, where the newly formed concepts are linked to a larger domain, as the concepts are applied within a problem-solving environment. Lesh and English (2005) also noted that there is evidence that models and modelling research has helped students 'develop powerful models for describing complex systems that depend on only new uses of elementary mathematical concepts'. This suggests that the models used for examining mathematical concepts and mathematical learning are not just restricted to understanding the phenomenon of learning the concepts but should also be part of the process of learning.

3.10.4 Concepts and Misconceptions in Primary Mathematics

Some of the basic problems that most primary school students face in learning mathematics are related to misunderstandings of the concepts that are presented to them, especially in the field of geometry. Geometry includes many concepts that have only slight differences, which may be the source of misunderstanding for the students in learning geometrical shapes.

Lovett and Forbus (2010) explained that people utilise either an abstract or a detailed representation of shapes. Lovett and Forbus also argued that the abstract representation, which is a qualitative representation of the shape, establishes the special relationship of the shape's parts and that it is an important means for comparing the shapes.

Lovett and Forbus (2010) model fits well with Gärdenfors' Conceptual Spaces model, where a new shape is learned through integrating its quality dimensions into a concept of a particular shape. Once the new concept is formed cognitively, it can serve as a model (prototype) of a singular concept through which it can be compared with other similar representations. The prototype will be updated as the individual learns more and updates the original concept to integrate the newly learned information (Moravčík, 2005).

Education (2005), guidance to mathematics instructors states that the effective understanding of the geometrical properties of two- and three-dimensional shapes first requires the concepts to be displayed visually in their respective two and three-dimensional figures. This is followed by providing their geometric details and finally demonstrating locations, movements, rotations, reflections and translations of the geometric properties. These steps are critically important when modelling a concept using Conceptual Spaces, and to that end, the properties as outlined in this text are discussed in detail.

Modelling the natural properties and quality dimensions of complex concepts can provide insight into developmental categories that may inhibit a student's ability to understand a concept. Difficulty in mastering a concept may be attributable to a missing dimension or skill. Learning activities can then be focused on stimulating the development of the missing or incomplete natural property and its connections to other relevant dimensions in the domain. This served as the foundation of this study in developing of a cognitive framework that identifies the missing dimension or skill by adopting a Conceptual Spaces cognitive theoretical framework within an e-learning system. For this work, the aim was to develop an e-learning system that bases its agent-oriented

model on a sound cognitive theory to identify the missing dimensions and skills of students and to fill the gaps in their skills.

Several researchers (Biber, Tuna and Korkmaz, 2013; Marchis, 2012; Öksüz, 2010) have examined misconceptions and have discovered that students have misconceptions of understanding different forms (symbolic, visual, etc.) of the same geometric concepts. For example, Marchis (2012) found that two-thirds of the 36 primary school students they investigated could not define basic geometrical shapes correctly. Students were also unaware of the properties of these shapes. Furthermore, Biber, Tuna and Korkmaz (2013) conducted a study involving angles in geometry on thirty students aged fourteen. The author found that students focused on the physical appearance of the geometrical shapes without focusing on the properties of the shape.

Other researchers examined Piaget's developmental theory (Ojose, 2008) that focused on learning mathematical concepts. Ojose indicated that Piaget identified the concrete operations stage (ages seven to eleven) in which children master striation and classification. Both of these abilities are prerequisites for making cognitive connections within a conceptual space. Classification is necessary for constructing and updating prototypes, and serialisation is related to making complex comparisons and evaluations on more than one dimension or natural property. This illustrates the need to understand not only the quality dimensions and natural properties of a particular complex shape concept but also how the dimensions connect. Difficulty in mastering a concept may be attributable to a missing, albeit minor, dimension or skill.

Clements et al. (1999) noted that students in the United States struggle with learning basic geometry, primarily because the lessons require rote learning. They found that these students 'do not recognize components, properties, and relationships between properties' (Clements and Battista, 1992). They recommended that lessons should be built upon the foundation of previously known concepts and discussed van Hiele's theory, which suggests that

learning geometry is a guided progression through levels of thought. Furthermore, the thinking process develops from an initial, Gestalt-like visual level and progresses through increasingly sophisticated levels that include descriptive and analytical, formal deductive, abstract and relational and mathematically rigorous levels (Clements et al., 1999).

3.11 Summary

The purpose of this chapter has been to review the related work that is relevant for the work discussed in subsequent chapters. First of all, the notion of learning was described in Section 3.2. Then, the various pedagogical methods used in our e-learning system were discussed in Section 3.3. In Section 3.4 geometric concepts teaching at primary school was presented. Then, Section 3.5 presented a comparison of e-learning systems for teaching mathematics. Sections 3.6, 3.7, 3.8 and 3.9 respectively described the Cognitive models, Theory of concepts, Conceptual spaces and Agent-based modelling. Finally, Section 3.10 presented the main mathematical concepts taught in primary schools.

It was proposed that geometry is a form of a language and that teaching shapes and concepts for primary school children is best done through choral repetition and pronunciation practice concepts that could be reiterated to their minds making them understand it and re-enact through symbols as a great way to reinforce understanding.

Visual, auditory and kinesthetic methods make for effective tools to teach shapes, (Arntsen, 2013). This because children are not always visual learners and some will not learn through looking at pictures as this would be overwhelming for them, (Constantinidou and Baker, 2002). They could be linguistic learners in the class as well and following a text based learning methodology would work for them very well. Having the class go through the pictures and having them say the shapes and write them down is a great way of doing this. Once the children understand the pictures, texts could be written below those images denoting which shapes those are and have the

children also to write them. The teacher can also include activities where the children make sentences (written) denoting what shapes they see around them. This will ensure that they understand what is being taught. Also, it could be universally agreed that games are the best way to teach children any concept which could be complex faster as they go a long way in reiterating a concept more strongly in their minds, (SG, 2013).

This chapter shows that if these methods had to be juxtaposed to find a superior one, then the chances are there would be no conclusion as there are different types of learners in a class and it is very hard to identify that right away for teachers. Therefore, the best pedagogical method to make everyone learn a concept effectively is by using multiple learning styles, (Project, 2013) and it is proposed that this work attempt to deliver an agent based e-learning system that exploits the above pedagogical method in the delivery of learning.

The following chapters presents the early development work before implementing the e-learning system. This work was based on exploring aspects that are related to an agent-based e-learning system that employs the cognitive modelling of Gärdenfors Conceptual Spaces Theory.

Chapter 4

Exploring Agent-Based E-Learning Systems for Teaching Mathematics

4.1 Introduction

The aim of this chapter is to present the efforts undertaken to explore issues related to or arising from the design of an agent-based e-learning system that employed the cognitive modelling of Gärdenfors Conceptual Spaces Theory. It describes the early developmental work that led to the final design of the e-learning system that is described in Chapter 6.

It was critical for the project to mitigate the risks arising from an inadequate presentation of the concepts and to develop the organisation and the flow of the interaction with the students in a way that facilitated the students' conceptualisations of the mathematical concepts. Potential challenges were identified that required clarification to eliminate their risks and to increase the success of the project goals. The specific goals/objectives for the research have been stated in section 1.3.

The following points summarise the challenges that were identified early in the design phase of the project:

1. selecting software to develop the e-learning system;
2. designing a user interface that displays the concepts in an assimilating orientation;
3. designing an intuitive interaction;

4. identifying sources of misconceptions (errors) that affect special conceptualisations of mathematical concepts;
5. prototyping a baseline for the e-learning system.

Part of the work presented in this chapter contributed to a poster and a paper that were respectively submitted to the 7th Saudi Arabia Students Conference at Edinburgh University (UK) and to The international Conference Competitiveness of Learning: Innovation and Opening new Markets, Bielsko-Biala (Poland). The poster presented an e-learning model that was developed based on Conceptual Spaces cognitive modelling for teaching geometric shape concepts (Alghurabi and Teahan, 2014).

The paper presented an approach for investigating the cognitive issues that are key in developing an effective e-learning system that would improve instructional outcomes (Alghurabi and Teahan, 2014). The investigation focused on the nature of misconceptions (errors) made by primary school children in geometry and whether these misconceptions were consistent with Gärdenfors Conceptual Spaces cognitive model, which represents objects and abstract concepts based on their quality dimensions (features that describe the concept).

The remainder of this chapter describes the diagnosis for maximising the effectiveness and usability of the e-learning system. Section 4.2 details the process of selecting a software platform that enabled the development of an e-learning system that could functionally and institutively be usable at many schools in Saudi Arabia and in the UK. Section 4.3 details the methodology used to compose a set of lessons and questions that were intended to identify the misconceptions of the geometric shapes, which is the core of this thesis project. Section 4.4 presents an initial pilot prototype experiment of the e-learning system that included a test of mathematical concepts as well as the results and the outcome of the pilot experiment. Section 4.5 describes the extended pilot experiment with Saudi Arabian students, its results and its outcome. Finally, section 4.6 summarises the chapter and explains the direction of the project work in building an effective e-learning system that

is based on Conceptual Spaces cognitive modelling to enhance student understanding of mathematical geometric shape concepts.

4.2 Design Considerations for an Agent-based E-Learning System

The author of this dissertation feels that designing an agent-based e-learning system entails a number of important considerations, which are summarised as follows:

1. The agents involved with the system.
 - a. Who are the agent?

2. Agent aspects of the e-learning system for primary students.
 - a. How and when should the e-learning system act on behalf of the users?
 - b. How should the system respond to the requests and proactively help the users?

3. Interaction / Dialogue between the e-learning system and the students.
 - a. What form should the interaction take between the e-learning system and the user?
 - b. What form of 'conversational' dialogue is most appropriate for teaching mathematical concepts to students in primary schools?

4. User-modelling and personalisation for teaching mathematical concepts.
 - a. How should the system (cognitively) model the users, e.g. represent their knowledge and understanding?

5. Evaluation of the agent-based mathematical e-learning system.
 - a. How should the effectiveness of an automated agent-based system be evaluated?

4.2.1 Selecting the Development Environment

There are several different programming languages that are used in developing various e-learning systems. Aside from their popularity among software developers, certain programming languages are operating system platform dependent. The main operating systems are Windows, iOS and Android platforms; however, the choice for this project was based primarily on trends within schools and with students. Apple computers and tablets have emerged as a dominant choice among educational systems, including primary schools and students. Therefore, Apple iOS was adopted as the platform in this study.

Other criteria were also considered when selecting the programming language. These criteria included the ease of use, organising ideas to adapt to models, structuring the flow and interaction, flexibility in developing models that build on an agent-based approach and the use of an open source language, for example. Based on these criteria the NetLogo programming language was selected for the implementation of this project mainly as it is an environment that allows rapid prototyping of interfaces and code compared to other languages such as Java and Javascript.

NetLogo is an open source software platform designed by Uri Wilensky, director of Northwestern University's Center for Connected Learning and Computer-Based Modelling. This language provides a programmable agent-based modelling environment that is 'well suited for modelling complex systems' and that easily facilitates the development of new models or modifications of existing models (Tisue and Wilensky, 2004). Programmes developed in NetLogo are called 'models', which will be used as a naming convention in the upcoming chapters.

NetLogo was useful for the implementation of this project because it is a multi-agent programming language that has been used for simulating complex phenomena that are related to the field of education (Tisue and Wilensky, 2004). According to Tisue and Wilensky (2004), NetLogo was

designed for both education and research, and it has also been used within a wide range of education levels. Hence, it can be used as a tool for research as well as for teaching at various levels of education.

NetLogo enables the users and e-learning system designers to create and to open simulations and collaboratively run models and to develop them simultaneously to achieve an optimum outcome from the development (Tisue and Wilensky, 2004). Furthermore, this programming environment can also allow the user to explore the behaviour of the open simulations under different conditions (Tisue and Wilensky, 2004; Wilensky and Resnick, 1999). In addition, NetLogo is simple to use and enables students, researchers and unseasoned programmers to easily create their models.

4.2.2 Design Considerations for the User Interface

An important aspect concerning the development of the e-learning system is the user interface that is to be adopted by the system. A description of some of the metaphors used in user interaction design is presented in Table 4.1. Increasingly, user interfaces are driven by an agent interaction metaphor, where the e-learning system both aids the user and acts on his/her behalf. For this project, a specific type of agent interaction metaphor was investigated, which is called the conversation metaphor, and was deemed to be appropriate for the objective of the user interface.

According to this type of metaphor, the interaction design focuses on the dialogue between the user and the system. Essentially, the system becomes a proxy for the system developer in the conversation.

Furthermore, with NetLogo, the user interface can include the menu on any location of the screen. The choice of the location of the menu is partially influenced by the desired user interaction but is heavily dependent on the type of programme. Examples of NetLogo interfaces showing different orientations of the menu options are presented in Figures 4.1 and 4.2, and the interfaces enable the user to focus on certain aspects of user-agent interactions. The models show the early interfaces that were explored in the context of this

Table 4.1: Some interface metaphors used for designing the user interaction of an e-learning system.

Environment	The user interface shall enable students to interact with the environment which they explore (such as a menu-based system). The direct user interaction is supported using a task domain model, in such a way that the interaction will be controlled by users. The standard HCI label for (Human Computer Interaction) covers these types of systems.
Agent	This treats the computer system as an intermediary that responds to user's requests. The emphasis is on the system to proactively help the user. A label used for these systems is 'HAI' which stands for 'Human Agent Interaction'.
Conversation	Whilst users are in a conversation or 'dialogue' with the system, there will be an interaction between the user and the system designer. Emphasis is on designing two-way feedback between the user and system. Systems that use the conversation interface metaphor are subsets of the systems that use the agent metaphor (since they must also employ an agent oriented approach to interaction). We coin the following label for this type of approach: 'HHI' for 'Human Human Interaction' since the conversation is between the user and the system designer(s) whose proxy is the system itself.

research project. The interface elements in Figures 4.1 and 4.2 convey conversational aspects to the user, such as 'Let me explain it to you' and 'Play with me'. The code for the 'Chat with me' element has not been completed; therefore, it is shown in red for this model.

These models provided essential insights into designing a user interface that is developed based on user-agent interactions for the purpose of teaching. Interface elements provide an impression of a dialogue and use a first-person agent perspective to encourage learning through situated cognition (the later of which is mentioned in (Teahan, 2010, pp.143-144)).

Figure 4.1 shows a model for learning area and perimeter concepts and demonstrates a vertical menu where user interactions are presented on the main window. It also provides an animation that is intended to explain the concepts. Figure 4.2 shows a model that has a user-agent interaction in a game setup, where the control of the game is placed horizontally at the bottom, and output from a chatbot is placed vertically on the right side of the main window to provide textual dialogue.

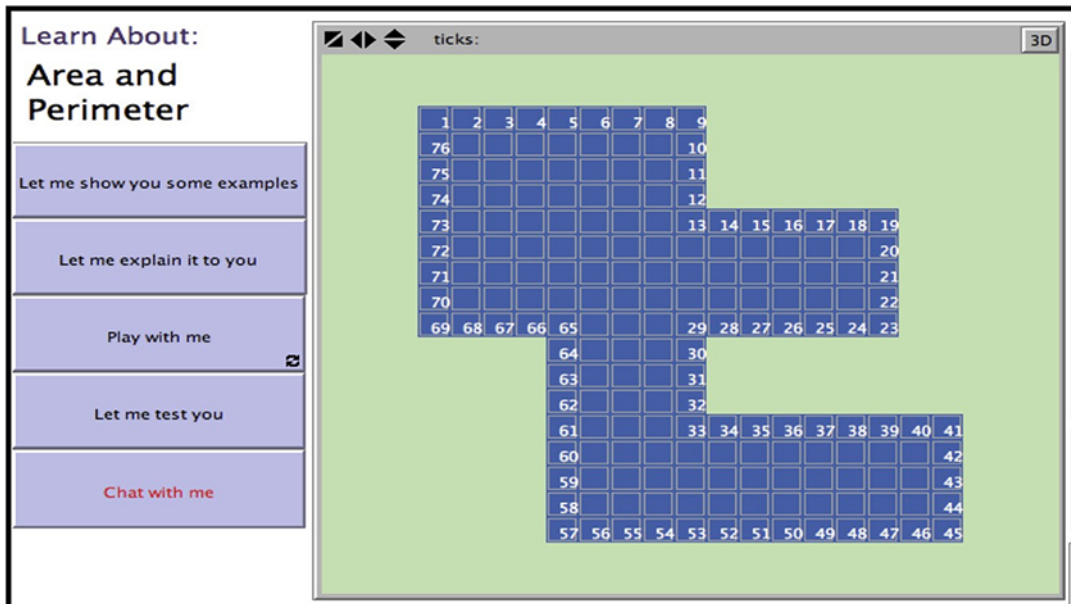


Figure 4.1: NetLogo model for learning area and perimeter.

These models were designed to aid teaching and to test mathematical concepts. Although four pedagogical methods—stories, games, chatbots and animation—were used to enhance the interaction of the user with the program, they do not resolve two fundamental issues related to user modelling and evaluation.

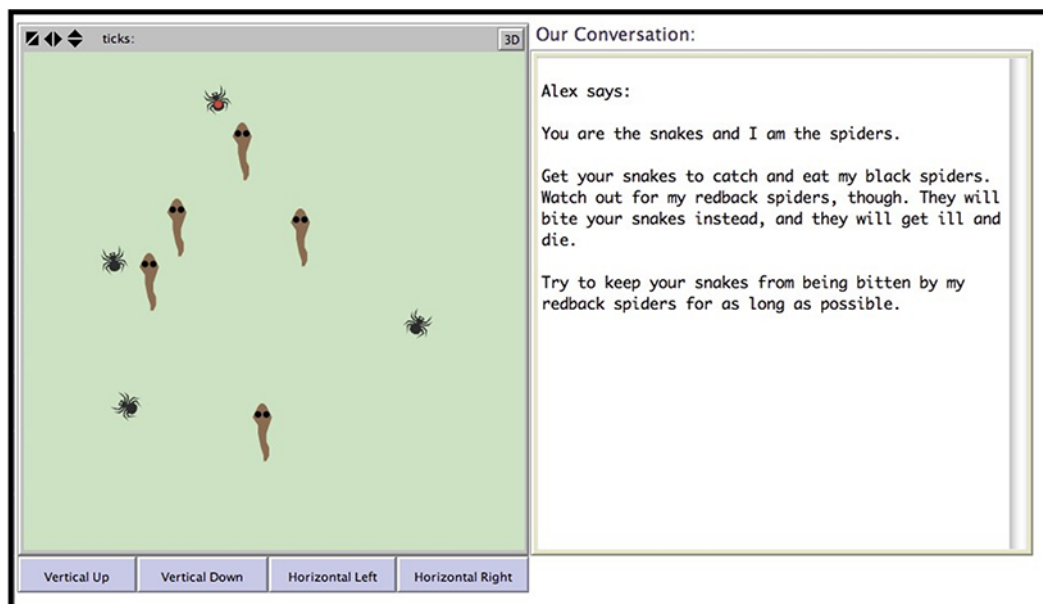


Figure 4.2: NetLogo based Snakes and Spiders game.

For user modelling, for example, there are various questions that need to be answered, such as: 'What does the student learn?' and 'How can we identify

the amount of knowledge they learnt or missed'? Furthermore, evaluating the effectiveness of the e-learning system requires answering additional questions, such as: 'Does the system really help the students to learn the subject matter'? and 'Does it do it better than other systems'?

4.2.3 Design Consideration for Agent-User Interactions

Teachers frequently face issues during the teaching process. Hence, they may ask themselves the following questions: 'What are the issues of the subject matter that students are aware of and/or that they understand'? and 'What are the issues of the subject matter that students are not aware of and/or that they don't understand'? If students have certain misunderstandings or a lack of knowledge about a given concept, then the following questions arise: 'What was the cause for this issue? Is this due to the complexity of the taught concept or to the inability of the teacher to teach'?

Additional questions include: 'How can we help the students to avoid mistakes? Is it the responsibility of the teachers to resolve this issue, or is it due to the current teaching method'? Therefore, these questions were the key factors and motivators in designing the e-learning system for the research project described in this dissertation. Hence, the aim of designing a user interface for this project was to develop an agent-based interaction that has flexible behaviour based on a Conceptual Spaces approach to cognitive modelling, which would help students conceptualise and understand mathematical concepts such as shapes. It also became evident that the user interface should employ a combination of pedagogical means that stimulate students' cognitive perceptions when learning mathematic shapes.

An initial prototype of the user interface and the interaction of students with the system was developed to achieve the objectives of this project as well as to address the issues and the concerns mentioned. Figure 4.3 shows the NetLogo based prototype model. This model adopts the agent-based metaphor that incorporates visual and audio interface elements for agent-user interactions. The visual and audio elements were intended to effectively aid

the students in conceiving the quality dimensions of the cognitive structure that is mentally represented for each shape. The interface for the developed model is described in more detail in section 4.4. The idea behind this design was that if the features describing a concept are better organised in the mind of the student, then this will aid the student in identifying, comparing and distinguishing between the various concept shapes.

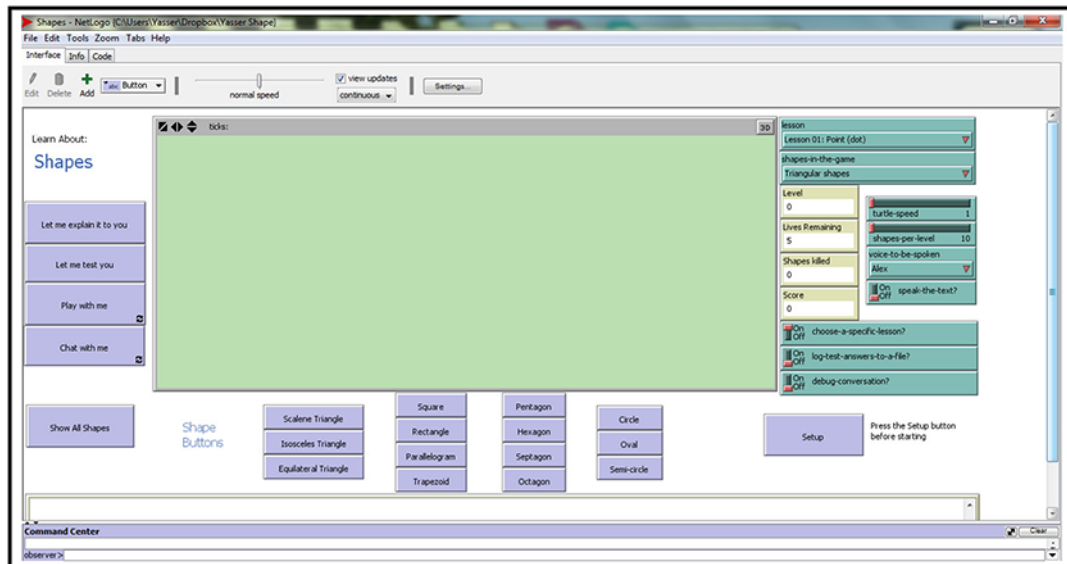


Figure 4.3: Screen shot of the prototype e-learning programme's main interface.

4.3 Methodology

This section details the methodology that was adopted for the exploratory research conducted, as described in this chapter (which is discussed in more detail in the following sections).

Before the programming of mathematical concepts in NetLogo, it was critical to focus on the curriculum of geometric shapes that is taught at primary schools. Formulating the questions from a standard curriculum would provide the means for identifying common misconceptions of the geometric shape concepts that are taught at primary schools. These misconceptions guided the design of the agent-based e-learning system, which is based on Conceptual Spaces cognitive modelling.

There are two main aspects of the properties of two-dimensional shapes and three-dimensional figures in primary schools that should be highlighted:

- The properties of two-dimensional shapes and three-dimensional figures help in their identification, comparison, sorting and classification.
- Representing two-dimensional shapes and three-dimensional figures to students in various sizes, forms and orientations should enhance their understanding of such properties.

The following section details the rigorous process that was undertaken to achieve the objective of understanding the Conceptual Spaces cognitive model used by primary students to conceptualise and organise the geometric concepts quality dimensions. The outcome of this process was used to build an effective e-learning system for mathematical shape concepts in this project, which is described in Chapters 6 and 7. These chapters also describe the design methodology for the interface for the e-learning system.

4.3.1 Analysis of the Content of Three National Curriculums

Three curricula were selected for analysing the mathematical concepts in geometric shapes that are taught at primary schools. The examined curricula were from the United Kingdom, New Zealand and the Kingdom of Saudi Arabia. Furthermore, a detailed analysis was performed with the aim to establish the years in which the concepts are taught to primary school students. (Appendix A shows the curriculum analysis of the three countries for contents geometric shape concepts.).

Hence, a total of twenty-five concepts in geometric shapes were identified and selected from the curricula. Moreover, an investigation was performed on various study levels in which these concepts were first introduced to the students as well as their continuation over the subsequent schooling years. Thus, primary schooling (e.g. year 1 to year 6) was examined across the curricula of the three countries. These concepts became the basis for investigating the common misconceptions when learning geometrical shapes that will be modelled in the e-learning system to make the latter better tailored to individual student needs. Based on the analysed curriculums, an initial set of questions were developed from the mathematical concepts of

geometric shapes taught at primary schools. These questions became the starting point for the next stage of the pilot project. It was necessary to build a prototype and to verify the level of difficulty of the lessons and the questions; however, the conceptual level of difficulty required verification and appraisal from the mathematical instructor community.

4.3.2 Verification of Conceptual Level of Difficulty

Before sending the initial set of questions for the appraisal of their level of difficulty, it was necessary to verify them by building a prototype e-learning system. This enabled addressing any issues that were uncovered during the verification of the conceptual testing. For example, some issues might be related to the programme that was created with NetLogo. Further issues might be linked to the user interaction as the lessons were followed and questions answered for each concept. Even though the lessons and the questions were formulated based on concepts taught in primary schools, their levels of difficulty were still not fully understood from the students' perspectives.

The initial pilot experiment was conducted with a self-selected sample of eight students from three schools in the UK that were contacted on an individual basis through their parents. In addition, the questions that the students answered incorrectly became the first step in determining general misconceptions of shape concepts.

4.3.3 Appraisal of the Sample Test for Concepts

Upon completion of the pilot experiment with the eight students, the content of the NetLogo programme, lessons and questions was sent to a mathematical instructor community. Based on the insights obtained from the pilot experiment, the set of questions were further expanded to include both visual and textual-based questions. The expanded set of questions was then sent to math teachers at schools as well as to university professors and lecturers to receive an appraisal of the conceptual level of difficulty and the questions. For this initial investigation, only the Arabic language was used

for several reasons. These reasons include the familiarity of the researcher with the Arabic language and the curriculum taught in Arabic as a previous teacher in an Arabic School as well as the flexibility of the environment and the offered collaboration in schools teaching in Arabic; however, the aim was to eventually extend the experiment using the English language.

In general, the feedback received from the mathematical instructor community was very encouraging, and it was indicated that the approach and the questions were appropriate for identifying the level of misconceptions in the concepts of geometric shapes for primary school students. Furthermore, some teachers provided suggestions for additional questions to be included in the testing of the shape concepts. There was a common theme in the comments received from the instructors. The majority of them (9 out of 12) were concerned about how to anticipate the outcome of the students' conceptualisations of concepts. The instructors were also concerned about how students would perceive the quality dimension of the concept and retain it to use the specific structure for future comparative mental exercises.

Other feedback suggested the simplification of the textual definitions of concepts and their presentation. The instructors also suggested that the concepts should be arranged in their order of difficulty so that the students would be introduced to and then tested on simple concepts first and then gradually progress to relatively more difficult concepts. Hence, a final refined set of questions was produced based on the appraisal process; however, before finalising these questions, cross checking between the curricula and the questions set was carried out by surveying primary school math teachers.

4.3.4 Survey of Primary School Math Teachers

The process of generating a set of questions from lessons that were identified from the analysed curricula required cross checking with primary school teachers. Because the project targeted students in Saudi Arabia (as was explained in section 4.3.3), primary school math teachers in Saudi Arabia were in a better position to further comment on the set of questions that were

included in the geometric concepts testing. The survey was carried out in the Arabic language using Google survey tools. The survey questionnaire contained the format and the presentation of the questions that were developed in the updated prototype e-learning system.

There were a limited number (4 teachers) of responses from the surveyed primary school math teachers, and the meaningful comments that were received from them were incorporated in the questions set. Appendix B shows a link to the Arabic survey designed for primary school math teachers. Appendix B.1 Shows Screenshot of the main Interface of the survey and the First page of the Survey. Appendix B.2 shows a list of all the concepts covered by the Arabic survey. Appendix B.3 shows a list of all the concepts covered by the survey translated to English.

4.3.5 Final Set of Questions for Geometric Shape Concept Testing

It was important to generate a set of questions that would achieve the objectives of the e-learning system, which is based on Conceptual Spaces cognitive model for teaching mathematics. The iterative process of generating the final set of questions was concluded after incorporating the feedback obtained through the survey distributed to primary school math teachers. These questions were then used in the remaining stages of the main project. The survey conducted on e-learning systems (as part of the Objective 1 phase of the research) led to the conclusion that two major issues, user modelling and evaluation, have not been adequately dealt with so far.

In the next section, the pilot experiment that was conducted to assist in developing an e-learning system that uses an effective method of user modelling based on the Conceptual Spaces cognitive model is described in detail.

4.4 Pilot Experiment 1: Evaluating a Prototype E-learning System

A pilot experiment that was based on a prototype model developed using the NetLogo programming language was conducted. The programme was designed to run either as a standalone e-learning system or as a system that could be accessed through the Internet. It was expected that the students would install the programme on their computers and then complete the lessons and the test questions at home. There were also some difficulties in obtaining required authorisations (such as obtaining consent, see below) and in the procedural steps for reaching out to primary students to conduct the pilot experiment.

4.4.1 Design of Experiment Using Prototype E-Learning System

To avoid delays in procedural issues and after obtaining consent from parents, a sample of eight primary school students in the UK were enrolled in a pilot experiment using the e-learning prototype model. The sample of the eight primary school students was composed of different ages (from 9 to 12 years old), genders (equal number of male and female) and classes.

In this experiment, forty concepts concerning geometrical shapes from 26 lessons were covered, with 78 questions that closely followed Standard UK, Saudi Arabia and New Zealand primary school curricula. Each student was expected to interact with the e-learning programme written in NetLogo 5.0.4. This programme consists of three main modes (see Figures 4.3):

Mode 1: This is initiated using the 'Let me explain it to you' button and provides students with a short explanation of each lesson on basic concepts in geometrical shapes.

Mode 2: This is initiated by the 'Let me test you' button, which presents a set of up to 6 multiple choice questions per lesson to test student understanding

from Mode 1. In addition, this mode provides a total result with a list of correct and wrong answers.

Mode 3: The 'Play with me' button allows the student to play a multi-level game that helps the student gain better understanding by reinforcing the concepts learned in Mode 1.

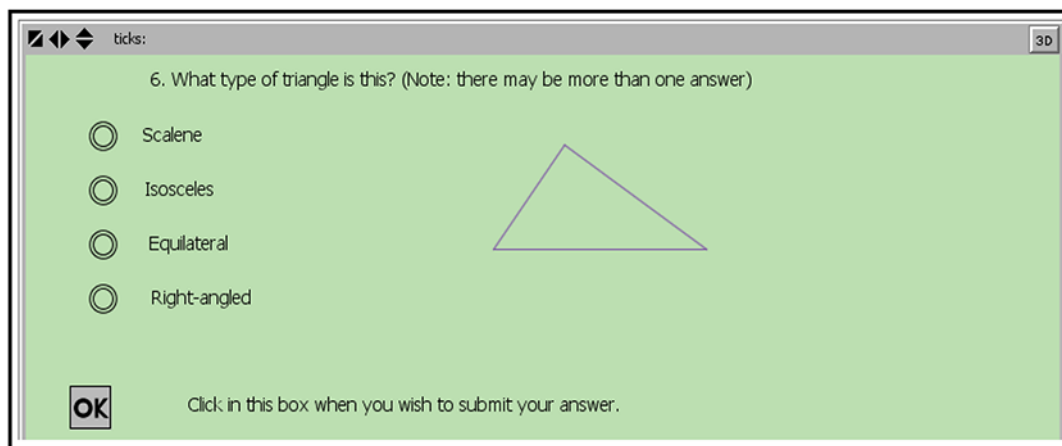


Figure 4.4: Question for the Triangle concept.

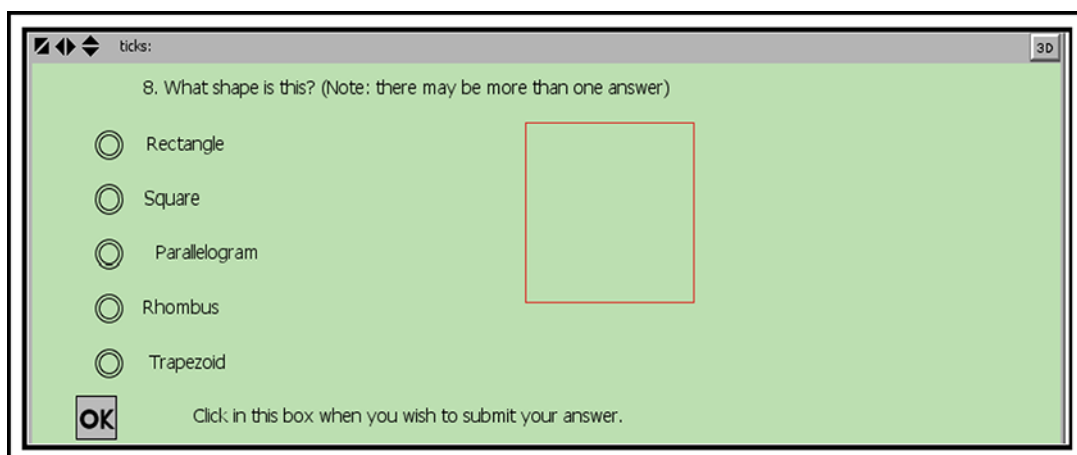


Figure 4.5: Question for the Square concept.

The e-learning system recorded the results of students' answers to the questions. The data obtained from the pilot project with the initial prototype was used to determine lessons and concepts where the students made mistakes and therefore had misconceptions. Figure 4.4 and Figure 4.5 show sample questions that were presented to the students using the prototype e-learning system. These were examples of basic initial designs thought appropriate for the initial pilot experiment.

The identified misconceptions (as identified by the percentage of incorrect answers) were used to determine the correlation of the Conceptual Spaces cognitive model for the main project study. It was anticipated that at least two design cycles would be required during the process of obtaining a comprehensive set of misconceptions, and the questions would require further fine-tuning to cover multiple aspects of the geometric shape concepts.

4.4.2 Results of Experiment Using the Prototype E-Learning System

The results obtained from this experiment showed an average score of 92.6% for students who answered all questions. There were many students who did not have difficulties recognising most of the shape dimensions (names, number of sides, etc.); however, the mistakes on specific questions (see figure 4.6) revealed consistent misconceptions that are consistent with the model

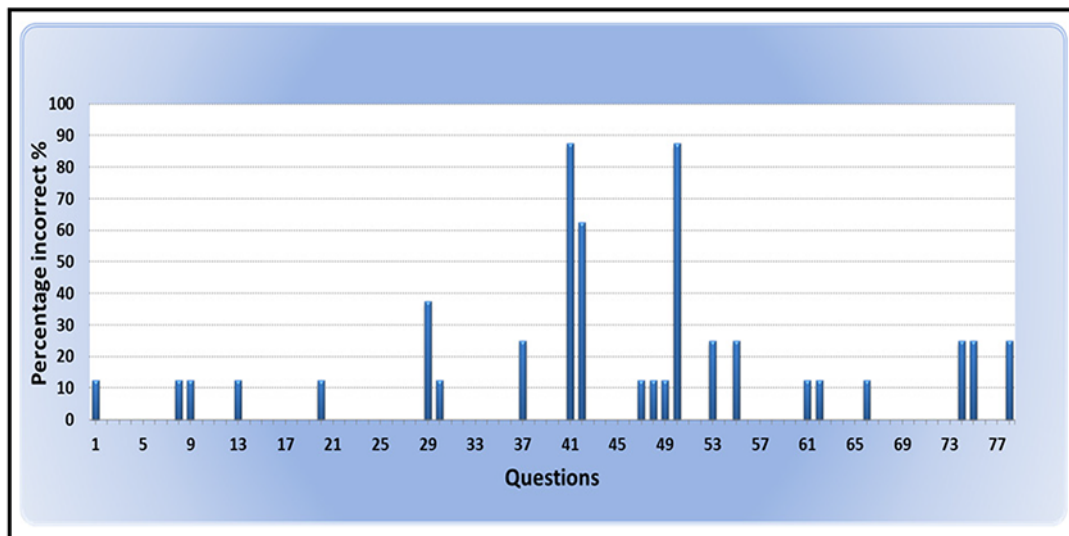


Figure 4.6: Percentages of incorrect answers (y axis) for each question (x axis).

to the Conceptual Spaces cognitive model. Some of the notable observations include:

1. The majority of students (87.5%) could only recognise a shape if it was shown with a standard orientation.
2. Very few students (1.6%) did not know specific shape names.
3. Many students (60.9%) did not realise that there may have been multiple labels for the same shape (e.g. a square is also a parallelogram as well

as a rhombus) and therefore chose a single label using the name for the narrowest concept only (e.g. square rather than parallelogram or rhombus).

4.4.3 Outcome of Experiment Using the Prototype E-Learning System

The results obtained from the pilot experiment using the standalone e-learning system are relevant for the Conceptual Spaces cognitive model when considering the possible quality dimensions (e.g. features) that students used to define the concepts. For example, most students gave an incorrect answer to the question shown in Figure 4.4 because they could not recognise the shape when it was rotated differently than the way it was originally taught. Similarly, most students knew the square shape and might also have known that it was a rhombus, but they overlooked the fact that the shape was also a parallelogram. These misconceptions point to a basic aspect of Gärdenfors' Conceptual Spaces cognitive model that defines quality dimensions as a building block for organising and structuring the mental perception of a shape.

Although the outcome of the pilot project was quite encouraging, there were still critical questions that required further study before fully accounting for the quality dimensions of the concepts included in the lessons and the questions. Critical issues included whether the different student groups had consistent misconceptions (mistakes) and determining which quality dimensions were used by the students for concept organisation in their mental perceptions. The answers to these questions were the basis of the critical decision to proceed with developing an e-learning system based on the Conceptual Spaces cognitive model, which would be effective for teaching mathematics, and more specifically, geometric shape concepts (see the poster that described this research that was presented at the 7th Saudi Arabia Students Conference at Edinburgh University (UK) in Appendix C).

4.5 Pilot Experiment 2: Further Evaluation Using a Paper-Based Test

To make objective progress towards building an effective agent-based e-learning system, a greater understanding of the cognitive issues related to mathematical geometric concepts was required. Hence, a second pilot experiment was conducted to perform further evaluations of student misconceptions and to determine which quality dimensions the students used to conceptualise the shape concepts. Therefore, a paper-based test was used, as it was not possible to use the prototype programme for many students, such as Arabic students, due to the lack of support for the Arabic language character set.

This second pilot experiment included an updated version of the question set that culminated the iterative process of generating comprehensive questions for testing geometric shapes concepts intended for primary students. The paper test enabled a thorough investigation of the nature of misconceptions made by primary school students regarding geometric shapes and concepts. Another aim of the experiment was to administer a test using a similar method that the students were accustomed to in their exams.

4.5.1 Design of Experiment 2: Using a Paper-Based Test

In general, the design details of the extended experiment were the same as the design described in section 4.4.1. The target subjects for this experiment were students at primary schools in the Kingdom of Saudi Arabia (as explained earlier in section 4.3.3). In this experiment, the lessons and the questions were ordered in an increasing level of difficulty for presenting geometric shape concepts to the students. Furthermore, the questions were arranged in such a way that the mathematical reasoning would not be negatively impacted due to an incomplete model of a specific concept. The design objective was that a student would gradually progress to more complex concepts, and that he/she would have the potential to integrate all the relevant quality categories or natural properties for the advanced concepts.

In this experiment, there was a concern over the length of time required to take the test. Therefore, the lessons covered and the questions asked in the previous e-learning system prototype were updated for this experiment. There were two additional reasons for this design change:

- (a) The scope of the experiment that focused on students' mistakes when answering the questions. This was the basis for identifying the students' misconceptions. There was no longer a need to repeat the same experiment.
- (b) A quick turnaround for the experiment could only be achieved with the use of a paper test rather than directly using the prototype e-learning system at this stage.

Furthermore, the paper-based questions provided an easier administration at multiple schools across the Kingdom of Saudi Arabia. The details and the format of the paper-based questions are included in Appendix D, also Appendix D.1 translation of D and will use in chapter 5.

The questions for this experiment can be classified into four types: Diagrammatic, Description, Symbolic and Combination. This classification of questions depends on whether the main part of the question involves diagrams, mainly text descriptions, tests on the understanding of symbolic representations or a combination of techniques to test student understanding (e.g. descriptions combined with diagrams).

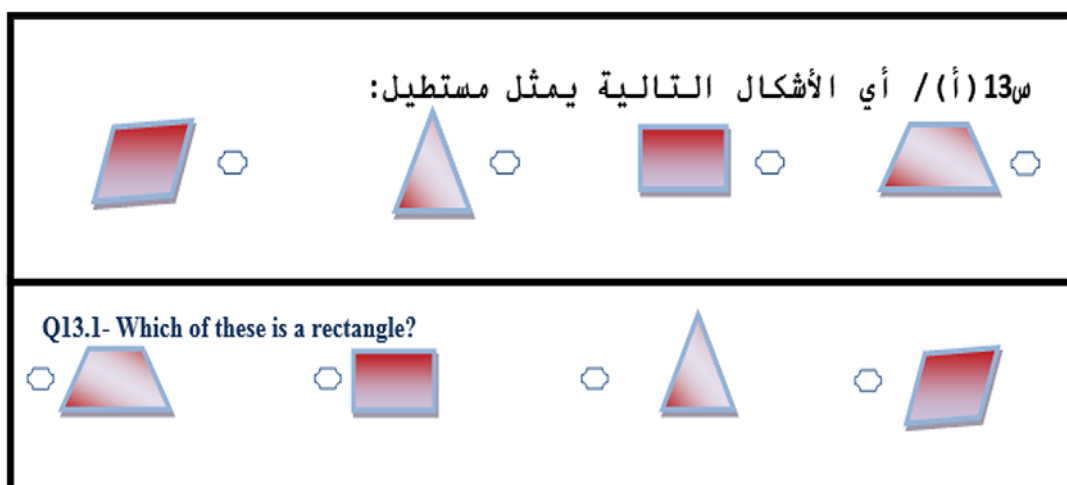


Figure 4.7: An example of a diagrammatic type question in English and Arabic.

Examples of the question classifications are shown in Figure 4.7 (diagrammatic question), Figure 4.8 (a description question), Figure 4.9 (a symbolic question) and Figure 4.10 (a combination question).

Although the questions were presented in Arabic for this experiment, translations are provided in English for the four figures. Translations were not included in the test that was given to the Saudi Arabia students.

<p>س2(أ) / أي من العبارات التالية يمثل الخط المستقيم:</p> <p><input type="radio"/> ليس له بداية وليس له نهاية ويمتد على إستقامة واحدة.</p> <p><input type="radio"/> له بداية وليس له نهاية.</p> <p><input type="radio"/> ليس له بداية وليس له نهاية ولا يمتد على إستقامة واحدة.</p> <p><input type="radio"/> له بداية وله نهاية.</p>
<p>Q2.1 Which of these is a line?</p> <p><input type="radio"/> Something that has no beginning and no end and goes straight in both directions.</p> <p><input type="radio"/> Something that has a beginning point but no end point.</p> <p><input type="radio"/> Something that has no beginning and end points.</p> <p><input type="radio"/> Something that has both a beginning and an end point.</p>

Figure 4.8: An example of a description type question in English and Arabic.

<p>س2(ج) / أي من الرموز التالية يمثل رمز الخط المستقيم:</p> <p><input type="radio"/> \overleftarrow{AB} <input type="radio"/> \overleftrightarrow{AB} <input type="radio"/> \overline{AB} <input type="radio"/> A</p>
<p>Q3.2 Which of these is the symbol of a line segment?</p> <p><input type="radio"/> \overleftarrow{AB} <input type="radio"/> \overline{AB} <input type="radio"/> A <input type="radio"/> \overleftrightarrow{AB}</p>

Figure 4.9: An example of a symbolic type question in English and Arabic.

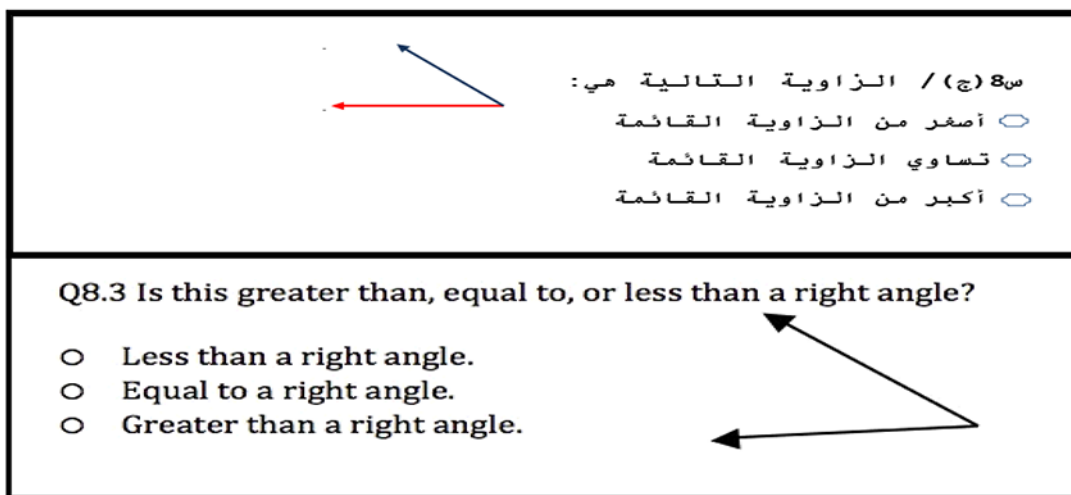


Figure 4.10: An example of a combination type question in English and Arabic.

4.5.2 Results of Experiment 2: Paper-Based Test

The paper-based experiment was conducted with a sample of 224 primary school students from Saudi Arabia aged 9 to 12 years old. This experiment covered thirty-three concepts about geometrical shapes with 65 questions from 21 lessons. Overall, the obtained results showed that the average score of all students for all questions was 65.2%. The results also showed that 189 out of 224 (84.4%) students received a score higher than 50% on the test.

The distribution of the question types and the percentage of errors made by the students are shown in Table 4.2. This table shows that students had more difficulties with Description and Symbolic type questions compared to Diagrammatic type questions, as they made roughly twice as many errors.

Table 4.2: Distribution of question types and percentage of incorrect answers for shape concepts.

Type of question	Diagrammatic	Description	Symbolic	Combination
Number of questions	16	17	6	26
Incorrect answers (%)	28.7	45.1	45.4	37.1

Table 4.3 lists the percentage of errors made by the students for all concepts arranged in descending order. These results have also been plotted in Figure 4.11. The results show that most errors made involved line concepts (e.g. 'line segment', 'straight line' and 'ray'). The highest error percentage was obtained for the 'Line segment' concept, where all students made nearly 60%

of errors. These results prove that it was difficult for students to discriminate between similar concepts as well as their mathematical symbols.

Table 4.3: Percentage of incorrect answers for all concepts arranged in descending order.

Concept(s)	Incorrect %	Concept(s)	Incorrect %
Line segment	59.82	Perpendicular lines	36.16
Straight line	54.17	Angles	33.68
Trapezium	52.46	Triangles	33.20
Ray	48.21	Oval	27.68
Rectangle	45.31	Pentagon	25.00
Polygons	45.03	Rhombus	23.44
Parallelogram	38.84	Dot	13.17
Circle	38.21	Square	11.83
Parallel lines	37.28	Open and closed shapes	9.60
Quadrilaterals	36.90	Curved line	6.25
Hexagon	36.83	-	-

The next highest error percentage was for the trapezium concept, which was approximately 52%. A possible reason for this high percentage is that firstly, the trapezium can be considered an advanced concept for some classes. Therefore, students may have not yet encountered the concept or they may not fully understand the concept and therefore face difficulties in conceptualising the distinguishing attributes of the concept. Secondly, another reason might be due to the confusion between the trapezium concept and the parallelogram and rhombus concepts because these concepts have approximately the same shape and number of sides.

The rectangular shape was one of the most straightforward and familiar shapes to students, and the error percentage was nearly 45%. For this shape, students were confused by the question asking about the number of pairs of equal length sides in a rectangle. The polygon concept resulted in a similar number of errors (45%). The incorrect answers can be summarised as follows: difficulty in naming the vertices of a polygon; difficulty in recognising that every side in a polygon consists of two vertices; not knowing that every angle in a polygon consists of three vertices or two sides with the middle vertex at

the head of the angle; difficulty in recognising different types of polygons; and difficulty in distinguishing between regular and irregular polygons.

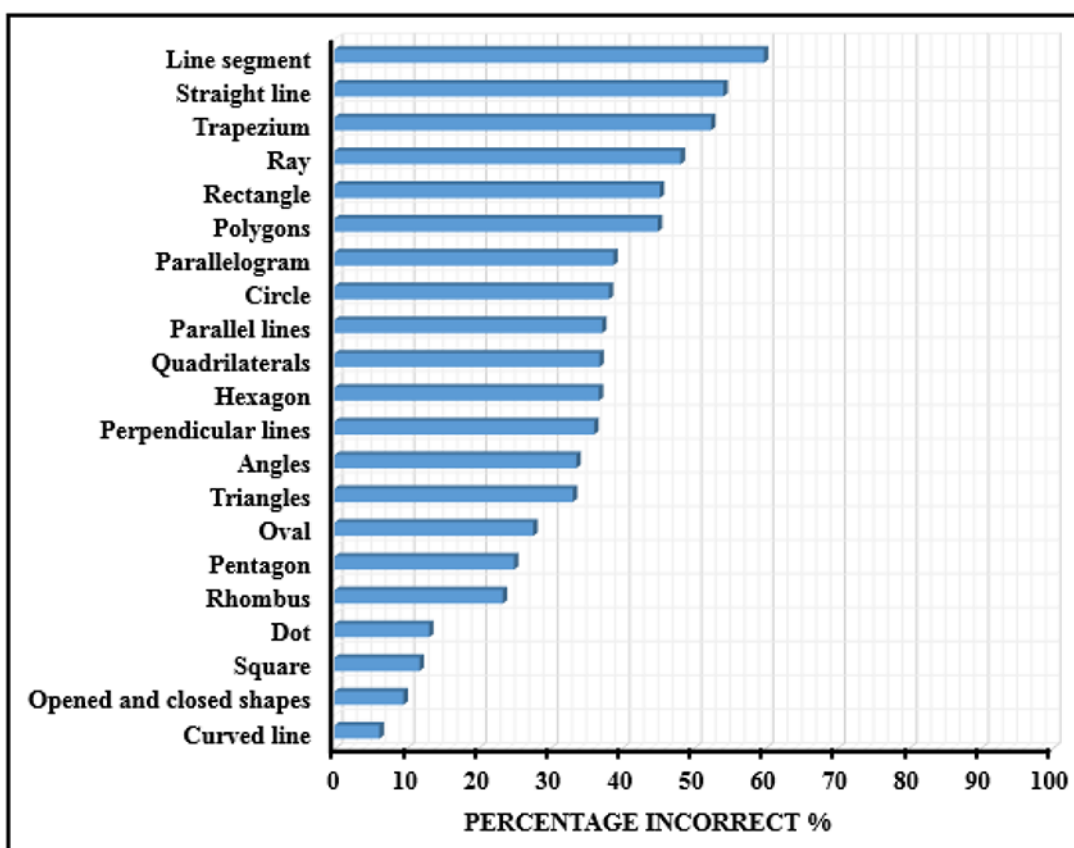


Figure 4.11: Percentage of errors made by students (x axis) on the paper test for each of the concepts (y axis)

Furthermore, the findings of the experiment showed that the parallelogram, circle, parallel lines, quadrilaterals, hexagon and perpendicular line concepts had almost the same error percentages with nearly one-third of the questions answered incorrectly. The most common error among these concepts was the lack of understanding of the characteristics of the concepts. For instance, for circles, students were confused between centre point, radius and diameter.

The concept that was best understood by students was the concept of a curved line (e.g. this produced the lowest percentage of errors within this sample of students). Only 6% of students provided incorrect answers related to this concept, which is statistically close to what is considered a normal error percentage.

4.5.3 Outcome of Experiment 2: Using a Paper-Based Test

The mistakes on specific questions revealed a consistent pattern of misconceptions that are consistent with the Conceptual Spaces cognitive model. For example, it was found that 64.8% of students could recognise a geometric shape only when the shape was shown to them in its standard orientation. In addition, for a non-standard orientation, it was found that 58.3% of students were not aware of the shape's name. Furthermore, 68% of students misunderstood the properties of some shapes. In relation to this aspect, the findings are similar to Biber, Tuna and Korkmaz (2013), Blanco (2001) and Lehrer, Jenkins and Osana (1998), which leads to the conclusion that the students' errors are attributed to difficulties in conceptualising the properties of the geometric concepts.

The results appear to contradict Clements and Battista (1992), who argued that children learn geometric shapes by memorisation and rote; however, Stigler and Perry (1988), showed that children are frequently not able to recognise the properties of shapes or the relationships between these properties when relying on routine mental processes. This was clear when the geometric shapes were presented differently from the typical form with which students were familiar. The previous opinion is supported by Lehrer, Jenkins and Osana (1998), who raised concerns about the required existence of a geometric thinking level (precognitive) prior to visual presentation. As well as concerning now, the geometric concepts are visually seen, young students face difficulties in understanding these concepts until they develop geometric thinking through the visualisation of the geometric concepts.

4.6 Pilot Experiment 3: Further Evaluation Using Interviews

The outcome of the previous experiments necessitated further evaluations to understand the students' conceptualisations when questioned on a computer screen or on paper versus being asked the same questions verbally. The objective of conducting this experiment was to evaluate whether the basic

concepts that led to the common misconceptions found in the previous experiments also existed among students if they were tested in individual interviews within the classroom. In this experiment, verbal questions about the basic concepts of lines and geometric shapes that were predefined in the previous two experiments were asked directly by teachers.

The oral questions were intended to account for the potential challenges for certain students who have difficulty in reading (for example, due to dyslexia) or have difficulty in fully comprehending the written questions in the experiments with the paper-based test. In addition, the interviews could be conducted in a local variant of the language that the students were accustomed to during the presentation and testing of the concept to help mitigate linguistic problems. The idea was that the interview-based experiment would complement the previous computer- and paper-based experiments. This complementarity should lead to identifying a more effective means to deliver concept learning and to overcome common misconceptions of mathematical lines and geometric shapes taught to primary students.

4.6.1 Design of Experiment 3: Using an Interview

This experiment was conducted using the same set of concepts detailed in the previous experiments (section 4.4.1). Math teachers in primary schools were recruited as volunteers to participate in the process of conducting the interview-based testing for the students. The participating teachers were selected based on their experience in teaching at schools in Saudi Arabia and experience teaching the mathematical concepts that were included in the interviews. All selected teachers had more than 10 years of teaching experience and were qualified to conduct the interviews objectively.

The teachers were given the concepts that were included in the interview as well as an example question to clarify the type of question for each concept. Furthermore, the teachers were given guidelines regarding the minimum number of questions for the interview; however, they were free to select the concepts and questions to consider. The minimum number of questions for an

interview was set to three questions for three different concepts. These three questions covered three main areas: drawing, identification and features of the concept. The guideline that was given to the teachers is included in Appendix (E). The interviewer was expected to take notes about student understanding and misunderstanding during the interview. In addition, the interviewer was required to video tape or audio record the interview session for later analysis.

Upon the completion of interviews, each participating math teacher attended a post-interview session with the researcher to provide subjective general observations and specific comments about the concepts that caused difficulties and misconceptions. The actual interview outcomes were obtained through post-analyses of the taped sessions.

4.6.2 Results of Experiment 3: Using an Interview

The experiment was conducted at primary schools in Saudi Arabia, and students were selected from the school years 4, 5 and 6. Six volunteer math teachers from five different primary schools were recruited to carry out the interviews. The distribution of the students and the results of the interviews varied among the different class levels, which is typically associated with the age group (9 to 12 years old) that participated in the interview-based experiment. Table 4.4 documents the outcome of the post-analysis of interviews that were conducted during this experiment.

In this experiment, the total number of interviewed students was 72. The total number of questions that the interviewers asked the students was 399 questions. Table 4.4 shows the distribution of the sample of students in each class year and their percentage in the experiment sample size. The table also details the number of questions and their percentages for the three types of questions that were used in the interview-based experiment.

The analysis of the recorded interviews showed that the students encountered a range of difficulties regarding the concepts that were tested during the interview.

Table 4.4: Distribution of the types of questions and the number of students in the interview.

Description	The number of students per class			The number of questions for each type		
	Year4	Year 5	Year 6	Drawing	Definition	Features
Number	19	23	30	143	127	129
The percentage %	26%	32%	42%	36%	32%	32%
Total of the number	72 students			399 questions		

For the questions that caused the highest number of misconceptions, the students simply provided the answer 'I don't know' without providing any reasoning. These questions included defining a particular line or shape, such as the definition of a straight line, line segment, ray, parallelogram, rhombus and some features and properties of quadrilaterals. Furthermore, the interviews showed that the students misunderstood the properties of concepts such as a rectangle, square, rhombus and quadrilateral and the concepts of a straight line, line segment and ray. Furthermore, some students had some knowledge about the features and definitions of geometric shapes, but they could not provide a logical explanation, which might be due to the fact that these students had a good memorisation of the definitions without grasping the concepts or visual applications.

In addition, some of the students could not determine the desired representation for concepts such as the parallelogram, rhombus or line segment; however, as a general observation of the interviews, the question that required students to draw a particular line or shape resulted in the students drawing it correctly or close to the correct form of the expected answer. This indicates that students had some cognitive structure of the quality dimension for the concept tested.

4.6.3 Outcome of Experiment 3: Using an Interview

The individual interviews showed that some students had difficulty in understanding some of the basic concepts of lines and geometric shapes and that they were not able to orally explain or identify them. It was

also noted that some of the misconceptions overlapped with the previously identified misconceptions found in the computer and written questionnaire experiments. This was encouraging, as it consistently identified a set of common misconceptions that primary students encounter during the learning process. Therefore, the research could focus on improving the conceptualisation of the concepts underlying these misconceptions.

In addition, the freedom given to the interviewing teachers to ask questions in any order of difficulty underscored the importance of presenting the concepts to students in an increasing order of difficulty. The random selection of the level of difficulty of the concepts and their questions could have resulted in some of the difficulties that the students experienced when they were conceptualising or effectively using their mental structure of the quality dimensions of the concepts that had been previously learned. Hence, the increasing order of difficulty of concepts seems to assist the students in properly identifying more complex concepts.

Another important outcome of this experiment was the need for students to have a conceptual understanding of the line and geometric shape concepts that is based on both visual and oral representations that relate the shapes to real-life examples. For example, the student should be able to point to a window or a door to identify a line segment or a quadrilateral shape. In essence, learning geometric concepts through a combination of traditional methods and physical properties could help in future evaluations of similarities between quality dimension symbols of complex concepts (Aisbett and Gibbon, 2001).

4.7 Shapes Conversion Game

The pilot prototype and the extended pilot experiment enabled the improvement of the modelling of shape concepts during the early stage of the development. The orientation problem led to the development of a 'Shapes Invasion' game. In this game, students must 'shoot down' different shapes as they 'invade' across the screen from right to left. The game has

various levels that increase the speed of the shapes across the screen, and upon successful completion of a level, the student transitions to the next level. More importantly, the orientation and size of the shapes are completely random. This should aid the students in learning non-standard orientations of shape concepts to overcome the orientation misconceptions outlined. The increasing speed and randomness of shapes are intended to emulate an increasing level of difficulty in comprehension.

4.7.1 Key Features of the Game

The game is organised in five possible playing modes and five levels of game stages. The playing modes are intended to enable the user to either focus on certain closely related geometric shape concepts or to simultaneously play with all 14 geometric shapes in the game, which emulates relatively advanced stages of the game. The following summarises the five modes of playing the game:

1. Triangle shapes (three concepts).
2. Rectangle shapes (four concepts).
3. Circle shapes (three concepts).
4. Shapes with more than four sides (four concepts).
5. Randomised shapes (all fourteen concepts).

The game has five levels of an increasing speed intensity of the shapes that the student must 'shoot down' as they traverse the screen. Figure 4.12 shows the setting interface for the conversion game. In each level and for each mode of playing the game, the shape orientation and the colour of the shape randomly change. In addition, the sequences of the shapes are randomised to help students correctly identify and conceptualise the shapes even when their presentation changes. The entire game is also randomised each time the game is played to ensure that students do not become familiar with the game sequence rather than focusing on the shape concepts for each game he/she plays.

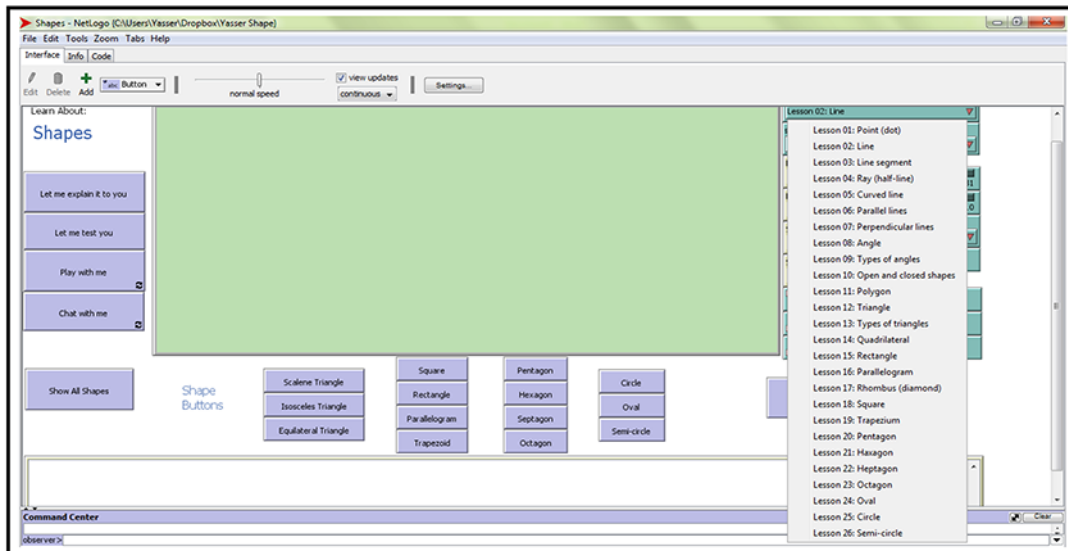


Figure 4.12: Interface for setting the initial configuration for the conversion game.

4.7.2 How to Play the Game

The game is integrated with the e-learning system, which was developed using NetLogo. During the initial configuration of the default setting of the e-learning programme, the player selects one of the five modes of the game. The game begins by pressing one of the main options on the right side of the main window of the e-learning system. After pressing 'play with me', audio instructions are provided, and then the game begins immediately. The audio instructions are played once, and they are not repeatable without restarting the game.

As the shapes invade across the screen from right to left, the player is expected to press one of the corresponding buttons from the fourteen shape concepts at the bottom of the main window to 'kill' the shape. When the player makes a correct selection for the shape on the window, e.g. the shape is 'killed', a music tone is played to indicate the correct selection, and the game moves on to the next shape. Figure 4.13 shows four monitor boxes (towards mid-right of the figure) that display the level, lives remaining, shapes killed and current score of the player.

When the student makes an incorrect selection, the game plays an annoying tune. The player is given up to three chances to make an incorrect selection, e.g. failure to kill an invading shape, for each speed level before the game

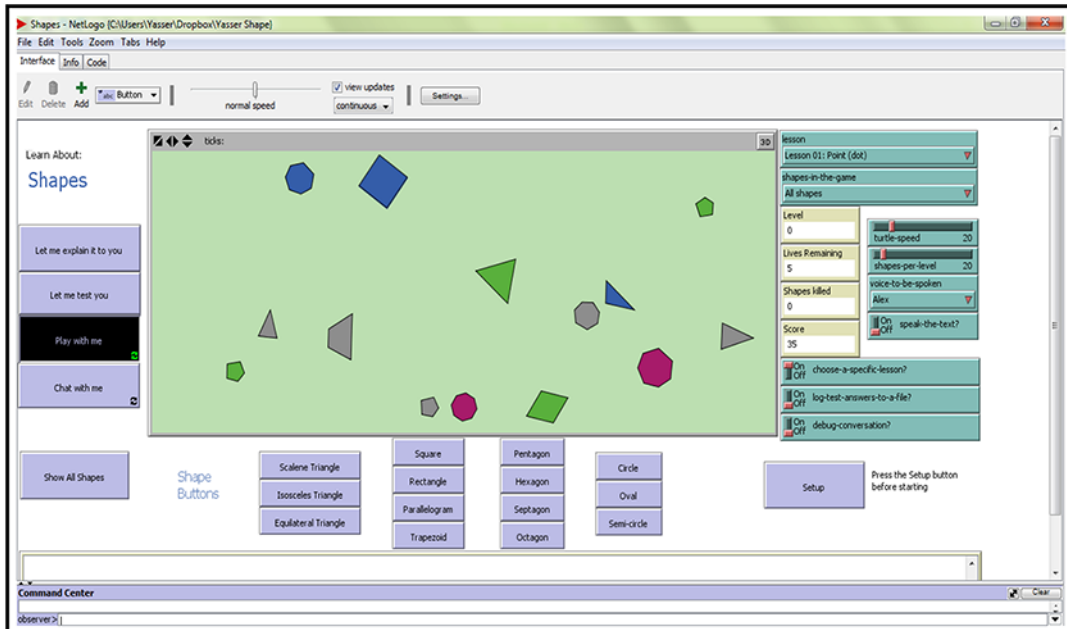


Figure 4.13: The level of progress and current score for the shape conversion game.

terminates. Figure 4.14 shows the termination of the game when no chances remain for the player at the current level of the game; however, a successful transition from one level to the next faster level will restore the three chances to make an incorrect selection. When the game terminates, the overall score for the game is displayed on the screen.

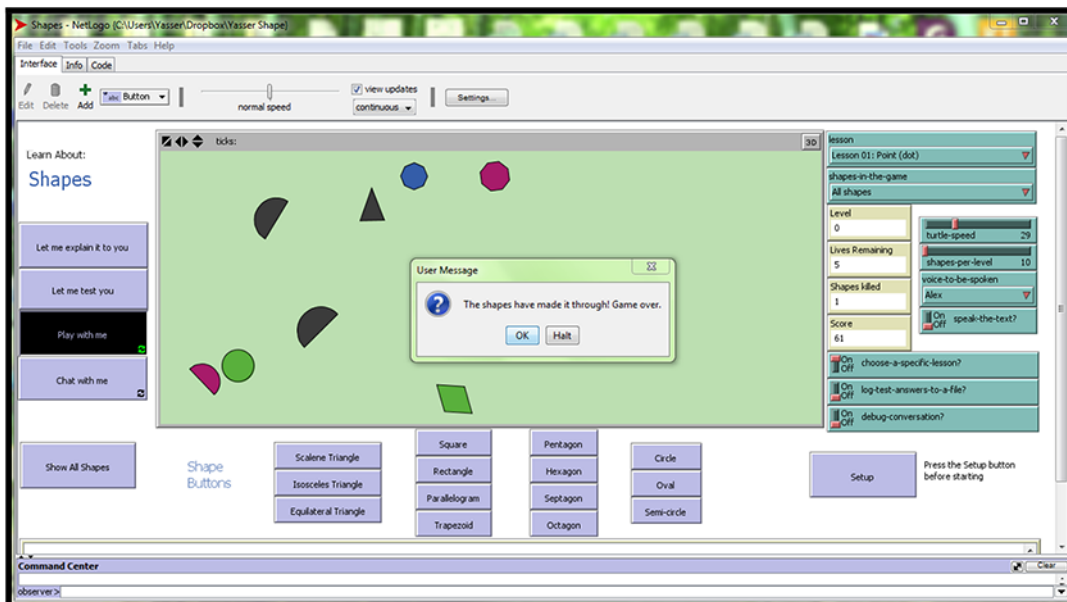


Figure 4.14: The termination of the shape conversion game.

4.8 Summary and Discussion

This chapter described the design and the early developmental work for an agent-based e-learning system. In addition, it describes the methodology of deriving and formulating a comprehensive set of questions that allowed for identifying common misconceptions of geometric shape concepts. The chapter also presents the results and outcomes of the pilot prototype experiment and the extended pilot experiment that was conducted with eight students in the UK. The extended experiment with 224 students in Saudi Arabia was also described.

To better understand the way students represent concepts and where they are likely to have misconceptions as a result, a prototype e-learning system was developed. In this system, a conversational interface metaphor was implemented, and the emphasis was on maintaining a two-way dialogue between the student and the system. The chapter also describes the process that was undertaken in developing an effective means for user modelling for an e-learning system based on Gärdenfors' Conceptual Spaces cognitive model. Therefore, modelling the natural properties and quality dimensions of the geometric concepts provided insight into developmental categories that might inhibit students' abilities to conceive the concept.

The difficulty in mastering a concept may be attributable to a missing dimension or skill. Learning activities can then focus on stimulating the development of the missing or incomplete natural property and its connections to other relevant dimensions in the domain. This aspect created the foundation of the approach, and a cognitive framework to identify the missing dimension or skill was developed by adopting a Conceptual Spaces cognitive theoretical framework within an e-learning system. Hence, this was the motivation behind the focus on student mistakes when answering questions about geometric shape concepts and the resultant misconceptions.

The results obtained from the pilot prototype and the extended pilot experiments described in this chapter are consistent with the Conceptual

Spaces cognitive model due to the possible quality dimensions (e.g. features) that students used to define the concepts. Consequently, the results obtained were considered promising.

In addition, this chapter has contributed to addressing Objectives 1 and 2, which aimed to develop an agent-based e-learning system. The process involved background research and an investigation of Saudi Arabian student misconceptions concerning shape concepts.

There are still critical concerns related to the commonality of the misconceptions across different countries. In essence, this refers to whether student misconceptions (mistakes) were consistent for the student groups from Saudi Arabia and from the UK as well as the determination of which quality dimensions the students used to organise the concepts. These aspects are examined in more detail in the next chapter.

Chapter 5

A Comparative Study of Student Understanding in Saudi Arabia and in the UK

5.1 Introduction

The design of the prototype agent-based e-learning system used to aid Saudi Arabia primary schools student understanding of mathematical geometric concepts has been described in Chapter 4. In this chapter, student understanding is discussed and examined from two perspectives: in Saudi Arabia and in United Kingdom primary schools. This chapter compares the understanding of students in the Kingdom of Saudi Arabia (KSA) and in the United Kingdom (UK) as well as how they conceptually represent mathematical concepts (lines and shapes). This chapter includes the following main sections: methodology, description and statistics of the data used in the experiments and the discussion of the results and the findings of the study.

To perform a statistical analysis of the collected data, a mathematical geometric exam was designed in two languages (Arabic and English). Then, two samples of students were chosen from Saudi Arabia and from the UK, with a total of 346 students from three different class levels (year 4, 5 and 6). In this study, the statistical analysis was performed using several techniques of summary information, such as frequency tables including percentages of success in each question, descriptive statistics such as mean values and

standard deviations and a summary of the 65 questions included in the designed mathematical test.

5.1.1 Why comparing Saudi Arabia and UK Students?

Comparative studies between groups of countries are considered as one of the strategic studies in the modern era. Therefore, developed countries show more interest in comparing their education systems with other countries' systems, in order to assess the level of teaching that is offered to their students based on international standards. For example, there were two studies conducted in the United States which compared the teaching of science and mathematics to other fifty states (TIMSS, 1995; TIMSS, 1999). Also, Japan, Britain and other countries are periodically conducting similar studies.

“Without comparing your teaching methods, approaches and results to other countries, it will be difficult to determine the extent of your success or your failure in your educational programs” (Fitz-Gibbon and Morris, 1987, p.26).

Comparing an educational system or part of it with another country is not an easy aspect due to many reasons. The first reason is that each nation often has a different and independent educational system from other nations. The second reason is that there are some educational concepts that are interpreted and understood in the context of a specific country in a different way from other countries, which requires a great effort to unify the aspects on which the comparison will be based (MacKinnon, Newbould and Zeldin, 1997).

From another side, and if the industrialised countries which were better off trying to take advantage of other countries through these comparative studies, then developing countries are more in need for these studies to advance their educational level, handle errors and benefit from the experiences of other countries. Hence, Saudi Arabia is in need for such studies, especially that the Arab arena devoid of such studies and because of its large development projects and its quantum leap in education in general and particularly in e-learning.

When the comparison is performed between countries that have completely different languages, customs, traditions and laws and regulations, and that are completely spaced from the geographical point of view, then such comparison seems to be more difficult as well as more interesting and vigorous. From another side, comparing a renewed and changing subject such as the subject of e-learning seems to be more difficult and complex. This is due to the fact that many of its terminology has not stabilised yet, and that its teaching and presenting methods to learners is a highly controversial aspect among educators and technicians. Therefore, the Kingdom of Saudi Arabia has funded a large number of scholarship students to complete their graduate studies and benefit from the experiences of countries, such as the UK, in the field of e-learning and its applications.

From what was said above, the necessity and importance of selecting these two countries for our study, e.g. Saudi Arabia and the UK, became clear, in order to find the most common and difficult concepts for students from the two countries, which should help in building an effective e-learning system that is supported by a realistic diagnosis of more than one country.

5.1.2 How does this Comparative Study link to our Thesis Objectives?

The e-learning and its application in teaching mathematics is considered amongst the most important areas that are given special attention by countries, and this is what was proven by the previous studies in the field of mathematics teaching. Therefore, the comparison between countries often includes the field of science and mathematics teaching. Hence, the importance of our study is revealed to be a gateway for future studies that compare teaching mathematics for any other concepts or other teaching subjects, between many other countries, to show and determine the international position in a more comprehensive manner.

Furthermore, the main aim of the comparative study performed in the current chapter is to assess and to validate our developed e-learning system within the educational environment, by comparing and assessing students'

achievement scores, regardless of their background, language, etc. More specifically, the goal is to determine if there are any significant differences between Saudi Arabia and UK students in relation to the misconceptions of mathematical concepts.

5.2 Methodology

To perform the comparison between Saudi Arabia and the United Kingdom students in relation to their understanding of geometric concepts, it was essential to use a quasi-experimental design that tests causal hypotheses, which is similar to other experimental designs, such as clinical trials. Although quasi-methods lack the random assignment of subjects to the experimental and control groups, it was not necessary to follow the entire procedure of the quasi-experimental design in this study. This is mainly because a quasi-experimental design identifies a comparison group that has similar characteristics, whilst the purpose of this study was to compare Saudi Arabian students with the United Kingdom students with the same background characteristics (e.g. age and class). Indeed, a quasi-experimental design has been adopted in several studies to investigate the understanding of geometric concepts (Bhagat and Chang, 2015; Idris, 2009). These studies compared the experimental groups with the control groups who belonged to the same community; however, this study compared two groups of students from two different countries (Saudi Arabia and the UK).

Furthermore, answering the research question of Chapter 1 (sub-question 5) regarding the existence of significant differences between Saudi Arabian and United Kingdom students in relation to the misconceptions of mathematical concepts required the use of statistical measurements, such as Spearman's correlation coefficient, between the factors of interest and the total test score. In addition, several statistical tests were performed to determine whether there were statistically significant differences between the two samples in the variables of concern based on the test scores of the students. Several statistical tests are discussed in this chapter, including two independent *t*-tests and an Analysis of Variance (ANOVA) when comparing a variable with

three or more categories (e.g. classes of students). Moreover, non-parametric tests, such as the Mann-Whitney test and the Kruskal-Wallis test, were used in the case of binary data for some concepts that only included one question. All mentioned tests were performed at a significance level of 0.05. Before reporting the results, each question was renamed using a simple, meaningful and short name so that the listed tables could be more easily organised. The various code meanings are illustrated in Appendix F.

The statistical software that was adopted in this study was IBM SPSS Statistics version 17.0, which was relevant to the nature of the collected data and provided accurate results that assisted in achieving the study objectives.

The main results of the statistical analysis are listed at the end of this chapter, which will assist stakeholders who are interested in improving the level of student understanding in the mathematical areas under study.

In the next section, the collected data is described, and an overview of the sample participants for the exam is presented.

5.2.1 Sample Size and Distribution

The analyses presented in this section answer the following research questions:

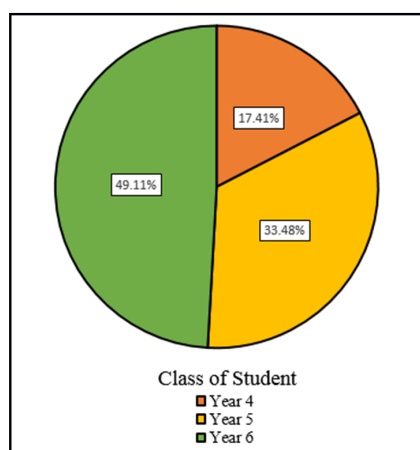
- The ability of students (in Saudi Arabia and in the UK) to understand the main concepts of basic geometrical shapes.
- The percentages of success and difficulty that students experienced (in Saudi Arabia and the UK) when answering the 65 questions related to basic geometrical knowledge.

The selected sample was composed of a total of 346 students of which 224 students were from Saudi Arabia, and 122 students were from the UK. Students from class year 6 represented the highest proportion within the sample in each country. These students correspond to 49.1% of the Saudi Arabia sample compared to 36.1% of the UK sample and 44.5% of the overall sample.

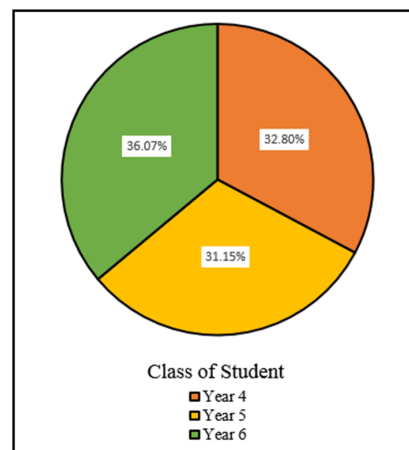
On the other hand, students from class year 4 represented the lowest proportion of the sample at 22.8%. This structure of study sample was expected mainly because some of the concepts in the exam could be more advanced compared to others. More details about the percentages of the various class year levels within the sample are listed in Table 5.1.

Table 5.1: Distribution of study sample according to student's country and class level.

Variable	Level	Statistics	Country		Total
			KSA	UK	
Class of Student	YEAR 4	Count	39	40	79
		% within Country	17.4%	32.8%	22.8%
	YEAR 5	Count	75	38	113
		% within Country	33.5%	31.1%	32.7%
	YEAR 6	Count	110	44	154
		% within Country	49.1%	36.1%	44.5%
Total		Count	224	122	346
		% within Country	100%	100%	100%



(a) Distribution of students in the sample according to class level in Saudi Arabia.



(b) Distribution of students in the sample according to class level in the UK.

Figure 5.1

The results in Table 5.1 are also shown graphically in Figure 5.1 using a pie chart. Figure 5.1.a and Figure 5.1.b show the distribution of students according to the class year level in Saudi Arabia and in the UK, respectively.

5.2.2 Descriptive Statistics

A. Internal Consistency

The internal consistency of the 65 questions selected for the exam was computed using the Cronbach's alpha measurement by SPSS software, which was equal to 0.801, indicating a 'good' level of internal consistency.

B. Success Percentage

The next step in describing the collected data was calculating the percentages of success for each question within each country regardless of the class level and hence the percentage of difficulty when answering each question.

The success rate for each question according to a student's country are presented in Table 5.2. As can be observed, there are variations in the success percentages between questions. For example, in question Q2_1, which is related to the *Line* concept, the difference in the success percentage was only 0.1%, while in general, both samples had a low percentage of success, indicating a high level of difficulty (about 68% of both supplied incorrect answers). Similarly, in question Q12_3, which is related to the *Quadrilateral shapes* concept, the difference in the success percentage was only 0.1%, while in general, both samples had a very low percentage of success, indicating a very high level of difficulty (about 80% supplied incorrect answers). In question Q8_4, which is related to the *Angle* concept, the difference in the success percentage was only 0.1%, while in general, both samples had a moderate percentage of success, indicating a low level of difficulty (about 21%).

On the other hand, in question Q8_2, which is also related to the *Angle* concept, the difference in the success percentage was 26.6%, and Saudi Arabian students had a success percentage of 74.1% compared to 47.5% for the UK students. In addition, another remarkable variation was in question Q10_3, which is related to the *Polygon* concept. For this question, the

Table 5.2: The percentage of success in each question according to the student's country.

Code of question	KSA	UK	Difference	Code of question	KSA	UK	Difference
Q1_1	80.4%	91.8%	11.4%	Q12_1	79.9%	67.2%	12.7%
Q1_2	93.3%	91.0%	2.3%	Q12_2	89.7%	76.2%	13.5%
Q2_1	32.1%	32.0%	0.1%	Q12_3	19.6%	19.7%	0.1%
Q2_2	61.2%	58.2%	3.0%	Q13_1	78.6%	90.2%	11.6%
Q2_3	44.2%	24.6%	19.6%	Q13_2	30.8%	66.4%	35.6%
Q3_1	27.2%	37.7%	10.5%	Q14_1	74.6%	59.8%	14.8%
Q3_2	53.1%	29.5%	23.6%	Q14_2	47.8%	57.4%	9.6%
Q4_1	54.0%	36.1%	17.9%	Q15_1	70.5%	57.4%	13.1%
Q4_2	49.6%	68.9%	19.3%	Q15_2	82.6%	82.0%	0.6%
Q5_1	93.8%	92.6%	1.2%	Q16_1	95.1%	94.3%	0.8%
Q6_1	72.3%	83.6%	11.3%	Q16_2	81.3%	83.6%	2.3%
Q6_2	53.1%	69.7%	16.6%	Q17_1	49.6%	77.9%	28.3%
Q7_1	74.1%	77.9%	3.8%	Q17_2	45.5%	26.2%	19.3%
Q7_2	53.6%	48.4%	5.2%	Q18_1	75.0%	74.6%	0.4%
Q8_1	87.9%	84.4%	3.5%	Q18_2	75.0%	85.2%	10.2%
Q8_2	74.1%	47.5%	26.6%	Q19_1	81.3%	55.7%	25.6%
Q8_3	48.2%	65.6%	17.4%	Q19_2	45.1%	80.3%	35.2%
Q8_4	78.6%	78.7%	0.1%	Q20_1	56.7%	60.7%	4.0%
Q9_1	88.8%	88.5%	0.3%	Q20_2	60.7%	60.7%	0.0%
Q9_2	92.0%	93.4%	1.4%	Q21_1	30.4%	43.4%	13.0%
Q10_1	84.4%	68.0%	16.4%	Q21_2	61.2%	63.9%	2.7%
Q10_2	24.6%	25.4%	0.8%	Q21_3	73.7%	52.5%	21.2%
Q10_3	72.8%	12.3%	60.5%	Q21_4	42.0%	52.5%	10.5%
Q10_4	23.7%	49.2%	25.5%	Q21_5	52.7%	40.2%	12.5%
Q10_5	93.8%	93.4%	0.4%	Q21_6	34.8%	41.0%	6.2%
Q10_6	39.7%	32.8%	6.9%	Q21_7	53.1%	44.3%	8.8%
Q10_7	46.0%	69.7%	23.7%	Q21_8	62.9%	63.9%	1.0%
Q11_1	99.6%	97.5%	2.1%	Q22_1	72.3%	91.8%	19.5%
Q11_2	89.7%	86.1%	3.6%	Q23_1	96.9%	94.3%	2.6%
Q11_3	62.1%	64.8%	2.7%	Q23_2	67.0%	86.9%	19.9%
Q11_4	77.7%	78.7%	1.0%	Q23_3	47.8%	34.4%	13.4%
Q11_5	64.3%	76.2%	11.9%	Q23_4	43.3%	67.2%	23.9%
				Q23_5	54.0%	52.5%	1.5%

difference in the success percentage was 60.5%, and Saudi Arabian students had a success percentage of 72.8% compared to 12.3% for the UK students.

The UK students had higher success percentages in other questions of the exam, such as in question Q13_2, which is related to the *Rectangle* concept. For this question, the difference in the success percentage was 35.6%. The UK students achieved a success percentage of 66.4% compared to 30.8% for Saudi Arabian students. In addition, for question Q19_2, which is related to the *Hexagon* concept, the difference in the success percentage was 35.2%. The UK students had a success percentage of 80.3% compared to 45.1% for Saudi Arabian students. The final remarkable result in Table 4.2 is for question Q20_2, which is related to the *shape properties* concepts. There was no difference in the success percentage of the samples (60.7% each).

C. Mean Values and Standard Deviations

After analysing the success percentages, the mean values as well as the standard deviations were computed for each question. Table 5.3 shows the mean values and standard deviations for each question for the Saudi Arabian sample. The level of students in each question can be classified into one of three levels according to the mean value as follows:

- from 0.0 to 0.33: low,
- from 0.34 to 0.66: moderate,
- from 0.67 to 1.0: high.

The main questions that resulted in a low student level for Saudi Arabian students are:

- Q2_1 (related to straight line) with a mean value of 0.32;
- Q3_1 (related to line segments) with a mean value of 0.27;
- Q10_2 and Q10_4 (related to polygons) with mean values of 0.25 and 0.27, respectively;
- Q12_3 (related to quadrilaterals) with a mean value of 0.20;
- Q13_2 (related to rectangles) with a mean value of 0.31; and
- Q21_1 (related to shape properties) with a mean value of 0.30.

Table 5.3: Mean values and standard deviations for each question for the Saudi Arabia sample.

Code of question	Mean	Standard deviation	Code of question	Mean	Standard deviation
Q1_1	0.80	0.398	Q12_1	0.80	0.402
Q1_2	0.93	0.251	Q12_2	0.90	0.304
Q2_1	0.32	0.468	Q12_3	0.20	0.398
Q2_2	0.61	0.488	Q13_1	0.79	0.411
Q2_3	0.44	0.498	Q13_2	0.31	0.463
Q3_1	0.27	0.446	Q14_1	0.75	0.437
Q3_2	0.53	0.500	Q14_2	0.48	0.501
Q4_1	0.54	0.499	Q15_1	0.71	0.457
Q4_2	0.50	0.501	Q15_2	0.83	0.380
Q5_1	0.94	0.243	Q16_1	0.95	0.217
Q6_1	0.72	0.448	Q16_2	0.81	0.391
Q6_2	0.53	0.500	Q17_1	0.50	0.501
Q7_1	0.74	0.439	Q17_2	0.46	0.499
Q7_2	0.54	0.500	Q18_1	0.75	0.434
Q8_1	0.88	0.326	Q18_2	0.75	0.434
Q8_2	0.74	0.439	Q19_1	0.81	0.391
Q8_3	0.48	0.501	Q19_2	0.45	0.499
Q8_4	0.55	0.391	Q20_1	0.57	0.497
Q9_1	0.89	0.316	Q20_2	0.61	0.489
Q9_2	0.92	0.272	Q21_1	0.30	0.461
Q10_1	0.84	0.364	Q21_2	0.61	0.488
Q10_2	0.25	0.431	Q21_3	0.74	0.441
Q10_3	0.73	0.446	Q21_4	0.42	0.495
Q10_4	0.24	0.426	Q21_5	0.53	0.500
Q10_5	0.94	0.243	Q21_6	0.35	0.477
Q10_6	0.40	0.490	Q21_7	0.53	0.500
Q10_7	0.46	0.499	Q21_8	0.63	0.484
Q11_1	1.00	0.067	Q22_1	0.72	0.448
Q11_2	0.90	0.304	Q23_1	0.97	0.174
Q11_3	0.62	0.486	Q23_2	0.67	0.471
Q11_4	0.40	0.317	Q23_3	0.48	0.501
Q11_5	0.42	0.372	Q23_4	0.43	0.497
			Q23_5	0.54	0.499

The questions with the the highest student mean values are:

- Q5_1 (related to curves) with a mean value of 0.94;
- Q10_5 (related to polygons) with a mean value of 0.94;
- Q11_1 (related to triangles) with a mean value of 1.00;
- Q16_1 (related to squares) with a mean value of 0.95; and
- Q23_1 (related to circles) with a mean value of 0.97.

Other questions ranged between a moderate to high level with mean values that ranged between (0.35 – 0.93).

For the UK sample, the mean values and the standard deviations were also computed for each question, as shown in Table 5.4.

The following questions resulted in the lowest students' level in the UK sample:

- Q2_1 (related to lines) with mean value of 0.32;
- Q2_3 (related to lines) with a mean value of 0.25;
- Q3_2 (related to line segments) with a mean value of 0.30;
- Q10_2, Q10_3 and Q10_6 (related to polygons) with a mean value of 0.25, 0.12, and 0.33 respectively;
- Q12_3 (related to quadrilaterals) with a mean value of 0.20; and
- Q17_2 (related to trapeziums) with a mean value of 0.26.

The questions with the highest students' mean values are:

- Q11_1 (triangle) with a mean of 0.98;
- Q16_1 (square) with a mean of 0.94; and
- Q23_1 (circle) with a mean of 0.94.

Other questions ranged between the moderate to high level with mean values ranging between (0.34 – 0.93).

It should be noted that both the Saudi Arabia and UK samples shared some high and low mean level questions. For example, both samples had low mean

Table 5.4: Mean values and standard deviations for each question for the UK sample.

Code of question	Mean	Standard deviation	Code of question	Mean	Standard deviation
Q1_1	0.92	0.275	Q12_1	0.67	0.471
Q1_2	0.91	0.288	Q12_2	0.76	0.427
Q2_1	0.32	0.468	Q12_3	0.20	0.399
Q2_2	0.58	0.495	Q13_1	0.90	0.299
Q2_3	0.25	0.432	Q13_2	0.66	0.474
Q3_1	0.38	0.487	Q14_1	0.60	0.492
Q3_2	0.30	0.458	Q14_2	0.57	0.497
Q4_1	0.36	0.482	Q15_1	0.57	0.497
Q4_2	0.69	0.465	Q15_2	0.82	0.386
Q5_1	0.93	0.262	Q16_1	0.94	0.234
Q6_1	0.84	0.372	Q16_2	0.84	0.372
Q6_2	0.70	0.462	Q17_1	0.78	0.417
Q7_1	0.78	0.417	Q17_2	0.26	0.442
Q7_2	0.48	0.502	Q18_1	0.75	0.437
Q8_1	0.84	0.364	Q18_2	0.85	0.356
Q8_2	0.48	0.501	Q19_1	0.56	0.499
Q8_3	0.66	0.477	Q19_2	0.80	0.399
Q8_4	0.66	0.421	Q20_1	0.61	0.491
Q9_1	0.89	0.320	Q20_2	0.61	0.491
Q9_2	0.93	0.249	Q21_1	0.43	0.498
Q10_1	0.68	0.468	Q21_2	0.64	0.482
Q10_2	0.25	0.437	Q21_3	0.52	0.501
Q10_3	0.12	0.330	Q21_4	0.52	0.501
Q10_4	0.49	0.502	Q21_5	0.40	0.492
Q10_5	0.93	0.249	Q21_6	0.41	0.494
Q10_6	0.33	0.471	Q21_7	0.44	0.499
Q10_7	0.70	0.462	Q21_8	0.64	0.482
Q11_1	0.98	0.156	Q22_1	0.92	0.275
Q11_2	0.86	0.348	Q23_1	0.94	0.234
Q11_3	0.65	0.480	Q23_2	0.87	0.339
Q11_4	0.53	0.378	Q23_3	0.34	0.477
Q11_5	0.48	0.330	Q23_4	0.67	0.471
			Q23_5	0.52	0.501

levels for question Q12_3 (related to quadrilaterals) with a mean value of 0.20, whilst both samples had a high mean level for question Q11_1 (related to triangles), Q16_1 (related to squares) and Q23_1 (related to circles), which correspond to (0.98; 1), (0.95; 0.94) and (0.97; 0.94), respectively. A graphical comparison between the Saudi Arabia and the UK sample mean values for the 23 exam concepts are displayed in Figure 5.2.

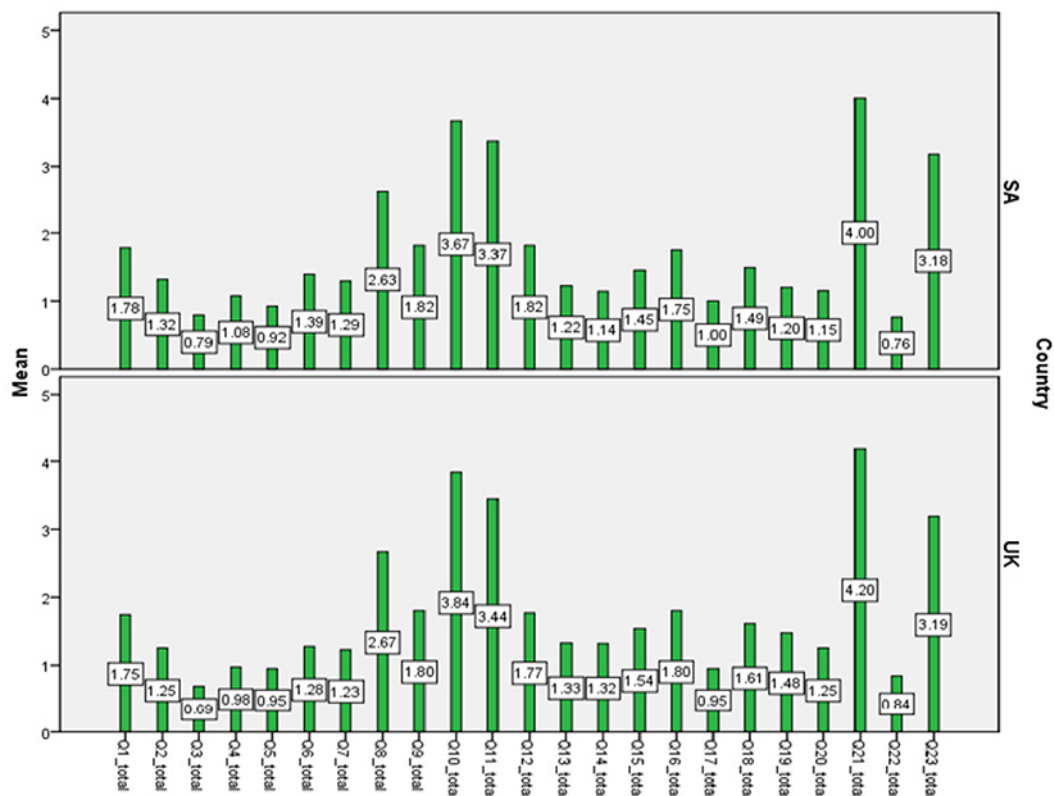


Figure 5.2: Mean concept scores for students in the KSA and the UK.

As shown in Figure 5.2, concept 21 (related to shape properties) obtained the highest mean value, which corresponds to 4.20 and 4.00 in the UK and Saudi Arabia samples, respectively; however, concept 3 (related to line segments) obtained the lowest mean value, which corresponds to 0.68 in the UK sample. Concept 22 (related to ovals) had the lowest mean value of 0.76 in the Saudi Arabia sample.

In general, the results that were obtained from the statistical analysis did not show a trend or a relationship between the total test scores and the country variable or with the students' class levels, meaning that there was no relationship between test scores and the country variable and no relationship

between test scores and class level. This might be due to the variability in responses amongst all 65 questions.

The correlation between variables can be tested using either Pearson's correlation coefficient (quantitative variables) or Spearman's correlation coefficient (at least one qualitative variable) to justify the strength and the direction (negative or positive) of the correlation. Hence, the strength of correlation should reflect the degree of relationship between variables, e.g. to which extent they are linearly related, where a correlation value of +1.00 or -1.00 means that there is a perfect relationship between variables, whilst a correlation value of 0 means that there is no relationship between variables. The direction of correlation reflects whether the variables tend to move in the same direction (a positive correlation value), or in an opposite direction (a negative correlation value).

Thus, in the case that the country variable and the student class level variable are classified as categorical variables, Spearman's correlation coefficients were calculated at a significance level of 0.05. In this case, there was no correlation between the total test score and the country variables, and the p-value was 0.219 (which is greater than the 0.05 significance level). Moreover, there was no correlation between the total test score and the student class level, and the p-value was 0.113 (which is greater than 0.05). Details of the obtained results are illustrated in Table 5.5.

Table 5.5: Spearman's rho correlation between test total scores and demographical factors (country and class level)

Dependent Variable	Spearman's rho	Country	Class of Student
Total scores	Correlation Coefficient	0.066	-0.085
	Sig. (2-tailed)	0.219	0.113
	N	346	346

The next section compares the mean values to determine whether there are statistical differences in the mean values of the total test score between the SA sample and the UK sample, as described in the next hypotheses testing section of this chapter.

5.3 Detailed Discussion of the Results

The analysis in this section will answer the following research questions:

- Is there a significant difference between students within different countries (e.g. Saudi Arabia and the UK) regarding the total test score mean values?
- Is there a significant difference between students' class levels (e.g. year 4, 5 and 6) regarding the total test score mean values?
- Is there a significant difference between students within different countries (e.g. Saudi Arabia and the UK) regarding each concept score's mean values?
- Is there a significant difference between students' class levels (e.g. year 4, 5 and 6) regarding each concept score's mean values?

To answer these questions, various hypotheses were formulated, which are discussed in the following sections.

The null hypothesis for each test states that '*there are no significant differences in the total test score mean values (e.g. the mean values are equal for all samples under comparison)*'. Furthermore, the alternative hypothesis states that '*a significant difference exists*'. Hence, the null hypothesis will be rejected if the corresponding p-value of each test is less than 0.05 of the significance level, and therefore the claim will not be true. In other words, statistically significant differences exist in the total test score mean values.

Moreover, in the case of factors that have more than two categories, such as the student class levels, another test was performed to check whether statistically significant differences existed or not. This corresponds to the ANOVA (Analysis of Variance) test at a significance level of alpha 0.05. In case of an ANOVA test, the null hypothesis for each test states that '*there are no significant differences in the total test score mean values (e.g. the mean values are equal for all samples under comparison)*'. The alternative hypothesis states that '*at least one significant difference exists between categories*'. Hence, the null hypothesis will be rejected if the corresponding p-value of each test is less than 0.05 the significance level, and therefore the claim will not be true.

5.3.1 Hypothesis 1

Hypothesis 1 states that there are no significant differences between students within Saudi Arabia and the UK countries regarding the total test score mean values.

Table 5.6 shows the results of two independent t-tests that were performed to compare the Saudi Arabia sample with the UK sample based on the mean values of the total test score. As shown, the null hypothesis should be accepted because the obtained p-value is 0.54, which is greater than 0.05. Therefore, no significant difference was found in the total test score mean values between the Saudi Arabia and UK samples.

Table 5.6: Two independent t-tests for comparing mean total scores between countries.

Country	Mean	Mean difference	Levene's test	T value	degrees of freedom	P-value
KSA	40.38	-0.53	0.076	-0.62	344	0.54
UK	40.92					

Moreover, as shown in Table 5.6 and in Figure 5.3, the mean values of the total test score for students in the Saudi Arabia sample is 40.38 compared to 40.92 for students in the UK sample, with a mean difference of -0.53. This difference is considered to be insignificant, as discussed previously.



Figure 5.3: Mean total scores for students in the KSA and the UK.

The obtained results reflect that the achieved scores by students were not affected by the country variable. This means that our e-learning system can be used independently from the environment of the student.

The next step was to determine whether significant differences in the total score mean values existed between the student class levels using the ANOVA test, as shown in Table 5.7.a.

Table 5.7.a: ANOVA for comparing mean total scores between class levels.

Source of variation	Sum of squares	degrees of freedom	Mean Squares	F value	P-value
Between Groups	1098.667	2	549.333	9.862	0.000
Within Groups	19106.664	343	55.705		
Total	20205.330	345			

5.3.2 Hypothesis 2

Hypothesis 2 states that there are no significant differences between students of the class years 4, 5 and 6 in the total test score mean values.

The results presented in Table 5.7.a support the claim that at least two mean values differ, and hence the null hypothesis was rejected (with F-value = 9.862, p-value 0.000) where the significance level was 0.05. This means that there was a difference between class levels. To specifically determine the classes that have significant differences between them, an LSD post hoc test was performed based on multiple comparisons, as shown in Table 5.7.b.

Table 5.7.b: LSD Multiple Comparisons for mean total scores between class levels.

(I) Class of Student	(J) Class of Student	Mean Difference (I-J)	Sig.
Year 4	Year 5	4.86100*	0.000
	Year 6	2.84410*	0.006
Year 6	Year 5	2.01691*	0.030

The results in Table 5.7.b illustrate the resulting p-value 0.000 in Table 5.7.a because the three class levels were completely different regarding the mean

of the total test scores, with p-values ranging between (0.00 – 0.030), which are less than the significance level of 0.05. For example, students in year 4 had a different mean value of test scores when compared to students in year 5, with a mean difference of 4.86 and a p-value of 0.000. Moreover, students in year 4 had a different mean value of test scores when compared to students in year 6, with a mean difference of 2.84 and a p-value of 0.006.

After comparing the mean values of the total test scores regarding the country and the class level variables, Table 5.8.a displays the results of comparing the mean values of each of the 23 concepts between the Saudi Arabia and UK samples in general. Two independent t-tests were used at a significance level of 0.05, taking into consideration that the assumption of variance equality should be satisfied before performing the test. Therefore, the values of Levene's test were listed, and if the assumption was not satisfied, the p-values corresponding to variances that were assumed not to be equal in the SPSS output were adopted.

5.3.3 Hypothesis 3

Hypothesis 3 states that there are no significant differences between students within Saudi Arabia and the UK regarding each concept score's mean value.

Note that tests that included concept 5 and concept 22 were not included in Table 5.8.a because the nature of the data of these concepts requires the use of non-parametric tests to determine the differences between students according to their countries.

The results in Table 5.8.a show that there is a significant difference between the Saudi Arabia sample and the UK sample in the mean value of concept 19 (hexagons), with a mean difference of -0.28 and (T-value = -3.715 , p-value 0.000). Furthermore, a significant difference exists between the Saudi Arabia sample and the UK sample in the mean value of concept 14 (parallelograms), with a mean difference of -0.18 and (T-value = -2.112 , p-value 0.036). On the other hand, there were no significant differences found between the Saudi Arabia sample and the UK sample in the mean values of the total test score

Table 5.8.a: Two independent t-tests for comparing mean concept scores between countries.

Concept	Country	Mean	Mean difference	Levene's test	T value	degrees of freedom	P-value
Concept 1	KSA	1.78	0.03	0.311	0.646	344	0.519
	UK	1.75					
Concept 2	KSA	1.32	0.07	0.004	0.586	344	0.558
	UK	1.25					
Concept 3	KSA	0.79	0.10	0.797	1.294	344	0.197
	UK	0.69					
Concept 4	KSA	1.08	0.90	0.449	1.291	344	0.198
	UK	0.98					
Concept 6	KSA	1.39	0.11	0.807	1.452	344	0.147
	UK	1.28					
Concept 7	KSA	1.29	0.06	0.000	0.904	344	0.367
	UK	1.23					
Concept 8	KSA	2.63	-0.03	0.145	-0.309	344	0.758
	UK	2.67					
Concept 9	KSA	1.82	0.02	0.564	0.276	344	0.783
	UK	1.80					
Concept 10	KSA	3.67	-0.17	0.073	-1.229	344	0.220
	UK	3.84					
Concept 11	KSA	3.37	-0.07	0.088	-0.665	344	0.507
	UK	3.44					
Concept 12	KSA	1.82	0.05	0.216	0.530	344	0.597
	UK	1.77					
Concept 13	KSA	1.22	-0.11	0.507	-1.332	344	0.184
	UK	1.33					
Concept 14	KSA	1.14	-0.18	0.011	-2.112	344	0.036
	UK	1.32					
Concept 15	KSA	1.45	-0.09	0.140	-1.230	344	0.220
	UK	1.54					
Concept 16	KSA	1.75	-0.05	0.023	-1.100	344	0.272
	UK	1.80					
Concept 17	KSA	1.00	0.05	0.178	0.595	344	0.552
	UK	0.95					
Concept 18	KSA	1.49	-0.12	0.016	-1.813	344	0.071
	UK	1.61					
Concept 19	KSA	1.20	-0.28	0.772	-3.715	344	0.000
	UK	1.48					
Concept 20	KSA	1.15	-0.10	0.200	-1.230	344	0.220
	UK	1.25					
Concept 21	KSA	4.00	-0.20	0.493	-0.960	344	0.338
	UK	4.20					
Concept 23	KSA	3.18	-0.01	0.105	-0.073	344	0.942
	UK	3.19					

regarding the remaining 21 concepts. As shown in Table 5.8.b, concept 5 and concept 22 have p-values of 0.077 and 0.341, respectively, which are greater than the significance level 0.05.

Table 5.8.b: Results of Mann-Whitney test for comparing mean concept scores between countries.

Concept	Country	Mean rank	Z value	P-value
Concept 5	KSA	171.87	-0.952	0.341
	UK	176.49		
Concept 22	KSA	168.57	-1.768	0.077
	UK	182.56		

The above results confirm our hypothesis that there are no significant differences between Saudi Arabia students and UK students regarding their achievement scores based on concept score mean value.

After comparing the mean values of the total test scores, Table 5.9.a displays the results of comparing the mean values of each of the 23 concepts between the samples according to class level (e.g. year 4, 5 and 6) in general using ANOVA at a significance level of 0.05.

5.3.4 Hypothesis 4

Hypothesis 4 states that there are no significant differences between students in class years 4, 5 and 6 for each concept's score mean values.

Note that the tests on concept 5 and concept 22 were not included in Table 5.9.a because the nature of the data for these concepts requires the use of non-parametric tests to determine the differences between students according to class levels.

The results in Table 5.9.a show that there are significant differences between students' class levels in the mean values of:

- concept 1 (Points) with (F-value = 3.372, p-value 0.035);
- concept 2 (Lines) with (F-value = 6.410, p-value 0.002);
- concept 4 (Rays) with (F-value = 4.311, p-value 0.014);
- concept 6 (Parallel lines) with (F-value = 7.348, p-value 0.001);

Table 5.9.a: ANOVA for comparing mean concept scores between class levels.

Concept	Source of variation	Sum of squares	degrees of freedom	Mean Squares	F value	P-value
Concept 1	Between Groups	1.572	2	0.786	3.372	0.035
	Within Groups	79.931	343	0.233		
	Total	81.503	345			
Concept 2	Between Groups	12.391	2	6.195	6.410	0.002
	Within Groups	331.540	343	0.967		
	Total	343.931	345			
Concept 3	Between Groups	3.137	2	1.568	2.981	0.052
	Within Groups	180.470	343	0.526		
	Total	183.607	345			
Concept 4	Between Groups	4.056	2	2.028	4.311	0.014
	Within Groups	161.377	343	0.470		
	Total	165.434	345			
Concept 6	Between Groups	6.943	2	3.471	7.348	0.001
	Within Groups	162.040	343	0.472		
	Total	168.983	345			
Concept 7	Between Groups	4.767	2	2.383	5.251	0.006
	Within Groups	155.696	343	0.454		
	Total	160.462	345			
Concept 8	Between Groups	2.393	2	1.197	1.251	0.288
	Within Groups	328.177	343	0.957		
	Total	330.570	345			
Concept 9	Between Groups	0.071	2	0.035	0.182	0.834
	Within Groups	66.718	343	0.195		
	Total	66.789	345			
Concept 10	Between Groups	13.711	2	6.856	4.831	0.009
	Within Groups	486.751	343	1.419		
	Total	500.462	345			
Concept 11	Between Groups	2.714	2	1.357	1.513	0.222
	Within Groups	307.530	343	0.897		
	Total	310.244	345			
Concept 12	Between Groups	4.116	2	2.058	3.441	0.033
	Within Groups	205.124	343	0.598		
	Total	209.240	345			
Concept 13	Between Groups	14.662	2	7.331	16.336	0.000
	Within Groups	153.928	343	0.449		
	Total	168.590	345			
Concept 14	Between Groups	1.983	2	0.992	1.864	0.157
	Within Groups	182.448	343	0.532		
	Total	184.431	345			
Concept 15	Between Groups	3.992	2	1.996	4.807	0.009
	Within Groups	142.404	343	0.415		
	Total	146.396	345			
Concept 16	Between Groups	0.437	2	0.218	1.054	0.350
	Within Groups	71.066	343	0.207		
	Total	71.503	345			
Concept 17	Between Groups	2.170	2	1.085	2.025	0.134
	Within Groups	183.726	343	0.536		
	Total	185.896	345			
Concept 18	Between Groups	3.773	2	1.887	4.891	0.008
	Within Groups	132.311	343	0.386		
	Total	136.084	345			
Concept 19	Between Groups	0.922	2	0.461	1.031	0.358
	Within Groups	153.416	343	0.447		
	Total	154.338	345			
Concept 20	Between Groups	2.309	2	1.154	2.123	0.121
	Within Groups	186.480	343	0.544		
	Total	188.789	345			
Concept 21	Between Groups	4.933	2	2.467	0.715	0.490
	Within Groups	1183.113	343	3.449		
	Total	1188.046	345			
Concept 23	Between Groups	3.615	2	1.808	1.235	0.292
	Within Groups	501.914	343	1.463		
	Total	505.529	345			

- concept 7 (Perpendicular lines) with (F-value = 5.251, p-value 0.006);
- concept 10 (Polygons) with (F-value = 4.831, p-value 0.009);
- concept 12 (Quadrilaterals) with (F-value = 3.441, p-value 0.033);
- concept 13 (Rectangles) with (F-value = 16.336, p-value 0.000);
- concept 15 (Rhombuses) with (F-value = 4.807, p-value 0.009); and
- concept 18 (Pentagons) with (F-value = 4.891, p-value 0.008)

As shown in Table 5.9.b, class levels that have significant differences between them are specifically clarified. For example, in concept 7 (Perpendicular lines), there is a significant difference between the mean value of class level 4 and class level 5, with a mean difference of -0.311 and a p-value of 0.002. Moreover, a significant difference exists between the mean value of class level 5 and class level 6, with a mean difference of 0.185 and a p-value of 0.028. Additional details about multiple comparisons can be found in Table 5.9.b.

Table 5.9.b: Significant multiple comparisons for mean concept scores between class levels.

Variable	(I) Class of Student	(J) Class of Student	Mean Difference (I-J)	Sig.
Concept 1	Year 4	Year 6	-0.173^*	0.010
Concept 2	Year 4	Year 5	-0.414^*	0.004
	Year 5	Year 6	0.398^*	0.001
Concept 4	Year 4	Year 5	-0.295^*	0.004
Concept 6	Year 6	Year 4	0.348^*	0.000
		Year 5	0.208^*	0.015
Concept 7	Year 4	Year 5	-0.311^*	0.002
	Year 5	Year 6	0.185^*	0.028
Concept 10	Year 6	Year 4	-0.380^*	0.022
		Year 5	-0.414^*	0.005
Concept 12	Year 5	Year 6	0.250^*	0.010
Concept 13	Year 6	Year 4	0.462^*	0.000
		Year 5	0.372^*	0.000
Concept 15	Year 6	Year 4	-0.193^*	0.031
		Year 5	-0.229^*	0.004
Concept 17	Year 4	Year 6	-0.204^*	0.045
Concept 18	Year 4	Year 5	0.284^*	0.002
		Year 6	0.183^*	0.034

For concept 5 (curves) and concept 22 (ovals), and as mentioned in subsections 5.3.3 and 5.3.4, the non-parametric Kruskal-Wallis test was performed to compare the concept mean values regarding the three class levels, as presented in Table 5.9.c. As shown, a significant difference was found between class levels regarding the mean value of concept 22 with a p-value of 0.002 at a significance level of 0.05. The mean ranks were 159.13, 163.57 and 188.16 for year 4, 5 and 6, respectively. From these results, it is clear that all class levels differed from each other.

Table 5.9.c: Kruskal Wallis test for comparing mean concept scores between class levels.

Concept	Country	Mean rank	Chi square	Degrees of freedom	P-value
Concept 5	Year 4	171.86	4.486	2	0.106
	Year 5	180.41			
	Year 6	169.27			
Concept 22	Year 4	159.13	12.237	2	0.002
	Year 5	163.57			
	Year 6	188.16			

From what was obtained in the above tables, it is obvious that there is a significant difference between students in the various class levels (e.g. year 4, 5 and 6) for each concept score mean values, especially with the concept 22.

The result obtained from the detailed statistical analysis data presented in this chapter led to the most important conclusions regarding the comparisons of the levels of students in Saudi Arabia and in the United Kingdom in primary schools (year 4, 5 and 6) related to studying geometric concepts.

5.4 Discussion of the Study Findings

This section presents the outcomes of the analysis of the results of the study as well as discusses them in light of the underlying theory. The key findings are as follows:

1. Both samples (Saudi Arabia and the UK) had a low mean value for Q12_3 (quadrilaterals), whilst both samples had a high mean value for Q11_1 (triangles), Q16_1 (squares) and Q23_1 (circles). There was no correlation

- between total test score and country nor between the total test score and the student class level.
2. There was no significant difference found in the total test score mean values between the Saudi Arabia and the UK samples.
 3. The three class levels (year 4, 5 and 6) were completely different regarding the mean total test scores, with p-values ranging between (0.00 – 0.030).
 4. There was a significant difference between the Saudi Arabia sample and the UK sample in the mean value of concept 19 (hexagons) and concept 14 (parallelograms).
 5. There was a significant difference between students' class levels in the mean values of: concept 1 (points), concept 2 (lines), concept 4 (rays), concept 6 (parallel lines), concept 7 (perpendicular lines), concept 10 (polygons), concept 12 (quadrilaterals), concept 13 (rectangles), concept 15 (rhombuses), concept 18 (pentagons) and concept 22 (ovals).

The results also showed that the Saudi Arabia students were more likely to understand the straight line and polygon concepts than their United Kingdom counterparts. In relation to the quadrilateral concept, the Saudi Arabia students were more likely to make more mistakes (errors) than the United Kingdom students. A possible explanation for the differences between the two groups is that teachers in the Saudi Arabia schools follow a traditional method of teaching in which students have no role in the learning and teaching processes. In her qualitative study on 13 and 14-year-old students in Malaysia, Idris (2009) found that words such as 'square' and 'rectangle' did not belong to students' normal vocabulary (terms). Olive (2000) attributed this to the fact that students are increasingly taught abstract concepts and experiencing only relational operators such as 'similar', 'equal' and 'congruent'. As mentioned, teachers in the Saudi Arabia schools depend upon memorisation rather than visualisation in teaching geometric concepts. Indeed, learning geometry does not appear to be easy when many students fail to develop appropriate skills in geometrical concepts, such as understanding, reasoning and problem solving (Battista, 1999).

On the other hand, teachers in the United Kingdom schools use more contemporary teaching methods that depend upon student's previous constructed knowledge and experience. Moreover, teachers in the United Kingdom schools focus on the interaction between students in the class and participation in discussions. In this case, students can use mental modelling to recognise the relationship among concepts; however, this requires continuously learning to restructure geometric concepts. According to Van Hiele's model of the development of geometric thoughts, students move from observing and identifying a shape to recognising its properties and understanding the relationship of properties (Usiskin, 2003). This depends upon how students learn the geometric concepts as well as the methods of teaching.

5.5 Summary

This chapter has compared students' understanding in primary schools in the Kingdom of Saudi Arabia and the UK with a focus on how they conceptually represent mathematical geometric concepts (shapes).

Therefore, a mathematical geometry exam was designed in two languages (Arabic and English). The exam included 65 questions, whose internal consistency was computed using the Cronbach's alpha measurement by SPSS software (value of 0.801). Furthermore, the percentages of success, the mean value and the standard deviation were calculated for each question within each country and for each class level.

Two student samples were chosen to take the designed exam from Saudi Arabia and from the UK with a total of 346 students from years 4, 5 and 6. The data that were collected and used in this study were statistically analysed using the statistical analysis software IBM SPSS. Hence, various statistical tests were performed (t-test, ANOVA and non-parametric tests). Furthermore, four hypotheses were formulated to study the statistical significance between various variables, including the students' countries and class levels, the total scores mean and the concepts score mean.

The results obtained from this statistical analysis showed that there were no significant differences between countries or between students' class levels either for the total test scores or for the means of specific concepts. These findings highlight the effectiveness of the e-learning system in overcoming social barriers (language, culture) as well as demographic differences (age, gender).

After discussing this comparative study of primary school students' understanding of mathematical concepts in the UK and Saudi Arabia, the next chapter presents the conceptual agent-based e-learning system regarding its design and methodology.

These results are relevant for the design and development of the e-learning system discussed in the next chapter since they suggest that a common design may be effective across countries.

Chapter 6

Design and Methodology of a Conceptual Agent-Based E-learning System

6.1 Introduction

This chapter presents the overall design and methodology employed for the conceptual agent-based e-learning system that was developed and used as the cornerstone of this research. There are three main aspects covered in this chapter.

Firstly, a detailed description of the architectural approach that was followed for the design and development of the e-learning system is provided. It details the functional requirements that had a significant impact on the chosen architecture, implementation and realisation of the functions in a layered approach, as well as the purpose of each of the layers and their concurrency aspects. Furthermore, it discusses the critical user interface decisions for the application subsystems, performance issues and constraints.

Secondly, and in parallel to the previous aspects, the chapter presents a general overview of the testing philosophy for implementing the subsystems. It also details the integration strategies employed for the application realisation. Furthermore, it highlights certain flexibilities that were inherent in the development environment that facilitated a simplified regression testing as the application evolved into its final form and shape. The decision to

maximise the reusability of the code in certain units greatly simplified the testing effort.

Thirdly, the chapter presents the design and methodology employed to carry out the main study project. It details the design of the experiments and the level of planning that was required to execute the project in the primary schools. It was important to make the right selection regarding the location and the time frame of the experiment, which facilitated the schools' and students' enrolment in a non-interfering manner that was suitable to all the parties. Moreover, the chapter provides the protocol that was followed when performing the experiments in Saudi Arabian schools, in addition to certain recruitment limitations that arose from the use of English as the experiment language in an Arabic speaking country.

Furthermore, it was critical to scope the design based on the project aims in order to eliminate the risks that could arise from out of scope elements that might tamper with the conceptual space theory. It was also important to adopt a standard design reference for realising the e-learning system that was guided by the objectives. The rest of the chapter is organised as follows: section 6.2, design scope; section 6.3, agent-based design for CABELS; section 6.4 application design and implementation; section 6.5, application testing and integration; section 6.6, project experiment design and methodology; section 6.7, study population; section 6.8, experiment logistics; and finally, section 6.9 for a chapter summary.

6.2 Design Scope

The design details presented in this chapter had two components that focused on the two main aspects of the project. The first aspect was the design approach taken to implement the e-learning system that embodied the theoretical foundations of conceptual space theory. The e-learning system design incorporated all the insights gained from experiments that were conducted in primary schools to determine students' visual and oral misconceptions of the geometric line and shape concepts. The second

aspect of the design was the details of the experiment that focused on the methodology that was intended to enhance student conceptualisation of the common misconceptions that were determined during the multiphase pilot experiments presented in Chapter 4 and Chapter 5. Hence, it was critically important to design an e-learning system that embodied conceptual space theory and interacted with the students in a manner that could be expected to aid and stimulate their cognition. These two design aspects were the means to achieve the project objectives defined in Chapter 1.

6.2.1 Design Aim and Objectives

A conceptual agent-based e-learning system (CABELS) was designed and implemented (as described in this chapter). It is a research application developed to verify how the teaching of mathematical concepts to children at primary school could be enhanced in terms of the accuracy of their cognition and long-term conceptualisation. The aim of the system is to use an agent-based approach in order to interpret students' answers, individually provide proper feedback to students, give them further clarification that will help them to understand the correct answer, and finally, to encourage pupils to maintain their concentration in order to produce correct answers.

The specific design objectives are as follows:

- adopt an easy-to-use e-learning system;
- develop an agent-based system (proactive and flexible);
- provide a flexible system tailored specifically for each individual learner;
- effectively deliver learning modules with which the student can interact;
- motivate and stimulate the student's mental engagement;
- provide sequential and incremental learning for building up of complex concepts;
- combine visual, audio and text to enhance conceptualisation.

6.2.2 Design Rationale

The design rationale for the e-learning system was to build an e-learning system that was capable of delivering visual-based learning with complementary audio- and text-based learning that embodied the conceptual space theory in order to enhance student cognition. The system's design was intended to be flexible in order to run on all major computing platforms with preference for the most popular platforms that have already been widely adopted in schools and within academia. In addition, it was critical to implement a mature software development product that was reliable and stable in executing the application. It was equally important that the chosen application should be easily downloadable and should be built offline without any restrictions from the developers (Tisue and Wilensky, 2004). NetLogo met all the above objectives, and with its rapid prototyping environment, it was the ideal choice for agile software development that can emulate the conceptual space theory. NetLogo used a development environment in which the user interface design, the documentation and code were all integrated. Additionally, it allowed for greater freedom during the design phase, especially with regard to the interface design considerations for the e-learning system.

The challenges in today's schools in regard to the teaching of math in general and geometry in particular is to ensure a thorough understanding of the subject as it is evident that most of the incredible innovations came through mathematics. In an earlier chapter 3, it was argued that the best way to make everyone learn a concept effectively is by using multiple learning styles. Explaining shapes is hard through text as there is no representation whether a round is actually round and how it looks. Visually it could be shown but without telling them that it is a circle, they wouldn't know. They must go hand-in-hand to ensure that learning is complete (Project, 2013).

The purpose of the system will be to evaluate the research questions as stated in section 1.4.

6.2.3 Design Assumptions

A set of assumptions was consciously made in order to achieve the objectives of the design of the application, mainly to realise an e-learning system that would help the student subjects that were going to participate in the experiment. The assumptions were intended to account for the flexibility of the suitable location for conducting the experiment and to outline any anticipated limitations of the conditions set forth for the experiment. It was necessary to assume that the enablers of the experiment would be readily present at the selected location for the experiment to be successful. These enablers primarily included the following:

- students able to comprehend basic English;
- students that had access to PC or Mac devices or Android;
- students that were selected from among primary school children aged between 9 and 12 years old;
- students that were interested in learning mathematical concepts.

6.3 Agent-based design for CABELS

The design objective of the CABELS was well suited to the adoption of an agent-based model to capture the actions and interaction of the users with the system. It was critical for the agent-based modelling to achieve an individual-centric approach for the implementation of the Conceptual Space theory. There are two agents being model in the system: the student agent and a supervisor or a tutor agent. Another critical aspect about an agent-based system was to achieve a flexible and proactive e-learning system.

One deliberate choice that was made for the agent-based modelling was to make it a semi-autonomous system rather than a fully autonomous agent as one would expect from traditional agent-based modelling. This decision was dictated by the fact that the system is intended to enhance the student conceptualisation and provides guidance within a certain limited set of interactions rather than open-ended interactions.

User-specific information related to each user were maintained. Hence this user-modelling aspect is an essential aspect of the agent-oriented design. The user-specific information was based on models of concepts rather than qualities. This served the purpose of the exercises that were designed with Gärdenfors theory of Conceptual Spaces in mind, for example, through the search and map capabilities of the CABLES system. Furthermore, all these exercises and overall behaviour of the system at any given point in time is personalised and provides individual feedback for the individual agent instances. This is achieved by using the standard Euclidean metric proposed by Gärdenfors to measure the distance between the concepts or qualities of a student's individual responses and the correct mathematical concepts or qualities. This can be done in order to find out which concepts or qualities are poorly understood (by listing those which are the furthest away according to the Euclidean distance metric). In this way the system provides specific feedback for each student and fulfills the flexible and semi-autonomous criteria required by an agent-based e-learning system.

6.4 Application Design and Implementation

This section discusses the design implementation of the CABELS system that was arrived out through trial and error and insight gained from the initial prototypes previously discussed in chapter 4.

The application, designed to achieve a conceptual agent-based e-learning system, has two interdependent stages for the enhanced geometric line and shape concepts learning objectives, namely the Learning stage, and the Testing and Verification stage. The Learning stage focuses on the introduction to, and the explanation of, line and geometric shape concepts. In this stage, text and audio or visual and audio options are used to convey concepts. The Testing and Verification stage focuses on the assessment of the students' conceptualisation of the line and geometric shape concepts that were presented to them. Both stages contain modular components that help realise the overall objectives of each stage. These components are detailed

in the high-level design process presented in Figure 6.1, and then explained in detail in the following sections.

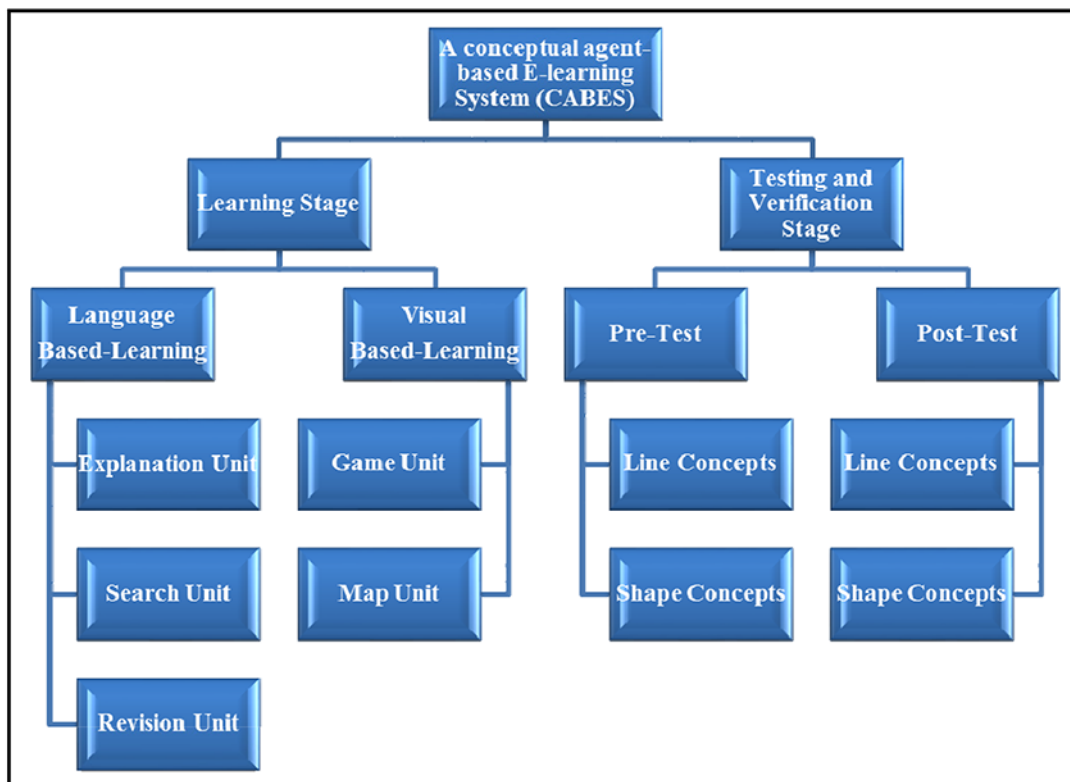


Figure 6.1: The design for CABELS.

The implementation of our system was facilitated by the process outlined in Figure 6.1 in which the actual coding of certain units and modules was carried out in a parallel and overlapping manner. The tasks could interweave without impacting each other in such a way that it enabled efforts to learn the development environment. For example, the Language-based learning module was implemented while the implementation of the Visual-based learning module was also in progress. Similarly, the Pre-test and Post-test modules leverage the same code that maximised code reusability across the modules.

6.4.1 Learning Stage

The Learning stage has two modules:

- a) Language-based learning (system A) that uses text and audio to convey learning concepts.

- b) Visual-based learning (system B) that uses pictorial representations and audio to convey learning concepts.

Each of the two modules has units that realise the module objectives. The combination of textual and audio or visual and audio was intended to facilitate the formation of the mental structure of the quality dimensions of the mathematical concepts that will be easier to retain for future conceptualising of the same concepts.

Several mathematic concepts were selected and implemented in our e-learning system since they were the basic shapes that are taught in primary schools. These concepts are straight lines, line segments, rays, parallelogram, trapezium, rectangle, square and rhombus.

Stimulating the student's cognitive engagement with the lessons in incremental steps that systematically introduced concepts and then tested the basic concepts before moving on to the more complex concepts was key to the realisation of the Learning stage. The implementation of this stage involved the organisation of the lessons in gradually increasing levels of complexity that relied and built on the previously constructed mental perception of the concepts that formed the basis for new concepts.

6.4.1.1 Use-case for the E-learning System

As part of realising the objective of the e-learning system, which was to embody the theory of conceptual space in order to enhance and further the student's conceptualisation of the geometric line and shape concepts, detailed attention was paid to the order and the way in which students interact with the system. To achieve this critical aspect of the system, a user interface was designed and developed which became centre stage to facilitate user interaction with the e-learning system. The primary interface of the e-learning system was also used as a template for completing the system integration. Figure 6.2 shows the primary interface of the e-learning system.

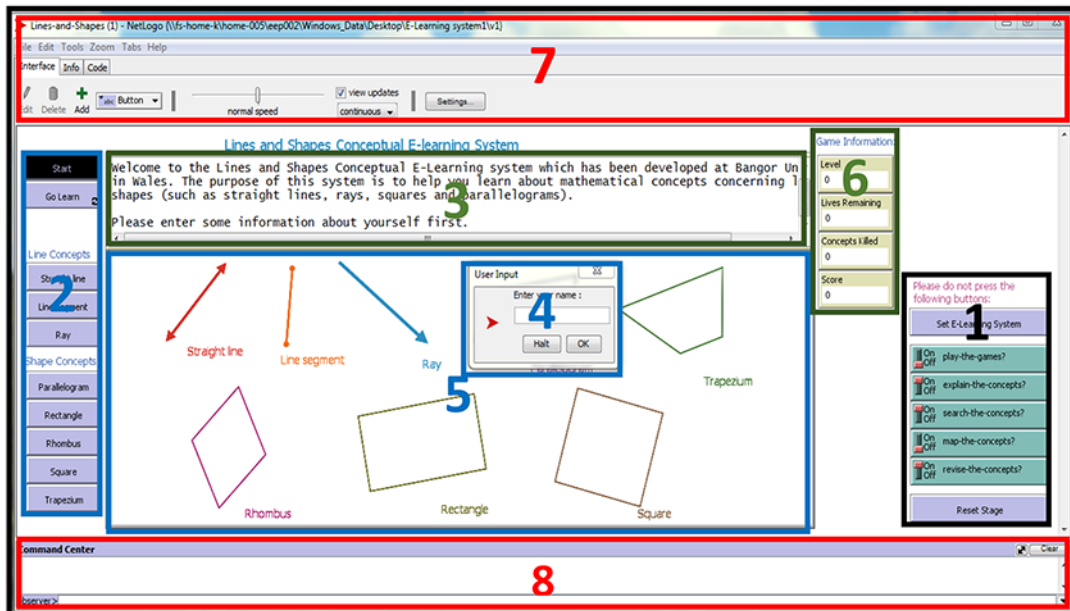


Figure 6.2: The primary interface of the developed application.

The primary interface is split into interface elements highlighted by their relevant section number in Figure 6.2 (and described below). The users are guided through the system in order to complete the data collection for the purposes of the experiment. The remainder of this section presents the typical scenario for use case interaction with the system.

6.4.1.2 E-learning System Interface

The interface elements, or screen sections, shown in Figure 6.2 are documented below:

1. **Screen section 1** is where the parent or teacher decides on the desired learning approach, e.g. System A – Language-based learning, or System B – Visual-based learning. This initial setting is not intended for students. The user interface for selecting either System A or System B is shown in Figure 6.3.

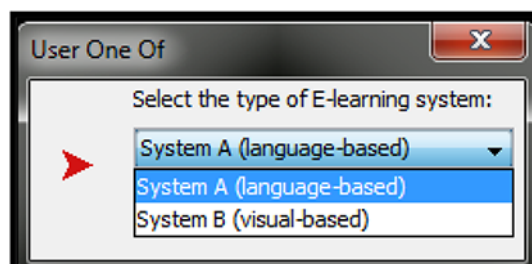


Figure 6.3: The setting for the learning approach (selecting System A).

Once the teacher or the parent chooses the desired path for the student's learning, only the relevant unit switches will be enabled. Unit switches control which units of the system will be run. The system was designed so that different configurations of the units are possible. The five switches (as seen in Figures 6.4 and 6.5) control whether the Game, Explain, Search, Map or Revision units are run during a session. However, for the types of e-learning that were tested for this project, only two configurations were chosen: System A and System B.

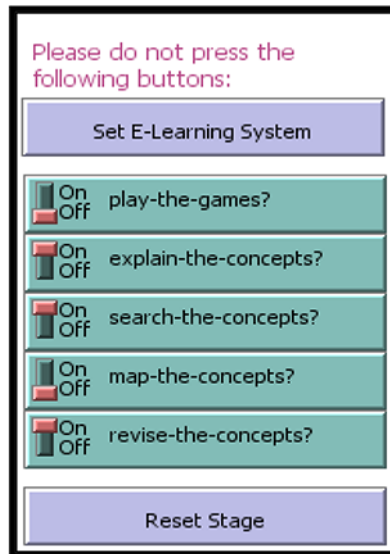


Figure 6.4: The configuration for System A (Language-based).

For example, when System A (Language-based) is chosen, the unit switches for explaining the concepts, searching the concepts and revising the concepts are enabled. Other unit switches that are not related to System A, like playing the Games and the Map units, are disabled.

On the other hand, when System B (Visual-based) is chosen, the unit switches for playing the Games and running the Map units are enabled, while the unit switches for explaining the concepts, searching the concepts and revising the concepts are disabled. This configuration of the user interface for System B is shown in Figure 6.5.

In order to correct any errors that prevent the user interaction with either System A or B from progressing, the teacher or the parent supervising the session is able to reset the system and re-start from a particular stage.

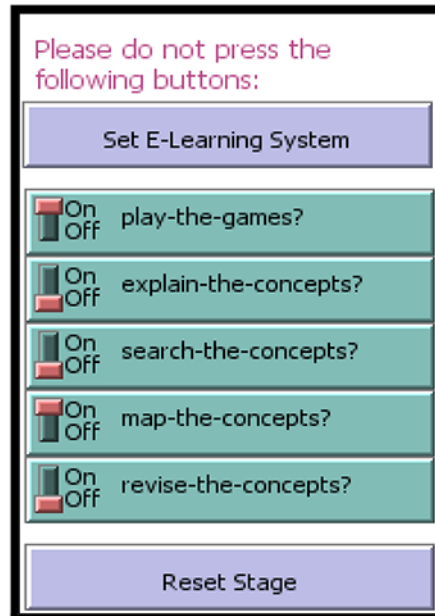


Figure 6.5: The configuration for System B (Visual-based).

Potential errors include the user inadvertently pressing the Halt key and the program freezing due to a run time error. Upon pressing the Reset Stage button, the teacher or parent is allowed to determine the stage to

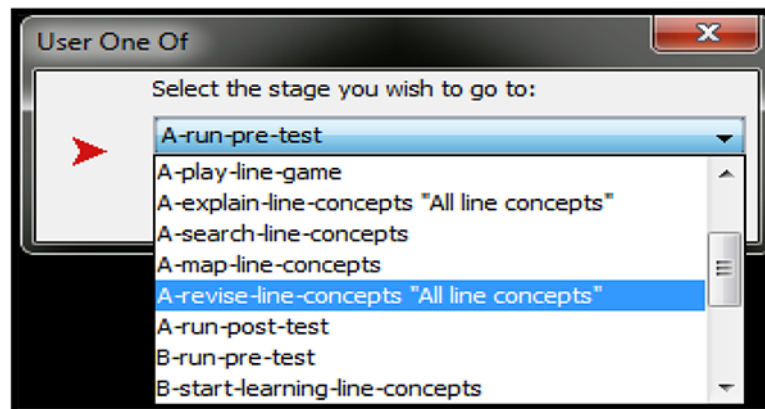


Figure 6.6: The procedure to reset and to select the desired stage.

which they want to return, and have the student continue their progress towards completing the learning system. This error correction interface for the supervising person is shown in Figure 6.6.

2. **Screen section 2** is where the student interacts with the application through the following options:

- A. The Start button allows the student to start the application setup.
- B. The Go Learn button allows the student to start the learning process.

- C. The Line Concepts buttons enable the student to select the relevant line concept during game play.
 - D. The Shape Concepts buttons enable the student to select the relevant shape concept during game play.
3. **Screen section 3** shows the information screen that provides the student with information through text (combined with voice) that is relevant to every stage of the learning process or during testing, or when providing the student's results.
 4. **Screen section 4** shows an example of an input dialog box. This one pops up at the beginning of the session and requests the name of the user. Other dialog boxes will also pop up at this stage and ask about the student's age, gender, language and the file location for logging the user interaction.
 5. **Screen section 5** is used for displaying mainly visual elements, such as images and diagrams, and for showing Game output in the visual learning layer.
 6. **Screen section 6** provides monitors that report game-based data, such as the level, lives remaining, number of kills and the score.
 7. **Screen section 7** is used by NetLogo for various functions (such as controlling the speed of the model or switching between the interface, documentation ['Info'] and code ['Code'], and is standard in NetLogo models.
 8. **Screen section 8** shows an area in the main NetLogo interface that is reserved for the developer.

After describing the main interface of our e-learning system, the Language-based learning module and its components are described in the following sections.

6.4.2 Language-based Learning Module (System A)

The design of the language-based learning module involved three units, namely: Explain, Search and Revision. Each unit was designed to achieve an integral aspect of the Language-based learning module. First, the design defines the objective of achieving an assimilated learning environment that interacts with the student's mental perceptions of real-world objects. Second, it takes an explanatory approach that motivates the student. Finally, the design of the Language-based learning module attempts to define the features of the mathematical concepts that are to be studied. The design and the implementation of the Language-based learning module define the interdependence of the three units of the module. This interdependence is shown in Figure 6.7.

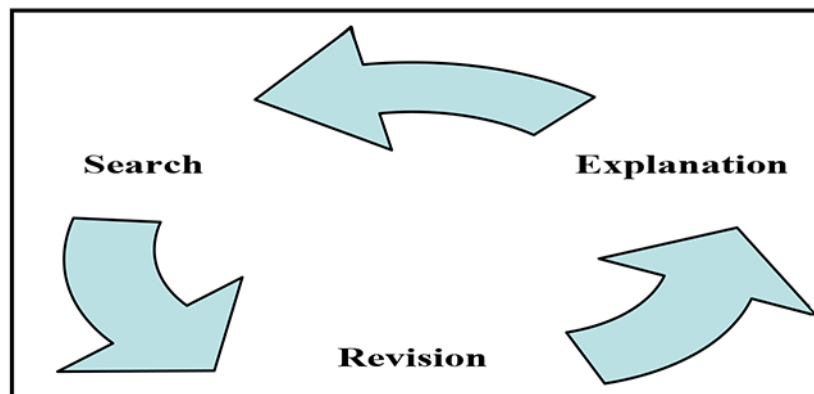


Figure 6.7: Unit interdependence in the Language-based learning module.

6.4.2.1 Explain Unit

The explanation of mathematical concepts is done through the use of text explanations displayed alongside the visual representation of the concept and spoken out loud to the student so that they can follow the examples. The examples provided make use of real-life and everyday objects that the student can observe or is already familiar with, such as a boat, etc. This design approach is intended to aid the conceptualisation of the students and then help with long-term persistence in their mental perception of the concepts. The design of the Explain unit in the Language-based learning module is shown in Figure 6.8.

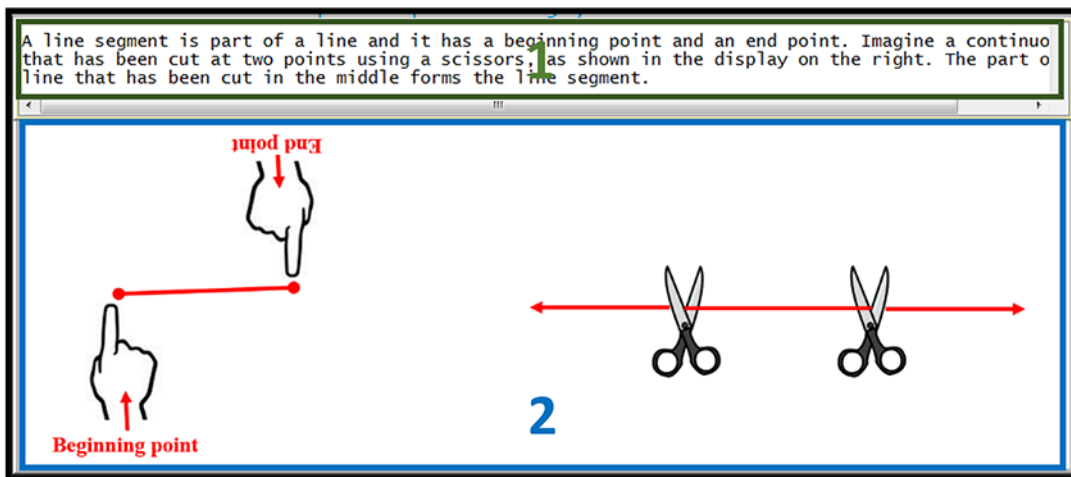


Figure 6.8: Explanation of a line concept by the Explain unit.

Hence, when the student selects the Explain option (either in Figure 6.4 or Figure 6.5), the system starts to define the concept being studied, and then provides detailed text and visual information about the features of the geometric line or shape concept that was chosen by the student. Text is displayed, as in screen section 1 in Figure 6.8, and spoken aloud to the student, alongside a visual representation of the concept. In the example shown in Figure 6.8 (screen section 2), the line segment concept is illustrated by scissors being used to cut a long line in order to create a line segment, and hands pointing to the required beginning/end points of the line segment.

The student is then presented with a number of other examples that are related to the mathematical concept he/she is studying, as shown below in Figure 6.9. Examples of the line segment concept are shown to the student here with various coloured line segments angled differently to emphasise to the student that a line segment is not necessarily horizontally oriented or always a single colour. This is important to young children to ensure that they do not learn the concept wrongly or in one single conceptual dimension only.

Finally, a real-life example is presented to the student where he/she will be able to identify the mathematical concept (line segment in this example) in nature or real life objects. In the example provided in Figure 6.10, a boat is shown and the spoken text explains to the student how he/she can identify line segments in this boat. The system then encourages the student to

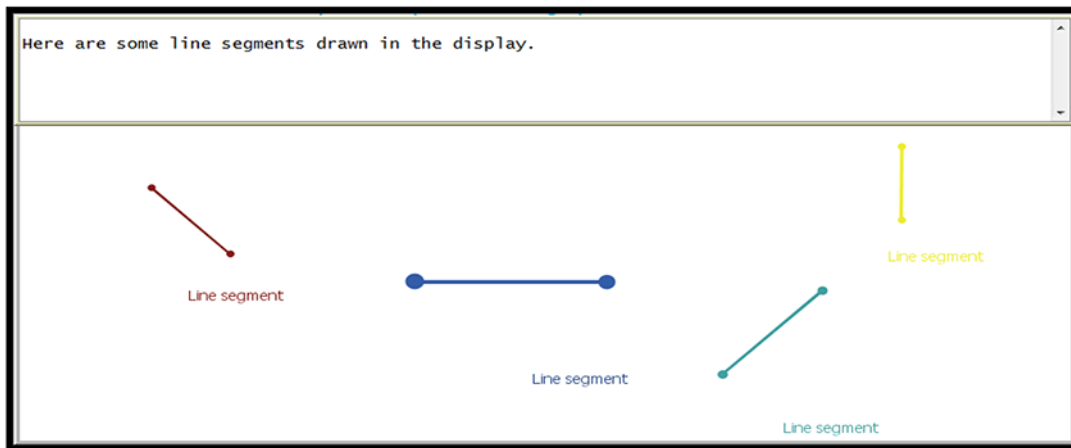


Figure 6.9: More examples of line segment concepts shown to the student by the Explain unit.

identify the geometric concept that he/she just learned in other places, such as the room in which the student is currently sitting.

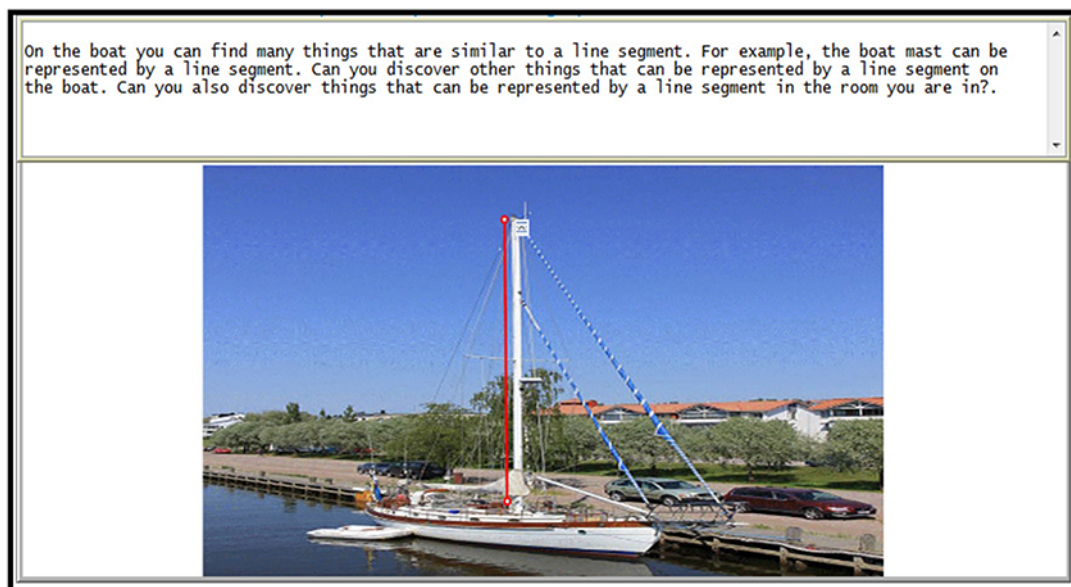


Figure 6.10: A real-life example provided by the Explain unit.

When the student has finished going through the explanation of a particular concept, e.g. a line or a shape, the system will display to the student a dialog box to request a repeat of the explanation as many times as he/she likes. This was a very important aspect of the e-learning system's aim to enhance the student understanding and conceptualisation of the concepts, because the ability to freely repeat the lessons alleviates any emotional stress that the student may have, especially if time was a limiting factor. Furthermore, while a quick learning student may not require a repeat, a slow learner will develop the same level of understanding by repeating the process until he/she feels

comfortable and confident with the concept. Moreover, the student is able to choose whether he/she wishes to have a single aspect of the concept (e.g. straight line, line segment, or ray) explained further, or to continue to the next stage of learning, as shown in Figure 6.11.

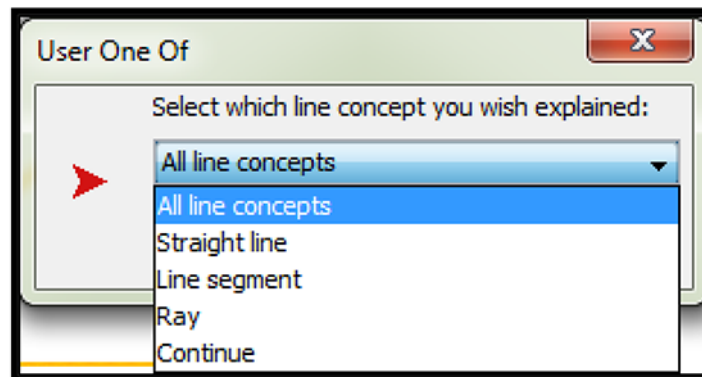


Figure 6.11: A dialog box for either repeating or progressing to the next concept.

6.4.2.2 Search Unit

The Search unit is designed to enable the student to seek out the features that are associated with the mathematical concept that is under study through a search able interface. Hence, the Search unit presents a list of select able features for a given mathematical concept. The system returns all objects that match the features selected by the student. This was intended to foster curiosity and motivate the student to explore the features of the concept under study, which will help with the conceptualisation of the qualitative dimensions of the features of that concept. This also relates to the notes of a concept as a region in n-dimensional space, according to conceptual space theory. Therefore, it makes sense to provide a tool to allow students to explore that space.

As shown in Figure 6.12.a, the student has searched for concepts that are straight, have beginning and end points, and can also be a side of a shape. The result of this search that is returned to the student is a single concept: a line segment (Figure 6.12.b). Using a dialog box, the student is then invited to carry out another feature search.

The system's ability to search for specific features was intended to increase the level of student interaction and engagement with the system in order to

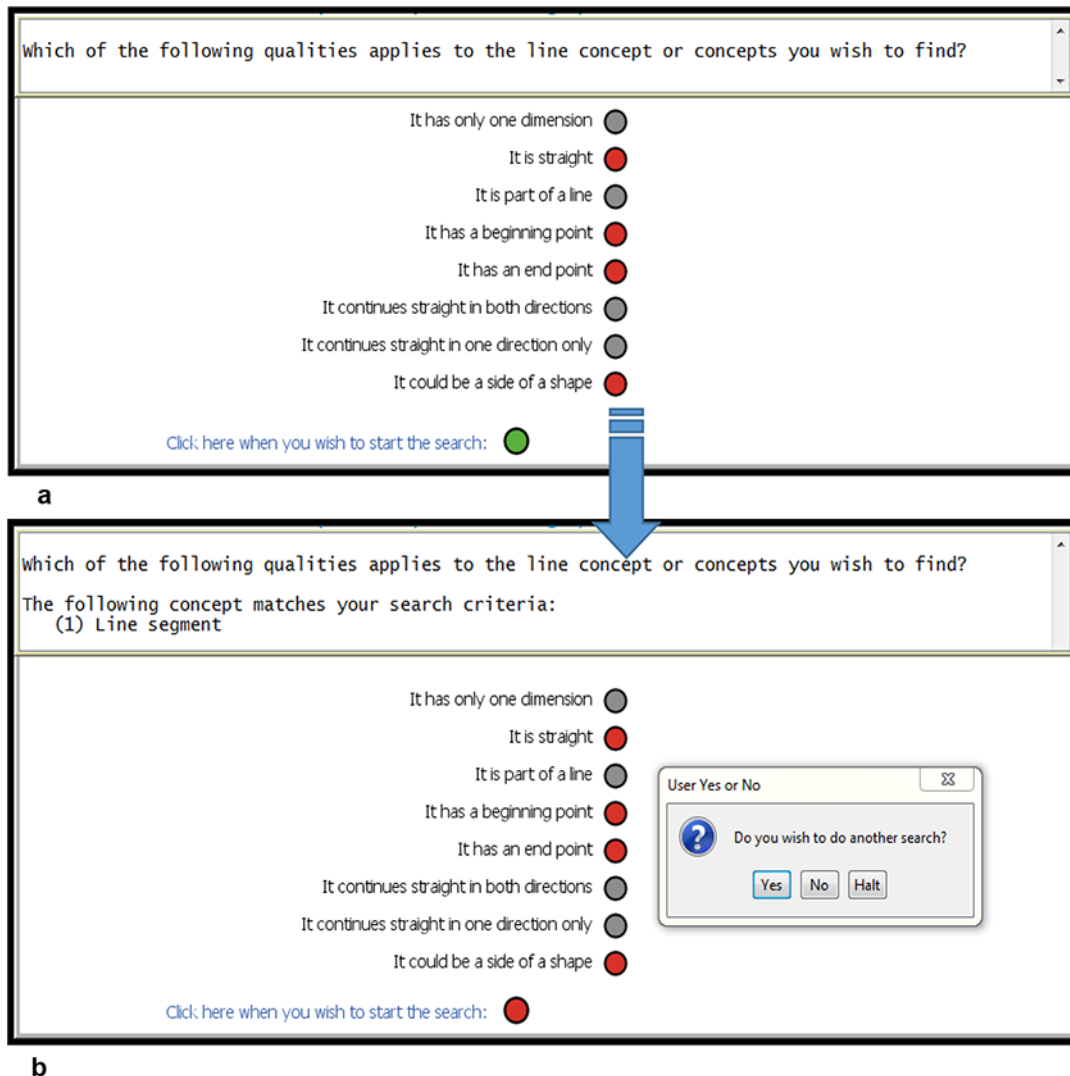


Figure 6.12: The Search unit interface.

deepen the student's understanding of the features of a particular concept, which relate to the quality dimensions in conceptual space theory.

6.4.2.3 Revision Unit

The Revision unit is designed to enable the student to review the features that were presented to him/her in the Explain unit that covered mathematical geometric line and shape concepts. The Revision unit is implemented to present the student with examples of the features of the mathematical concept. This unit covers all the aspects of the Explain unit in a sequential manner, and in increasing complexity regarding the features of the concepts. Moreover, the design of this unit allows the student to select either a complete review of the concept or a particular aspect of the concept. The design of

the Revision unit is illustrated in Figure 6.13, and explained in the following paragraphs.

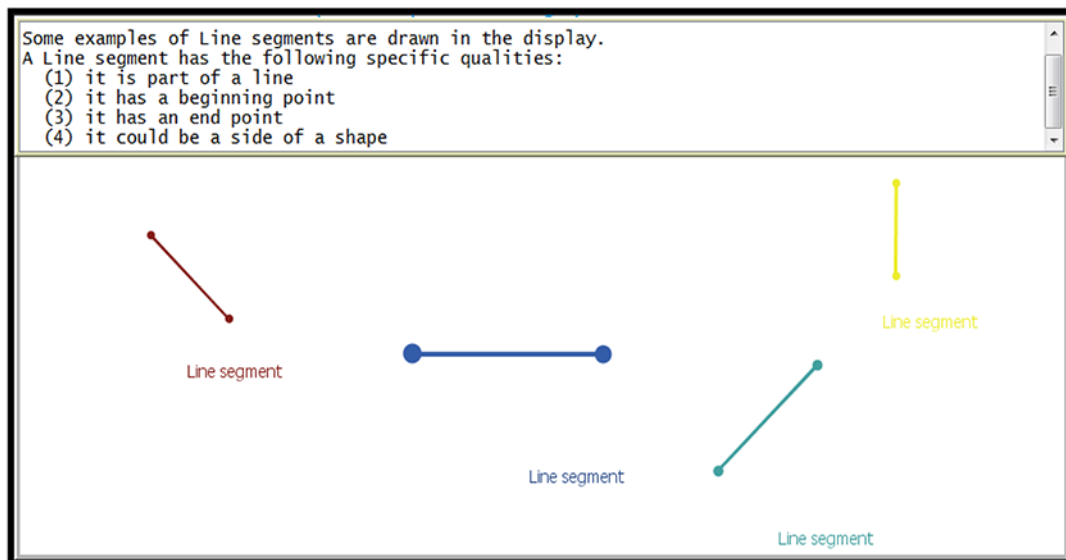


Figure 6.13: The Revision unit interface.

When a student completes a learning module for a specific concept, the system goes one step further to refresh the specific quality dimensions of the concept through a revision process. The e-learning system attempts to review with the student all the features of the relevant geometric line or shape concept. It presents to the student the features that are the basis for the distinctive quality dimensions of the concept by providing examples. To illustrate this aspect of the e-learning system, the revision of the line segment is shown in Figure 6.13. This is again achieved through a combination of speech and text. The revision itself covers all aspects of the explanation regarding the concept, but it is up to the student to decide on the depth and breadth of the revision of the concerned concept.

After completing the revision step, the system then allows the student (via a dialog box) to request a repeat of the revision as many times as is desired. As was explained earlier, this flexibility is intended to eliminate the inequality between slow learners and fast learners. Furthermore, the student is able to choose a single aspect of the concept for revision, as shown in Figure 6.14 (such as straight line, line segment, or ray), or choose to continue to the next stage of learning, e.g. shape concepts.

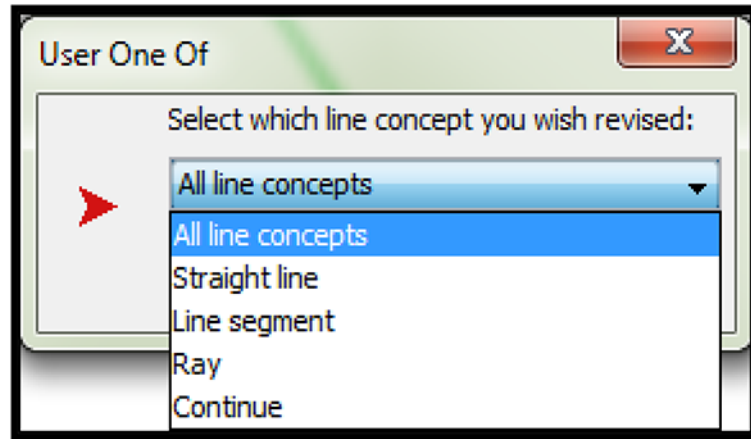


Figure 6.14: The dialog box for concept revision.

After describing the various components of the Language-based learning module, the Visual-based learning module and its components are presented in the following sections.

6.4.3 Visual-based Learning Module (System B)

The Visual-based learning module was designed to include two units, namely, the Games and Map units. These two units effectively aided the student's conceptualisation through a visual presentation of the concepts in a manner that also increased the student's eagerness to learn and to understand the concepts in the learning module (as evidenced by observation of the students in experiments described in chapter 7). The design goal for this module was to further increase the level of required engagement and interaction from the student in an enjoyable manner. The intensity of the interaction increased as the student progressed through the learning module and complex concepts were expected to become relatively intuitive as the student advanced in the module units.

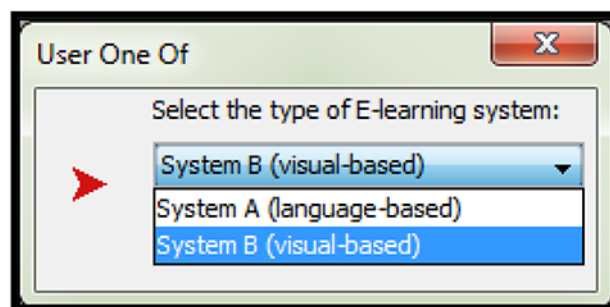


Figure 6.15: The setting for the learning approach (Selecting System B).

When the user selects the Visual-based learning module (Figure 6.15), there are two possible options: the Games unit or the Map unit. At this stage, the emphasis is placed on the visual representation of the concepts but the audio component is still played. The student is free to choose which of the two possible options he or she does first, because the two options are expected to complement one another. The Maps unit, in particular, aims to ensure that the student is able to distinguish between different mathematical concepts sharing similar features. Again, the idea is to provide an analogy of concepts being represented in a space that can be mapped, which is analogous to conceptual space theory. The following sections elaborate on each of these units and provide examples.

6.4.3.1 Game Unit

The Game unit was designed to teach the student geometric line and shape concepts through a visual representation of the concepts in various animated forms. The design approach of the Game unit was based on matching a pictorial representation of a concept with its name. Matching the quality dimension defining a mental structure of a concept to the geometric lines or shapes that are pictorially displayed on the screen is an integral aspect of conceptual space theory. The Game unit is intended to emulate the process of mental cognition that relies on successful matching of the mental perception and the visual input from the screen animation (the Game unit was initially described in Chapter 4, section 4.7).

Figure 6.16 illustrates the interface that is displayed to the student upon selecting the Game unit. The screen for playing the Game unit is divided into different sections that are labelled above. Screen section 1 presents the students with spoken text explaining what they need to do to play the game. The students do not interact with this box and it is for their information only. Screen section 2 is where the shapes are randomly displayed. Screen section 3 is where the students are able to select the corresponding button for the pictorial representation of the concept on the screen. Screen section 4 displays the current level and the score of the game that is in progress.

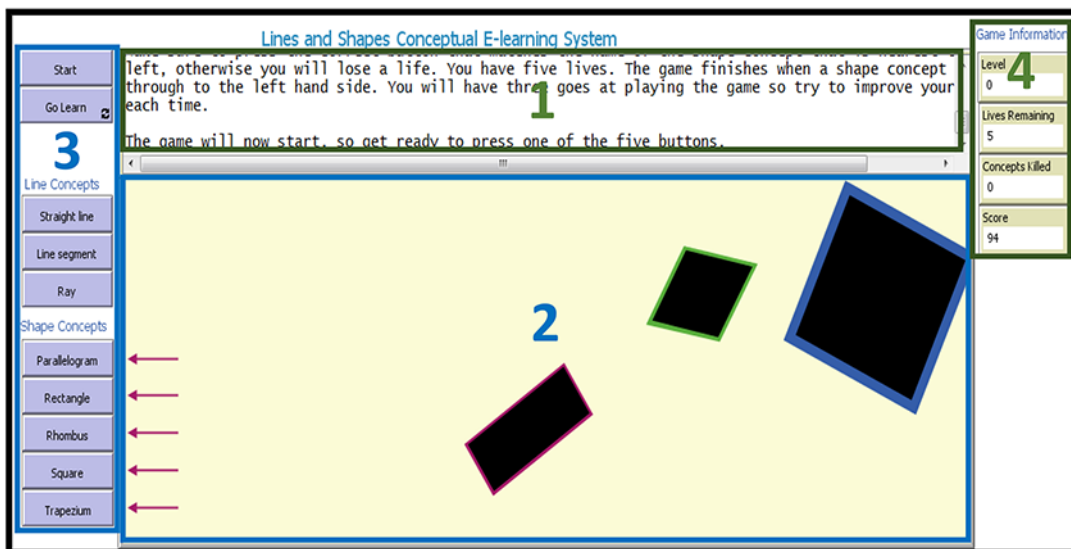


Figure 6.16: The Game unit interface.

The score is displayed in screen section 4. At every level, the student encounters 10 random shapes. The game speeds up as the player progresses to each new level and reaches a point at which it is even quite challenging for an advanced adult to score. The student is given five lives (e.g. chances to make mistakes) and when these are used up, the game is over.

Upon selecting the Game unit, an animation of the concepts is displayed. Geometric line or shape concepts start appearing randomly on screen section 2 as the game begins. The line or shape concepts start moving to the left of the screen and the student must ensure that no concept makes it through to the left side otherwise the game will end. The student must press the concept button that corresponds to the leftmost concept on screen section 3, in order to remove it from the screen before it gets through. The student is given five chances to make a correct button selection. If the student clicks the correct button that corresponds to the visual representation of the concept on the screen, then a musical note is played that indicates a positive selection. If, on the other hand, the student selects an incorrect button, an annoying sound is played to indicate a negative selection, and this decreases the number of errors that the student is allowed to make in the future when playing at this game level (e.g. it reduces the number of 'lives' a student has). The speed at which the new shapes are generated increases as the student transitions from one game level to the next level up. The student can also manually

increase the speed of screen transitions from the initial level before he/she successfully completes the current level of the game.

When the game is over, the student is presented with a performance summary that includes the total score, the highest level reached, and how many times the student made a mistake per shape type. This helps the student to self-evaluate so that he/she can self-correct in an effort to achieve a higher score or reach a higher level in the game. The overall purpose of the game is to enhance the student's mental perception of concepts that will facilitate future cognitive understanding of the concepts.

6.4.3.2 Map Unit

The Map unit was designed to give the student the opportunity to distinguish between different mathematical concepts that have similar features. The Map unit was a critical aspect of visually aiding the student in their judgment of the similar quality dimensions of related concepts. The idea was to help the student to identify and recognise the similarities and differences of the quality dimensions in their mental perception. This goes back to the core of the conceptual space theory that is envisioned to enhance the student's cognition of the geometric line and shape concepts. The design of the map is expected to facilitate the mental process of conceptualising the geometric line or shape concepts presented.

The presentation of the map itself is based on the Venn diagram that groups similarly shaped objects together. The design and implementation of the map, which is part of the realisation of the goal of the visual module, was expected to help the students to recognise and to describe the grouping of geometric line or shape concepts according to their degree of similarity or difference. The design also allows the student to modify the speed of the map formation if so desired. The speed is intended as an element of creating a user-driven process for the e-learning system that ultimately enhances the student's understanding of the concepts and eliminates any misconceptions

because of their improved overall mental retention through the interactive learning environment.

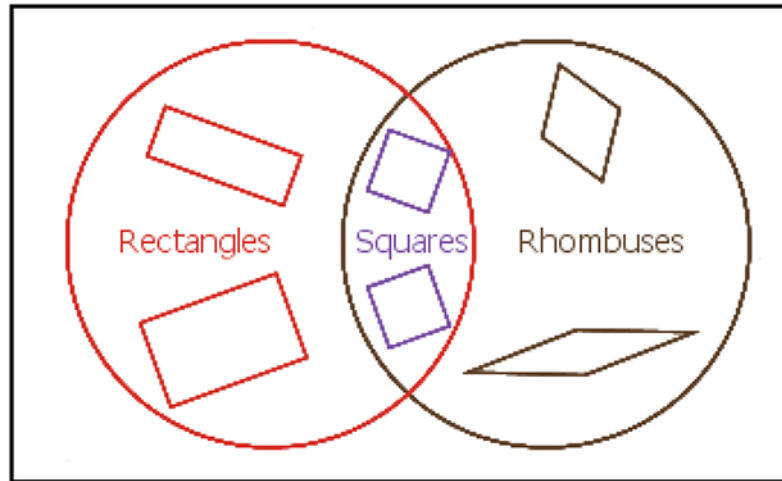


Figure 6.17: Example map for rectangle, square, and rhombus concepts.

Upon selecting the Map module, the e-learning system will start presenting shapes that are grouped in a Venn diagram, as shown in Figure 6.17. The student is expected to recognise the concept’s representation that is presented within the grouping, and be able to differentiate between the quality dimensions that characterise their geometric description. To illustrate this point, square shapes share the grouping with rhombuses and rectangles, as shown in Figure 6.17.

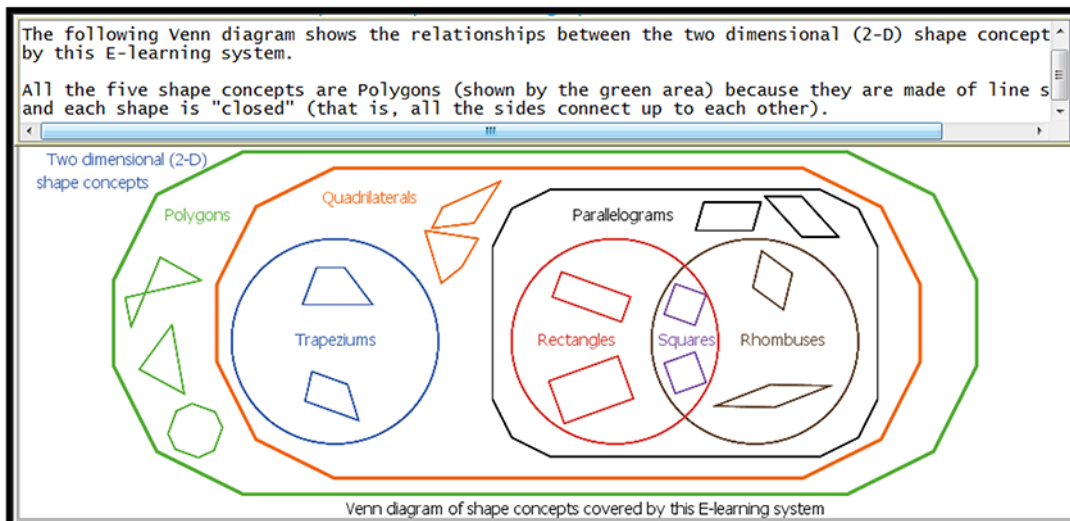


Figure 6.18: Example of a complex map.

The student is expected to properly identify the shapes and then recognise the relationship between them. In this example, square concepts are shown to be at the intersection between rectangle concepts and rhombus concepts,

since squares can be seen as rectangles and at the same time as rhombuses. In essence, this is a mental exercise of organising each shape in a manner similar to conceptual space theory, so that similar concepts are close together and unrelated concepts are far apart. The composition of the Venn diagram becomes progressively more complex in the map as the student advances in the Map unit. An example of a more complex map is shown in Figure 6.18.

6.4.4 Testing and Verification Stage

In this stage, we attempt to verify the ability of the students to recognise the different line and shape concepts presented to them. The e-learning process is organised into systematic steps that will initially require the student to go through a Pre-test unit in order to complete a set of questions that pertain to either line or shape concepts. After completing the Learning stage, the e-learning system will ask the same set of questions to the student in the Post-test unit. The questions asked are the same for both System A (Language-based learning) and System B (Visual-based learning).

Hence, the two modules composing the Testing and Verification stage are:

- a) Pre-test module: uses textual representation and audio to convey learning concepts.
- b) Post-test module: uses pictorial representation and audio to convey learning concepts.

Even though the two modules have complete overlap with the module's units, their respective objectives are different. The e-learning system is designed in such a way that the two testing modules (Pre-test and Post-test modules) are encapsulated within each learning module. The Pre-test and Post-test present the same set of questions to evaluate the progress made by the student after going through the relevant learning process. The testing application design approach is highlighted in the following logical diagram in Figure 6.19.

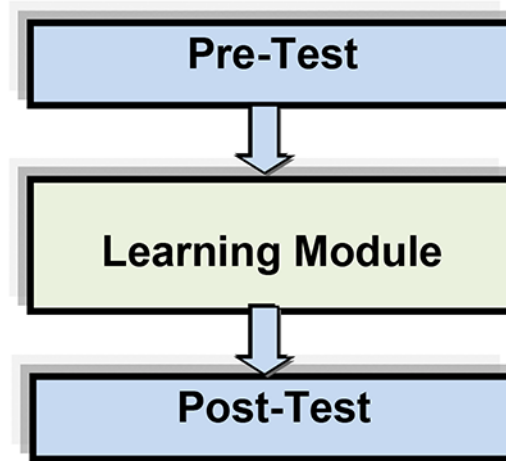


Figure 6.19: Design of the testing stage.

6.4.4.1 Pre-test Module

The Pre-test module contains two set of questions that are related to either the line or the shape concepts. These are intended for a pre-assessment of the student's understanding of the concepts to be covered in the learning module. The questions are derived from the common misconceptions that the students exhibited in the pilot experiments (detailed in Chapter 4 and Chapter 5). They are focused on the learning module implemented in the e-learning system that will follow after completing the questions. For example, if the student is going to learn concepts related to geometric shapes, then the Pre-test will contain questions that assess the student's conceptualisation of the pertinent geometric shape concepts.

The design of the presentation and the format of the questions shown to the student are key to the intended aim of aiding the student's mental perception of the concepts. In particular, the questions are presented according to an increasing level of difficulty, and they are organised sequentially in order to facilitate the recognition of the quality dimensions as they become more complex. Keeping the student interested in the e-learning system was also part of the concerted effort that was put into making the design of the Testing and the Learning modules engaging for the student. The selection of the colours and the blending of visual, audio and textual elements all contributed to making interaction with the system a stimulating experience.

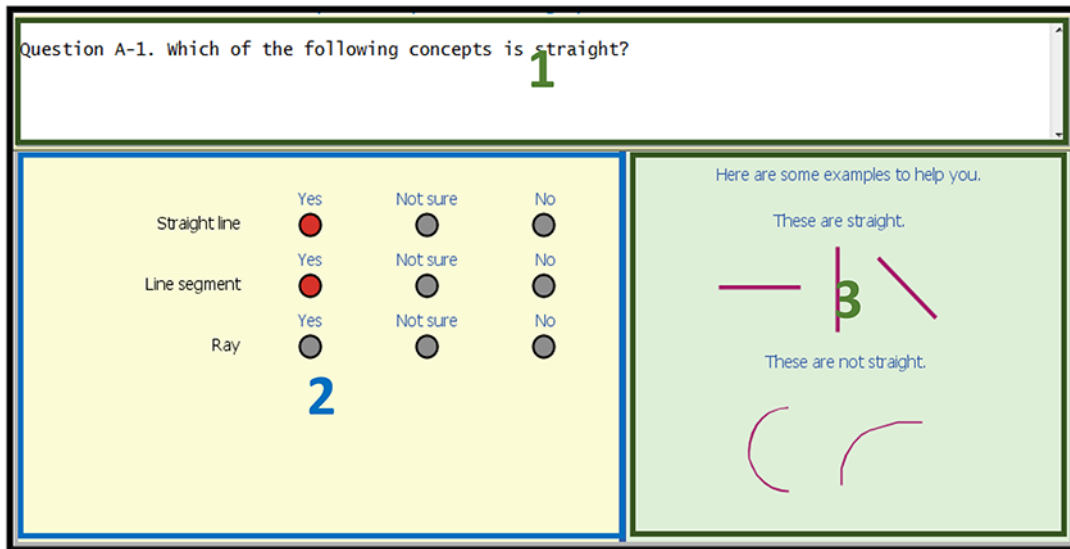


Figure 6.20: The Line Pre-test interface.

The Pre-test module, shown in Figure 6.20, presents a number of questions to the student and invites him/her to answer by selecting the button corresponding to the correct response. The screen presented to the student is split into three sections, as shown in Figure 6.20:

1. **Screen section 1** gives textual as well as audio information to present the question to the student.
2. **Screen section 2** gives a set of possible answers from which the student can select.
3. **Screen section 3** gives some examples to help the student to conceptualise the concept that is being assessed in the question.

The colour red in the selection box does not indicate the accuracy of the answer given. Once the student provides an answer to the question posed, the system automatically passes him/her to the next question without providing feedback on the accuracy of the answer since the feedback step is not implemented in the Pre-test. Feedback is eventually given once the Post-test questions have been asked. Figure 6.21 provides an example of a shape-based Pre-test question.

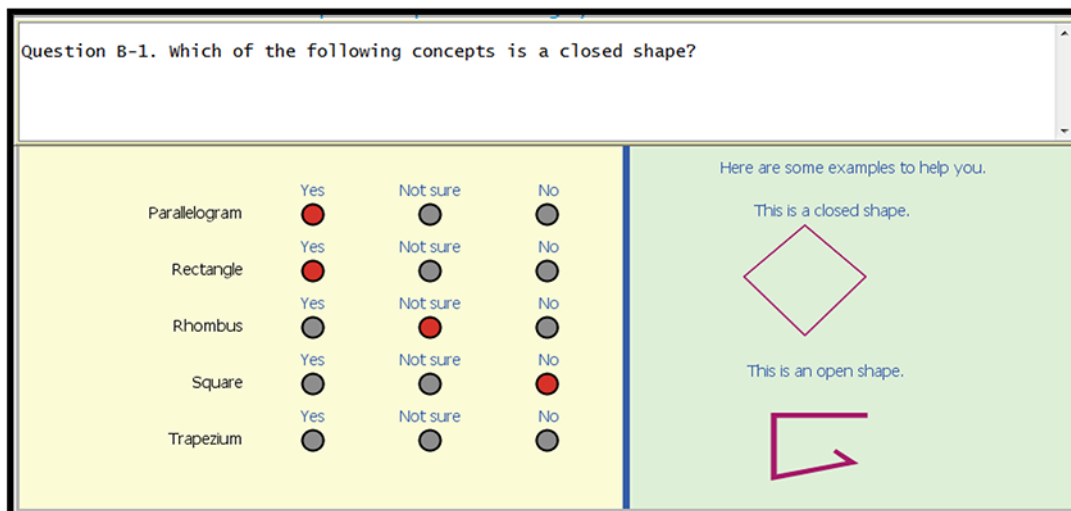


Figure 6.21: The Shape Pre-test interface.

6.4.4.2 Post-test Module

The Post-test module is a mirror image of the Pre-test module, but has a different objective. In this second testing module, the aim is to assess whether the student's level of understanding has increased and whether the learning module has helped the student's cognition of the concepts that were presented during the learning module. This sandwiched approach for testing the learning module and its experimental outcome after the student completes the sessions was a critical step in assessing whether the project objectives outlined in Chapter 1 had been achieved. The design details for the Post-testing were the exact replica of the Pre-test design and implementation.

As indicated earlier, the Post-test module presents the same questions to the student as in the Pre-test module. However, in this second testing module, feedback on the accuracy of the student's answers is given as he/she progresses from one question to another. The final result is also given to the student at the end of the test. Figure 6.22 illustrates how the feedback is provided to the students.

To better understand the information arranged on the feedback screen, it should be noted that each section plays a role in conveying information to the student or the supervising individual. Hence, in screen section 1 in Figure 6.22, the application gives feedback on the answers specifically given by the

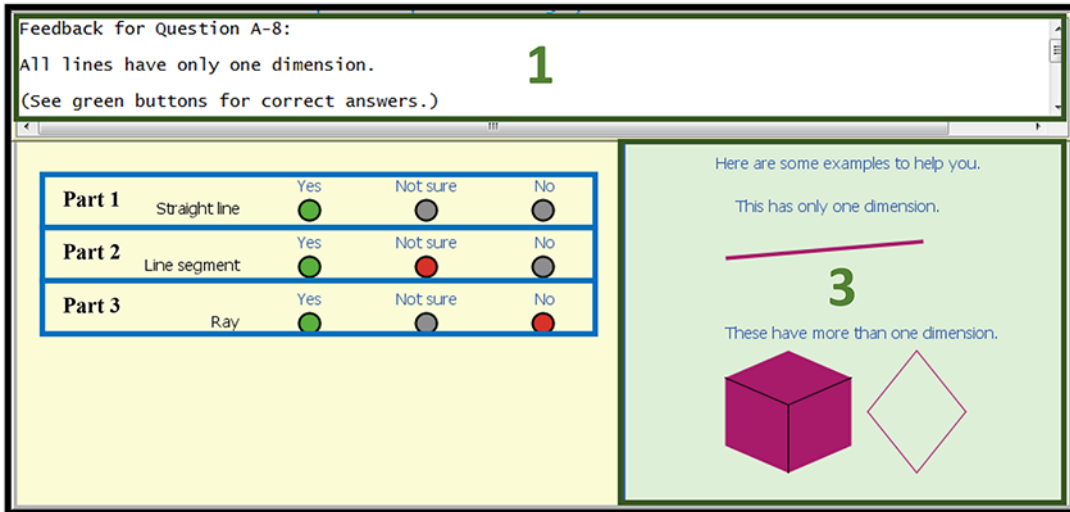


Figure 6.22: The Post-test feedback interface layout.

student. In screen section 2, the answers chosen by the student and the system's correction, as well as the feedback, are presented. In screen section 3, the system provides examples of a single- and multi-dimensional object (this help was also provided in the Pre-test module).

In this example, the student has given the wrong answer to parts 1 and 3 of the question (straight line and ray, as shown in the sub-boxes of screen section 2), but correctly answered part 2 of the question (line segment). The application indicates the correct answer in green and uses red to show where the student has answered incorrectly.

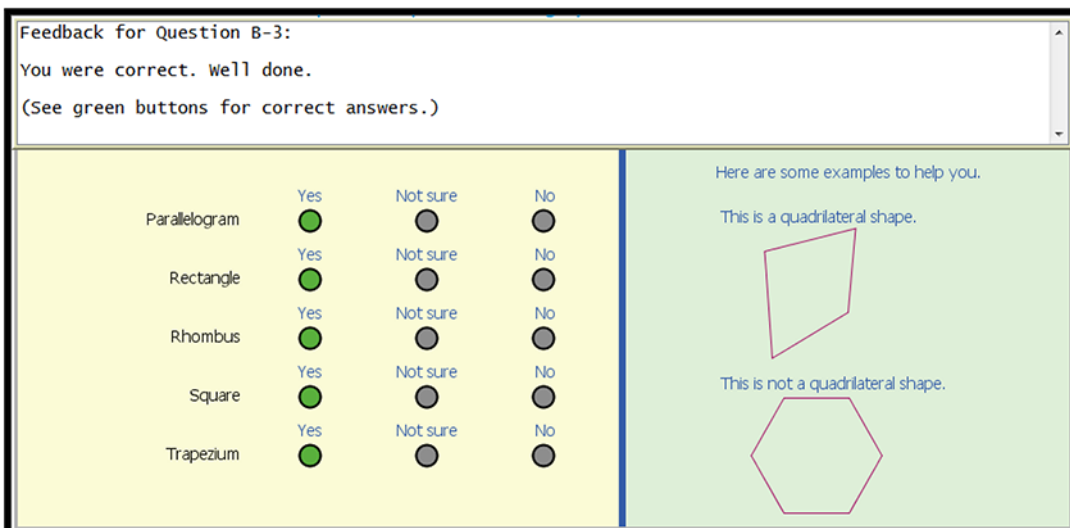


Figure 6.23: Example of a Post-test feedback message.

In another example, the system provides encouraging feedback to the student by using a text message and an audio message, pronouncing “Well Done”, for giving the correct answer (as highlighted in Figure 6.23).

6.5 Application Testing and Integration

In order to complete our work in a timely manner, the testing and integration effort followed a process that was envisioned to carry out the task most efficiently. Every unit was implemented and tested as a standalone unit. This step ensured that issues were addressed while they were still easily traceable to their source unit level. Even though the coding of the units and the modules under both stages were developed and tested through parallel tasks, their integration followed an organised process that aligned with how it was envisioned that the e-learning system would typically be used from start to finish.

The top-down process of integrating the modules of the e-learning system is shown in Figure 6.24.

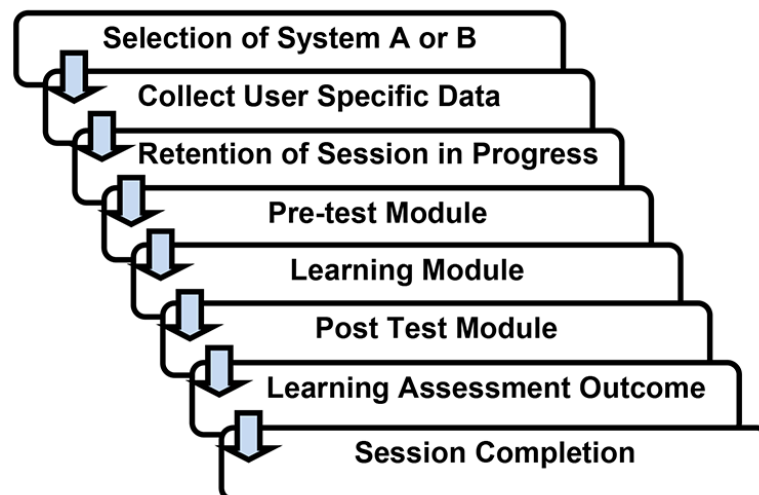


Figure 6.24: The top-down approach that was adopted for testing the application implementation.

The unit level testing was carried out upon the completion of each unit. Therefore, if there were no dependent units or modules that needed to be completed before integrating them into the system, then they were immediately integrated and the tasks were marked as complete. It was

easier to modify and enhance the features and functions of units or modules that had no dependencies because their codes could be changed without impacting any other part of the system. Moreover, upon verifying the updated code, it was possible for the unit to be added back into the system without requiring significant system level regression testing. On the other hand, if the unit or the module had any dependencies, then their integration was delayed until all the dependencies were eliminated, and regression testing was required when their code was modified, depending on the number of other dependent units.

While testing was flexible to a certain extent, the integration process was tightly controlled based on the flow chart shown in Figure 6.24. Initially, the user interface was completed and it formed the skeleton of the system integration shown in Figure 6.25, in the next section, and it was also used to detail the system level use case scenario. Based on how the system was expected to work, the code for the selection of System A (Language-based learning) or System B (Visual-based learning) was completed and integrated into the system.

Additionally, every student who used the system was required to enter some personal data. The information was intended to form the basis of synthesising the student's learning outcomes, and included basic information regarding the student's age, year of study, gender, etc. The code for prompting the user to enter the personal data was integrated into the system. Each session was required to be uniquely identified, which called for the implementation of code that would enable the logging of the session from the beginning to the end. The information that was collected was written as text to a log file.

The e-learning system uses Mac iOS text-to-speech software to read aloud the text that is related to a particular step in the program. The initial prototype was developed to support multiple types of speakers that included male and female voices. During the initial prototype experiment, it became clear that the students started to become preoccupied with altering the speaker's voice. In order to keep the students focused on the learning content, their

ability to select the speaker's voice was removed from the final version of the e-learning system. Instead, one default speaker voice code named "Alex" was selected from the Mac iOS text-to-speech software, since that was the platform that was available.

The next step involved integrating the Pre-test module into the system. This module contained the implementation of assessing the student's understanding and conceptualisation of the geometric line and shape concepts. This assessment forms the baseline of the student's understanding of the concepts before being exposed to the learning module that is based on conceptual space theory.

The learning module was the next task that was integrated into the system. Along with the Pre-test and Post-test, these were the core modules of our e-learning system. The learning module was the most important aspect of the e-learning system because it aimed to help the student to enhance their level of understanding of the geometric concepts that was determined through the baseline step in the Pre-test module. The learning module incorporated visual representations of the concepts, text that described each concept and audio explanations of the same concepts. The threefold use of visual, textual and audio elements was intended to stimulate the student's effective structuring of the qualitative dimensions of the concepts and help with their mental perception that could be easily reused for future cognitions.

The Game module and the Map module were also integrated into the e-learning system as part of the learning module. Each of these two modules was also intended to increase the student's interest in learning mathematics, and motivate him/her to remain focused during the interaction with the e-learning system.

The Post-testing involved a complete re-use of the Pre-test code because the objective was to assess the impact of the learning module on the student's understanding of the concepts that had been previously determined in the baseline assessment of the Pre-test. Therefore, the learning assessment

module was integrated into the system after the Post-testing module. This involved the recording and the presentation of the test score and general progress information.

Other important aspects of the e-learning implementation included the procedure to reset the running session to a desired stage. This was required to mitigate the problems caused by the errors that could possibly occur during the session execution while the students interact with the system. If such errors were to occur, then the design anticipated a means to reset the progress of the session to return to an earlier stage that can be determined by the supervising individual.

As a standard practice for implementing any computer program that realises a system, the e-learning system goes through the process of program initialisation at the start of every program execution. This involves setting up the environment variables and their initial values and states.

Ultimately, each e-learning system session had to be closed upon the user's completion of the session. This involved the graceful closure of the logging session and was critical to conducting a post-session analysis of the completed sessions as a means of conducting the study's experiment.

6.6 Experiment Design and Methodology

In this research study, quasi-experimental design was used due to the nature of the problem studied and the procedural decision followed during the execution of the experiment (Gribbons and Herman, 1997). One defining decision was the deliberate selection of the student subjects to use either System A or System B. This enabled the study to compare the two groups and the e-learning system's causal effect on the students' understanding of the geometric line and shape concepts.

6.6.1 Design

The experimental design was chosen to be quasi-experimental where a sample was assumed to be representative of the overall student population. It was practically impossible to test all the primary school students, although this would have provided a conclusive causal effect of the e-learning system (AlKilani and Alsharifin, 2005, p.74). The design was intended to evaluate the effect of the independent variable (e-learning application) on the dependent variable (understanding of the geometric concepts). The design used a Pre-test of the concepts before they were taught to the student and a Post-test after the learning module was complete in order to determine the causal effect. Figure 6.25 shows the design of the experiment.

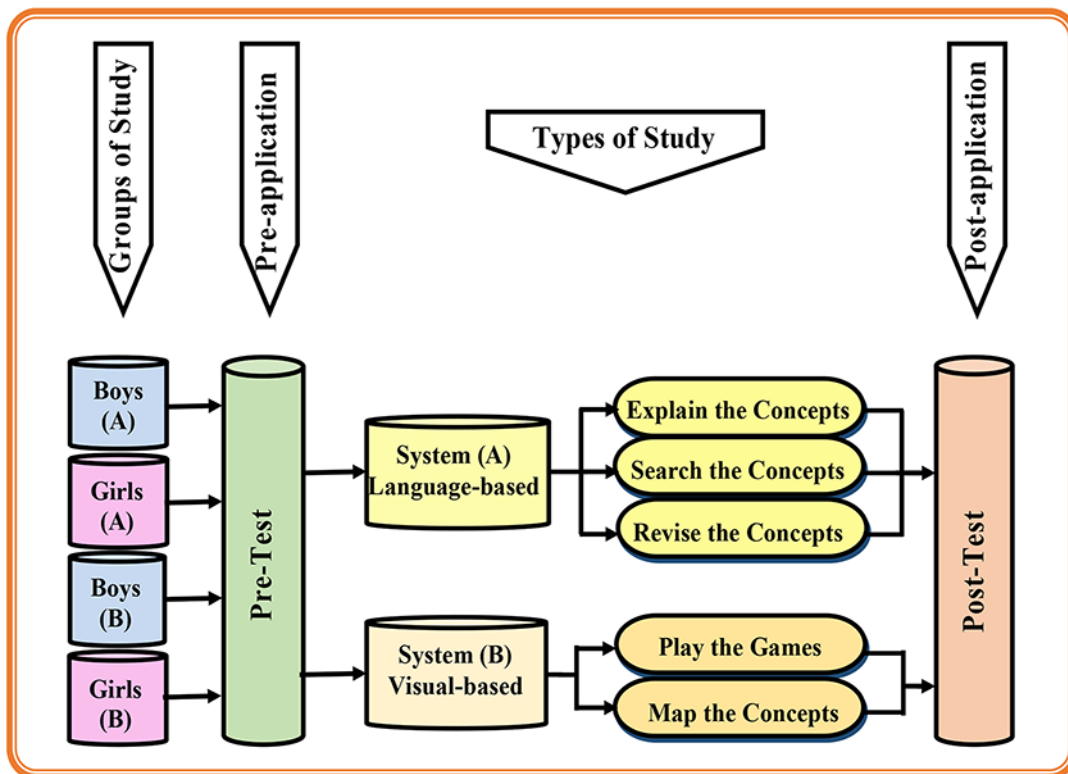


Figure 6.25: Quasi-experimental design of the study.

The student subjects of the experiment were divided into two separate groups: boys and girls. The two groups were further subdivided into two additional groups based on the types of learning the students would encounter when using the e-learning system, e.g. either Language-based (System A) or Visual-based (System B). This resulted in four groups of boys and girls being studied.

System A applied a Language-based approach that was built on concept explanation, a search of the characteristics and features of the concept and a review of the concepts. System B applied a Visual-based approach that was built on playing a game and providing a map of the concepts using a Venn diagram. Both approaches used aspects of conceptual space theory to structure the learning and focus on the enhancement of the student's understanding of the geometric concepts.

The experimental setup that was used when the e-learning system was tested was limited to no more than three students of the same group at the same time. This aimed to normalise the different requirements of the participating primary schools but also served to make it more comfortable for the students using the system. The small number of students in a given times lot also facilitated the orderly execution of the experiment in a primary school. Figures 6.26 and 6.27 show the setup of a girls' group and a boys' group, respectively.

Each of the four groups of students completed all the steps that were required in their particular e-learning system type. In addition, all the questions were multiple choice and the students were required to select their answers from among the choices provided for each question.

6.6.2 Design of System A (Language-based)

System A involved the completion of the Language-based module and the related testing modules in the following sequence that began with line concepts and was followed by shape concepts.

The sequence for line concepts is as follows:

1. The Pre-test unit begins at the start of the session. Each of the questions has examples that are designed to aid the student's understanding of the question. The first eight questions concern line concepts only. When the student completes the test, the student is made aware of the percentage of correctly answered questions without knowing exactly which ones were answered correctly.



Figure 6.26: Girls using the system during the experiment.

2. The Explain unit immediately follows the completion of the Pre-test module, and the e-learning system starts providing detailed information on the three line concepts that were involved in the Pre-test questions. The information includes definitions and features of the concepts, then the e-learning system gives typical examples of the concepts. The e-learning system also follows with more examples taken from real life in which the concept is related to phenomena in the environment. Upon completion of the explanation process, the e-learning system gives the student the opportunity to repeat the explanation of the entire line concept or parts of the line concept that were covered in the Explain unit. When the student reaches a satisfactory level, in terms of their understanding of the line concept, then the system transitions to the next unit of searching concepts.
3. The Search unit then starts and it enables the student to begin exploring the qualities of the line concepts that were studied in the Explain unit. The student is able to select a specific quality to explore or simultaneously



Figure 6.27: Boys using the system during the experiment.

explore all the qualities of the line concepts. The Search unit continues until the student reaches a satisfactory level.

4. The Revision unit revisits the qualities of the line concepts that were explored in the Search unit. The e-learning system in the Revision unit gives comprehensive information in a detailed format for each line concept. The e-learning system gives the student the opportunity to repeat the review of the entire line concept or part of the line concept that was covered in the Revision unit. When a student reaches a satisfactory level, then the system transitions to the next unit, which is the Post-test unit for line concepts.
5. The Post-test unit on line concepts begins as soon as the Revision unit is complete. Similar to the Pre-test, the questions provide examples that are designed to help the student focus on the question. The questions for the line concepts contain the same eight questions that were used in the Pre-test unit. The system provides feedback after each question is answered, and gives the correct answer if the student makes a mistake. When the

student correctly answers the questions, the system gives encouraging feedback. Once the student has completed the entire test, the e-learning system provides the results and gives the outcome of the concepts that were tested. The student is made aware of the percentage of correctly answered questions and the system also provides the correct answers to any incorrect responses. Additionally, the e-learning system lists the concepts and their qualities with which the student struggled during the test stage.

The e-learning system then switches from line concepts to shape concepts. The sequence for shape concepts follows exactly the same steps as it does for line concepts:

1. The Pre-test unit on shape concepts begins at the start of the session, with examples accompanying each question. The 17 questions concern shape concepts only. When the student completes the test, the e-learning system provides the results and the outcome of the concepts that were tested. The student is made aware of the percentage of correctly answered questions without knowing exactly which ones were answered correctly at this stage.
2. The Explain unit immediately follows the completion of the Pre-test module, and the e-learning system starts providing detailed information on the five shape concepts that were involved in the Pre-test questions. The information includes definitions and features of the concepts, then the e-learning system gives typical examples of the concepts. The e-learning system also follows with more examples taken from real life in which the concept is related to phenomena in the environment. Upon completion of the explanation process, the e-learning system gives the student the opportunity to repeat the explanation of the entire shape concept or parts of the shape concept that were covered in the Explain unit. When the student reaches a satisfactory level, in terms of their understanding of the shape concept, then the system transitions to the next unit of searching concepts.
3. The Search unit starts and it enables the student to begin exploring the qualities of the shape concepts that were completed in the Explain unit.

The student is able to select a specific quality to explore or simultaneously explore all the qualities of the shape concepts. The Search unit continues until the student reaches a satisfactory level.

4. The Revision unit revisits the qualities of the shape concepts that were explored in the Search unit. The e-learning system in the Revision unit gives comprehensive information in a detailed format for each shape concept. The e-learning system gives the student the opportunity to repeat the review of the entire shape concept or part of the line concept that was covered in the Revision unit. When the student reaches a satisfactory level then the system will transition to the next unit, which is the Post-test unit for shape concepts.
5. The Post-test unit for shape concepts begins as soon as the Revision unit is complete. Similar to the Pre-test, the questions provide examples that are designed to help the student focus on the question. The questions for the shape concepts contain the same 17 questions that were used in the Pre-test unit. The system provides feedback after each question is answered, and gives the correct answer if the student makes a mistake. Hence, when the student correctly answers questions, the system gives positive encouragement. After the student completes the entire test by answering all the questions, the e-learning system provides the results and the outcome of the concepts that were tested. The student is made aware of the percentage of correctly answered questions and the system also provides the correct answers to any incorrect responses. The e-learning system also lists the concepts and their qualities with which the student struggled during the test.

6.6.3 Design of System B (Visual-based)

System B involved the completion of the Visual-based module and the related testing modules in the following sequence that begins with line concepts and then finishes with shape concepts.

The sequence for line concepts is as follows:

1. The Pre-test unit of line concepts is the same as in System A.
2. The Game unit immediately follows the completion of the Pre-test module in which the student plays the game described in section 6.3.3.1 but with only the three line concepts that were involved in the Pre-test questions. The speed of the game gradually increases as the student progresses from one game level to the next. The game terminates when the student uses up all their lives in the current level. At the end of the game, the e-learning system provides the results of the concepts that were played. The system then transitions to the next unit.
3. The Map unit then starts and its purpose is to provide the student with opportunities to integrate the qualities of the line concepts that were completed in the Game unit. The system explains the relationships of the one-dimensional qualities using a Venn diagram, in combination with a text and voice explanation of each of the qualities of the line concepts that are displayed on the map. Upon completion of this unit, the system will transition to the next unit for testing the line concepts.
4. The Post-test unit of line concepts is the same as in System A.

The e-learning system then switches from line concepts to shape concepts. The sequence for shape concepts follows exactly the same steps as the line concepts:

1. The Pre-test unit of shape concepts is the same as in System A.
2. The Game unit immediately follows the completion of the Pre-test module, and the game is played using only the five shape concepts that were involved in the Pre-test questions. The speed of the game gradually increases as the student progresses from one game level to the next. The game terminates when the student uses up their lives in the current level. At the end of game, the e-learning system provides the results of the concepts that were played. The system then transitions to the next unit.
3. The Map unit starts and again its purpose is to provide the student with opportunities to integrate the qualities of the shape concepts that were completed in the Game unit. The system explains the relationships of the two-dimensional qualities using a Venn diagram and a text and voice

explanation of the qualities of the shape concepts that are displayed on the map. Upon completion of this unit, the system will transition to the next unit for testing the shape concepts.

4. The Post-test unit of shape concepts is the same as in System A.

6.7 Study Population

The total study population was 845 students from 11 international primary schools in Taif city in Saudi Arabia in the 2015/2016 academic year. These students included 428 girls and 417 boys. The language of instruction in these schools is primarily English and both Saudi and foreign students attend. This closely followed the pilot experiments that were conducted with Saudi students and UK students. (See Appendix G All the documents of the study procedures: approval letter to conduct the experiment in school in the UK and Saudi Arabia, request for permission to conduct research in school and letter of consent from parents).

6.7.1 Study Sample

To compose the four groups in the experiment, the study deliberately selected a sample group of students from years four, five and six. Two schools were selected based on their use of UK curriculum and resources, which facilitated the process of conducting the experiment. The two schools had diverse student populations that included various nationalities, cultures and native languages. Table 6.1 shows the composition of the four groups.

Table 6.1: The distribution of the four study samples.

Type of study	Number of students		Overall	Percentage of sample
	Boys	Girls		
System (A) – Language-based	18	23	41	55%
System (B) – Visual-based	16	18	34	45%
Total	34	41	75	100%

The table shows that the size of the sample that was selected was composed of 75 students, of whom 34 were boys and 41 were girls. Forty-one of the

students used the Language-based system (System A), whilst 34 of them used the Visual-based system (System B).

6.8 Experiment Logistics

After describing the process of conducting the experiment, the study was carried out in the following order:

1. The approval of the schools that were asked to participate in the study involving their students in years 4, 5 and 6 was obtained.
2. Each of the schools was visited to meet the head teachers, the math teachers, and the responsible person for the IT department. During the meeting, the objectives, scope and steps involved in the execution of the project were presented to them. After the presentation, a readiness review of the project requirements was conducted and the school representatives provided a tour of the room where the experiment was to be conducted.
3. Any issues identified during the tour within the location were resolved and as a result, the schools were deemed to be ready.
4. The groups of students subjected to System A or System B were also selected from the schools.
5. It was agreed with the schools that no more than three students would sit in the room at any given time during the experiment.
6. The e-learning system was downloaded and installed on the machines in the experiment room.
7. The selected students were given an introduction to the e-learning system; instructions and steps for using the e-learning system were provided.
8. The experiment started on 23 August 2015.
9. Individual student interviews were conducted after each student completed the allocated session, in order to ascertain their opinions on the learning system.
10. The experiment was completed on 9 September 2015.

6.9 Summary

This chapter presented the design of our e-learning system, as well as the design of the thesis study experiment. Concerning the e-learning system, this chapter detailed the evolution of an e-learning system that leverages conceptual space theory. The design was intended to enhance the learning environment of primary school students to increase their understanding of geometric line and shape concepts. The design involved comparing two distinct systems, Language-based and Visual-based, that used text combined with audio to present the content, and visual effects combined with audio to create an effective learning environment.

The Language-based system employed three units. First, the Explain unit provided detailed explanations and examples of each concept that ultimately related to a real-life object in the environment that helped the student to retain long-term conceptualisation of the idea. Second, the Search unit provided a means for the student to organise and structure the quality dimensions of the concept that enabled him/her to cognitively organise the features of the concept. Third, the Revision unit allowed the student to revisit the quality dimensions of the concept by re-exploring its qualities.

The Visual-based system employed two units. The Game unit provided a conceptualisation exercise that gradually increased the intensity of the exercise while simultaneously accessing the perceived mental structure of the emulated concept and matching it with the quality dimensions of the geometric line or shape concepts on display. The Game unit was designed to aid cognition of the quality dimensions of a concept that is an integral aspect of conceptual space theory. The Map unit used Venn diagrams to provide a means of organising the concepts in two-dimensional space in order to reproduce the geometric aspects of conceptual space theory.

A key aspect of the e-learning system was building up the student's confidence in order for them to obtain a satisfactory level of understanding of the concepts. It was important to enable the student to repeat all the Language-

based modules until he/she reached a comparable level of understanding to other students. Similarly, the Visual-based module provided perception enhancement exercises for concepts that helped the students to make better judgments about the quality dimensions defining the different concepts.

Concerning the thesis study design, the chapter detailed the methodology behind setting up and executing the experiment. Since earlier pilot experiments determined that language was not a factor in the common misconceptions, English was chosen as the language of our experiment in Saudi Arabia. The international schools were a desirable fit for the experiment because not only did they use English as their language of instruction, but they also had a diverse student population that included Saudi Arabian and foreign students. The students had different native languages, cultures and customs that have cognitively influenced their psychology and background.

Beyond the advantage of accessing a diverse population of primary school students, the selection of these international primary schools enabled the experiment to normalise the differences between certain students that tended to prefer rote memorisation as a method of studying and others that were inclined to prefer a practical or visual method of studying. The methodology of executing the experiment also emphasised the comfort of the setup that the student subjects would experience during their engagement with the e-learning system. The methodology called for no more than three students to sit together in a room at any given time when using the e-learning system. The small group setup also facilitated the quick and subjective individual feedback interview that was conducted once the e-learning experience was complete.

The execution of the experiment in Taif, Saudi Arabia, was generally successful, with the exception of the number of students that ultimately participated in the experiment. The experiment logistics were carefully planned and due diligence with the candidate primary schools was conducted in person. It was expected that the number of student participants would be in the range of 200 students. This was a number range that closely mirrored the sample

sizes of the pilot experiments. However, due to individual school policies and school IT rules, this experiment's sample size was affected for both boy and girl participants. Upon completing the execution of the experiment and the data collection, the project progressed to the data analysis and synthesis of the results stage, which forms the topic of the next chapter.

The data obtained from the experiments that were conducted in this chapter are used in the next chapter in order to evaluate the effectiveness of our conceptual space agent-based e-learning system.

Chapter 7

Evaluating the Conceptual Agent-based E-learning System

7.1 Introduction

This chapter presents a detailed analysis and discussion of the results of the data obtained from the experiment conducted following the methodology detailed in Chapter 6. The aim of this chapter is to answer the following question: “How effective is a conceptual space agent-based e-learning system when teaching basic geometric concepts in primary school classes?” The design of this system, which was inspired by Gärdenfors’ conceptual space cognitive theory, was discussed in the previous chapters. The project also makes an effort to demonstrate the potential of the CABELS e-learning system to positively affect students’ cognitive outcomes. To address this question, statistical methods are used to analyse the data obtained from the experiment and the outcome is used to discuss and interpret it.

7.2 Statistical Methods Used in the Study

The IBM SPSS statistical tool was used for the analysis of the data. The results of the Pre- and Post-test stages and the objectives of the study and its limits were determined according to the following parameters:

1. **Mean:** to determine the average of the sample values in the study;
2. **Standard deviation:** to determine the dispersion of sample values from the mean; that is, to determine the homogeneity of the study sample;
3. **t-test:** to compare the averages of the groups in the Pre- and Post-tests;

4. **Level of improvement i** : to measure any increase in the performance of the study sample after testing, calculated using the following equation:

$$i = \frac{\bar{x}_2 - \bar{x}_1}{\bar{x}_1} * 100$$

Where: \bar{x}_1 = mean of values in the first set and \bar{x}_2 = mean of values in the second set.

5. **Two-way analysis of variance**: to determine the effect of the interaction between the study variables;

6. **One-way analysis of variance (ANOVA)**: to assess the implications of the differences between the posterior averages of the study groups.

Moreover, it was ensured that the requirements for variance were met in this study; they are as follows:

A- Independent: The study sample was obtained randomly and thus meets the requirement for independence;

B- Moderation: The Kolmogorov-Smirnov test was used in order to verify whether the groups followed a normal distribution; Table 7.1 illustrates the results of this test.

Table 7.1: Kolmogorov–Smirnov test values.

Lessons	Kolmogorov–Smirnov Test						
	Gender	Number of samples	Type of system	Pre-test		Post–test	
				Kolmogorov–Smirnov	p -value (2-tailed)	Kolmogorov–Smirnov	p -value (2-tailed)
Line	Boys	18	A	0.82	0.51	0.54	0.94
Shape				1.01	0.26	0.99	0.28
Line	Boys	16	B	1.06	0.22	0.94	0.35
Shape				0.70	0.72	0.59	0.87
Line	Girls	23	A	1.15	0.14	1.09	0.19
Shape				0.82	0.51	0.57	0.90
Line	Girls	18	B	0.76	0.61	0.76	0.62
Shape				0.83	0.50	0.65	0.79

It is evident from Table 7.1 that the statistical significance of the values of the four groups was greater than 0.05. This indicates that there was no difference between the population and the sample, which means that the four groups

followed a normal distribution; thus, the condition of moderation of the four study groups was verified.

7.3 Results of the Study

The study sample of primary school students was divided into four groups that were subjected to the CABELS system in order to find out whether their cognition of basic geometric concepts improved upon completing the experiment. Table 7.2 shows the Pre-test and Post-test results of the four groups.

Table 7.2: Descriptive statistics for the study sample.

Lessons	Statistical methods									
	Gender	Number of participants	Type of system used	Pre-test			Post-test			Level of Improvement i %
				Mean	Std. deviation	Percentage %	Mean	Std. deviation	Percentage %	
Line	Boys	18	A	2.11	1.75	26	4.72	2.11	59	124
Shape				2.83	3.97	17	7.67	3.99	45	171
Line	Boys	16	B	2.31	1.62	29	3.81	1.83	48	65
Shape				3.56	3.46	21	8.19	3.17	48	130
Line	Girls	23	A	2.13	2.51	27	4.61	2.02	58	116
Shape				4.17	4.40	25	8.43	4.30	50	102
Line	Girls	18	B	2.50	2.46	31	4.78	2.02	60	91
Shape				5.83	3.94	34	10.28	3.23	60	76

Table 7.2 also lists the relationship between the percentages of the mean Pre-test and the Post-test for line and shape concepts in each group. This is also depicted graphically in Figure 7.1.

Table 7.2 and Figure 7.1 show that the values of the mean for all study groups were greater in the Post-test than in the Pre-test. This indicates that using the system improved the conceptualisation of the basic concepts of lines and geometric shapes for the students in years 4, 5 and 6. Furthermore, there was a high level of improvement in the answers given by both boys and girls who had used either e-learning System A (Language-based learning), or e-learning System B (Visual-based learning), where the level of improvement in the line

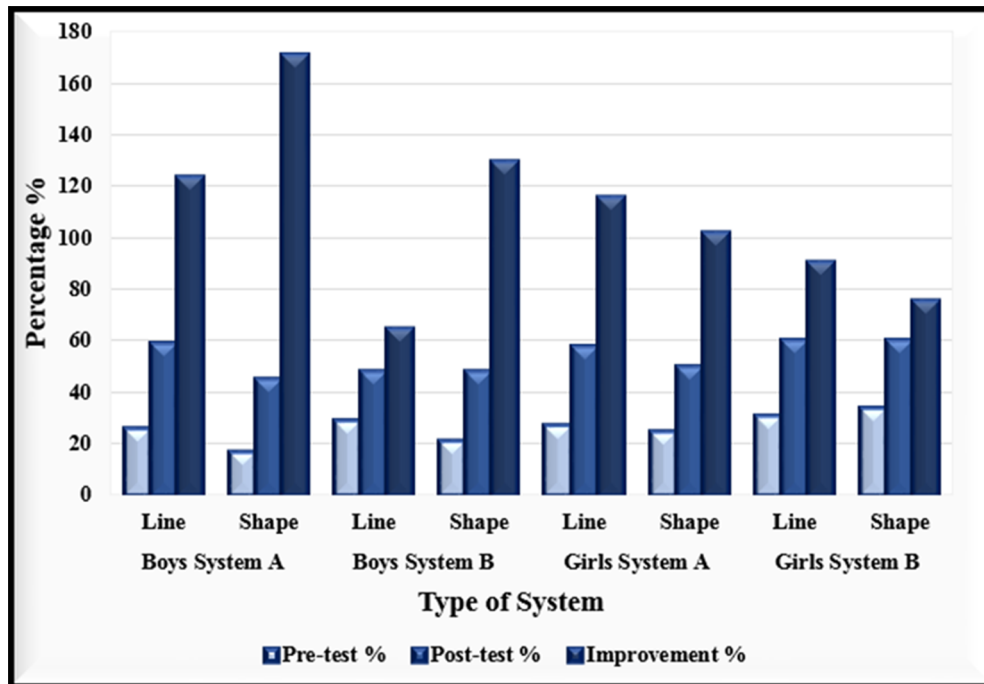


Figure 7.1: Relationships between the mean Pre-test and Post-test results and the level of improvement for all study samples.

concepts was between 65–124% and was between 76–171% in the shape concepts.

7.4 Detailed Discussion of the Results

A set of hypotheses is used to evaluate the statistically significant difference at the level of $\alpha \leq 0.05$ between the performance of the students' tests before and after completing the experiments using the CABELS system. The evaluation is conducted to measure the performance of the four groups (both individually as well as collectively) that were subjected either to CABELS System A (Language-based learning) or System B (Visual-based learning), and that were selected from school years 4, 5 and 6. Furthermore, two comparative studies were carried out between male and female students, and between students from the three class years 4, 5 and 6, respectively.

7.4.1 Hypothesis 1

Hypothesis 1 states that there will be no statistically significant difference between the averages of the understanding of geometric line and shape concepts for male students in the first group before and after being subjected

to CABELS System A (where ‘understanding’ is measured by the performance of the students in the tests as demonstrated by the percentage of correct answers given).

To test this hypothesis, a barometric test using a *t*-test was employed to observe the difference between the averages. The test results are presented in Table 7.3.

Table 7.3: Results of the *t*-test to determine the differences between the averages of the male students who used the Language-based Learning.

Lessons	Statistical methods					
	Type of system	Mean Difference	Std. deviation	Calculated t-value	Degrees of freedom	Sig. (2-tailed)
Line	A	2.61	1.82	6.09	17	0.00
Shape		4.83	3.22	6.36	17	0.00

Table 7.3 shows that the *t*-value for line concepts was 6.09, and that the significance value (2-tailed) was less than 0.05. This means that there was a statistically significant difference between the averages of understanding of the male students in group A for the line concepts before and after being subjected to CABELS System A. Moreover, the *t*-value for shape concepts was 6.36 and the significance value (2-tailed) was less than 0.05. This also means that there was a statistically significant difference between the averages of the boys’ understanding of the shape concepts before and after being subjected to CABELS System A. The results also indicate the presence of statistically significant differences at the level of $\alpha \leq 0.05$ and 17 degrees of freedom in favour of the averages obtained by this male group in the Post-test stage. These outcomes lead to the rejection of Hypothesis 1, and acceptance of the opposing view that CABELS System A improved the average understanding of the boys in group A with regard to basic geometric line and shape concepts.

7.4.2 Hypothesis 2

Hypothesis 2 states that there will be no statistically significant difference between the averages of the understanding of geometric line and shape

concepts for male students in the second group before and after being subjected to CABELS System B.

To test this hypothesis, a barometric test using a *t*-test was employed to observe the difference between the averages. The test results are presented in Table 7.4.

Table 7.4: Results of the *t*-test to determine the differences between the averages of male students who used Visual-based Learning.

Lessons	Statistical methods					
	Type of system	Mean Difference	Std. deviation	Calculated t-value	Degrees of freedom	Sig. (2-tailed)
Line	B	1.50	1.21	4.95	15	0.00
Shape		4.63	2.75	6.72	15	0.00

Table 7.4 shows that the *t*-value for lines was 4.95, and the significance value (2-tailed) was 0.00 which is less than 0.05. This means that there was a statistically significant difference between the averages of understanding of the male students in group B for the line concepts before and after being subjected to CABELS System B. Moreover, the *t*-value for shapes was 6.72, and the significance value (2-tailed) was 0.00, which is less than 0.05. This also means that there was a statistically significant difference between the averages of the boys' understanding of the shape concepts before and after being subjected to CABELS System B. The results also indicate the presence of statistically significant differences at the level of $\alpha \leq 0.05$ and 15 degrees of freedom in favour of the averages obtained by this male group in the Post-test. These outcomes lead to the rejection of Hypothesis 2 and acceptance of the opposing view that CABELS System B improved the average understanding of the boys in this group with regard to basic geometric line and shape concepts.

7.4.3 Hypothesis 3

Hypothesis 3 states that there will be no statistically significant difference between the averages of the understanding of the geometric line and shape concepts for the boys who are either in group A or group B of male students that were subjected to either CABELS System A or CABELS System B.

To test this hypothesis, a barometric test using a *t*-test was employed to observe the difference between the averages. The test results are presented in Table 7.5.

Table 7.5: Results of the *t*-test to determine the differences between the averages of male students who carried out Language-based Learning and those who carried out Visual-based Learning.

Lessons	Statistical methods					
	Type of system	Mean Difference	Std. deviation	Calculated t-value	Degrees of freedom	Sig. (2-tailed)
Line	A and B	0.91	3.18	1.33	32	0.19
Shape		0.52	3.58	0.42	32	0.68

Table 7.5 shows that the *t*-value for lines was 1.33, and the significance value (2-tailed) was 0.19, which is greater than 0.05. This means that there was no statistically significant difference between the averages of the two male groups A and B for their understanding of the geometric line concepts after using either the Language-based learning System A or the Visual-based learning System B. Moreover, the *t*-value for shapes was 0.42, and the significance value (2-tailed) was 0.68, which is greater than 0.05. This also means that there was no statistically significant difference between the averages of the two male groups A and B in their understanding of the geometric shape concepts after using either the Language-based Learning System A or the Visual-based Learning System B. The results also show that there were no statistically significant differences at the level of $\alpha \leq 0.05$ and 32 degrees of freedom between the two male groups that used either System A or System B. These outcomes lead to the acceptance of the hypothesis that there was no statistically significant difference between the averages of the understanding of the geometric line and shape concepts among the two male groups that were subjected to either CABELS System A or CABELS System B.

7.4.4 Hypothesis 4

Hypothesis 4 states that there will be no statistically significant difference between the averages of the understanding of geometric line and shape concepts for the female students in the first group before and after being subjected to CABELS System A.

To test this hypothesis, a barometric test using a *t*-test was employed to observe the difference between the averages. The test results are presented in Table 7.6.

Table 7.6: Results of the *t*-test to determine the differences between the averages of the female students who carried out the Language-based Learning.

Lessons	Statistical methods					
	Type of system	Mean Difference	Std. deviation	Calculated t-value	Degrees of freedom	Sig. (2-tailed)
Line	A	2.48	1.88	6.32	22	0.00
Shape		4.26	2.96	6.91	22	0.00

Table 7.6 shows that the *t*-value for lines was 6.32, and the significance value (2-tailed) was 0.00, which is less than 0.05. This means that there was a statistically significant difference between the averages of understanding of the female students in group A for the line concepts before and after being subjected to the CABELS System A. Moreover, the *t*-value for shapes was 6.91, and the significance value (2-tailed) was 0.00, which is less than 0.05. This also means that there was a statistically significant difference between the averages of the girls' understanding of the shape concepts before and after being subjected to CABELS System A. This indicates the existence of statistically significant differences at the level of $\alpha \leq 0.05$ and 22 degrees of freedom in favour of the averages obtained by this female group in the Post-test. These outcomes lead to the rejection of Hypothesis 4 and acceptance of the opposing view that CABELS System A improved the average understanding of the girls in group A with regard to the basic geometric line and shape concepts.

7.4.5 Hypothesis 5

Hypothesis 5 states that there will be no statistically significant difference between the averages of understanding of the geometric line and shape concepts for the female students in the second group before and after being subjected to CABELS System B.

To test this hypothesis, a barometric test using a *t*-test was employed to observe the difference between the averages. The test results are presented in Table 7.7.

Table 7.7: Results of the *t*-test to determine the differences between the averages female students who carried out Visual-based Learning.

Lessons	Statistical methods					
	Type of system	Mean Difference	Std. deviation	Calculated t-value	Degrees of freedom	Sig. (2-tailed)
Line	B	2.28	1.64	5.90	17	0.00
Shape		4.44	2.77	6.81	17	0.00

Table 7.7 shows that the *t*-value for lines was 5.90, and the significance value (2-tailed) was 0.00, which is less than 0.05. This means that there was a statistically significant difference between the averages of understanding of the female students in group B for the line concepts before and after being subjected to CABELS System B. Moreover, the *t*-value for shapes was 6.81, and the significance value (2-tailed) was 0.00, which is less than 0.05; this also means that there was a statistically significant difference between the averages of the girls' understanding of the shape concepts before and after being subjected to CABELS System B. The results also indicate the presence of statistically significant differences at the level of $\alpha \leq 0.05$ and 17 degrees of freedom in favour of the averages obtained by this female group in the Post-test. These outcomes lead to the rejection of the hypothesis and accepting the opposing view that the CABELS System B improved the average understanding of the girls in this group for the basic geometric line and shape concepts.

7.4.6 Hypothesis 6

Hypothesis 6 states that there will be no statistically significant difference between the averages of the understanding of the geometric line and shape concepts for the female students who were subjected to either CABELS System A or CABELS System B.

To test this hypothesis, a barometric test using a *t*-test was employed to observe the difference between the averages. The test results are presented in Table 7.8.

Table 7.8: Results of the *t*-test to determine the differences between the averages of female students who carried out the Language-based Learning and those who carried out Visual-based Learning.

Lessons	Statistical methods					
	Type of system	Mean Difference	Std. deviation	Calculated t-value	Degrees of freedom	Sig. (2-tailed)
Line	A and B	0.17	2.79	0.27	39	0.79
Shape		1.84	6.33	1.51	39	0.14

Table 7.8 shows that the *t*-value for lines was 0.27, and the significance value (2-tailed) was 0.79, which is greater than 0.05. This means that there was no statistically significant difference between the averages of the two female groups A and B in their understanding of the geometric line concepts after using either Language-based learning (System A) or Visual-based learning (System B). Moreover, the *t*-value for shapes was 1.51, and the significance value (2-tailed) was 0.14, which is greater than 0.05. This also means that there was no statistically significant difference between the averages of the two female groups A and B in their understanding of the geometric shape concepts after using either System A or System B. The results also showed that there were no statistically significant differences at the level of $\alpha \leq 0.05$ and 39 degrees of freedom between the two female groups that used either System A or System B. These outcomes lead to the acceptance of the hypothesis that there was no statistically significant difference between the averages of the understanding of the girls regarding the geometric line and shape concepts among the two female groups who used either CABELS System A or CABELS System B.

7.4.7 Hypothesis 7

Hypothesis 7 states that there will be no statistically significant difference between the averages of the understanding of the geometric line and shape concepts for the male students and the female students in the first set of groups who were subjected to CABELS System A.

To test this hypothesis, a barometric test using a *t*-test was employed to observe the difference between the averages. The test results are presented in Table 7.9.

Table 7.9: Results of the *t*-test to determine the differences between the averages of the male and female students who carried out Language-based Learning.

Lessons	Statistical methods					
	Type of system	Mean Difference	Std. deviation	Calculated t-value	Degrees of freedom	Sig. (2-tailed)
Line	A	0.11	0.08	0.18	39	0.86
Shape		0.77	0.54	0.59	39	0.56

Table 7.9 shows that the *t*-value for lines was 0.18, and the significance value (2-tailed) was 0.86, which is greater than 0.05. This means that there was no statistically significant difference between the averages of the understanding of the male and female students in the first set of groups A for the line concepts before and after using Language-based learning System A. Moreover, the *t*-value for shapes was 0.59, and the significance value (2-tailed) was 0.56, which is greater than 0.05. This also means that there was no statistically significant difference between the averages of the understanding of the male and female students in the first set of groups A for the shape concepts before and after using Language-based learning System A. The results also showed that there were no statistically significant differences at the level of $\alpha \leq 0.05$ and 39 degrees of freedom between the male and female students in the first set of groups that used CABELS System A. These outcomes lead to the acceptance of the hypothesis that there was no statistically significant difference between the averages of the understanding of the male and female students in the first set of groups A for the geometric line and shape concepts among the two male and female student groups who used CABELS System A.

7.4.8 Hypothesis 8

Hypothesis 8 states that there will be no statistically significant difference between the averages of the understanding of the geometric line and shape concepts for the male students and the female students in the second set of groups who were subjected to CABELS System B.

To test this hypothesis, a barometric test using a *t*-test was employed to observe the difference between the averages. The test results are presented in Table 7.10.

Table 7.10: Results from the *t*-test to determine the differences between the averages of male and female students who used Visual-based Learning system.

Lessons	Statistical methods					
	Type of system	Mean Difference	Std. deviation	Calculated t-value	Degrees of freedom	Sig. (2-tailed)
Line	B	0.97	0.66	1.45	32	0.16
Shape		2.09	1.10	1.90	32	0.07

Table 7.10 shows that the *t*-value for lines was 1.45, and the significance value (2-tailed) was 0.16, which is greater than 0.05. This means that there was no statistically significant difference between the averages of the understanding of the male and female students in the second set of groups B for the line concepts before and after using the Visual-based learning System B. Moreover, the *t*-value for shapes was 1.90, and the significance value (2-tailed) was 0.07, which is greater than 0.05. This means that there was no statistically significant difference between the averages of the understanding of the male and female students in the second set of groups B for the shape concepts before and after using Visual-based learning System B. The results also showed that there were no statistically significant differences at the level of $\alpha \leq 0.05$ and 32 degrees of freedom between the male and female students in the second set of groups that used CABELS System B. These outcomes lead to the acceptance of the hypothesis that there was no statistically significant difference between the averages of the understanding of the male and female students in the second set of groups B for the geometric line and shape concepts among the two male and female student groups who used CABELS System B.

7.4.9 Hypothesis 9

Hypothesis 9 states that there will be no statistically significant difference between the averages of the understanding of the geometric line and shape concepts for the male students and the female students in the first set of

groups selected from year 4, 5 and 6 who were subjected to CABELS System A.

To test this hypothesis, a barometric test using a one-way ANOVA was used to test and calculate the F -value because three year groups were being compared. The test results are presented in Table 7.11.

Table 7.11: Results of the ANOVA test and calculated the F -value for the female and male students who used Language-based Learning system.

ANOVA test							
Lessons	Type of system	Source of variation	Sum of squares	Degrees of freedom	Mean square	Calculated F -value	Statistical significance
Line	A	Between Groups	10.57	2	5.29	1.30	0.29
		Within Groups	154.65	38	4.07		
		Total	165.22	40			
Shape	A	Between Groups	71.82	2	35.91	2.23	0.12
		Within Groups	611.79	38	16.10		
		Total	683.61	40			

Table 7.11 shows that the F -value for lines was 1.30, and the value of statistical significance was 0.29, which is greater than 0.05. This means that there was no statistically significant difference between the averages of the understanding of the male and female students in the first set of groups selected from years 4, 5 and 6 for the line concepts before and after using Language-based learning System A. Moreover, the F -value for shapes was 2.23, and the value of statistical significance was 0.12, which is greater than 0.05. This means that there was no statistically significant difference between the averages of the understanding of the male and female students in the first set of groups selected from years 4, 5 and 6 for the line concepts before and after using Language-based learning System A. The results also showed that there were no statistically significant differences at the level of $\alpha \leq 0.05$ between the male and female students in the first set of groups selected from years 4, 5 and 6 that used CABELS System A. These outcomes lead to the acceptance of the hypothesis that there was no statistically significant difference between the averages of the understanding of the male and female students in the first set of groups selected from years 4, 5 and 6 for the

geometric line and shape concepts among the two male and female students groups selected from years 4, 5 and 6 who used CABELS System A.

7.4.10 Hypothesis 10

Hypothesis 10 states that there will be no statistically significant difference between the averages of the understanding of the geometric line and shape concepts for the male students and the female students in the second set of groups selected from years 4, 5 and 6 who were subjected to CABELS System B.

To test this hypothesis, a barometric test using a one-way ANOVA was used to test and calculate the F -value because three year groups were being compared. The test results are presented in Table 7.12.

Table 7.12: Results of the ANOVA test and calculated the F -value of male and female students who have used the Visual-based learning system.

ANOVA test							
Lessons	Type of system	Source of variation	Sum of squares	Degrees of freedom	Mean square	Calculated F -value	Statistical significance
Line	B	Between Groups	3.17	2	1.59	0.34	0.71
		Within Groups	143.21	31	4.62		
		Total	146.38	33			
Shape	B	Between Groups	11.54	2	5.77	0.32	0.72
		Within Groups	547.52	31	17.66		
		Total	559.06	33			

Table 7.12 shows that the F -value for lines was 0.34, and the value of statistical significance was 0.71, which is greater than 0.05. This means that there was no statistically significant difference between the averages of the understanding of the male and female students in the second set of groups selected from years 4, 5 and 6 for the line concepts before and after using Visual-based learning System B. Moreover, the F -value for shapes was 0.32, and the value of statistical significance was 0.72, which is greater than 0.05. This means that there was no statistically significant difference between the averages of the understanding of the male and female students in the second

set of groups selected from years 4, 5 and 6 for the line concepts before and after using the Visual-based learning system. The results also showed that there were no statistically significant differences at the level of $\alpha \leq 0.05$ between the male and female students in the second set of groups selected from years 4, 5 and 6 that used CABELS System B. These outcomes lead to the acceptance of the hypothesis that there was no statistically significant difference between the averages of the understanding of the male and female students in the second set of groups selected from years 4, 5 and 6 for the geometric line and shape concepts among the two male and female student groups selected from years 4, 5 and 6 who used CABELS System B.

7.4.11 Hypothesis 11

Hypothesis 11 states that there will be no statistically significant difference in relation to gender between the averages of the understanding of the geometric line and shape concepts between the male and female students in all groups who were subjected to either CABELS System A or CABELS System B.

To test this hypothesis, a barometric test using the Kolmogorov-Smirnov test was used to verify the normal distribution between the four study groups. The test results are presented in Table 7.13.

Table 7.13: The Kolmogorov–Smirnov test values between groups.

Lessons	Kolmogorov–Smirnov Test						
	Gender	Number of samples	Type of system	Post-test			
				Kolmogorov–Smirnov Z	p-value (2-tailed)	Mean	Std. deviation
Line	Boys	34	A and B	1.06	0.21	4.29	2.01
Shape				1.10	0.18	7.91	3.58
Line	Girls	41	A and B	0.94	0.34	4.68	1.99
Shape				0.53	0.94	4.24	3.94

Table 7.13 shows that the *p*-value (2-tailed) for boys at the lines was 0.21, and at the shapes it was 0.18, while for girls, the *p*-value (2-tailed) was 0.34 at the lines and 0.94 at the shapes. These values are greater than 0.05, indicating that there was no difference between the samples. Thus, the results followed

a normal distribution. Therefore, to test this hypothesis, a barometric test using a *t*-test was employed to observe the differences between the averages. The test results are presented in Table 7.14.

Table 7.14: Results of the *t*-test to determine the differences between the averages of the male and female students in all groups who used Language-based Learning and Visual-based Learning that are related to gender.

Lessons	Statistical methods					
	Type of system	Mean Difference	Std. deviation	Calculated t-value	Degrees of freedom	Sig. (2-tailed)
Line	A and B	0.39	0.27	0.84	74	0.41
Shape		1.33	0.94	1.52	74	0.13

Table 7.14 shows that the *t*-value for lines was 0.84, and the significance value (2-tailed) was 0.41, which is greater than 0.05. This means that there was no statistically significant difference between the averages of the understanding of all the groups related to gender for the line concepts before and after being subjected to the CABELS systems. Moreover, the *t*-value for shapes was 1.52, and the significance value (2-tailed) was 0.13, which is greater than 0.05. This means that there was no statistically significant difference between the averages of the understanding of all the groups related to gender for the shape concepts before and after being subjected to the CABELS systems. The results also showed that there were no statistically significant differences at the level of $\alpha \leq 0.05$ and 74 degrees of freedom between all groups related to gender that used the CABELS systems. These outcomes lead to the acceptance of the hypothesis that there was no statistically significant difference between the averages of the understanding of all the groups related to gender for the geometric line and shape concepts among all the groups that used the CABELS systems.

7.4.12 Hypothesis 12

Hypothesis 12 states that there will be no statistically significant difference between the averages of the understanding of the geometric line and shape concepts between the male and female students in all groups who were subjected to either CABELS System A or CABELS System B with respect to their years or gender.

To test this hypothesis, a barometric test using the two-way ANOVA was tested and the F -value was calculated to ensure that there was interaction between the parameters of years and gender. The test results are presented in Table 7.15.

Table 7.15: Results of two-way ANOVA for the interaction between years and gender.

Tests of between-subjects effects						
Lessons	Source of variation	Type III Sum of squares	Degrees of freedom	Mean square	Calculated F -value	Statistical significance
Line	Gender	2.55	1	2.55	0.69	0.41
	Year	15.72	2	7.86	2.12	0.13
	Gender * Year	19.81	2	9.91	2.68	0.08
	Error	255.49	69	3.70		
	Corrected Total	294.75	74			
Shape	Gender	31.62	1	31.62	2.26	0.14
	Year	57.43	2	28.72	2.06	0.14
	Gender * Year	22.11	2	11.05	0.79	0.46
	Error	964.15	69	13.97		
	Corrected Total	1075.28	74			

Table 7.15 shows that the F -value for lines in the gender variable was 0.69; and in the year variable, the F -value was 2.12. For the interaction between the year and gender variables, the F -value was 2.68; the significance value for lines in the gender variable was 0.41; the year variable was 0.13; and for the interaction between the year and gender variables it was 0.08, which is greater than 0.05. This means that there were no statistically significant differences for the interaction between year and gender in terms of understanding the basic concepts of lines for all groups. Moreover, the F -value for shapes in the gender variable was 2.26; in the year variable the F -value was 2.06; for the interaction between the year and gender variables the F -value was 0.79; the significance value for shapes in the gender variable was 0.14; in the year variable it was 0.14; and for the interaction between the year and gender variables it was 0.46, which is greater than 0.05. This means that there were no statistically significant differences for the interaction between year and gender in the understanding of the basic concepts of geometric shapes for all groups. Thus, there was no statistically significant difference at the level of $\alpha \leq 0.05$ for the interaction between year and gender

in all groups regarding their understanding of the basic concepts of lines and geometric shapes. These outcomes lead to the acceptance of the hypothesis that there was no statistically significant difference between the averages of the understanding between all the groups for the geometric line and shape concepts who used the CABELS systems that were related to the interaction between year and gender.

7.5 Observations and Discussion of the Study Findings

This section presents the outcome of the analysis of the results of the study and discusses them in light of the underlying theory. The following highlights the key findings:

1. After using CABELS, the students demonstrated significant improvements in their overall understanding of the basic concepts of lines and shapes (as measured by their test performances). This highlights the positive impact of the CABELS e-learning system in enhancing the students' conceptualisation of lines and geometric shapes.
2. The CABELS system equally enhanced both male and female students' cognition of line and geometric shape concepts, irrespective of their cultural and background differences in the international schools.
3. The CABELS system had a positive impact on students from three different year levels. The results showed that students from years 4, 5 and 6 all had their conceptualisation improved after using the system. More importantly, the age factor in their year was not relevant in their achievements after using the CABELS system.
4. The students who used the CABELS system that was limited to either Language-based learning or Visual-based learning achieved similar outcomes.

Since these findings provide strong evidence about the effectiveness of the CABELS system in teaching the basic concepts of lines and geometric shapes to primary school students, the factors that enabled the rapid improvement

of the students' conceptualisation are deemed important to be initially noted. CABELS was designed to present the concepts in a gradual, easy to follow, and interlinked way using some similar and opposite features. This enabled the students to construct the concepts in their brains with orderly organised quality dimensions for the features of the concepts. Furthermore, the possibility of remodelling concepts several times gave the students an opportunity to fine-tune the mental building blocks of the distinctive quality dimensions that were unique to each concept. A solid formation of the mental perception of the features of the qualitative dimensions for each concept is believed to have led to the rapid improvements in the students' conceptualisations of the geometric line and shape concepts.

The CABELS system used several stimuli, such as sound, motion, images and text to keep the students fully engaged. These external triggers meant that the students had to use more than one sense while constructing and organising the mental representation of the features and the quality dimensions of the concepts. Hence, the results implied that this aspect of the CABELS system also contributed to the rapid enhancement of the perception of the line and geometric shape concepts amongst the students.

The results did not show any significant differences among female and male students in their respective groups, whether they were subjected to Language-based learning (e.g. CABELS System A) or Visual-based learning (e.g. CABELS System B). Even though the two subsystems used different approaches for their delivery of the explanation of the concepts, their core objectives remained the same, and focused on aiding student cognition. This may have been due to the use of the sequential display of the explanations of the interrelated concepts and the presentation of their properties in several ways, which included real-life objects, and was intended to inspire student reflection and to make them easier to understand.

The CABELS system was also designed to lead the students through a gradual transition from general to specific, which is one of Piaget's strategies in the formation and learning of concepts. However, Piaget's strategy recognised

the growth of a child's intelligence in the direction of learning mathematical concepts (Ghoneim, 1973). In this study, the students from three different years showed the same level of intelligence in their ability to conceptualise the concepts that were explained to them. It was expected that the CABELS system would emphasise the properties of each concept by defining the quality dimensions of their distinctive features. These were enhanced through the presentation of a comprehensive view of the whole concept that was tied to a real-life experience, and then reviewed by means of a search engine to reinforce the elements that the student had not fully understood, thereby individualising the learning objective.

The results highlighted an improvement in the understanding of the concepts presented in the learning modules across all four groups that used either System A (e.g. Language-based learning) or System B (e.g. Visual-based learning). This achievement can be attributed to two critical success factors for the realisation of the CABELS agent-based e-learning system that was based on conceptual space theory. The first factor was obtaining the undivided attention of the students while the concepts were presented to them. The second factor was enabling the students' immediate reflection on the concepts that were presented to them in a playfully challenging way. Hence, the first factor increased the interest of the students through the use of suspense and motivation, while the second factor increased the students' mental engagement through induced self-competition.

Other factors that could be attributed to the realised improvement include the objective of the CABELS system in searching, gaming and mapping of the features of the concepts. For example, there is a close relationship between children and games; thus, games represent an important way to improve students' abilities, develop their intelligence and increase their motivation to learn faster. Searching and mapping achieved the same effect on the children. Therefore, the CABELS system used these techniques to enable the students to understand and discover concepts' properties and their relationships with each other. There are a number of psychological theories related to the study and interpretation of children's play and the activities associated with

it. These theories include Piaget's cognitive theory, wherein he linked the playing activity to intelligence, defining it as "[t]he organization of the reality at the level of action or thought [and] not just a reproduction". Therefore, the objectives had the following goals:

- a. To link concepts to images in the mind of the child that enabled him or her to retain the cognition effect of learning.
- b. To present the concepts and their properties in a coherent way that helped the students to distinguish between the concept and its properties and its relationships to other concepts, as well as to recognise common features between the concepts.
- c. To provide a mental perception of the structure and the operations of the composition of the concepts, as well as the formation of higher cognitive processes that would inspire students to continue searching the structure of knowledge to find the right concept.
- d. To enable the students to develop the appropriate thinking skills over knowledge that would enable their ability to sort, order and judge the features of the concepts in future mental exercises.

An important question that arises from the findings of the study is what distinguishes the outcome of this study from several other studies that showed improvement in students' learning of mathematical concepts through the use of computers and multimedia? While improvement in the student learning outcomes with regard to the concepts is a common denominator, there is a fundamental distinction that separates CABELS from other e-learning systems. Essentially, CABELS aims to aid the student's understanding by forming a reflection of the concepts that clearly defined and organised the quality dimensions of features that were unique to the concepts. This served to achieve both an unambiguous conceptualisation of the concepts and long-term retention of the cognitive perception of the concepts. It is expected that the students that showed improvement in their understanding of the concepts after being subjected to the CABELS system will perform similarly over a longer period of time if they are tested again on the concept in any shape or form. Retaining cognitive perception of the concepts will enable

future sorting, ordering and judging of the quality dimensions of the unique features of the concepts that facilitate the retention of the clear understanding of the concepts.

For example, the concept of a line is introduced before the concept of an angle that is formed by joining two lines. The findings in this research highlight that the introduction of any new concept (stimulus) must include all the required prerequisite concepts that should have been introduced to the student beforehand in order to facilitate the formation of a cognitive structure of the newly acquired concept with clearly defined quality dimensions. It is presumed that only in this orderly fashion can the students effectively categorise the newly introduced concept and form a cognitive perception that can effectively recognise and judge similarities and differences with other concepts that were already introduced to them.

Piaget points out that some concepts cannot be conceived by a child until she/he reaches and attains sufficient neurological maturation (Gärdenfors, 2011). Furthermore, the findings of this research also point out that the ordered fashion of the concept introduction can facilitate the mental process of conceptualisation. This means that a younger student has the potential to conceive and conceptualise a new concept, while an older student that has supposedly already attained the required mental maturation but lacks all the prerequisites of the concept will fail to conceive it.

The research community found that the continuity in the build-up of the progressive concepts is more important than the developmental milestone that Piaget (Hampton, 1999; Hearn and Hildebrand, 2010) contends. While the difficulty exhibited by the students to conceptualise relatively complex mathematical geometric shape concepts appears to support Piaget's argument about constraints in processing some stimuli before the students reach certain developmental milestones, it did not explain the failure of the students to conceptualise a previously introduced concept after a gap in their syllabus for the particular concept.

For example, students who learned a geometric shape concept for the first time in year 2 but did not encounter it until (depending on the curriculum) year 4 or 5 showed difficulty in conceptualising or distinguishing the attributes of the concepts that they were supposedly familiar with from previous years. This is one area in which the CABELS system promises to alleviate the effect of curriculum gaps in education, because the system consistently organizes the concepts and the quality dimensions for each concept are processed all at the same time and examples are also provided from real life.

The research also found that the existence of a common level of general misconceptions and difficulty in sorting out quality dimensions of either relatively simple or complex concepts was not influenced by the student's culture and/or the environment in which the child was raised. It was found that students in the Kingdom of Saudi Arabia that grew up in a culture with an emphasis on memorisation and exhibited similar misconceptions as the students in the United Kingdom that grew up in a culture with artefacts of tangibility and practicality. CABELS again was able to improve the understanding and conceptualisation of international students that had different native languages and came from different cultural backgrounds.

With respect to mathematical geometry (Dasen, 1994; Rogoff, 1990) which contends that a child's development is heavily influenced by their culture and other elements of their environment, these findings seem to contradict that interpretation. At least from a mathematical geometry point of view, concept formation and retention of previously introduced concepts have no relationship to either culture or environment. Nevertheless, it should be noted that the students must meet the prerequisite of any new concepts before they are introduced to them, regardless of the environment in which they were raised. Then the concept should have an organised cognitive structure and perception in order to use its quality dimensions to compare and to judge it in the future.

Finally, the question of whether a teacher standing in front of the class and explaining mathematical geometric concepts in the traditional manner is

more effective than an e-learning system like CABELS that is able to adapt to the individual students' needs, should be discussed. It is hard to argue against the traditional teaching methods that have helped produce the current body of knowledge. However, it is reasonable to state that CABELS, which is built using agent-based conceptual space cognitive modelling, will enhance student conceptualisation through the use of stimuli through structured learning pathways which aid in the construction and organisation of the mental representation of mathematical geometric concepts. This aspect of the CABELS system will effectively fulfill the needs of each individual student, which will lead to normalising the students' differences in comprehending and learning concepts while enabling the building of confidence until each student reaches a satisfactory level. In contrast, the teacher in the traditional setting delivers the lesson to the class at a predetermined and fixed time without repeating it to meet an individual student's needs.

Since the participating students were used to the teacher delivering lessons and explaining concepts in the traditional way, their feedback from first-hand experience in using CABELS provides an indication of the usefulness of the system. The following section presents the findings of the survey that was conducted on all subject students upon completion of their sessions with the CABELS e-learning system.

7.6 Student Satisfaction Rating for the CABELS System

In order to further evaluate the e-learning system, an interview was conducted at the end of each session to get immediate feedback from the student about his/her experience with the e-learning system. The interview questions were intended to ascertain the personal experience and opinions of the student, and more importantly, to determine how well the e-learning system was perceived by the students as a learning medium for mathematics. Two categories of questions were presented to the students in the eight-question survey. The first category asked the students to rate the e-learning system on a scale of 1 to 5, where 1 was the lowest rating and 5 was the highest rating.

The second category asked the students to give their opinions regarding the use of the e-learning system as a learning platform. The questionnaire was structured as follows:

Q1: How would you rate the experience of using this system?

Q2: Which aspect did you like the most in the e-learning system?

Q3: Which aspect you did not like in the e-learning system?

Q4: How would you rate the usefulness of the system to your learning?

Q5: How would you rate the level of ease in using the system?

Q6: What would you like to add to the system?

Q7: What other subjects would you like to learn through a system like this?

Q8: How would you rate whether you would like to apply this system to your school?

If the CABELS system does not appeal to the students then they might not use it, which could be a setback to the potential of the e-learning system. The rest of this section presents the findings of the survey that underscores a favourably high rating and satisfaction from the students that were subjected to the CABELS system.

Table 7.16: Students' ratings of their satisfaction about the use of the e-learning system CABELS.

Rating Questions						
Number	Question	Rating				
		1 0-20%	2 21-40%	3 41-60%	4 61-80%	5 81-100%
Q1	How would you rate your experience with using this system?	0%	1%	5%	4%	90%
Q2	How would you rate the usefulness of the system for your learning?	0%	1%	9%	5%	85%
Q3	How would you rate the level of easiness in using the system?	0%	0%	4%	4%	92%
Q4	How would you rate your desire to adopt this system at your school?	0%	1%	5%	3%	91%

Table 7.16 shows the students' responses to questions on how satisfied they were with using the CABELS e-learning system. The students indicated a satisfaction rate of 90% or higher for the overall experience, ease of use and

desire to adopt it in their schools. The students provided a lower satisfaction rating of 85% for the usefulness of CABELS to them. However, their opinion contradicts their performance measured by the Post-test outcome after the concepts were explained to them through the CABELS lessons.

Table 7.17: Students' opinions of their satisfaction regarding the use of the e-learning system CABELS.

Opinion Questions								
Number	Question	CABELS						
		Explain Unit	Search Unit	Revision Unit	Game Unit	Map Unit	Test Unit	All systems
Q5	What aspect of the e-learning system you did you like the most?	18%	5%	4%	20%	4%	9%	40%
Q6	What aspect of the e-learning system you did you not like?	<ul style="list-style-type: none"> - 85% responded there was nothing to dislike about it. - 15% had different responses that included: system took a long time; there were too few examples or there was too much audio component. 						
Q7	What would you like to add to the system?	<ul style="list-style-type: none"> - 83% responded no additional suggestion. - 17% wanted to add one or more of the following: more explanation of the test questions, stories, games, intelligent questions and videos. 						
Q8	What other subjects would you like to learn through a system like this?	<ul style="list-style-type: none"> - 47% wanted to apply it to learning all subjects. - 27% wanted to apply it to learning more math lessons. - 17% wanted to apply it to learning sciences. - 9% wanted to apply it to learning languages. - 1% wanted to apply it to learning geography. - 1% did not want to apply it to any subject except geometric shapes. 						

Table 7.17 presents the responses to the rest of the survey and provides further evidence in support of the potential implementation of the CABELS system. When the students were asked what they did not like about CABELS, 85% of them indicated that they had nothing to dislike about it, and that they thought the program was great. However, 15% of the students provided mixed responses. Some of the students thought that CABELS took a long time, but this was a function of the experiment and the setup was designed for it. It was possible for the student to pause at any stage, and then return the next day to continue from that point. But this feature was not part of the experimental setup.

Other students indicated that there were too few examples or there was too much audio. When asked what other subjects they would have preferred to use for CABELS, it was interesting to discover that close to half of the students (47%) would like to use it for all their subjects, 27% would like it for learning more math lessons, 17% would like it for their science courses, 9% would prefer to use it in their language courses and 1% would like to use it in their geography course. More interestingly, only 1% of the students stated that they did not want to apply the CABELS system to any subject other than mathematical geometric shapes. The outcome of the survey provides insight into the practical aspects of the CABELS system and its appeal to the intended end-users.

7.7 Summary

This chapter presented the analysis of the data that was obtained from the application of CABELS, as described in Chapter 6. In addition, the outcome of the analysis provided the basis for answering the main research question of 'How effective is a conceptual space agent-based e-learning system when teaching basic geometric concepts in primary school classes?' Clearly, the CABELS system was effective in enhancing students' understanding of the mathematical geometric concepts that were identified as the source of common misconceptions for primary school students in the UK and in Saudi Arabia.

CABELS is an e-learning system that was intended to stimulate the student's interest in the study subject and the concepts that were explained in the lessons. CABELS engages all the student's senses by using simultaneous stimuli. The expectation was that the concepts would be clearly constructed in the student's mind, and the unique quality dimensions that completely describe the features of each concept would have a lasting mental perception that would enable future identification, sorting of the features and judging them during comparison exercises. Therefore, the key distinctive features of the CABELS e-learning system are its ability to help form a mental archive

of perceptions of the concepts and facilitating ease of access for future use. These aspects of the CABELS system were detailed in the above discussions.

The next, and final, chapter summarises the work carried out in this thesis and its outcomes. It also reviews the thesis' aims and objectives, presents the limitations of the study and proposes future research and perspectives on this study.

Chapter 8

Study Outcomes and Conclusions

8.1 Introduction

This chapter presents the summary of the research, the key outcomes and conclusions of all the experiments that were conducted as groundwork in the study. The study outcomes and conclusions are highlighted with respect to the importance of e-learning utilisation in academic learning in general, and in primary schools in particular. Furthermore, this chapter presents a detailed description of the study's limitations as well as the factors that led to these limitations, and reviews the aims and objectives of our research. The chapter concludes by presenting opportunities for future work, along with personal and professional recommendations on how to move forward in the quest to realise an effective e-learning system for teaching mathematics at primary schools.

8.2 Research Summary

The main aim of this study was to establish and evaluate an effective agent-based model that can help in teaching basic mathematics at primary schools, and that would be based on the cognitive theory of Gärdenfors' Conceptual Spaces (Gärdenfors, 2004b).

In order to achieve this aim, several steps were undertaken. First, and as it was shown in Chapter 4, a prototype user interface for the interaction of the students with the system was developed using NetLogo 5.0.4. An interface was developed that integrates visual and audio elements that can aid the

students in conceiving the quality dimensions of the cognitive structure of mathematic geometric shapes.

Second, a set of questions was developed for our study experiments based on an extensive procedure that included an investigation of three national curricula (the United Kingdom, New Zealand and the Kingdom of Saudi Arabia) in order to choose the most relevant geometric concepts for our study. Furthermore, an investigation was conducted into the years in which the selected concepts are taught to primary school students, as well as the various study levels in which the selected concepts were first introduced to the students. Therefore, primary school students in years 4, 5 and 6 were chosen as the subjects of our study.

Third, a pilot experiment was performed using our prototype e-learning system in order to work towards devising an e-learning system that is based on the Conceptual Spaces cognitive model. This experiment was conducted on a sample of eight primary school students from the UK, presenting a total of 78 questions from 26 lessons covering 40 concepts concerning geometric shapes. Additionally, a paper-based experiment was conducted with the aim of performing further evaluations of student misconceptions, and establishing the quality dimensions used by students to conceptualise the shape concepts. This paper-based experiment was conducted on a sample of 224 primary school students from Saudi Arabia aged from 9 to 12 years old. This experiment included 65 questions from 21 lessons, covering 33 geometric shape concepts. A third pilot experiment was conducted, which was an interview-based test, with the objective of evaluating whether the common misconceptions found in the previous two experiments also existed in this verbal experiment. Hence, a total of 399 verbal questions on the basic concepts predefined in the previous two experiments were asked directly by 6 teachers to 72 students at primary schools in Saudi Arabia.

Based on the insights obtained from the pilot experiments, the set of questions was further expanded to include both visual- and textual-based questions. Upon completion of the pilot experiments, our developed user interface,

lessons and questions were reviewed by a community of mathematics instructors. Based on this appraisal process, a final refined set of questions was generated, and then used to implement our CABELS system, which embodied the Conceptual Spaces model theory. The CABELS system included two parallel learning modules: Language-based (system A) and Visual-based (system B). CABELS involved three learning stages: a Pre-test of concepts, lessons explaining the concepts, and then a Post-test to determine the improvement in student understanding.

In order to evaluate the performance and effectiveness of the CABELS system in teaching mathematic geometric shapes at primary schools, several evaluations were made. First, a comparison of students' understanding at primary schools in Saudi Arabia and in the UK, respectively, was conducted, where the data collected and used in our study was statistically analysed (Chapter 5). This analysis was based on studying the statistical significance between various variables, such as student country and class levels on the one hand, and the total scores' mean and the concepts scores' mean on the other hand. Second, the effectiveness of the CABELS system in teaching basic geometric concepts was evaluated by statistically analysing the performance of the students before and after completing the tests when using the CABELS system (see Chapter 7). This statistical analysis examined various variables, mainly the e-learning module being used (e.g. Language-based learning (system A) or Visual-based learning (system B)), the students' genders and the students' levels (years). Third, further evaluation of the effectiveness of CABELS was performed based on student satisfaction. Hence, an interview was conducted with the students at the end of each e-learning session to get their immediate feedback about their experience with our e-learning system (see Chapter 6 and Chapter 7). The interview questions were intended to ascertain the personal experience and opinions of the student, and more importantly, how well the e-learning system was perceived by the students as a learning medium for the maths subject.

8.3 Study Outcomes

Our CABELS e-learning system was developed using a user-agent approach with the goal of individualising the learning experience for conceiving mathematical geometric concepts. The implementation of the e-learning system focused on providing a clear and reflective perception of the concept quality dimensions that were presented through an audio-visual display.

8.3.1 Students' Understanding

The results obtained from the pilot experiments conducted in Chapter 4 were consistent with the Conceptual Spaces cognitive model if quality dimensions used by the students to define the concepts are taken into account. Furthermore, the results obtained from the paper-based experiment showed that student's recognition of shape concepts increases with the visual display. It also showed that a significant proportion of students had difficulties with recognising shape names or with understanding their properties, especially when the shape was displayed in non-standard orientations.

The findings of the interview-based test experiment showed that there were difficulties in understanding, explaining or identifying some of the basic geometric shape concepts. It was also noted that some of the misconceptions overlapped with misconceptions identified in previous experiments. Another important outcome of this experiment was the need for the students to have a conceptual understanding of the line and geometric shape concepts that is based on both visual and oral representations, which related the shapes to real-life examples (e.g. a window or a door). Hence, combining traditional methods and physical properties of the concepts during the learning process would help in the future judgment of similarities between quality dimensions of complex concepts.

8.3.2 Students' Satisfaction with CABELS

The statistical tests performed in Chapter 5 in order to compare students' understanding at primary schools in Saudi Arabia and the UK did not highlight

any significant difference between student countries or between students' class levels, either for the total test scores, or for specific concepts' means, which means that the measured variables did not have a significant effect on students' scores.

The evaluation of the effectiveness of the CABELS system in teaching basic geometric concepts (performed in Chapter 7) showed that our CABELS system improved students' understanding of basic concepts of lines and shapes. Moreover, there were no significant effects caused by student gender, student class year, the cultural and background differences between students, or the chosen learning method (Language-based (system A) or Visual-based (system B)).

Furthermore, the evaluation of the CABELS system based on student satisfaction showed their overall satisfaction, which was due (according to them) to its ease of use, usefulness, etc. Moreover, there were different percentages of students wishing to adopt CABELS in other learning subjects (47% for all subjects, 27% for more math lessons, 17% for sciences, 9% for languages, 1% for geography, and 1% did not want to apply it to any subject other than mathematical geometric shapes).

All these evaluation results promote the effectiveness and efficiency of the CABELS system in improving students' learning, and support the possibility of extending the e-learning system to other subjects learned at primary school, in countries beyond Saudi Arabia and the UK.

8.4 Conclusions

Teaching mathematical geometry to primary school students requires a great deal of effort on the part of school teachers in order to achieve their students' uniform grasp of knowledge and for them to comprehend visually presented concepts in mirroring mental perception. Each student is unique in how he/she prefers to learn. Additionally, there might be some particular needs for individual students in their learning ability within a classroom cohort.

Therefore, the burden falls on the teacher to ensure that the interest and the attention of the students is maintained throughout the teaching sessions. In addition, students have different levels of understanding in respect to their learning abilities.

The main aim of this project was to establish and evaluate an effective agent-based model that can help in teaching mathematical geometric concepts at primary school, mainly in the UK and Saudi Arabia. Therefore, a prototype e-learning system (CABELS) was developed and used in our experiments. Moreover, it was determined through pilot experiments that there exist certain common misconceptions among primary school students with regards to geometric line and shape concepts. Traditionally, both UK and Saudi schools have taught their students about these concepts in their respective curricula. However, the students' mental perception and cognition of these concepts has remained fuzzy at best, and they were confused about the specific and unique features of the concepts. Such issues highlighted a shortcoming in the traditional method of teaching mathematical geometric concepts to primary school students, irrespective of their native language, culture or country.

The opportunity to engage the students' cognitive processes and to increase the conceptualisation of mathematical geometric concepts was facilitated by the application of Gärdenfors' conceptual space theory (Gärdenfors, 2004b) in our system. CABELS carefully selected stimuli to aid understanding and to iterate through the steps that enable individual students to construct a solid mental perception of the concepts through guided e-learning sessions. Furthermore, the evidence supports the hypothesis that the CABELS system was able to increase student interest in the learning process. This was mainly achieved by changing the student from a passive individual in a teacher-led classroom to an active student that interacts with the lesson in his/her preferred learning style, in order to reach a satisfactory level of understanding. This interactivity made the students fully attentive during the learning sessions and it continued to impact their cognitive processes. Thus, it helped in the construction of the quality dimensions of the specific concept in their brains. Indeed, the level of student interest and attentiveness during

the study experiment sessions was obvious from the student responses to the post-session survey (described in Chapter 7).

Through the CABELS system, and as shown in Chapter 7, we achieved an approach that increased the students' interest and attentiveness throughout the learning sessions. Hence, combining the tasks of searching, gaming and mapping of concept features improved the students' motivation to learn faster and compete to demonstrate their understanding. CABELS' potential was validated in primary school students in years 4, 5 and 6. Their scores in the Pre-test and Post-test showed a significant improvement in their learning. Furthermore, CABELS demonstrated an effective e-learning system to teach basic mathematical shape concepts to primary school students. Additionally, it was proven to be an appealing educational environment for the students. Hence, we believe that CABELS' effectiveness in enhancing students' cognitive processes in a way that increased their conceptualisation of concepts promises a novel approach for teaching mathematical geometry.

Moreover, our research highlights the importance of providing computer-based programs that involve interaction that introduce stimuli that aid in the construction of an effective cognitive structure in the mind of the user. Such a process of modelling educational computer-based programs will help in situations that involve competition or retention of acquired learning lessons. Using such programs will also enable the student's cognitive processes when they are used at the stage when the student is conceptualising previously learned concepts or conceiving new concepts.

8.5 Limitations

This project concentrated on the possibility of enhancing the cognitive process of students' conceptualisation of mathematical geometric concepts at primary school, in particular, in years 4, 5 and 6. The use of CABELS successfully enhanced the students' conceptualisation of concepts that were previously misunderstood.

However, it is not clear whether the achievement of CABELS that is based on Gärdenfors' conceptual space theory could be generalised to all years. This is mainly because if earlier class years (e.g. years 1, 2 and 3) were to be included in the study experiments, then the students' ages might be a limiting factor. This is due to Piaget's argument that some concepts cannot be conceived by a child until he/she attains sufficient neurological maturation (Gärdenfors and Warglien, 2012). However, an interesting aspect of our study was that the students in our experiments comprised of three different age groups based on their academic year (e.g. 4, 5 and 6). Hence, it was not possible to determine whether these age groups had similar neurological maturations. Furthermore, if this study were to include students in year 1 or 2, then they would lack the required prerequisite for the quality dimensions of the concepts being studied. Such a gap in their understanding could undermine the effective completion of the mental structuring process that was intended to enhance student conceptualisation. Similarly, the effectiveness of CABELS in enhancing student conceptualisation in advanced levels (e.g. middle or secondary schools) has not been explicitly verified.

One of the key successes of the CABELS system was effectiveness of both the visual and language approaches for learning. However, it is not known whether the same success rate can be achieved for other mathematical courses, like algebra. This is because the quality dimensions of the concepts are unknown in other areas.

8.6 Review of Aims & Objectives

In this section, the objectives and achievements of our research study are reviewed by comparing them to our study objectives as presented in Chapter 1. Significant progress has been made towards accomplishing the aims of the research which were to build and evaluate an effective e-learning model (CABELS) based on the cognitive theory of Gärdenfors' Conceptual Spaces (Gärdenfors, 2004a), intended to investigate the level of enhancement that can be achieved for teaching mathematical geometry concepts in primary schools.

Our main research objectives and achievements were the following:

1. **Objective one:** To conduct a comprehensive review of the most recent research in order to discover the best way to design an agent-based system suitable for young pupils, with a particular focus on learning basic geometric shapes.
 - **Achievement one:** A substantial literature review was carried out and described in Chapter 2 and 3 in order to identify the main characteristics that should be present in our e-learning system, such as visual/audio descriptions, and so on.

2. **Objective two:** To build a novel e-learning model based on conceptual space theory that helps to teach basic geometric shape concepts to pupils in primary school.
 - **Achievement two:** As shown in Chapter 6, a conceptual agent-based e-learning system (CABELS) was built which embodied the Conceptual Spaces model theory in order to enhance the student conceptualisation of mathematic geometric shapes. Prior to that, and as was shown in Chapter 4, a prototype user interface for the interaction of the students with the system was developed, comprising a conversational interface metaphor with visual and audio elements that aid the students' conceptions of the quality dimensions of the cognitive structure of the studied shapes.

3. **Objective three:** To validate the developed software within the educational environment, by comparing and assessing the student achievement scores and by collecting feedback.
 - **Achievement three:** The effectiveness of our developed CABELS system in teaching basic geometric concepts was evaluated based on a statistical analysis of its data and results (see Chapter 7). First, the statistically significant difference between the student's performance before and after completing the tests using the CABELS system was measured based on relevant variables (mainly the system being used, e.g. Language-based learning (system A) or Visual-based learning (system B), the student's gender and the student's year). Second,

the evaluation of the CABELS system based on student satisfaction was performed through post-session interviews with the students to measure their personal experience and opinions concerning the e-learning system and its usefulness as a learning medium for mathematical geometric shapes (see Chapter 6 and Chapter 7). Furthermore, and in order to study the effectiveness of our CABELS system regardless of the student environment, the understanding of students in Saudi Arabia and the UK and how they conceptually represent mathematical concepts, were compared based on a summary of the data gathered by our study.

8.7 Future Work

The psychology of learning, or teaching concepts, is also critical to the development of a successful curriculum for teaching mathematical geometry throughout primary school. This is due to the fact that a student's success in the later years depends on the progressive structure of the syllabus and the continuity of the critical foundational concepts that are required for the conception of future advanced concepts. Hence, the gradual introduction of the concepts, from basic and simple ones to interrelated concepts, is critical, and it is very important to gradually build up the quality dimensions for each concept in the student's mental perception. Consequently, the student will attain the desired level of understanding and form a solid cognition of mathematical geometric concepts.

The current research suggests that a conceptual space cognitive approach would increase the effectiveness and efficiency of a student's natural abilities related to the mental processes that are used for building a long lasting perception of learnt concepts. This could offer more opportunities and possibilities for disadvantaged students in particular, and to effective teaching in general. It is unclear whether CABELS also has the potential to enhance the learning process of students who have learning difficulties, like dyslexia or other learning impediments. However, the effectiveness of our CABELS system, or any other similar e-learning system that is based on conceptual space theory, is an opportunity to be explored in this regard.

The long-term retention of concepts and the effective execution of future mental exercises of previously learnt concepts are also important aspects in student learning. If our CABELS system, or any other similar e-learning system, enhances the mental construction of the learnt concepts using clear organisation and judgment, then students' mental states will be translated into a solid memory of these concepts for long-term memorisation.

It would also be interesting to perform long-term studies to see if the underlying theoretical model that CABELS was built on would lead to the long-term understanding of the concepts. The potential of CABELS in this regard, which makes it different from other commercially available e-learning systems, was not practically proven and represents a theoretical expectation.

Another area for research is whether CABELS, or any e-learning program that is based on conceptual space theory, can be used to complement the traditional classroom teacher. In essence, if the teacher can lead the class using an e-learning system like CABELS to review the lessons and help students to complete their homework, then this approach might be more effective than using the CABELS system alone.

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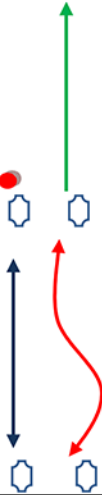



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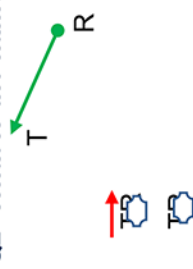

Appendices




Appendix A

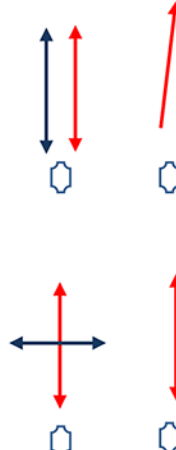
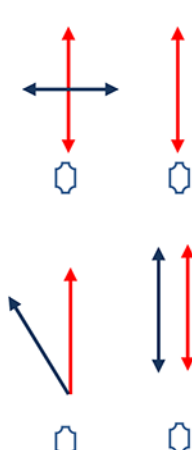

The curriculum analysis of the three countries (United Kingdom, New Zealand and Saudi Arabia) for contents geometric shape concepts.



Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
1	1	A dot and a line	A dot (point)	<ul style="list-style-type: none"> - Define the concept of a dot on a plane (point in geometry). - Denote the dots (point) using a symbol (e.g. A). 	<p>Q1- Which of these make a dot (point)?</p> <p><input type="checkbox"/> Your pen <input type="checkbox"/> Your Finger</p> <p><input type="checkbox"/> Your ruler <input type="checkbox"/> Your book</p> <p>Q2- Which of these is a dot (point)?</p> <p><input type="checkbox"/> A  </p> <p><input type="checkbox"/>  B</p>	1	1	1 2 3 4
2	1	A dot and a line	A line	<ul style="list-style-type: none"> - Define the concept of a line on a plane. - Denote the line using a symbol (e.g. \overleftrightarrow{AB}). 	<p>Q1- Which of these is a line?</p> <p><input type="checkbox"/> Has no beginning and no end and extends on a single straightness on the both directions.</p> <p><input type="checkbox"/> Has a beginning point and no end point</p>	1	1 4	1 2 3 4 6


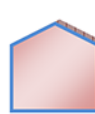

Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
				- Draw a line using a ruler.	<p>Q2- Which of these is a straight line?</p>  <p>Q3- Which of these is the symbol of a line?</p> 			
3	1	A dot and a line	A line segment	<ul style="list-style-type: none"> - Define the concept of a line segment on a plane. - Denote the line segment using a symbol (e.g. AB). - Draw a line segment using a ruler. 	<p>Q1- Which of these a line segment?</p>  <p>Q2- Which of these is the symbol of the line segment?</p> 	1 2	1	1 2 4


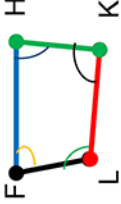
Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
4	1	A dot and a line	A ray (half-line)	<ul style="list-style-type: none"> - Define the concept of a ray on a plane. - Denote the ray using a symbol (e.g. \overrightarrow{AB}). - Draw a ray using a ruler. 	<p>Q1- What is this?</p>  <p>Q2- What is the name of this figure?</p> 	1	1 4	1 3 4
5	1	A dot and a line	A curved line	<ul style="list-style-type: none"> - Define the concept of a curved line on a plane. - Draw a curved line. 	<p>Q1- What is this?</p> 			1



Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
6	2	Parallel and perpendicular lines and angles	Parallel Lines	<ul style="list-style-type: none"> - Define the concept of Parallel Lines on a plane. - Denote the Parallel Lines using a symbol (e.g. $AB \parallel CD$). - Draw parallel lines. 	<p>Q1- Are these lines Parallel?</p>  <p><input type="checkbox"/> Yes <input type="checkbox"/> No</p> <p>Q2- What is the name of this figure?</p>  <p><input type="checkbox"/> $AB \parallel CD$ <input type="checkbox"/> $AC \parallel BD$ <input type="checkbox"/> ABCD <input type="checkbox"/> $A \parallel D$</p>	3	5	4 5
7	2	Parallel and perpendicular lines and angles	Perpendicular lines	<ul style="list-style-type: none"> - Define the concept of Perpendicular lines on a plane. - Denote the Perpendicular lines using a symbol (e.g. $AB \perp CD$). 	<p>Q1- What is this?</p>  <p><input type="checkbox"/> Perpendicular <input type="checkbox"/> Parallel <input type="checkbox"/> Lines <input type="checkbox"/> Ray</p>	3	5	4 5

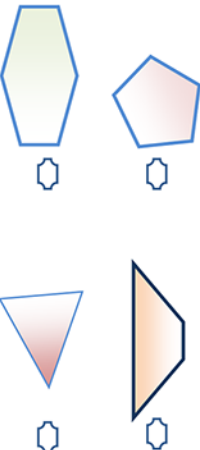
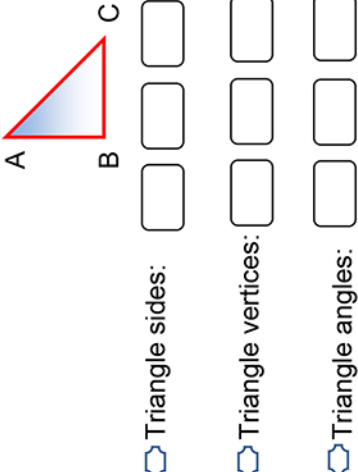
Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
8	2	Parallel and perpendicular lines and angles	Angle	<ul style="list-style-type: none"> - Draw parallel lines. 	<p>Q2- What is the figure of this symbol? ($AB \perp DC$)</p>  <p>Q1- Which of these is an angle?</p>  <p>Q2- What is the name of this figure?</p>  <p> <input type="checkbox"/> $\angle DXW$ <input type="checkbox"/> $\angle DWX$ <input type="checkbox"/> DXW <input type="checkbox"/> DX </p>	1 2 3 4 5 6	1 4 5	1 2 3 4 5 6

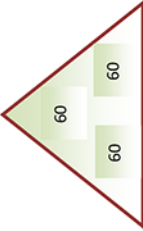
Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
9	2	Parallel and perpendicular lines and angles	Types of angle	<ul style="list-style-type: none"> - List the types of angles. - Define the concept of the types of angles. - Identify different types of angles. - Measure angles using a protractor. - Draw angles using a protractor with a ruler. 	<p>Q1 - Is this angle greater than, equal to, or less than a right angle?</p>  <p><input type="checkbox"/> Less than a right angle</p> <p><input type="checkbox"/> Equal to a right angle</p> <p><input type="checkbox"/> Greater than a right angle</p> <p>Q2- Put each name in front of the right figure?</p>  <p><input type="checkbox"/> Right angle</p> <p><input type="checkbox"/> Acute angle</p> <p><input type="checkbox"/> Obtuse angle</p>	2 3 4 5 6	1 4 5	2 3 4 5 6

Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
					<p>Q3- Using a protractor what is the measurement of this angle?</p>  <p><input type="checkbox"/> 120</p>			
					<p>Q1- Is this shape open or closed?</p>  <p><input type="checkbox"/> Open <input type="checkbox"/> Closed</p>			
10	3	Open and closed shapes, polygons, triangles and types	Open and closed shapes	<ul style="list-style-type: none"> - Define the concept of an open and closed shapes. - Draw open and closed shapes. 	<p>Q2- Which of these is an open shape?</p>  <p><input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/></p>	3		1 2 3



Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
	3	Open and closed shapes, polygons, triangles and types.	A Polygon	<ul style="list-style-type: none"> - Define the concept of a Polygon. - Determine the sides, vertices and angles of the Polygon . - Define the concept of the sides, vertices and angles of the Polygon . - Determine the number of sides, vertices and angles of the polygon. - Distinguish polygons from geometric shapes. - Draw the polygon using a ruler. 	<p>Q1- Is this a Polygon shape?</p>  <p><input type="checkbox"/> Yes <input type="checkbox"/> No</p> <p>Q2- Type the name of the sides, vertices and angles of this polygon:</p>  <p>Polygon sides: <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/></p> <p>Polygon vertices: <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/></p> <p>Polygon angles: <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/></p>	2 3 4 5 6	1 2 3 5	1 2 3 5 6







Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
					<p>Q3- How many sides this polygon have?</p>  <p> <input type="checkbox"/> 5 <input type="checkbox"/> 3 <input type="checkbox"/> 4 <input type="checkbox"/> 6 </p> <p>Q4- How many polygons names do you know?</p> <p>Q5- Is this shape a regular polygon or an irregular?</p>  <p> <input type="checkbox"/> Regular <input type="checkbox"/> Irregular </p>			


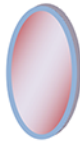
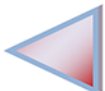
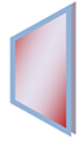

Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
11	3	Open and closed shapes, polygons, triangles and types.	A triangle	<ul style="list-style-type: none"> - Define the concept of a triangle. - Determine the sides, vertices and angles of the triangle . - Draw triangles using a ruler. 	<p>Q1- Which of these a triangle?</p>  <p>Q2- Type the name of the sides, vertices and angles of this triangle:</p> 	1 2 3 4 5 6	1 2 3 4 5 6	1 2 3 4 5 6






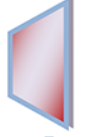
Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
12	3	Open and closed shapes, polygons, triangles and types.	Types of triangles	<ul style="list-style-type: none"> - List the types of triangles. - Define the concept of the types of triangle. - Identify different types of triangles. - Classify triangles by sides and angles. 	<p>Q3- You have a shape that has 3 sides and 3 angles. Which shape can you make?</p> <p><input type="checkbox"/> Triangle <input type="checkbox"/> Rectangle</p> <p><input type="checkbox"/> Square <input type="checkbox"/> Rhombus</p> <p>Q1- What kind of triangle is this?</p>  <p><input type="checkbox"/> Equilateral <input type="checkbox"/> Scalene <input type="checkbox"/> Isosceles</p>	5	4	4
						6	5	6



Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
				- Draw triangles.	<p>Q2- Put each name in front of the right figure?</p> <p><input type="checkbox"/> Equilateral</p> <p><input type="checkbox"/> Scalene</p> <p><input type="checkbox"/> Isosceles</p> <p><input type="checkbox"/> Right-angled</p> <p>Q3- The shape have 2 equal angles and a 1 different angle. Which shape can you make?</p> <p><input type="checkbox"/> Isosceles triangle</p> <p><input type="checkbox"/> Right-angled triangle</p> <p><input type="checkbox"/> Scalene triangle.</p> <p><input type="checkbox"/> Equilateral triangle</p>			

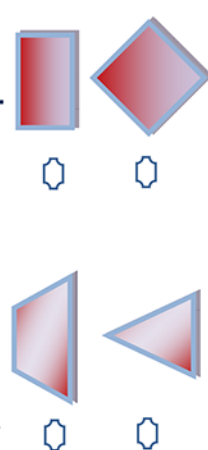


Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
13	4	Quadrilatera Is and types	A Quadrilateral	<ul style="list-style-type: none"> - Define the concept of a quadrilateral. - Determine the sides, vertices and angles of the quadrilaterals . - Determine the number of sides, vertices and angles of the quadrilaterals. - Classify quadrilaterals by their characteristics as of rectangles, squares, Parallelograms, rhombuses, and trapezoids. - Draw the quadrilaterals using a ruler. 	<p>Q1- Is this a Quadrilateral shape?</p>  <p><input type="checkbox"/> Yes <input type="checkbox"/> No</p> <p>Q2- How many angles this Quadrilaterals have?</p>  <p><input type="checkbox"/> 4 <input type="checkbox"/> 5 <input type="checkbox"/> 6 <input type="checkbox"/> 3</p>	5 6	2 3 5	2 3 4 5 6

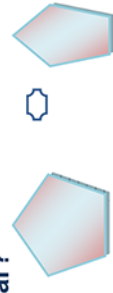
Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
					<p>Q3- Which names describes this shape?</p>  <p> <input type="checkbox"/> Parallelogram <input type="checkbox"/> Rectangle <input type="checkbox"/> Trapezoid <input type="checkbox"/> Rhombus </p> <p>Q4- Put each name in front of the right shape?</p>      <p> <input type="checkbox"/> Rectangle <input type="checkbox"/> Square <input type="checkbox"/> Parallelogram <input type="checkbox"/> Rhombus <input type="checkbox"/> Trapezoid </p>			



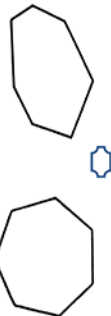
Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
					<p>Q5- Your brother drew a shape that had 4 sides, with opposite sides parallel. Which shape could he have drawn?</p> <p><input type="checkbox"/> Parallelogram <input type="checkbox"/> Isosceles triangle</p> <p><input type="checkbox"/> Oval <input type="checkbox"/> Triangle</p>			
14	4	Quadrilaterals and types	A rectangle	<ul style="list-style-type: none"> - Define the concept of a rectangle. - Determine the sides, vertices and angles of the rectangle . - Classify rectangles by its characteristics . 	<p>Q1- Which of these is a rectangle?</p> <p><input type="checkbox"/>  <input type="checkbox"/>  <input type="checkbox"/>  <input type="checkbox"/> </p> <p>Q2- How many sides equal this rectangle have?</p> <p><input type="checkbox"/> 2 <input type="checkbox"/> 3</p> <p><input type="checkbox"/> 0 <input type="checkbox"/> 4</p> <p></p>	1 2 3 4 5 6	1 2 3 4 5	1 2 3 4 5 6

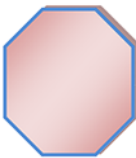
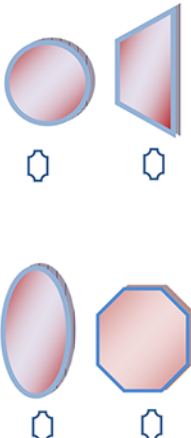
Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
15	4	Quadrilaterals and types	A parallelogram	<ul style="list-style-type: none"> - Define the concept of a parallelogram. - Determine the sides, vertices and angles of the parallelogram . - Classify parallelograms by its characteristics . 	<p>Q1- Is this a parallelogram shape?</p> <p><input type="checkbox"/> Yes <input type="checkbox"/> No</p>  <p>Q2- How many pairs of parallel sides of this parallelogram have?</p>  <p><input type="checkbox"/> 1 <input type="checkbox"/> 2 <input type="checkbox"/> 3 <input type="checkbox"/> 4</p>	5 6	2 3 5	4 5
16	4	Quadrilaterals and types	A rhombus (diamond)	<ul style="list-style-type: none"> - Define the concept of a rhombus. - Determine the sides, vertices and angles of the rhombus . 	<p>Q1- Which of these is a rhombus?</p>     <p><input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/></p>	5 6	1 2 3 5	3 4 5

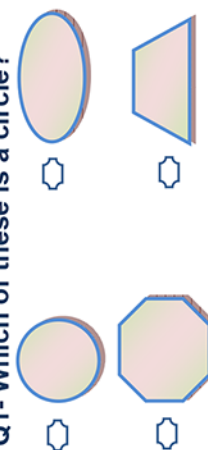
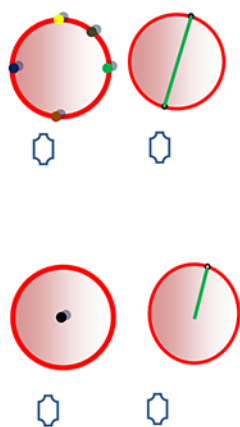
Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
				- Classify rhombus by its characteristics .	<p>Q2- How many sides equal this rhombus have?</p> <p><input type="checkbox"/> 4 <input type="checkbox"/> 0</p> <p><input type="checkbox"/> 3 <input type="checkbox"/> 2</p>			
17	4	Quadrilatera Is and types	A square	<ul style="list-style-type: none"> - Define the concept of a square. - Determine the sides, vertices and angles of the square. - Classify square by its characteristics . 	<p>Q1- Is this a square shape?</p> <p><input type="checkbox"/> Yes</p> <p><input type="checkbox"/> No</p>  <p>Q2- How many angles equal this square have?</p> <p><input type="checkbox"/> 4 <input type="checkbox"/> 0</p> <p><input type="checkbox"/> 3 <input type="checkbox"/> 2</p> 	1 2 3 4 5 6	1 2 3 4 5 6	1 2 3 4 5 6

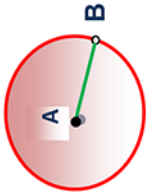
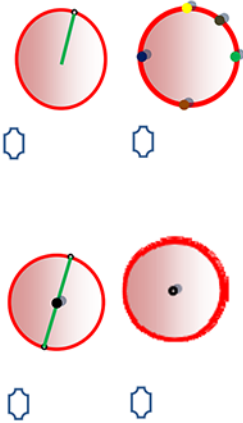
Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
18	4	Quadrilatera Is and types	A trapezium	<ul style="list-style-type: none"> - Define the concept of a trapezium. - Define the concept of an Isosceles trapezoid. - Determine the sides, vertices and angles of the trapezium. - Classify trapezium by its characteristics . 	<p>Q1- Which of these is a trapezium?</p>  <p>Q2- Which of these is Isosceles trapezoid?</p>  <p>Q3- How many angles equal this Isosceles trapezoid have?</p>  <p> <input type="checkbox"/> 2 <input type="checkbox"/> 0 <input type="checkbox"/> 3 <input type="checkbox"/> 4 </p>	5 6	2 3 5	5 6

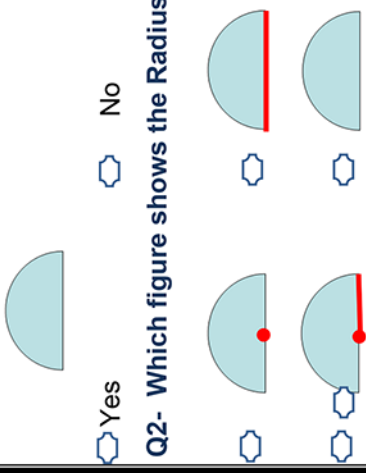
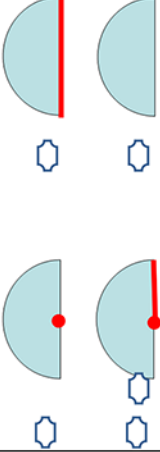
Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
19	5	Pentagons, Hexagons, Heptagons and Octagons	A Pentagons	<ul style="list-style-type: none"> - Define the concept of a Pentagons. - Determine the sides, vertices and angles of the Pentagons. - Distinguish between the pentagons regular and irregular. 	<p>Q1- You have a shape which has 5 sides and 5 angles. Which shape can you make?</p> <p><input type="checkbox"/> Pentagon <input type="checkbox"/> Trapezium</p> <p><input type="checkbox"/> Hexagons <input type="checkbox"/> Parallelogram</p> <p>Q2-Which of these is a pentagon irregular?</p>  <p><input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/></p>	2	1 2 3	3 4
20	5	Pentagons, Hexagons, Heptagons and Octagons	A Hexagons	<ul style="list-style-type: none"> - Define the concept of a Hexagons. - Determine the sides, vertices and angles of the Hexagons. - Distinguish between the Hexagons regular and irregular. 	<p>Q1- You brought a shape with 6 angles, What is called?</p> <p><input type="checkbox"/> Hexagons <input type="checkbox"/> Pentagon</p> <p><input type="checkbox"/> Parallelogram <input type="checkbox"/> Pentagons</p> <p>irregular</p>	2 3	1 2 3	3 4

Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
21	5	Pentagons, Hexagons, Heptagons and Octagons	A Heptagons		<p>Q2- Which of these is a Hexagons regular?</p>  <p>Q1- Which of these is a Heptagons?</p>  <p>Q2- Which of these is a Heptagons irregular?</p> 			
				<ul style="list-style-type: none"> - Define the concept of a Heptagons. - Determine the sides, vertices and angles of the Heptagons. - Distinguish between the Heptagons regular and irregular. 			2	3 4

Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
22	5	Pentagons, Hexagons, Heptagons and Octagons	An octagon	<ul style="list-style-type: none"> - Define the concept of an octagon. - Determine the sides, vertices and angles of the octagons. - Distinguish between the octagons regular and irregular. 	<p>Q1- You have a shape that has 8 equal sides and 8 equal angles. Which shape can you make?</p> <p><input type="checkbox"/> Octagons regular <input type="checkbox"/> Pentagon</p> <p><input type="checkbox"/> Hexagons <input type="checkbox"/> Octagons irregular</p> <p>Q2- Is this an octagon shape?</p>  <p><input type="checkbox"/> Yes <input type="checkbox"/> No</p>	2	2	4
23	6	Oval shape and A circle	Oval shape	<ul style="list-style-type: none"> - Define the concept of an oval on a plane. - Draw the oval. 	<p>Q1- Which of these is an Oval?</p>  <p><input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/></p>	1	2	1

Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
24	6	Oval shape and A circle	A circle	<ul style="list-style-type: none"> - Define the concept of a circle. - Determine the Center point of the circle. - Determine the Circumference of the circle. - Determine the Radius of the circle. - Determine the diameter of the circle. - Draw a circle using calipers. 	<p>Q1- Which of these is a circle?</p>  <p>Q2- Which figure shows the center point?</p> 	1 2 3 6	1 2	1 2 3 4 5

Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
					<p>Q3- What is AB ?</p>  <p> <input type="checkbox"/> Radius <input type="checkbox"/> Diameter <input type="checkbox"/> Circumference <input type="checkbox"/> Center point </p> <p>Q4- Which figure shows the diameter?</p> 			

Lesson No	Level	Unit Title	Concept	Learning Outcome: On successful completion of the module, students will be able to:	Sample questions	The Concept is applied at (Year/level) in		
						UK	NZ	KSA
					<p>Q5- Which of these is a Circumference?</p> <p><input type="checkbox"/> Regular curved line <input type="checkbox"/> Line</p> <p><input type="checkbox"/> Parallel Lines <input type="checkbox"/> Line segment</p>			
25	6	Oval shape and A circle	A semicircle	<ul style="list-style-type: none"> - Define the concept of a semicircle. - Determine the center point, Radius and diameter of the semicircle. 	<p>Q1- Is this is a semicircle shape?</p> <p><input type="checkbox"/> Yes <input type="checkbox"/> No</p>  <p>Q2- Which figure shows the Radius?</p> 	6	2	2

Appendix B

Link to the Arabic Survey designed for primary school math teachers

The survey questionnaire was designed to get a cross checking of the lessons and questions that were intended to identify the misconceptions in the geometric shape concepts from math teachers at the primary schools. The survey is in Arabic language and located at Google facility in the following link:

https://docs.google.com/forms/d/1WGe8EFX5ZKQIrmYNxrXEqSgsrfPv5ITHo7krsG8SiQw/viewform?edit_requested=true

Screenshot of the First Bag of the Survey.

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

1

1- النقطه
الصفوف التي يستخدم فيها المفهوم

أسباب متوقعة لعدم فهم المفهوم

الأهداف التعليمية لتحقيق المفهوم

أسئلة مقترحة لتحقيق المفهوم

ملاحظات ترغب إضافتها

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Appendix B.2

A list of all the concepts covered by the Arabic survey.

المرحلة التدريسية:

عدد سنوات الخبرة:

الاسم اختياري:

ملاحظات تُرغب إضافتها	أسئلة مقترحة لتحقيق المفهوم	أسباب متوقعة لعدم فهم المفهوم	الأهداف التعليمية لتحقيق المفهوم	الصفوف التي يستخدم فيها المفهوم	المفهوم الرياضي	م
					النقطة	1
					الخط المستقيم	2
					القطعة المستقيمة	3
					نصف المستقيم (الشعاع)	4
					الخط المنحني	5
					الخطوط المتوازية	6
					الخطوط المتعامدة	7
					الزوايا	8
					زاوية قائمة	9
					زاوية حادة	10
					زاوية منفرجة	11
					الأشكال المغلقة والأشكال المفتوحة	12
					المضلعات	13
					المثلث	14
					مثلث متطابق الساقين	15
					مثلث	16

ملاحظات ترغب إضافتها	أسئلة مقترحة لتحقيق المفهوم	أسباب متوقعة لعدم فهم المفهوم	الأهداف التعليمية لتحقيق المفهوم	الصفوف التي يستخدم فيها المفهوم	المفهوم الرياضي	م
					مختلف الأشلاع	
					مثلث متطابق الأشلاع	17
					مثلث قائم الزاوية	18
					الأشكال الرباعية	19
					المستطيل	20
					المربع	21
					متوازي الأشلاع	22
					المعين	23
					شبه المنحرف +	24
					شبه المنحرف	
					متطابق الساقين	
					الشكل الخماسي	25
					الشكل السداسي	26
					الشكل الثماني	27
					الشكل البيضاوي	28
					الدائرة (مركز الدائرة - قطر الدائرة - نصف قطر الدائرة - نصف الدائرة) (29

Appendix B.3

A list of all the concepts covered by the survey translated to English.

Name (optional):

Experience years:

Teaching stage:

No	Mathematical concept	Which classes that teaching this concept?	What are the reasons from your point of view of the lack of students' understanding of this concept?	What are the learning outcomes to achieve this concept?	Write questions proposed to achieve this concept.	Notes want to add
1	Dot					
2	Straight line					
3	Line segment					
4	Ray					
5	Curved line					
6	Parallel lines					
7	Perpendicular lines					
8	Angles					
9	Right angle					
10	Type of angles					
11	Acute angle					
12	Obtuse angle					
13	Open and closed shapes					
14	Polygons					
15	Triangle					
16	Equilateral					
17	Scalene					
18	Isosceles					
19	Right-angled triangle					
20	Quadrilateral					

No	Mathematical concept	Which classes that teaching this concept?	What are the reasons from your point of view of the lack of students' understanding of this concept?	What are the learning outcomes to achieve this concept?	Write questions proposed to achieve this concept.	Notes want to add
20	Rectangle					
21	Square					
22	Parallelogram					
23	Rhombus					
24	Trapezoid – isosceles trapezoid					
25	Pentagons					
26	Hexagons					
27	Octagon					
28	Oval shape					
29	Circle (center point – radius – diameter - Semicircle)					

Appendix C

The poster that was submitted to the 7th Saudi Arabia Students Conference at Edinburgh University (UK)

A conceptual spaces agent-based model for e-learning of mathematics: Teaching basic concepts concerning geometric shapes to school children.



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Introduction

There is a need to find more effective e-learning methods especially in mathematics. In this work, we wish to find a useful way to teach mathematics to primary school students. Our method will be based on the “conceptual spaces” cognitive model that was suggested by Gärdenfors (2004).

Conceptual spaces is cognitive model in which objects and abstract concepts are represented by quality dimensions (features that describe the concept). A specific concept is a set of regions from the domains within an n -dimensional geometric space. Each domain is composed of quality dimensions, and the primary function of the domain is to represent various qualities of situations or objects.

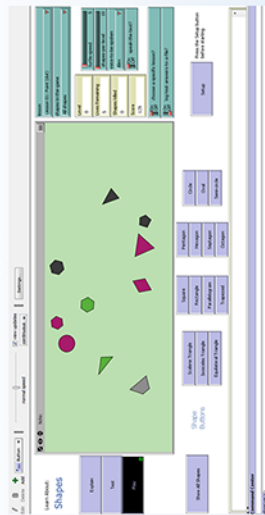
Method

A pilot experiment has been conducted by using a sample of 8 primary school students with different ages, genders (50% male and female) and classes. Forty concepts concerning geometrical shapes were covered by the experiment in 26 lessons with 78 questions which closely followed standard English and New Zealand primary school syllabi. Each student interacted with an e-learning program, written in Netlogo 5.0.4. The program consists of three main modes (see figures 1):

- Mode 1: This is initiated using the “Explain” button, and provides students with a short explanation about various concepts concerning basic geometrical shapes for each lesson.
- Mode 2: This is started by the “Test” button which asks a set of up to 6 multiple choice questions per lesson in order to test the student’s understanding from Mode 1. In addition, this part gives a total result with a list of correct and wrong answers.
- Mode 3: The “Play” button allows the student to play a game. This game is multi-level and helps the student to gain better understanding by reinforcing the concepts learned in Mode 1.

The initial purpose of the program is to record and then determine where students have made mistakes and therefore have misconceptions. The misconceptions will be used to determine the validity of the conceptual spaces cognitive model.

Figure 1: The e-learning program’s main interface.

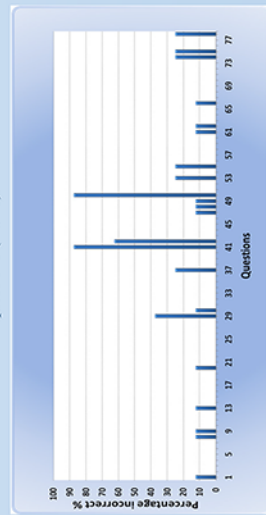


Results

The results show that the average score of all students for all questions is 92.6%. Many of the students had no problems with knowing most of the shape dimensions (names, number of sides, etc.). However, the mistakes on specific questions (see figure 2) reveal consistent misconceptions that confirm aspects of the conceptual spaces cognitive model. For example:

- Most students (87.5%) could only recognize a shape if it was shown with standard orientation.
- Very few students (1.6%) did not know specific shape names.
- Many students (60.9%) did not realise there may have been multiple labels for the same shape (e.g. a square is also a parallelogram as well as a rhombus) and chose a single label rather than parallelogram or rhombus).

Figure 2: Percentage incorrect score (y axis) for all questions (x axis).



These results are consistent with the conceptual spaces cognitive model if one considers the possible quality dimensions (i.e. features) that students are using to define the concepts. For example, most students answer the question in the following figure wrongly. One possible reason is that the students do not recognize the shape when it is rotated (they have learnt a specific orientation as one of the quality dimensions). Another possible reason is that the students thought that a right-angled triangle should have equal sides.



The following figure shows another question most students answered incorrectly. Most students know that the shape is square and may also know it is a rhombus but overlook the fact that the shape is also a parallelogram.



The results are promising. The next stage of the research will be to determine whether student misconceptions (mistakes) are consistent for different student groups and to determine which quality dimensions the students are using to organise the concepts. The aim will be to see whether teaching and learning can be improved by using unambiguous quality dimensions within the conceptual spaces cognitive model in order to teach the students to avoid the misconceptions and clarify the meaning of the concepts.

References: Gärdenfors, P. (2004). *Conceptual Spaces: The Geometry of Thought*. MIT press.

Appendix D

D.1 The paper-based test questions in the geometric shapes for Saudi students. (Arabic version).

بسم الله الرحمن الرحيم	
اسم الطالب/	اسم المدرسة/
الصف/	التاريخ/ / / 1435هـ
تعليمات قبل الإجابة عن أسئلة الاختبار:	
<p>1- هذا الاختبار في موضوعات وحدة الأشكال الهندسية ضمن مقرر الرياضيات للمرحلة الابتدائية ، استعن بالله واقراء الأسئلة بدقة لمعرفة المقصود من كل سؤال، ثم أجب عن جميع الأسئلة.</p>	
<p>2- اكتب بياناتك بخط واضح .</p>	
<p>3- لا تترك فقرة بدون إجابة.</p>	
<p>4- يوجد هناك أسئلة لها أكثر من خيار ، إختار جميع الإجابات الممكنة لهذه الأسئلة .</p>	
<p>5- الإجابة تكون على نفس ورقة الأسئلة وفي المكان المخصص لكل سؤال</p>	
<p>،،، هذا وبالله التوفيق والسداد ،،،</p>	

إختر الإجابة الصحيحة فيما يلي:

س1(أ) / أي من العبارات التالية يمثل نقطة:

- أثر رأس قلمك على الورقة كتابك
 مسطرتك أصبع يدك

س1(ب) / أي من الأشكال التالية يمثل نقطه:

- أ  ج  

س2(أ) / أي من العبارات التالية يمثل الخط المستقيم:

- ليس له بداية وليس له نهاية ويمتد على إستقامة واحدة.
 له بداية وليس له نهاية.
 ليس له بداية وليس له نهاية ولا يمتد على إستقامة واحدة.
 له بداية وله نهاية.





س2(ب) / أي من الأشكال التالية يمثل الخط المستقيم:

-    

س2(ج) / أي من الرموز التالية يمثل رمز الخط المستقيم:

- أ  أ  أ  أ 

س3(أ) / أي من الأشكال التالية يمثل القطعة المستقيمة:

-    

س3(ب) / أي من الرموز التالية يمثل رمز القطعة المستقيمة:

- أ  أ  أ  أ 

س4(أ) / ماذا يمثل الشكل التالي:

خط مستقيم قطعة مستقيمة نقطة نصف خط مستقيم (شعاع)

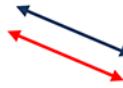
س4(ب) / يعبر عن نصف المستقيم بالرمز التالي:

س ص س ص س ص س ص



س5(أ) / ماذا يمثل هذا الشكل:

خط مستقيم قطعة مستقيمة خط منحنى شعاع



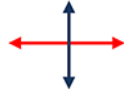
س6(أ) / هل الخطان التاليان يمثلان خطان متوازيان:

نعم لا



س6(ب) / يعبر عن الشكل التالي بالرمز:

أ || ب أ ب || ج د أ ج || ب د أ ب ج د



س7(أ) / ماذا يمثل هذا الشكل:

خطان متوازيان خطان متعامدان شعاع خطان مائلان

س7(ب) / ماهو الشكل الذي يمثل هذا الرمز (أ ب ⊥ ج د):



س8(أ) / أي من الأشكال التالية يمثل زاوية:



س8(ب) / يعبر عن الزاوية التالية بالرمز: أ ب ج أ ب ج أ ب ج أ ب ج

س8(ج) / الزاوية التالية هي:

أصغر من الزاوية القائمة

تساوي الزاوية القائمة

أكبر من الزاوية القائمة

س8(د) / أكتب رقم الشكل من العمود (أ) أمام ما يناسبه من العمود (ب):

(ب)

() زاوية قائمة

() خطين متوازيين

() زاوية منفرجة

() زاوية حادة

(أ)

-1

-2

-3



س9(أ) / هل الشكل التالي مغلق أم مفتوح:

مغلق

مفتوح

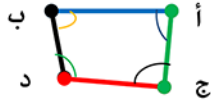
س9(ب) / أي الأشكال التالية شكل مفتوح:





نعم

لا

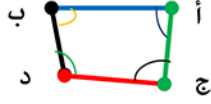


س10(ب) / ماهي أسماء أضلاع المضلع التالي:

أ ، ب ، ج ، د

أ ج ، ج د ، د ب ، ب أ

أ ج د ، ج د ب ، د ب أ ، ب أ ج

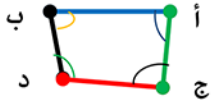


س10(ج) / ماهي أسماء رؤوس المضلع التالي:

أ ، ب ، ج ، د

أ ج ، ج د ، د ب ، ب أ

أ ج د ، ج د ب ، د ب أ ، ب أ ج



س10(د) / ماهي أسماء زوايا المضلع التالي:

أ ، ب ، ج ، د

أ ج ، ج د ، د ب ، ب أ

أ ج د ، ج د ب ، د ب أ ، ب أ ج



س10(هـ) / كم عدد أضلاع المضلع التالي:

6

5

4

3

س10(و) / أي من الأشكال التالية هو مضلع: (إختر جميع الإجابات الصحيحة):

شكل خماسي

دائرة

مربع

مستطيل



س10(ز) / هل الشكل التالي منتظم أو غير منتظم:

غير منتظم

منتظم

س11(أ) / أي من الأشكال التالية يمثل مثلث:



س11(ب) / لديك شكل له 3 أضلاع و3 زوايا, ماهو هذا الشكل :

مستطيل معين دائرة مثلث

س11(ج) / أي من أنواع المثلثات المثلث التالي:



متطابق الأضلاع متطابق الساقين

قائم الزوايا مختلف الأضلاع

س11(د) / أكتب رقم الشكل من العمود (أ) أمام إسمه من العمود (ب):

(ب)

() مثلث قائمة الزاوية

() مثلث متطابق الأضلاع

() منحنى مغلق

() مثلث مختلف الأضلاع

() مثلث متطابق الساقين

(أ)



-1



-2



-3



-4

س11(هـ) / مثلث له زاويتان متساويتان وزاوية واحدة مختلفة ,
ما هو نوع هذا المثلث: (يوجد أكثر من إجابة)

- مثلث متطابق الأضلاع مثلث متطابق الساقين
 مثلث قائم الزوايا مثلث مختلف الأضلاع



س12(أ) / هل الشكل التالي شكل رباعي:

- لا نعم

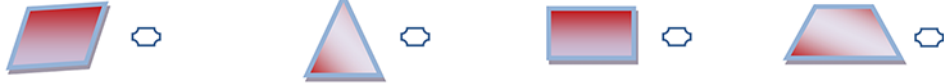
س12(ب) / كم عدد زوايا الشكل الرباعي:

- 2 3 4 5

س12(ج) / ما هو مجموع زوايا أي شكل رباعي:

- 360° 180° 90° 60°

س13(أ) / أي الأشكال التالية يمثل مستطيل:



س13(ب) / كم عدد الأضلاع المتساوية في الشكل التالي:

- 1 2 3 4



س14(أ) / ما هو إسم الشكل التالي:

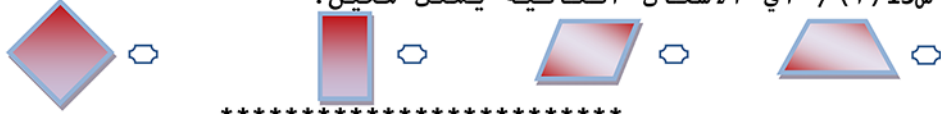
- مستطيل معين متوازي أضلاع مربع



س14(ب) / كم زوج من الأضلاع متوازية في الشكل التالي:

- 1 2 3 4

س15 (أ) / أي الأشكال التالية يمثل معين:



س15 (ب) / كم عدد الأضلاع المتساوية في المعين التالي:



4 3 2 1

س16 (أ) / هل الشكل التالي مربع:



لا نعم

س16 (ب) / كم عدد الزوايا القائمة في المربع التالي:



4 3 2 1

س17 (أ) / أي الأشكال التالية يمثل شبه منحرف:



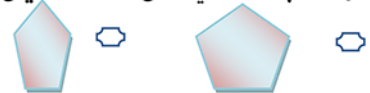
س17 (ب) / أي من الأشكال التالية يمثل شبه منحرف متساوي الساقين:



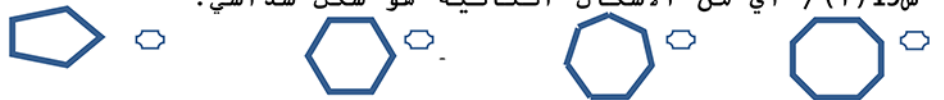
س18 (أ) / لديك شكل له خمسة أضلاع وخمسة زوايا, ما هو إسم هذا الشكل:

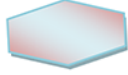
متوازي أضلاع شبه منحرف شكل خماسي شكل سداسي

س18 (ب) / أي من الشكلين التاليين هو شكل خماسي غير منتظم:



س19 (أ) / أي من الأشكال التالية هو شكل سداسي:





س19 (ب) / هل الشكل التالي هو شكل سداسي منتظم:

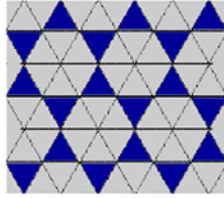
لا نعم

س20 (أ) / ماهي مجموعة الأشكال التي يتكون منها الشكل التالي:



- (مربع, مستطيل, مثلث)
 (مربع, مستطيل, شبه منحرف)
 (مربع, متوازي أضلاع, شبه منحرف)
 (مربع, معين, مستطيل)

س20 (ب) / ماهي مجموعة الأشكال التي يتكون منها الشكل التالي:



- (معين, شكل سداسي, مثلث)
 (معين, شكل خماسي, مستطيل)
 (معين, شكل ثماني, مثلث)
 (معين, مربع, مستطيل)

س21 (أ) / ضع علامة (✓) أمام العبارة الصحيحة وعلامة (X) أمام

العبارة الخاطئة فيما يلي:

- () كل مربع معين.
() كل مستطيل مربع.
() أي رباعي زواياه المتوازية متطابقة يكون متوازي أضلاع
() أي رباعي أضلاعه المتوازية متطابقة يكون مستطيل.
() الأضلاع المتوازية في شبه المنحرف متطابق الساقين متوازية
() جميع زوايا متوازي الأضلاع متطابقة.
() كل مستطيل متوازي أضلاع.
() كل معين مستطيل.

س22 (أ) / أي من الأشكال التالية هو شكل بيضاوي:



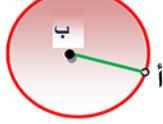
س23 (أ) / أي من الأشكال التالية هو دائرة:



س23 (ب) / أي من الأشكال التالية موضح عليها مركز الدائرة:



س23 (ج) / ماذا يسمى (أب) المرسوم داخل الدائرة التالية:



نصف قطر محيط مركز الدائرة قطر

س23 (د) / أي من الأشكال التالية يوضح قطر الدائرة:



س23 (د) / محيط الدائرة عبارة عن:

خط مستقيم خط منحنى منتظم

خطين متوازيين قطعة مستقيمة

D.2 The paper-based test questions in the geometric shapes for UK students. (English version)

Test in geometric shapes

Name: _____ Year: _____ Gender: Boy Girl




Choose the correct answer from the following:

Q1.1-Which of these make a point(dot)? (Note: there are two possible answers for this one! Select both of them):

Your pen as you write something with it. A needle as you prick your Finger

A drawing pin as You press it in. Your tennis racket as it moves while playing tennis.

Q1.2- Which of these is a dot (point)?

 A  B 

Q2.1- Which of these a line?





Something that has no beginning and no end and goes straight in both directions.

Something that has a beginning point but no end point.




Something that has no beginning and end point.

Something that has a beginning point and end point.





Q2.2- Which of these is a straight line?




Q2.3- Which of these is the symbol of a line?


 A  

Q3.1- Which of these is a line segment?

Q3.2- Which of these is the symbol of the line segment?


  A 

Q4.1- What is this? 

A line A line segment A dot A ray

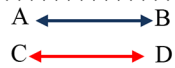
Q4.2- What is the name of this figure?

 TK TK TK TK

Q5.1- What is this?

 A line A line segment A curved line A ray



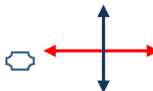

Q6.1- Are these lines Parallel?




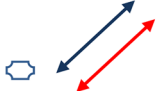
 Yes No

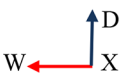
Q6.2- What is the name of this figure?

 A || B AB || CD AC || BD ABCD

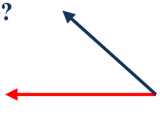
Q7.1- What is this?

 Parallel lines Lines that are curved Perpendicular lines A ray

Q7.2- What is the figure for this symbol (AB ⊥ DC)?
    

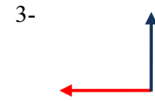
Q8.1- Which of these is an angle?
    

Q8.2- What is the name of this figure?

 DWX DW ∠DXW ∠XWD

Q8.3- Is this angle greater than, equal to, or less than a right angle?
 Less than a right angle Equal to a right angle Greater than a right angle


Q8.4. Type the number of the shape of the column (A) in front of what suits it from column (B):

(A)



(B)

Right angle

Parallel lines

Obtuse angle

Acute angle

Q9.1- Is this shape open or closed?

Open

Closed



Q9.2- Which of these is an open shape?



Q10.1- Is this a polygon shape?

No

Yes

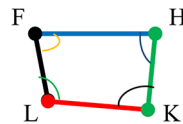


Q10.2-Which are the sides of this polygon?

F, H, L and K.

FH, HK, KL and LF.

FHK, HKL, KLF and LFH.

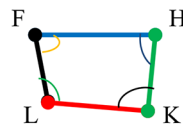


Q10.3. Which are the angles of this polygon?

F, H, L and K.

FH, HK, KL and LF.

FHK, HKL, KLF and LFH.

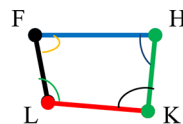


Q10.4. Which are the vertices of this polygon?

F, H, L and K.

FH, HK, KL and LF.

FHK, HKL, KLF and LFH.



Q10.5- How many sides dose this polygon have?



3

4

5

6

Q10.6. Which of the following are names of polygons? (Note: Choose all correct answers).

Rectangle

Square

Circle

Pentagon

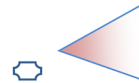
Q10.7- Is this shape a regular polygon or an irregular polygon?



Regular

Irregula

Q11.1- Which of these a triangle?



Q11.2- You have a shape that has 3 sides and 3 angles. Which shape can you make?

Rectangle

Square

Rhombus

Triangle

Q11.3- What kind of triangle is this?



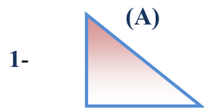
Scalene

Isosceles

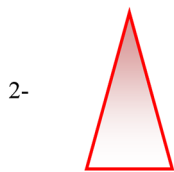
Equilateral

Right-angled

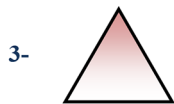
Q11.4 Type the number of the shape of the column (A) in front of what suits him from column (B):



(B) Right-angled triangle



Equilateral triangle



Curved line

Scalene triangle



Isosceles triangle

Q11.5- The shape have 2 equal angles and a 1 different angle. Which shape can you make? (Note: there are two possible answers for this one! Select both of them):

- Isosceles triangle
 Equilateral triangle
 Right-angled triangle
 Scalene triangle

Q12.1- Is this a quadrilateral shape?



- Yes
 No

Q12.2- How many angles do Quadrilaterals have?

- 2
 3
 4
 5

Q12.3. What is the sum of angles of any quadrilateral shape?

- 360°
 180°
 90°
 60°

Q13.1- Which of these is a rectangle?

-

Q13.2- How many pairs of equal length sides does this rectangle have?



- 1
 2
 3
 4

Q14.1- What is this shape?



- Rectangle
 Rhombus
 Parallelogram
 Square

Q14.2- How many pairs of parallel sides does this parallelogram have?



- 1
 2
 3
 4

Q15.1- Which of these is a rhombus?

-

Q15.2- How many equal sides does this rhombus have?



- 1
 2
 3
 4

Q16.1- Is this a square shape?



- Yes
 No

Q16.2- How many equal angles this square have?

1

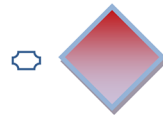
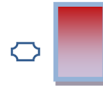
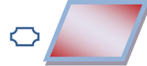
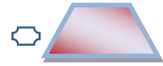
2

3

4



Q17.1- Which of these is a trapezium?



Q17.2- Which of these is in isosceles trapezoid?



Q18.1- You have a shape which has 5 sides and 5 angles. Which shape can you make?

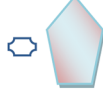
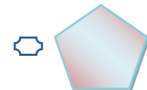
Parallelogram

Trapezium

Pentagon

Hexagons

Q18.2-Which of these is an irregular pentagon?



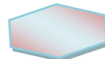
Q19.1- Which of these is hexagon?



Q19.2- Is this a regular hexagon shape?

Yes

No



Q20.1- What shapes make up the figure on the right?

(Square, Rectangle, Triangle)

(Square, Rectangle, Trapezoid)

(Square, Parallelogram, Trapezoid)

(Square, Rhombus, Rectangle)



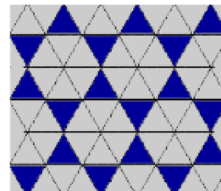
Q20.2- What shapes make up the figure on the right?

(Rhombus, Hexagons, Triangle)

(Rhombus, Pentagon, Rectangle)

(Rhombus, Octagon, Triangle)

(Rhombus, Square, Rectangle)



Q21- Tick (✓) for the correct sentence, and tick (X) for the incorrect sentence of the following:

- 1- () All squares are rhombus.
- 2- () All rectangles are square.
- 3- () Any quadrilateral that has opposite angles that are congruent is a parallelogram.
- 4- () Any quadrilateral that has opposite sides that are congruent is a rectangle.
- 5- () The opposite sides in a isosceles trapezoid are parallel.
- 6- () All the angles in a parallelogram are congruent.
- 7- () All rectangle are parallelogram.
- 8- () All rhombus are rectangle.

Q22- Which of these is an Oval?



Q23.1- Which of these is a circle?

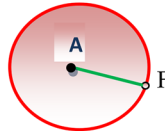


Q23.2- Which figure shows the center point?



Q23.3- What is the line \overline{AF} in this figure?

- A radius
- A circumference
- A diameter
- A center point



Q23.4- Which figure shows the diameter?



Q23.5- A circumference is which of the following?

- A straight line
- A regular curved line
- A parallel lines
- A line segment

""THANK YOU FOR YOUR COOPERATION""

Appendix E

The Interview guideline that was given to the maths teachers

بسم الله الرحمن الرحيم

أسئلة المقابلة الشخصية لوحدة الأشكال الهندسية

س1/ أرسم الأشكال التالية: (يرسم الطالب مجموعة من الأشكال التالية):

نقطة – خط المستقيم – قطعة مستقيمة – نصف خط مستقيم (شعاع) – خط منحنى – خطين متوازيين – خطين متعامدين – زاوية قائمة – زاوية منفرجة – زاوية حادة – مثلث – مثلث قائم الزاوية – مثلث متطابق الضلعين – مثلث متطابق الأضلاع – مثلث مختلف الأضلاع – مربع – مستطيل – معين – متوازي الأضلاع – الدائرة (حدد مركز الدائرة – نصف القطر – القطر.) – شكل بيضاوي.

س2/ عرف الأشكال التالية: (يعرف الطالب بكلمات من عنده ماذا تمثل الأشكال):

(مثال يسأل الطالب عن: ما هو المثلث؟) (تعريف الأشكال).

نقطة – خط المستقيم – قطعة مستقيمة – نصف خط مستقيم (شعاع) – خط منحنى – خطين متوازيين – خطين متعامدين – زاوية قائمة – زاوية منفرجة – زاوية حادة – مثلث – مثلث قائم الزاوية – مثلث متطابق الضلعين – مثلث متطابق الأضلاع – مثلث مختلف الأضلاع – مربع – مستطيل – معين – متوازي الأضلاع – الدائرة (حدد مركز الدائرة – نصف القطر – القطر.) – شكل بيضاوي.

س3/ ماهي خصائص الأشكال التالية: (يفسر/ يذكر الطالب خصائص الأشكال الهندسية التالية):

(مثال يسأل الطالب عن: لماذا الشكل التالي هو مستطيل؟) (خصائص الأشكال).

نقطة – خط المستقيم – قطعة مستقيمة – نصف خط مستقيم (شعاع) – خط منحنى – خطين متوازيين – خطين متعامدين – زاوية قائمة – زاوية منفرجة – زاوية حادة – مثلث – مثلث قائم الزاوية – مثلث متطابق الضلعين – مثلث متطابق الأضلاع – مثلث مختلف الأضلاع – مربع – مستطيل – معين – متوازي الأضلاع – الدائرة (حدد مركز الدائرة – نصف القطر – القطر.) – شكل بيضاوي.

"جزاك الله ألف خير على تعاونك معنا"

Appendix F

A list of test items and corresponding question codes

No.	Item	Code of question	No.	Item	Code of question
1	Dot (Point) q1	Q1_1	12	Quadrilateral q1	Q12_1
	Dot (Point) q2	Q1_2		Quadrilateral q2	Q12_2
2	Straight Line q1	Q2_1	13	Quadrilateral q3	Q12_3
	Straight Line q2	Q2_2		Rectangle q1	Q13_1
	Straight Line q3	Q2_3		Rectangle q2	Q13_2
3	Line segment q1	Q3_1	14	Parallelogram q1	Q14_1
	Line segment q2	Q3_2		Parallelogram q2	Q14_2
4	Ray q1	Q4_1	15	Rhombus q1	Q15_1
	Ray q2	Q4_2		Rhombus q2	Q15_2
5	Curve q1	Q5_1	16	Square q1	Q16_1
6	Parallel lines q1	Q6_1		Square q2	Q16_2
	Parallel lines q2	Q6_2	17	Trapezium q1	Q17_1
7	Perpendicular lines q1	Q7_1		Trapezium q2	Q17_2
	Perpendicular lines q2	Q7_2	18	Pentagon q1	Q18_1
8	Angles q1	Q8_1		Pentagon q2	Q18_2
	Angles q2	Q8_2	19	Hexagon q1	Q19_1
	Angles q3	Q8_3		Hexagon q2	Q19_2
	Angles q4	Q8_4	20	Shapes properties 1-q1	Q20_1
9	Open-Closed shapes q1	Q9_1		Shapes properties 1-q2	Q20_2
	Open-Closed shapes q2	Q9_2	21	Shapes properties 2-q1	Q21_1
10	Polygon q1	Q10_1		Shapes properties 2-q2	Q21_2
	Polygon q2	Q10_2		Shapes properties 2-q3	Q21_3
	Polygon q3	Q10_3		Shapes properties 2-q4	Q21_4
	Polygon q4	Q10_4		Shapes properties 2-q5	Q21_5
	Polygon q5	Q10_5		Shapes properties 2-q6	Q21_6
	Polygon q6	Q10_6		Shapes properties 2-q7	Q21_7
	Polygon q7	Q10_7		Shapes properties 2-q8	Q21_8
11	Triangle q1	Q11_1	22	Oval q1	Q22_1
	Triangle q2	Q11_2		Circle q1	Q23_1
	Triangle q3	Q11_3	23	Circle q2	Q23_2
	Triangle q4	Q11_4		Circle q3	Q23_3
	Triangle q5	Q11_5		Circle q4	Q23_4
			Circle q5	Q23_5	

Appendix G

The documents of the study procedures:

- approval letter to conduct experiment in school in the UK and Saudi Arabia;
- request for permission to conduct research in school; and
- letter of consent from parents.

Al Taif International School

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مدارس الطائف الدولية

تعليم المنهج البريطاني
تحت إشراف وزارة التربية والتعليم
ترخيص رقم (٣٩١ / س)

APPROVAL LETTER TO CONDUCT EXPERIMENT IN SCHOOL

I have granted permission for Mr. Yasser M. Alghurabi, a PhD. student in the School of Computer Science at Bangor University, from 23/08/2015 to 27/08/2015 to conduct research at Al-Taif International School(ATIS) as part of his PhD research project "Learning Lines and Shapes".

I understood that participants were asked to use e-learning software in order to learn about geometric shapes and lines and to obtain data needed for the study. The experiment was under the Academic supervisor Mr. Osama M . Kamel.

If you have any questions regarding permission, please do not hesitate to call.

Your Sincerely,

Name of headteacher
Abdul Nasser Al Talhi
Phone: 00966538036466
Email:osama_mohamed5667@yahoo.com



الطائف - ص.ب: ٨٣٠٢ الطائف رمز البريد : ٢٦٥١١ - تلفاكس : ١٠٨ / ٠١٢٧٤٤٢٤٢٨
Al Taif P.O.Box : 8302 Al Taif Zip Code : 26511 Tel/Fax : 012 7442428 Ext 108
Email : altaif.school@gmail.com Web : www.theatis.com

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مدارس الطائف الدولية

لتعليم المنهج البريطاني
تحت إشراف وزارة التربية والتعليم
ترخيص رقم (٣٩١ / س)

I have granted permission for Mr. Yasser M Alghurabi, a PhD. Student in the school of Computer Science at Bangor University, from 30/08/2015 to 09/09/2015 to conduct research at Al-Taif International School (ATIS) as part of his PhD research project "Learning Lines and Shapes".

I understood that participants were asked to use e-learning software in order to learn about geometric shapes and lines to obtain data needed for the study. The experiment as under the academic supervisor Mrs. Nadia Mahmoud and school Admin Officer Ms.Khawla Abdullah

If you have any questions regarding permission, please do not hesitate to call.

Your Sincerely,

Name of headteacher

Khawla Abdullah Alsokheri

Phone 00966538036466

Email:osama_mohamed5667@yahoo.com

Principal

Nadia Mahmoud

Nadia Mahmoud
8/10/2015



المطائف - ص.ب: ٨٣٠٢ المطائف رمز البريد : ٢٦٥١١ - تلفاكس : ٠١٢ ٧٤٤٢٤٢٨ / ١٠٨
Al Taif P.O.Box : 8302 Al Taif Zip Code : 26511 Tel/Fax : 012 7442428 Ext 108
Email : altaif.school@gmail.com Web : www.theatis.com


02/06/15

APPROVAL LETTER TO CONDUCT EXPERIMENT IN SCHOOL

I have granted permission for Mr. Yasser M. Alghurabi, a PhD. student in the School of Computer Science at Bangor University, on 10/02/2014 to conduct research at Cae Top School as part of his PhD research project "Learning Lines and Shapes". I understood that participants were asked simple questions about geometric shapes and lines in order to obtain data needed for the study. The test was under the supervision of mathematics teacher Mr Llew Davies and Miss Ceri Jones.

If you have any questions regarding site permission, please do not hesitate to call.

Yours sincerely



pp Head Teacher



llewdavies@gwynedd.gov.uk

HILLGROVE SCHOOL



Ffriddoedd Road,
Bangor,
Gwynedd. LL57 2TW

Tel : 01248 353568
Fax : 01248 353971
email : headmaster@hillgrove.gwynedd.sch.uk

Your Ref:
Our Ref:

May 5th.2015

Dear Sir,

APPROVAL LETTER TO CONDUCT EXPERIMENT IN SCHOOL.

I have granted permission for Mr.Yasser M Alghurabi, a PhD student in the School of Computer Science at Bangor University, on 10 / 02 / 2014, to conduct research at Hillgrove School as part of his PhD research project "Learning Lines and Shapes".

I understand that participants were asked simple questions about geometric shapes and lines in order to obtain data needed for the study. The test was under the supervision of Maths Teachers, Mr.John Holmes and Mr.Paul Gash.

If you have any questions regarding site permission, please do not hesitate to telephone.

I trust that this information is as required.

Yours sincerely,

James G.Porter

HILLGROVE PRIVATE SCHOOL
BANGOR LL57 2TW, GWYNEDD

Principals: Mr.J.G.J.Porter and Mrs S.P.Porter

Founded in 1934

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ



المملكة العربية السعودية

وزارة التربية والتعليم
(٢٨٠)

إدارة العامة للتربية والتعليم بمحافظة الطائف

الرقم : _____

التاريخ : / / ١٤

المشروعات : _____

إفادة بتطبيق دراسة علمية

من: مدير مدرسة أبو عمرو البصري

بشأن: تطبيق دراسة علمية للباحث/ ياسر بن محمد الغريبي

والسلام عليكم ورحمة الله وبركاته وبعد

بناءً على ما عرضه علينا الباحث ياسر بن محمد الغريبي عن موضوع بحثه (نحو فضاء مفاهيمي لوكيل قائم على نموذج للتعلم الإلكتروني في الرياضيات) وإيماناً منا بأهمية هذا الموضوع وفائدته لمعلمي الرياضيات والطلاب لذا من الأهمية تطبيقها للاستفادة من نتائجها مستقبلاً.

عليه فقد تم تسهيل تطبيق الدراسة وتمت متابعتها من قبل الأستاذ/ عائض بن عيد العتيبي وكيل المدرسة ومعلم رياضيات سابقاً بتاريخ 1435/2/20هـ.

أعطي هذا الخطاب بناءً على طلب الباحث لتوثيق عمل التجربة.

،، ولكم تحياتي وتقديري،،

اسم مدير المدرسة
عبدالله بن عبدالعزيز
[Signature]



الرقم :
التاريخ :
المشروعات :
الموضوع :



المملكة العربية السعودية
وزارة التربية والتعليم
إدارة التربية والتعليم بمحافظة الطائف
مدرسة الحاج بن يوسف الثقفي الابتدائية بالهدا
هاتف / ٠١٢٧٥٤١٩٣٠

إفادة بتطبيق دراسة علمية

من: مدير مدرسة الحاج بن يوسف الثقفي الابتدائية

بشأن: تطبيق دراسة علمية للباحث/ ياسر بن محمد الغريبي

ويعد

السلام عليكم ورحمة الله وبركاته

بناءً على ما عرضه علينا الباحث ياسر بن محمد الغريبي عن موضوع بحثه (نحو فضاء مفاهيمي لوكيل قائم على نموذج للتعليم الإلكتروني في الرياضيات) وإيماناً منا بأهمية هذا الموضوع وفائدته لمعلمي الرياضيات والطلاب لذا من الأهمية تطبيقها للاستفادة من نتائجها مستقبلاً. عليه فقد تم تسهيل تطبيق الدراسة وتمت متابعتها من قبل الأستاذ/ طلال بن هاشم الثقفي والأستاذ/ تركي بن خلف الثبيتي معلم الرياضيات في المدرسة بتاريخ ١٤٣٥/٢/٢٠هـ. أعطي هذا الخطاب بناءً على طلب الباحث لتوثيق عمل التجربة.

،، ولكم تحياتي وتقديري،،

اسم مدير المدرسة
تركي بن حماد النمري



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ



المملكة العربية السعودية

وزارة التربية والتعليم

(٢٨٠)

الإدارة العامة للتربية والتعليم بمحافظة الطائف

الرقم : _____

التاريخ : / / ١٤ هـ

المشروعات : _____

مشهد إفادة بتطبيق دراسة علمية

تشهد مدرسة عمر بن الخطاب الابتدائية بأن الباحث / ياسر بن محمد الغريبي قام بتطبيق الدراسة عن موضوع بحثه (نحو فضاء مفاهيمي لوكيل قائم على نموذج للتعلم الإلكتروني في الرياضيات) بالمدرسة تحت إشراف الأستاذ / حسن ناصر العصيمي والأستاذ / أمين سعد بن دريم (معلم رياضيات) ، وذلك بتاريخ 20 / 2 / 1435 هـ .
وعليه أعطي هذا المشهد لتوثيق عمل التجربة ، والله خير الشاهدين .

مدير المدرسة

تركي صالح المالكي



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ



المملكة العربية السعودية

وزارة التربية والتعليم
(٢٨٠)

الإدارة العامة للتربية والتعليم بمحافظة الطائف

الرقم : _____
التاريخ : / / ١٤١٤ هـ
المشروعات : _____

إفادة بتطبيق دراسة علمية

من: مدير مدرسة الأمير فيصل بن فهد الابتدائية

بشأن: تطبيق دراسة علمية للباحث/ ياسر بن محمد الغريبي

السلام عليكم ورحمة الله وبركاته
ويعد

بناءً على ما عرضه علينا الباحث ياسر بن محمد الغريبي عن موضوع بحثه (نحو فضاء مفاهيمي لوكيل قائم على نموذج للتعلم الإلكتروني في الرياضيات) وإيماناً منا بأهمية هذا الموضوع وفائدته لمعلمي الرياضيات والطلاب لذا من الأهمية تطبيقها للاستفادة من نتائجها مستقبلاً.

عليه فقد تم تسهيل تطبيق الدراسة وتمت متابعتها من قبل الأستاذ/ حسن بن فرحة الزهراني معلم الرياضيات في المدرسة بتاريخ 1435/2/20 هـ.

أعطي هذا الخطاب بناءً على طلب الباحث لتوثيق عمل التجربة.

،، ولكم تحياتي وتقديري،،

اسم مدير المدرسة

منصور بن محمد بن
الزهراني





المملكة العربية السعودية
وزارة التربية والتعليم
(٢٨٠)
الإدارة العامة للتربية والتعليم بمحافظة الطائف

الرقم : /
التاريخ : ٦ / ٦ / ١٤٢٦ هـ
المشروعات : /

إفادة بتطبيق دراسة علمية

من: مدير مدرسة ابن كثير الابتدائية

بشأن: تطبيق دراسة علمية للباحث/ ياسر بن محمد الغريبي

والسلام عليكم ورحمة الله وبركاته وبعد

بناءً على ما عرضه علينا الباحث ياسر بن محمد الغريبي عن موضوع بحثه (نحو فضاء مفاهيمي لوكيل قائم على نموذج للتعلم الإلكتروني في الرياضيات) وإيماناً منا بأهمية هذا الموضوع وفائدته لمعلمي الرياضيات والطلاب لذا من الأهمية تطبيقها للاستفادة من نتائجها مستقبلاً.

عليه فقد تم تسهيل تطبيق الدراسة وتمت متابعتها من قبل الأستاذ/ منصور بن معتوق النمري وكيل المدرسة ومعلم رياضيات سابق بتاريخ ٢٠/٢/١٤٢٥ هـ.

أعطي هذا الخطاب بناءً على طلب الباحث لتوثيق عمل التجربة.

،، ولكم تحياتي وتقديري،،

مدير المدرسة
محمد فايز الثبيتي



COLEG Y GWYDDORAU FFISEGOL A CHYMHWYSOL
COLLEGE OF PHYSICAL & APPLIED SCIENCES
YSGOL CYFRIFIADUREG
SCHOOL OF COMPUTER SCIENCES



Dr. William J Teahan
Director of Postgraduate studies
Email: w.j.teahan@bangor.ac.uk
Telephone: 01248 382686

REQUEST FOR PERMISSION TO CONDUCT EXPERIMENT IN SCHOOL.

Dear Ms. /Mr the Head of School,

We are writing to request permission to conduct research study as part of the requirements of PhD degree for Mr. Yasser M. Alghurabi, a PhD. student in the School of Computer Science at Bangor University.

He is currently studying e-learning – to establish and evaluate an effective agent-based model that could help in tutoring basic mathematics for children. Moreover, his task in this subject is to ask children aged from 8 to 13 years old to use e-learning software in order to learn about geometric shapes and lines (For example: line segment, ray, parallelogram, trapezium, rectangle, square and rhombus). The overall experiment should take no longer than 45 minutes.

The data/results of this study will remain absolutely confidential and to be used on educational purposes only.

Your approval to conduct this study in your school will be greatly appreciated. If you have any questions, please do not hesitate to call.

Your Sincerely,



Dr. William J Teahan	Ysgol Gwyddorau Cyfrifiadurol Prifysgol Bangor	School of Computer Science Bangor University
	Stryd y Deon, Bangor Gwynedd, LL57 1UT, DU	Dean Street, Bangor Gwynedd, LL57 1UT UK
	ffôn/Tel: E-bost/ E-mail :	ffôn/Tel: E-bost/ E-mail :
PRIFYSGOL BANGOR STRYD Y DEON, BANGOR, GWYNEDD, LL57 1UT, DU	BANGOR UNIVERSITY DEAN STREET, BANGOR, GWYNEDD, LL57 1UT, UK	DR SA'AD MANSOOR B.Sc, BEng, PhD, MBCS, MIET EBOST/EMAIL: cs@bangor.ac.uk
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RIF FFLUDEN GOBRESTREDIG/REGISTERED CLARITY NO. 1141565



Re: Letter of Consent

Dear parent,

My name is Yasser M. Alghurabi. I am a PhD. student in the School of Computer Science at Bangor University. I am conducting a research study as part of the requirements of my PhD degree, and I would like to invite your child to participate.

I am studying e-learning – to establish and evaluate an effective agent-based model that could help tutoring basic mathematics for children. If you decide to allow your child to participate, your child will be asked to use e-learning software in order to learn about geometric shapes (for example: line segment, ray, parallelogram, trapezium, rectangle, square and rhombus). The overall experiment will be take no longer than 45 minutes.

Participation is confidential. The results of the study may be published or presented at professional meetings, but your child’s identity will not be revealed.

We will be happy to answer any questions you have about the study. You may contact me or my supervisor (**Dr. William J Teahan, phone number: 01248 382686, and e-mail address: w.j.teahan@bangor.ac.uk**) if you have any questions.

Thank you for your consideration. Your help is greatly appreciated. If you would like to allow your child to participate, please write the name of your child and sign this letter and return it to the head of school.

Name your child:.....

Signed:.....

date: / / 2015

With kind regards,

Yasser M. Alghurabi

Phone number: 07926103211

E-mail address: abuamary@hotmail.com

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