# Financial Frictions and the Futures Pricing Puzzle

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**Economic Modelling**

*DOI:*
10.1016/j.econmod.2019.08.009

Published: 01/05/2020

Peer reviewed version

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**Abstract:**

In perfect capital markets, the futures price of an asset should be an unbiased forecast of its realized spot price when the contract matures. In reality, futures prices are often higher for some assets and lower for others. However, there is no stability in the relationship between futures prices and the realized spot prices. This instability has been a puzzle in the existing financial literature. The key to this puzzle may lie in the nature of the model and the lack of market imperfections. In this study, we take a theoretical approach in a dynamic multi-period environment. We incorporate competition between disparate economic agents and impose financial frictions (i.e., imperfections) that are in the form of hedging and borrowing limits on them. Our model gives rise to multiple equilibria, each with unique market clearing prices, with the market switching between these equilibria. Our analysis incorporates a comprehensive consideration of the risks faced by the futures markets participants (i.e., speculators and hedgers) and leads to a better understanding of the puzzle.



**JEL Classification Codes**: D58, D74, G13, N20

**Key Words**: Futures Pricing, Contango, Financial Frictions, Normal Backwardation.

## 1. Introduction

The futures pricing puzzle is one of the oldest asset pricing puzzles in finance and has been an object of study for over a hundred years (Rouwenhorst and Tang, 2012). In its most basic form, it signifies the discrepancy between the empirical behavior of commodity prices and theoretical assertions, which suggest that the futures price of an asset should be an unbiased forecast of its realized spot price when the contract matures. Our paper suggests that this puzzle arises due to the presence of multiple equilibria in the futures market, and the frequent shifting of market outcomes between these equilibria. This interpretation has important implications in that it implies that price risk is not always efficiently re-allocated by futures markets.

Futures markets provide opportunities for producers and end-users of commodities (henceforth, commonly termed hedgers) to mitigate the risk associated with holding or buying these assets by hedging against future changes in the spot price. Speculating counterparties to these transactions are willing to bear this risk and are compensated via an 'insurance' premium (Stoll, 1979; and Jebabli and Roubaud, 2018). Anderson and Danthine (1983) propose two views on the relationship between the spot and futures price. The first assumes that the futures prices are unbiased predictors of the future spot rate at contract expiration. The other view, consistent with Keynes (1930), suggests that the two prices are not equivalent. The difference in prices implies that hedgers pay an insurance premium to speculators, the size of which is a function of their relative levels of risk aversion and the net positions of the hedgers. This price deviation is termed as Normal Backwardation when the futures price lies below the expected spot price (Anderson and Danthine, 1983). In contrast, the deviation is termed as Contango when the futures price trades above expected spot.[1] In either case, the flow of profits between hedgers and speculators is akin to the premiums from an insured to the insurer (Lee, 2013).

Despite Keynes's confidence on the 'normality' of backwardation, the majority of the research displays mixed results on the reliability of this phenomenon. Numerous studies have arrived at inconsistent results despite examining the same commodities over similar time periods and employing extremely sophisticated statistical techniques (Houthakker, 1957; Rockwell, 1967; Chang, 1985; Rouwenhorst and Tang, 2012; Mishra and Smyth, 2016).

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1 In his seminal paper, Keynes (1930) did not strictly differentiate between discount and premium, and referred to them both as *Normal Backwardation*. However, contemporary literature differentiates between the two outcomes.

This discrepancy between the theory and empirics might arise as a result of the strong assumptions of perfect capital markets, without assuming any risk-shifting behavior on the part of economic agents entering into a futures contract. The novelty of our study consists of introducing frictions into the theoretical model of futures markets, in the form of borrowing and hedging constraints to enforce compliance or fulfilment of the futures contract.[2] These constraints differ for hedgers and speculators. Furthermore, they also differ between *short* hedgers (such as producers) and long *hedgers* (such as consumers).[3] Ignoring these constraints on futures markets participants can lead to erroneous futures pricing conditions, because financial constraints restricting risk-shifting confer market power to one or more agent type in the economy.

The main purpose of this paper is to illustrate the impact of these frictions on the pricing of futures. That is, to reconcile the conflicting results of futures prices in the empirical literature to the presence of multiple varying equilibria where the pricing parameters of futures (and hence its discount or premium to expected spot) are *not uniquely* determined. In this way, our paper provides a theoretical explanation for the futures pricing puzzle which has been well documented in the literature.

We model commodity futures in a simple overlapping generations model with Rational Expectations (RE).[4, 5] Competition between economic agents determines the *supply* and

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2 David Adler (2014, pp. xi) expounds that: "A friction is an impediment, obstruction, or constraint that prevents markets and economies from working smoothly. This approach to economics is a departure from the frictionless, idealized world of classical economic and financial theory, such as the Arrow-Debreu model of general equilibrium or the capital asset pricing model (CAPM) of Sharpe (1964) and others…..(F)rictions….are a central issue in today's revision of economics, one that economists had previously overlooked or had trouble formally modelling."

3 The strict assumption of perfect capital markets without mitigation of risk-shifting implies that the gyrations stemming from the financial sector of the economy will be allowed to permeate the real sector, where the transaction costs of offsetting (or changing) operations are prohibitive. Management Science literature classifies this as a reverse of the well-known Bull-Whip or the Whiplash Effect (Wang and Disney, 2016). This effect focuses on the economic inefficiencies caused by the amplification of information on the demand of an end-product up the supply chain, constituting of retailers and suppliers, leading to an increase in the variance of demand and hence cost of inventory. In the context of our study, storage operators of a commodity (and ultimately the consumers) are extremely vulnerable to cost overruns as they are downstream of the supply chain. This is the reason why we classify all producers and consumers as strictly hedgers.

4 Overlapping generations modelling is adopted for its rigor and strong following in the academic and policy communities (Weil, 2008). This framework also allows participants to stack up on open interests on futures (subject to their binding constraints). Within this setting, employing RE allows asset prices to aggregate and reveal private information in equilibrium (Biais et al., 2010). This is a consequence of the Efficient Market Hypothesis where capital market participants can easily decipher any private information held by a counterparty by observing their trading patterns (EMH – Bray, 1981; Jawadi et al. 2017; and Jebabli and Roubaud, 2018). This assumption is valid for the commodities sector as governmental and non-governmental organizations disseminate information to economic agents about future commodity price/ demand (Tang et al., 2015).

*demand* side relationships and consequently the equilibrium parameters of futures contract. Nonetheless, our model is distinct from the widely used models in the literature assuming perfect capital markets (Breeden, 1980; Jagannathan, 1985; and Kolb, 1996; Hirshleifer, 1988; De Roon et al., 2000; Arseneau and Leduc, 2013; Gorton et al., 2013; Hamilton and Wu, 2015). These models, based as they are on homogenous agents, do not reflect the real world where open positions in the futures market are concentrated in the hands of an elite group of financial market agents. In contrast, the approach taken in this paper is consistent with this empirical observation.

We initially study the case of a perishable commodity in an economy with three types of agents. There are two types of hedgers, commodity producers and consumers, along with a single type of speculators. We model competition between these economic agents in the presence of borrowing and hedging constraints which reflect the reality of futures markets. The commodity producers (as hedgers) are confined to shorting an amount of futures they are able to produce in the worst state of the economy. Likewise, the consumers (as hedgers) are constrained to taking positions that they can fulfill in the worst state of the economy. Finally, the speculators are constrained by internal risk management, which prevents them from taking excessive positions in the futures market. Thus the basic model yields results which shed light on the futures pricing puzzle.

After solving the basic model, with a perishable commodity, we extend our study to include a non-perishable good, which gives rise to the possibility of storage. We also extend our analysis to model the effects of institutional investors in futures markets, as a means of modelling the 'financialization of commodities' (Basak and Pavlova, 2016; and Aït-Youcef, forthcoming). We ignore the imposition of position limits as they are deemed to be counterproductive (ap Gwilym and Ebrahim, 2013).

Our model extends the framework of ap Gwilym and Ebrahim (2013), where random shocks of production (or yield risks) emanating from the supply side impact on the equilibrium pricing of the commodity, leading to price risks on the demand side. This has

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[5] Recent studies such as Sockin and Xiong (2015) and Singleton (2017) have emphasized on informational frictions in the commodity markets. Practitioners, however, refute this perspective as stated in Meyer and Hume (2016, p. 20): "*The advent of ship tracking software, satellite data on crop conditions and other services has also eroded the value of traders' inside knowledge of commodity flows. "Technology in the last 20 years has made information asymmetry a much less common thing," said Richard Payne of Accenture Consulting, who is a former Cargill executive."* Nonetheless, informational advantages added to our analysis will exacerbate the market power of one set of economic agents vis-à-vis others. This does not change the quality of our results.

credence in the real world as agricultural commodities are subject to the fluctuations of weather on the supply side, giving rise to changes in prices which affect the demand side.

We make the following assumptions in our analysis: First, the agents in our economy are heterogeneous across types, but homogenous within their type. In other words, producers in our study are strictly *short* hedgers, while consumers are strictly *long* hedgers. Furthermore, speculators take a position opposite to that of the aggregate hedging one (incorporating the netting of producers' and consumers' positions). Second, the short horizon of our study constrains the various classes of agents to enter into binding futures contracts, which cannot be offset as in a multi-period framework. Third, our study prices futures contracts in a nonlinear setting instead of a linear, 'cash and carry', one where arbitrage is not feasible (Varian, 1987). This yields futures prices as a function of risk aversion parameters of agents in the economy. Our study can be interpreted as one in a complete markets setting, entailing 'iron-clad' or 'firm commitment' contracting in the sense of Popescu and Seshadri (2013), by imposing constraints on agents to curtail risk-shifting, and thus defaults.

Our dynamic model, yielding a multitude of equilibria, sheds light on the futures pricing puzzle. The economic ramifications of our results are important, as the multiple equilibria characterize the extent to which the price risk of the commodity is being dissipated in the financial system. This issue is of interest to academics, practitioners, policy-makers as well as regulators as commodity price risk can instigate financial crises. The most efficient equilibrium involves the optimal allocation of the commodity price risk across the different agents. The further we go down the pecking order of equilibria, i.e., from the most efficient to the least efficient ones, the lower is the reduction in price risk. In other words, the decreasing order of Pareto-efficiency signifies an increase in the price risk. The traditional futures market view is that the role of hedgers is to disperse the underlying commodity price risk, while that of the speculators is to shoulder some risk in order to improve the return of their overall portfolio. Our multiple equilibria arise from the trade of financial claims between different hedgers and/ or speculator. This gives rise to erratic behavior in the differences between futures and expected spot. That is, it represents a change in the cost of insurance (in the form of a premium or discount) as the market moves from one equilibrium to another. This can even result in the migration of a premium to a discount or vice versa, thus rationalizing the futures pricing puzzle. This also implies that creating a perfect hedge with commodity futures is not guaranteed in our framework.[6]

Testing our theoretical results empirically is beyond the scope of this paper as the moment conditions, characterizing each equilibrium used for estimating the model based on the Generalized Method of Moments (GMM), are changing. The number of moment conditions is dynamically unstable and varies depending on whether the budget, input and hedging constraints are satisfied or not. Consequently, employing different moment conditions lead to different estimation results.

This paper is organized as follows: Section 2 reviews the literature, while Section 3 models futures contract of a perishable commodity and illustrates its solution. This result is extended to the case of storable commodity in Section 4. Finally, Section 5 concludes the study.

## 2. Related Literature

Recent work by Rouwenhorst and Tang (2012) (R&T) revisits the empirical literature to check the theory of normal backwardation. Their study examines: (i) commodities positions; (ii) risk premia; (iii) profit and speculation; and (iv) storage decisions. R&T found that with respect to net positions of commercials (aka hedgers), the directional hedges were consistent with normal backwardation. However, the tests of several empirical papers indicated that there is limited evidence of normal backwardation in explaining risk premia and that the explanations are more consistent with storage decisions. With respect to risk premia, R&T found results coherent with previous literature indicating that, at the individual commodity level, the evidence for normal backwardation was *not* statistically significant. However, at the aggregated portfolio level, the average basis for a majority of commodities is negative. This result implies that futures curves are upward sloping (contango) in most markets. Additionally, the work conducted by Chang (1985) is reexamined on a more recent dataset to identify the ability of non-commercials (aka speculators) to earn profits. Profitability of speculator's trades, a necessary condition for normal backwardation, yields inconsistent results. Analyses conducted across 28 commodities markets at the weekly and monthly

6 The quality of our results do not change when a set of institutional agents are allowed to invest in commodity markets in conjunction with financial ones. Our tweaked final model is able to: (i) help integrate the commodity and financial markets; and (ii) improve risk-sharing that is contingent on the risk-bearing capacity of financial investors. These two additional features lead to a commodity risk-premium, a subresult which is in agreement with that of Acharya et al. (2013), and Basak and Pavlova (2016). This subresult, however, does not influence our main result, which is the intricacy of pricing of futures under multiple equilibria.

levels, reveal that the market timing ability of speculators is *not* statistically significant in any of the 28 markets. At the individual asset level, timing is significant in 7 out of 28 markets, although the results are *unstable*. That is, markets with timing ability at the weekly level are not the markets with timing ability at the monthly level and vice versa. The authors found that these results conflict with Chang's findings, which suggest normal backwardation.

Given the lack of consistent results regarding normal backwardation, the theory of storage has also been used to understand risk premia in futures markets. This is largely attributable to the interaction that exists between the production side of the market (supply) and the consumption side (demand). Work has been done by a number of authors (Kaldor, 1939; Working, 1949), which rationalize contango as emanating from the difference between storage costs (costs of holding inventories) and the convenience yield (a real option that fluctuates in value and that is inversely related to the inventory level). Further research links these risk premia to inventory levels (Fama and French, 1987; Acharya et al., 2013; and Gorton, Hayashi and Rouwenhorst, 2013). The findings within these strands of research reveal fairly consistent patterns, which illustrate risk premia as declining with inventory levels. Ultimately, risk premium volatility is a function of the level of inventories. R&T suggest that the ambiguity of the findings may be a function of the temporal nature of the research conducted and the results may be sample and/or time specific.

Given the above inconsistencies, many researchers have opted for partial and general equilibrium models of commodities pricing and storage (Arseneau and Leduc, 2013; ap Gwilym and Ebrahim, 2013; Sockin and Xiong, 2015). Constrained optimization allows researchers to control the environment of the model in line with theory. This is however not consistent with the noisiness of actual data in the real world. For example, Arseneau and Leduc (2013) integrate a canonical rational expectations model into a general equilibrium framework. They find that the movement of interest rates in a general equilibrium context enhances the effects of competitive storage on commodity prices. Relative to fixed interest rate models, a non-stationary interest rate leads to higher persistence in commodity prices (and simultaneously lower volatility) with competitive storage. Many of the policy prescriptions offered by regulators been built upon conjecture that has been found to be erroneous empirically. Arseneau and Leduc (2013) provide evidence which refutes the assertion that institutional trading drives commodity prices away from fundamentals. They find that endogenous movements in interest rates, implied under general equilibrium, reinforce the impact of storage on commodity prices. This leads to persistence in commodity prices due to consumption smoothening. They develop the framework of the seminal work of Williams and Wright (1991) who use a partial equilibrium model in which the interaction of consumers, producers and storers endogenously determines prices. Their model incorporates a two period economy with households (making a static optimal labor supply decision and an optimal intertemporal savings decision) and a production sector (final goods sector and primary commodity sector). The commodity sector incorporates producers and risk neutral speculators who exist to smooth price volatility in the primary commodity sector. This volatility is smoothed via the competitive storage market for the commodity. Equilibrium in the model is achieved by way of three household optimizing conditions (labor supply, savings and primary commodity demand), two firm optimizing decisions (labor demand and industrial demand for the primary commodity), the profit maximizing condition for the commodity producer and the optimal, non-negative storage condition. A key outcome of this line of research is the ability of the household to consume smoothly over time given the level of storage, which is a function of interest rates and subsequently intertemporal savings decisions, in equilibrium. Not only do the results provide salient outcomes on commodity volatility and consumption, but also provide theoretically valid insights that can influence policy as it relates to commodity market intervention/control.

Additionally, Acharya et al. (2013) suggest that the impact of speculation on commodity prices is not zero as determined in many studies. They rationalize that increase in speculation has stabilizing impacts in the futures market. This emanates from the enabling of producers to more effectively manage inventories with short positions. The authors contend that demand for commodities is largely driven by actual physical demand, despite the price instability caused by speculators in the past. In this paper, the authors use an equilibrium model of commodity markets, where speculators are capital constrained and producers have hedging demand for commodities futures. This model incorporates both channels of the traditional theory of futures prices, hedging demand and inventory management. In addition to a simulated approach, there is also an empirically motivated component of the paper, i.e. producer’s default risk. The model is a two period model of commodity spot and future prices, which incorporates both optimal inventory demand (a la Deaton and Laroque, 1992) and hedging pressure demand (akin to Anderson and Danthine, 1980; and Hirschleifer, 1988). The structure of the model is similar to research mentioned previously (including Arseneau and Leduc, 2012; and ap Gwilym and Ebrahim, 2013). There are three agents in the model: (i) consumers who demand the commodity; (ii) producers who manage profits via inventory management and futures hedging; and (iii) speculators whose demand for spot and commodity futures jointly determine the futures price with producers hedging demand. They find that: (i) commodity producers' default risk is positively related to their hedging demand; (ii) an increase in the default risk of producers is predictive of an increase in the excess returns on short-term futures; (iii) the effect of default risk of producers on risk premia is greater in periods of higher volatility; (iv) the proportion of risk premium attributable to default risk is greater when the demand for assets from institutional investors is diminishing; and (v) as producers' default risk increases, they will hold less inventory, thereby depressing current spot prices.

The increase in the financialization of futures markets has been well documented in the literature (Gosh et al., 2012; Tang and Xiong, 2012; Algieri, 2016; Basak and Pavlova, 2016; and Aït-Youcef, forthcoming). Simultaneously, there has been a significant increase in the volatility in futures markets as well (Cheng et al., 2015; Roy and Roy, 2017; and Mo et al. 2018).[7] As a result of the highly volatile commodities pricing environment over the past eight years, many practitioners and regulators have proposed that restrictions be placed on commodities futures in the form of position limits (U.S. Senate Subcommittee Report, 2009). One of the most notable examples is the hedge fund manager Michael Masters. In his testimony before the U.S. Congress, Masters claimed that speculation in the commodities markets has led to the increase in price volatility, reducing liquidity and impeding upon the price discovery process. Masters suggested that the increase in prices was brought about by a significant increase in the demand side, creating a 'bubble' in the commodities markets. Empirically, however, there is very little evidence supporting his claim as markets are sufficiently liquid to absorb the large order flows (Hamilton and Wu, 2015). Thus, index funds' investment in commodities futures markets does not harm the price discovery mechanism. Regulatory support for position limits in futures markets 'appears to be ill conceived' (ap Gwilym and Ebrahim, 2013).

## 3. The Theoretical Model

We assume a simple overlapping generations economy with two goods, a commodity and a numeraire good, and three types of agent, Producers (**P**), Consumers (**C**) and Speculators (**S**). The commodity is produced solely by the Producers and consumed solely by

7 Cheng et al. (2015) illustrate that volatility in the futures markets, stemming from crisis, leads to financial traders [hedgers] to reduce their long [short] positions respectively.

the Consumers.[8] Speculators neither produce nor consume the commodity, but may choose to hold futures contracts relating to it as counter-parties to either Producers or Consumers.

Agents of generation ‘t’ are born in period t, endowed with an amount $e_{i_{t+1}}$ of the numeraire (where i ∈ {P,C,S} represents agent type). During this period, Producers decide how much of their endowment to devote to producing the commodity, and all agent types decide whether or not to engage in futures contracting. In the following period, t+1, agents of this generation are mature, the stochastic output of the productive process is realized and futures contracts are settled. The agents also trade and consume the goods.

The production process used for the commodity is subject to random shocks ($\tilde{\theta}_{t+1}$) stemming from exogenous forces such as weather or any idiosyncrasy of the production process. The distribution of $\tilde{\theta}_{t+1}$ is presumed to be bounded, and is known to all agents. Each producer converts $x_{t+1}$ units of the numeraire good into $\tilde{y}_{t+1}$ units of the commodity using the production function $\tilde{y}_{t+1} = g(x_{t+1}, \tilde{\theta}_{t+1})$. We assume that the production function is monotonically increasing in both arguments, and concave in the input x.

All agents are risk averse, maximizing the expectation of their respective strictly concave and twice continuously differentiable (Von Neumann-Morgenstern) utility functions.

### *3.1. Financial Frictions:*

The producers and consumers are hedgers in our framework. We assume that financial frictions imply that the positions that these agents hold in futures markets are constrained by their ability to settle their positions under the worst state of the economy. Hence, producers abstain from contracting futures beyond what they produce in the worst state of the economy:

$$q_{P_{t+1}} \leq \text{Min.}[\, g(x_{t+1}, \tilde{\theta}_{t+1})\,] \tag{1}$$

where $q_{P_{t+1}}$ is the position held by the representative producer in the futures market. Similarly, consumers are constrained to holding a position that they will be able to settle:

$$q_{C_{t+1}} \leq \text{Min.}[\, \tilde{\rho}_{C_{t+1}}] \tag{2}$$

where $q_{C_{t+1}}$ is the position held by the representative consumer in the futures market, and their demand for the commodity is represented by $\tilde{\rho}_{C_{t+1}} = \tilde{\rho}_c(\tilde{p}_{t+1}, e_{C_{t+1}})$, and $\tilde{p}_{t+1}$ is the stochastic price of the commodity in the spot market.

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8 Initially, we assume that the commodity is perishable. This assumption is not critical to our analysis, and is relaxed in Section 4 to demonstrate the invariance of our basic results.

We refer to equations (1) and (2) as hedging constraints. They are crucial in controlling risk-shifting in our simple economy. Both academics and practitioners (Rolfo, 1980; Lee, 2003) corroborate this assumption. These constraints are central to the internal risk management of firms participating in futures markets. A recent report by the counterparty Risk Management Policy Group III (2008), known as the Corrigan Report, ascribes weakness in such compliance systems for instigating 'rogue' trading.

We assume that the speculators' participation in the futures markets is limited by a resource (or capital) constraint on the speculator akin to that espoused in Acharya et al. (2013). The Corrigan Report (Counterparty Risk Management Policy Group III, 2008) recommends such a constraint. We term this the internal risk management constraint and model it as a symmetric constraint limiting the capital exposure of speculators:

$$|q_{S_{t+1}}| \leq \bar{q}_{t+1} \tag{3}$$

We further assume that financial frictions prevent producers from borrowing funds that they may be unable to repay. Hence, the following input constraint restrains the resources employed in the production process to the endowment of the producer:

$$x_{t+1} \leq e_{P_{t+1}} \tag{4}$$

*3.2. The Commodity Producer:*

The goal of each of the $n_{P_t}$ Producers of generation 't' is to optimally select the amount ($x_{t+1}$) of endowment to be employed in the production process and the amount ($q_{P_{t+1}}$) of the commodity to be pre-sold in the futures market (at a unit price f) in order to maximize their expected utility of consumption. That is,

$$\mathit{Max.}\ E_t\{U_P(\tilde{c}_{P_{t+1}})\}$$
$$(\mathit{in}\ c_{P_{t+1}}, x_{t+1}, q_{P_{t+1}})$$

subject to the hedging constraint (equation 1), input constraint (equation 4) and the following budget constraint:

$$\tilde{c}_{P_{t+1}} + x_{t+1} \leq e_{P_{t+1}} + q_{P_{t+1}} f_{t+1} + \tilde{p}_{t+1}\,[\tilde{y}_{t+1} - q_{P_{t+1}}] \tag{5}$$

where $\tilde{c}_{P_{t+1}}$ is the stochastic consumption of the numeraire good by the producer when they are mature.

This optimization gives rise to the following first-order conditions:[9]

9 In this and the following sections, we follow the standard step by step microeconomic procedure for optimization, details of which are not given for brevity.

$$E_t[U'_P(\tilde{c}_{P_{t+1}})\tilde{p}_{t+1}g(x_{t+1},\tilde{\theta}_{t+1})] = E_t[U'_P(\tilde{c}_{P_{t+1}})]+E_t(\Phi_{t+1})-E_t[\mu_{t+1}[\text{Min.}(g(x_{t+1},\tilde{\theta}_{t+1}))] \qquad (6)$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} + \frac{E_t(\mu_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} \qquad (7)$$

where $\Phi_{t+1}$ and $\mu_{t+1}$ are the Lagrangian multipliers on the input and hedging constraints respectively.

Equation (6) implies that, in the absence of binding input or hedging constraints (i.e., $\Phi_{t+1} = \mu_{t+1} = 0$), the marginal utility of consuming out of earned income equals the marginal utility of consuming from endowment. If the input constraint is binding (i.e., $\Phi_{t+1} > 0$) then the optimal level of production is at the corner (i.e. $x_{t+1} = e_{P_{t+1}}$) and the marginal utility of consuming out of earned income exceeds the marginal utility of consuming from endowment. However, if the hedging constraint is binding (i.e., $\mu_{t+1} > 0$), the marginal utility of consuming out of earned income is below that of the marginal utility of consuming from the endowment. This is because extra production relaxes the constraint.

Equation (7) represents the *supply* side relationship of $q_{P_{t+1}}$ units of output pre-sold (at a price $f_{t+1}$). In other words, it links the futures price with that of the expected future spot prices. The covariance term (i.e., $Cov_t(U'_P(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})$ is negative, as an increase in the expected price of the commodity increases the wealth of the producer thereby leading to a decline in the marginal utility of the producer. This implies that, in the absence of a binding constraint (i.e., $\mu_{t+1} = 0$), illustrated below, the producer will optimally pre-sell $q_{P_{t+1}}$ units of the commodity at a discount to the expected spot price. This discount represents the cost of avoiding the risk of selling at the future spot price. Here, the producers do not have any power to extract economic surplus from the futures market.

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} < 0 \qquad (7b)$$

If, however, the hedging constraint is binding (i.e., $\mu_{t+1} > 0$), as illustrated below, then the discount, and hence the cost of ‘insurance’, is reduced. Here producers have the power to extract economic surplus.

$$f_{t+1} - E_t(\tilde{p}_{t+1}) > \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} < 0 \qquad (7c)$$

*3.3. The Consumer:*

The goal of each of the $n_{C_t}$ Consumers of generation 't' is to optimally select the amount ($q_{C_{t+1}}$) of the commodity to pre-purchase in the futures market along with the amount $[\tilde{\rho}_{C_{t+1}} - q_{C_{t+1}}]$ in the spot market in order to maximize their expected utility of consumption. Here, $\tilde{\rho}_{C_{t+1}}$ represents the consumer's demand for the commodity. That is,

$$\mathit{Max.}\ E_t\{U_C(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{C_{t+1}})\}$$
$$(\mathit{in}\ \tilde{c}_{C_{t+1}}, \tilde{\rho}_{C_{t+1}}, q_{C_{t+1}})$$

subject to the hedging constraint (equation 2) and the following budget constraint:

$$\tilde{c}_{C_{t+1}} + q_{C_{t+1}}\,(f_{t+1}) + \tilde{p}_{t+1}\,[\tilde{\rho}_{C_{t+1}} - q_{C_{t+1}}] \le e_{C_{t+1}} \tag{8}$$

where $\tilde{c}_{C_{t+1}}$ and $\tilde{\rho}_{C_{t+1}}$ are the consumption by the representative consumer of the numeraire good and the commodity respectively, while the remaining notations have the same meaning as stated earlier.

This optimization gives rise to the following first-order conditions:

$$E_t[U'^{\rho}_C(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}})] = E_t[\tilde{p}_{t+1}\, U'^{C}_C(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}})] - E_t[\Phi'_{t+1} \frac{\partial[\mathrm{Min}.\tilde{\rho}_{c_{t+1}}]}{\partial \rho_{C_{t+1}}}] \tag{9}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{\mathrm{Cov}_t(U'^{C}_C(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c\,t+1}), \tilde{p}_{t+1})}{E_t(U'^{C}_C(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} - \frac{E_t(\Phi'_{t+1})}{E_t(U'^{C}_C(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} \tag{10}$$

where $\Phi'_{t+1}$ is the Lagrangian multiplier on the consumer's hedging constraint.

Equation (9) illustrates the relationship between the expected marginal utilities of the numeraire good and the commodity. When the hedging constraint is not binding (i.e., $\Phi'_{t+1} = 0$), then the expected marginal rate of substitution between the two goods is equal to the expected price ratio. On the contrary, when the constraint is binding (i.e., $\Phi'_{t+1} > 0$), then consumption of the commodity decreases, reflecting its lower expected marginal utility.

Equation (10) represents the *demand* side relationship of $q_{C_{t+1}}$ units of the commodity pre-bought at a price $f_{t+1}$. In other words, it links the futures price with that of the expected spot price. The covariance term (i.e., $\mathrm{Cov}_t(U'^{C}_C(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{C_{t+1}}), \tilde{p}_{t+1})$ is negative [positive] in our two good economy when the substitution effect dominates [is dominated by] the income effect. That is, an increase in the expected price of the commodity decreases the relative planned consumption of it thereby increasing [decreasing] the planned consumption of the numeraire. This results in the following relationship between futures and expected spot prices. When the hedging component is not binding, (i.e., $\Phi'_{t+1} = 0$), demonstrated below, the consumer will optimally pre-purchase $q_{c_{t+1}}$ units of the commodity (in the futures market) at a *discount* [*premium*] to the expected spot price. This discount compensates for the riskiness of the expected spot price. Here, the consumers have no power to extract economic surplus.

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{c_{t+1}}, \tilde{\rho}_{c\,t+1}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{c_{t+1}}, \tilde{\rho}_{c_{t+1}}))} < 0 \text{ when the substitution effect dominates} \tag{10bi}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{c_{t+1}}, \tilde{\rho}_{c\,t+1}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{c_{t+1}}, \tilde{\rho}_{c_{t+1}}))} > 0 \text{ when the income effect dominates} \tag{10bii}$$

In contrast, when the hedging constraint is binding (i.e., $\Phi'_{t+1} > 0$), as demonstrated below, then the above discount [premium] is higher [lower]. Here, the consumers enjoy the power to extract economic surplus.

$$f_{t+1} - E_t(\tilde{p}_{t+1}) < \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{c_{t+1}}, \tilde{\rho}_{c\,t+1}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{c_{t+1}}, \tilde{\rho}_{c_{t+1}}))} \tag{10c}$$

*3.4. The Speculator:*

The goal of each of the $n_{s_t}$ Speculators of generation 't' is to optimally select the amount ($q_{s_{t+1}}$) of the commodity to pre-purchase in the futures market in order to maximize their expected utility of consumption. That is,

$$\underset{(in\ c_{s_{t+1}},\ q_{s_{t+1}})}{Max.}\ E_t\{U_S(\tilde{c}_{s_{t+1}})\}$$

subject to the internal risk management constraint (equation 3) and the following budget constraint:

$$\tilde{c}_{s_{t+1}} + q_{s_{t+1}} f_{t+1} \leq e_{s_{t+1}} + q_{s_{t+1}} (\tilde{p}_{t+1}) \tag{11}$$

where $\tilde{c}_{s_{t+1}}$ is the consumption of Speculator at (t+1), while the remaining notations have the same meaning as stated earlier. It should be noted that the sign of $q_{s_{t+1}}$ may be positive [negative] representing a long [short] position in the futures contract. In other words, speculators arbitrage between futures and spot prices.

This optimization gives rise to the following first-order condition:

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_S'(\tilde{c}_{s_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{s_{t+1}}))} - \frac{E_t(\Phi''_{t+1})}{E_t(U_S'(\tilde{c}_{s_{t+1}}))} \tag{12}$$

where $\Phi''_{t+1}$ is the Lagrangian multiplier on the speculator's internal risk management constraint.

Equation (12) represents the *demand* [*supply*] side relationship for positive [negative] units of output ($q_{s_{t+1}}$) pre-purchased [pre-sold] at a price $f_{t+1}$. It also constitutes an arbitrage pricing condition for the Speculator. Here, the covariance term (that is, $Cov_t(U'_S(\tilde{c}_{s_{t+1}}), \tilde{p}_{t+1})$ is negative [positive] as an increase [decrease] in the expected price of the commodity implies an increase in the wealth of the Speculator, and thus, a decrease in the expected marginal utility of consumption. Thus, equation (12) can be interpreted as follows: The Speculator will optimally pre-purchase [or pre-sell] $q_{s_{t+1}}$ [$-q_{s_{t+1}}$] units of the commodity at a discount [premium] to expected spot price in the absence of the risk management constraints ($\Phi''_{t+1}$) as explained below. This discount compensates her/ him for bearing the risk of the spot price. Here speculators have no power to extract economic surplus.

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_S'(\tilde{c}_{s_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{s_{t+1}}))} < 0 \;\; \forall\, q_{s_{t+1}} > 0 \tag{12bi}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_S'(\tilde{c}_{s_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{s_{t+1}}))} > 0 \;\; \forall\, q_{s_{t+1}} < 0 \tag{12bii}$$

If the long risk management constraint (i.e. $\Phi''_{t+1} > 0$) is binding, the discount is widened. In contrast, if the short risk management constraint (i.e. $\Phi''_{t+1} < 0$) is binding the premium is widened. Here, speculators enjoy the power to extract economic surplus.

$$f_{t+1} - E_t(\tilde{p}_{t+1}) < \frac{Cov_t(U_S'(\tilde{c}_{s_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{s_{t+1}}))} \tag{12c}$$

*3.5. Market Clearing and Solution of the Model:*

A Rational Expectations Equilibrium (REE) is defined as the one in which all agents in the economy are aware of: (i) the probability distribution of the random shock ($\tilde{\theta}_{t+1}$) of the production process; along with the (ii) structure and parameters of the economic model.

A REE satisfies:

a) the first order conditions represented by equations 6, 7, 9, 10 and 12,

b) the production function,

c) all three budget constraints (i.e., equations 5, 8, and 11) with equality as they bind, and

d) the clearing of the market for the commodity (equation 13 below) along with that of futures market (equation 14), and the market for the numeraire good (equation 15).

$$n_{C_t}\,[\tilde{\rho}_{c_{t+1}}] = n_{P_t}\,[g^*(x_{t+1}{}^*, \tilde{\theta}_{t+1})] = n_{P_t}\,[\tilde{y}^*{}_{t+1}] \quad (13)$$

$$n_{P_t}\,q_{P_{t+1}} = n_{C_t}\,q_{C_{t+1}} + n_{S_t}\,q_{S_{t+1}} \quad (14)$$

$$n_{P_t}\,\tilde{c}_{P_{t+1}} + n_{C_t}\,\tilde{c}_{C_{t+1}} + n_{S_t}\,\tilde{c}_{S_{t+1}} = n_{P_t}\,e_{P_{t+1}} + n_{C_t}\,e_{C_{t+1}} + n_{S_t}\,e_{S_{t+1}} \quad (15)$$

**[Insert Table about Here]**

The model yields numerous solutions for generation 't' contingent on the market power of one or more agent to extract economic surplus. This is dependent on: (i) whether the input or the hedging constraints are binding; and (ii) whether one of the economic agents fails to participate in the futures market. A total of 56 solutions (including 16 double ones) are observed. The initial 28 (including 8 double ones) constitute as interior solutions (i.e., $x_{t+1} < e_{P_{t+1}}$), while the remaining 28 (including 8 double ones) comprise as its replicative corner solutions (i.e., $x_{t+1} = e_{P_{t+1}}$). The initial 28 (including the 8 highlighted double solutions in blue) are illustrated in Table 1. This result is an advancement relative to ap Gwilym and Ebrahim (2013) as we illustrate solutions (such as 2, 3, 4, 7, 8, 10, 12, 14 and 15) where one or more agents are priced out of the futures market. This is because the first order condition on the amount of commodity pre-purchased (or pre-sold) in the futures market (i.e., $\frac{\partial L_t}{\partial q_{t+1}}$) is not satisfied. This development relative to the previous study comes in spite of us employing a symmetric risk management constraint on the Speculator. The basic 28 solutions range from the highest ranking (or most efficient) equilibrium $(PCS)_{t+1}$ to the mid-ranking equilibria (including $PC_{t+1}$, $PS_{t+1}$, $CS_{t+1}$, $P'C'_{t+1}$, $P'S'_{t+1}$, $C'S'_{t+1}$) to the lower ranking ones (such as $P_{t+1}$, $C_{t+1}$, $S_{t+1}$, $(P')_{1\text{-}2_{(t+1)}}$, $(C')_{1\text{-}2_{(t+1)}}$, $(S')_{1\text{-}2_{(t+1)}}$). It should be noted that an equilibrium such as $PC_{(t+1)}$ denotes one where the pricing of futures is determined jointly by Producers and

Consumers, while Speculators are price-takers, extracting economic surplus. Finally, $(P')_{1_{(t+1)}}$ and $(P')_{2_{(t+1)}}$ represent equilibria where the pricing of futures is determined solely by the Producer with another agent as a price-taker and the last one being priced out of the futures market. The basic solutions include some that do not make economic sense. That is, (i) solutions such as 6, 11, 16, 18, 19, 22, 24, 27 and 28, where the binding hedging or risk management constraints conflict with the Kuhn-Tucker Conditions; or (ii) solutions such as 20, 23 and 26, where the economic agent, who is supposed to set the price of the futures contract does not participate in the market. When these inconsistent results are removed, we are left with 16 interior solutions and 16 corner ones. We elaborate on these feasible solutions in the Appendix.

Under the dynamics of the model, the solution for the next generation depends on the heterogeneous growth of the competing agents impacting on the market clearing condition of the numeraire good through their aggregate endowments (equations 13 and 15). If the endowments of the agents for period (t+2) change, it impacts on their budget constraints and their market clearing conditions (equations 4, 5, 8, 11, 13 and 15). If the technology of production changes or productivity improves, this impacts on the production function $[g_{t+2}(x_{t+2}, \tilde{\theta}_{t+2})]$ and subsequently the hedging constraints and the aggregate demand/ supply of the commodity in the subsequent period (equations 1, 2 and 11). Finally, changes in the probability distribution of the random shock $(\tilde{\theta}_{t+2})$ along with the structure and parameters of our model (described above) impacts on the Kuhn-tucker conditions of all agents leading to a change in equilibria. This implies that as generations advance from ‘t’ to ‘(t+1)’ to ‘(t+2)’ etc., we will observe movements between equilibria, rationalizing the Futures Pricing Puzzle.

### *3.6. Key Result of Model*

**Proposition 1:**

Hedging a perishable commodity’s price risks in a perfect and complete market with financial frictions (encompassing heterogeneous producers, consumers, and speculators in a simple one period overlapping generations economy) is feasible in multiple Rational Expectations Equilibria ranked in a pecking order of decreasing efficiency. The equilibria range from an unconstrained interior one, where none of the agents have any economic power; to corner ones, where one or more agents have market power. Normal backwardation or contango are feasible for the same commodity contingent on the equilibria at hand. Extending our results of Generation 't' to that of Generation '(t+1)' yield volatile movements between multiple equilibria.

This result helps to explain the futures pricing puzzle as follows. The multiple equilibria exemplify the extent to which the commodity price risk is diffused in the financial system. The most efficient equilibrium entails maximum dispersion of the price risk. In contrast, the lower tier equilibria contain some residual price risk. Thus, the further we go down the pecking order of equilibria, the less is the price risk diffusion. The movement between equilibria involves different hedgers and speculators, giving rise to changes in the cost of insuring the price risk (in the form of a premium or discount). It is this varying cost of insuring the price risk that rationalizes the futures pricing puzzle.

**Proof of Proposition:** See the Appendix for the elaboration of our results.

Thus, our model yields multiple equilibria, ranked in a decreasing order of economic efficiency, with the market fluctuating between normal backwardation and contango. This result, illustrating as it does the varying costs of insuring the underlying commodity price risk, explains the futures pricing puzzle. The result emanates from our non-linear (i.e., risk-averse) constrained framework and is different from the unconstrained perfect capital markets models in the literature such as the: (i) risk-neutral (i.e. linear) 'cash-and-carry' one, where arbitrage yields a unique equilibrium; and the (ii) partial equilibrium framework where the futures pricing mechanism is unique for each commodity and is contingent on its systematic risk and cross-market hedging pressures (Breeden, 1980; Jagannathan, 1985; Hirshleifer, 1988; Kolb, 1996; De Roon et al. 2000; Hull, 2006; Arseneau and Leduc, 2013; Gorton et al., 2013; Hamilton and Wu, 2015). The key difference is that our equilibrium involves consumption smoothing, and the market power of participants emanating from the borrowing and hedging constraints that are absent in the above frameworks.

Testing for multiple equilibria using real data is extremely challenging for empirical researchers and can lead to conflicting results for the same commodity studied. One possible way to run the test is as follows. Since each equilibrium implies distinct moment conditions, one can think about building tests based on the GMM approach, in particular using the Sargan's J test of over-identification/ identification (Sargan, 1958, 1975; and Hansen, 1982). However, this is supplemented with strong assumptions requiring inflexible (budget, input or hedging) constraints on economic agents, which may not be realistic. Changes in the constraints lead to the market fluctuating between equilibria, and consequently between their respective moment conditions, resulting in inconsistent inferences.[10]

The appendix of our paper summarizes the solutions of our dynamic model under different frictions (i.e., borrowing and hedging constraints). The solutions are characterized in terms of moment conditions that take the form of equalities and inequalities. The appendix also shows that the solutions of our dynamic model are described by a number of these equality/inequality moment conditions depending on the constraints of economic agents that change over time. This instability in the constraints of economic agents implies different equilibria given by different equality/inequality moment conditions. This instability in the moment conditions (movements between multiple equilibria) make the estimation/identification of the model and consequently of the futures pricing extremely challenging. In addition, the estimation results will depend on the moment conditions that we use. Using different moment conditions might lead to different estimation results. Gregoir (2002) argues that the estimation of models with non-unique equilibrium using a GMM-type approach is a demanding task. Furthermore, in contrast to the standard GMM approach, where all moment conditions take the form of equalities, Moon and Schorfheide (2009) argue that estimating models in which a subset of moment conditions take the form of inequalities [see for example our Solution 9] may introduce identification problems. That is, there is a non-singleton subset of the model parameter space that satisfies the equality/inequality moment conditions under consideration.

As mentioned earlier, the constraints of economic agents change over time. This makes the use of low frequency data for the estimation of our dynamic (futures pricing) model very complicated. This is because in low frequency data the constraints are likely to change at each data point, which makes the estimation extremely challenging. In contrast, there is less likelihood in high-frequency data (e.g., every millisecond, second, or minute) that these constraints will change with a large number of data points. It might therefore be reasonable to assume that the constraints are stable within a day. With high frequency data, the number of observations within a day are very large. This makes for stable moments and ensures that the models are correctly identified/ estimated. However, if within a day the constraints of

[10] One way to surmount this difficulty is to use high frequency (tick-by-tick) data and make the assumption that the constraints are stable over a small interval of time, such as a day. High or ultra-high frequency data could provide enough observations, within a day, to estimate the parameters of interest using GMM, while the constraints leading to the moment conditions are stable. High frequency trading (HFT) is now garnering the attention of contemporary researchers (Brogaard et al., 2014).

economic agents lead to a subset of moment conditions that take the form of inequalities, then as Moon and Schorfheide (2009) point out this might introduce identification problems for our model.

Thus, from the above, we can infer that frictions lead to futures markets oscillating between multiple equilibria, thereby rationalizing the futures pricing puzzle.

## 4. Extension of Model to Storable Commodities and Institutional Investors

In the previous section, we analyzed the pricing of futures contract based on the Theory of Normal Backwardation (Keynes, 1930). This was done in an overlapping generations setting assuming the perishability of the commodity. This section extends the same issue but from the framework of the Theory of Storage (Kaldor, 1939; Working, 1949 and Brennan, 1958). For the sake of brevity, we do not present the full model here. This is available from the corresponding author on request.

We relax our assumption of *perishability* by allowing the stockpiling of commodities in a multi-period overlapping generations economy (Chaturvedi and Martinez-de-Albéniz, 2016). Each time 't' generation consists of an identical number of agents ($n_{P_t}$ Producers (**P**), $n_{C_t}$ Consumers (**C**), $n_{S_t}$ Speculators (**S**) and $n_{I_t}$ Inventory Operators (denoted by suffix **I**)), who are born in period t, young in period (t+1), mature/old in period (t + 2), and dead in period (t + 3) and beyond. Our earlier assumption of fixed endowments for all agents is retained in the submodel of storable commodities in Section 4.1 below. We tweak this submodel in Section 4.2 below to incorporate the *financialization of commodity markets* (Basak and Pavlova, 2016; and Aït-Youcef, forthcoming) by changing the Inventory Operators to that of Institutional Investors (denoted by suffix **II**) and bestowing them with stochastic endowments (representing their stake in financial markets) in periods (t+1) and (t+2).

Since accumulation of commodities impact on the distribution of their spot prices (Chambers and Bailey, 1996), the theory stipulates that spot prices, and thus their underlying futures prices, are impacted by the management of inventory.

Our extended model encapsulates Storage Operators (in Section 4.1) and Institutional Investors (in Section 4.2), who are subject to capital/ resource constraint which limits their hedging ability. Since capital is the essential ingredient needed to expand capacity, a capital constraint is akin to a capacity constraint. In this extended model, the aggregate demand for the commodity stems from consumers in conjunction with storage operators/ institutional investors. In equilibrium, this should equal the aggregate supply stemming from Producers in conjunction with inventory saved by storage operators or institutional investors (net of wastage). Thus, the equilibrium conditions incorporating the capital constraint (on storage operator or institutional investors) include a function linking consumer demand (with stochastic yield) with inventory (incorporating replenishment and depletion).

The model yields an optimal inventory policy for the storage operator or institutional investors. This is evaluated in an open interval where the expected value of marginal utility on consumption multiplied by the stochastic price of inventory (net of wastage – adjusted for cost of carry) is zero. The lower bound of the open interval, which has a value of zero, is termed as the Gustafson (1958) condition signifying the impossibility of carrying forward negative inventories. The upper bound of the open interval is the optimal operational yield of the producer ($\tilde{y}^*_{t+1}$).

## *4.1. Key Result of the Model with Storable Commodities*

Our model solution illustrates that hedging a commodity's price risk in a complete market with frictions (comprising of heterogeneous producers, consumers, speculators and storage operators – in an overlapping generations economy) again involves movement between Rational Expectations Equilibria, ranked in a pecking order of decreasing efficiency, with the market alternating between normal backwardation and contango (or vice versa). The solution in this case comprises at most 2(165) = 330 equilibria of which 2(104) = 208 are not economically viable. The initial interior solution thus comprises of 61 equilibria, which are described in the decreasing order of economic efficiency as follows: $PCSI_{t+1}$, $PCS_{t+1}$, $PCI_{t+1}$, $PSI_{t+1}$, $CSI_{t+1}$, $P'C'S'_{t+1}$, $P'C'I'_{t+1}$, $P'S'I'_{t+1}$, $C'S'I'_{t+1}$, $PC_{t+1}$, $PS_{t+1}$, $PI_{t+1}$, $CS_{t+1}$, $CI_{t+1}$, $SI_{t+1}$, $(P'C')_{1\text{-}2(t+1)}$, $(P'S')_{1\text{-}2(t+1)}$, $(P'I')_{1\text{-}2(t+1)}$, $(C'S')_{1\text{-}2(t+1)}$, $(C'I')_{1\text{-}2(t+1)}$, $(S'I')_{1\text{-}2(t+1)}$, $P''C''_{t+1}$, $P''S''_{t+1}$, $P''I''_{t+1}$, $C''S''_{t+1}$, $C''I''_{t+1}$, $S''I''_{t+1}$, $P_{t+1}$, $C_{t+1}$, $S_{t+1}$, $I_{t+1}$, $(P')_{1\text{-}3(t+1)}$, $(C')_{1\text{-}3(t+1)}$, $(S')_{1\text{-}3(t+1)}$, $(I')_{1\text{-}3(t+1)}$, $(P'')_{1\text{-}3(t+1)}$, $(C'')_{1\text{-}3(t+1)}$, $(S'')_{1\text{-}3(t+1)}$, $(I'')_{1\text{-}3(t+1)}$.[11]

## *4.2. Key Result of the Model with Institutional Investors*

Here the futures pricing model incorporates covariance terms involving the marginal utility of consumption of Institutional Investors with the stochastic spot prices of commodities (akin to an extended equation (12)). The smoothing of future consumption, which depends

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[11] Here the $(P'')_{1(t+1)}$ $(P'')_{2(t+1)}$ and $(P'')_{3(t+1)}$ represent three separate equilibria where the pricing of futures is determined solely by the Producer with another agent as a price-taker and the remaining two agents being priced out of the futures market.

upon payoffs from both financial and commodity markets, increases the correlation between the two markets. This result integrates the two markets, leading to portfolio diversification, depending on the capacity of financial investors as discussed in Acharya et al. (2013) and Cheng et al. (2015).[12] This leads to time varying commodity risk-premia as discussed in Bask and Pavlova (2016). However, this subsidiary result does not affect our main result, that futures markets are characterized by movements between multiple equilibria.

A dynamic economic environment again implies movement between equilibria, with the market alternating between normal backwardation and contango. This is difficult to analyze, generating irreconcilable results. This strengthens our earlier result rationalizing the futures pricing puzzle.

## 5. Concluding Comments

Futures contracting is employed to alleviate commodity production and price risks. The pricing of futures, however, has confounded academics, practitioners, policymakers and regulators for decades. This issue has led to researchers querying whether normal backwardation or contango is really the norm for commodities. This paper, however, takes a completely different approach to this crucial issue of asset pricing. We model the rivalry between agents in an overlapping generations (i.e., dynamic) economy with frictions in the form of borrowing and hedging constraints. These frictions alternately endow market power to the various market participants, leading to movements between equilibria. These equilibria can be ranked in a decreasing order of Pareto-efficiency, reflecting a diminution in the efficiency sharing the commodity price risk.[13] We attribute the inconsistent results in the empirical literature to the movement of the market between multiple equilibria, giving rise alternately to both normal backwardation and contango as it accrues from the exchange of financial claims between different hedgers and/ or speculator. This yields varying costs of insuring the commodity price risk (in the form of premium or discount).[14]

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[12] This result, implying that the shocks from either the financial or the commodity markets can spill over the other market, is in agreement with the observation of Cheng et al. (2015).

[13] Our basic result remains intact if one set of economic agents have privileged access to information. This is because informational advantage aggravates the market power of the privileged set vis-à-vis the remaining sets of agents.

[14] This result is true even in the presence of institutional investors who own mixed (financial plus commodity) asset portfolio.

We caution empirical investigators on drawing inferences from the time series data, which reflect movements between multiple equilibria, and hence pose significant challenges and give rise to conflicting results. This is because each equilibrium implies satisfaction of distinct moment conditions. This is hard to evaluate under a dynamic setting as the underlying strict assumption of the rigidity of constraints on economic agents breaks down. This rationale explains the futures pricing puzzle. One possible way to overcome the issue of changes in the constraints and consequently in the moment conditions is to use high frequency (i.e., tick-by-tick) price data and assume that the constraints are stable over a small interval of time, such as a day. High frequency data will provide enough observations within a day to estimate the parameters of interest so long as the constraints yielding the moment conditions are stable.

## APPENDIX

**Proof of Proposition:**

Our model solutions are described as follows. The first equilibrium constitutes as the unconstrained highest ranking one. The second group of equilibria constitutes of mid-ranked ones, which are subject to a single binding constraint. Finally, the third group of equilibria constitutes of the lowest ranked ones, which are subject to two binding constraints.

The Economically Highest Ranked Equilibrium ($PCS_{t+1}$ – **Solution 1**, see Table)

This signifies an interior equilibrium, where the hedging or risk management constraints on agents are not binding (i.e. $\mu_{t+1} = \Phi'_{t+1} = \Phi''_{t+1} = 0$). In other words, the futures contracting of all agents are in the satiation region. This equilibrium is evaluated by superimposing the demand-supply financial sector (i.e., futures) constraint (equation 14) on the respective pricing functions of various agents derived in Sections 3.2-3.4. Since this equilibrium involves *four endogenous* variables ($f_{t+1}$, $q_{P_{t+1}}$, $q_{C_{t+1}}$, $q_{S_{t+1}}$), four *independent* equations (7), (10), (12) and (14) are sufficient to yield a *unique* normal backwardation solution. In this case:

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} = \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} = \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))} < 0 \qquad (16)$$

Here the marginal utility of each agent adjusts in such a way that no agent can extract any economic surplus from the other. Deviation of futures price from expected spot price is given in terms of a covariance term (of marginal utility of stochastic consumption with price risk) divided by the expectation of marginal utility of consumption. The stochastic consumption parameter of all agents is impacted jointly by the operational (i.e., yield) and price risks, as illustrated in equations (5), (8) and (11). We note that the above pricing function (along with the others given below such as equations (17–22), and (23–31)) reflect the risk profile of the agents in the economy. This is different from that derived from the ″cash–and–carry″ arbitrage, which is free of risk aversion parameters.

The Economically Mid-Ranked Equilibria

In general, non-satiation of futures contracting of either agent in the economy leads to a strictly binding hedging or risk management constraint. The binding constraint endows the respective agent with market power. Since constraints lead to a reduction in welfare, the agent subject to it gains market power. S/he can take the price of futures determined by the competing agents at a favorable level.

To elaborate this point further:

(i) Equilibrium **$PC_{t+1}$ (Solution 5)**:

If the risk management constraint on the Speculator is binding ($\Phi''_{t+1} > 0$; $q_{S_{t+1}} = \bar{q}_{t+1}$), then the futures pricing is determined by both Producers and Consumers, while the economic surplus is retained by the Speculator. That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} = \frac{Cov_t(U'^C_C(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^C_C(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} < 0, \text{ and}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) < \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))} \tag{17}$$

Equilibrium **$PS_{t+1}$ (Solution 9)**:

Here, the Consumer's hedging constraint binds ($\Phi'_{t+1} > 0$; $q_{C_{t+1}} = \text{Min.}(\tilde{\rho}_{c_{t+1}})$). Futures pricing is thus determined by both Producers and Speculators, while the economic surplus is retained by the Consumer. That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} = \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))} < 0, \text{ and}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) < \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} \qquad (18)$$

Equilibrium **CS**$_{t+1}$ **(Solution 13)**:

Here, the Producer's hedging constraint binds ($\mu_{t+1} > 0$; $q_{P_{t+1}} = \text{Min.}(\tilde{y}^*_{t+1})$). Futures pricing is determined by both Consumers and Speculators, while the economic surplus is retained by the Producer. That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} = \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))} >/< 0, \text{ and}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) > \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} \qquad (19)$$

(ii) The equilibria given below are an improvement over the results of ap Gwilym and Ebrahim (2013).

Equilibrium **P'C'**$_{t+1}$ **(Solution 2)**:

Here the futures pricing is determined jointly by the Producers and Consumers, while the Speculator is priced out of the market ($q_{S_{t+1}} = 0$). That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} = \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} < 0 \qquad (20)$$

Equilibrium **P'S'**$_{t+1}$ **(Solution 3)**:

Here, the futures pricing is determined jointly by the Producers and Speculators, while the Consumer is priced out of the market ($q_{C_{t+1}} = 0$). That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} = \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))} < 0 \qquad (21)$$

Equilibrium **C'S'**$_{t+1}$ **(Solution 4)**:

Here, the futures pricing is determined jointly by the Consumers and Speculators, while the Producer is priced out of the market ($q_{P_{t+1}} = 0$). That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} = \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))} >/< 0 \qquad (22)$$

<u>The Economically Lowest Ranked Equilibria</u>

If two of the three constraints on the agents are binding, then the futures price is determined by the remaining agents' reservation price. Here the market power is extricated by the agents facing the constraint, as their futures pricing conditions hold as strict inequalities.

(i) <u>Equilibrium **P**$_{t+1}$ **(Solution 17)**:</u>

Here, the futures pricing is determined solely by the Producers, while the Consumers and Speculators are price-takers. That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} < 0,$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) < \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))}, \text{ and}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) < \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))} \qquad (23)$$

<u>Equilibrium **C**$_{t+1}$ **(Solution 21)**:</u>

Here, the futures pricing is determined solely by the Consumers, while the Producers and Speculators are price-takers. That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} >/< 0,$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) > \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))}, \text{ and}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) < \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))}. \qquad (24)$$

<u>Equilibrium **S**$_{t+1}$ **(Solution 25)**:</u>

Here, the futures pricing is determined solely by the Speculators, while the Producers and Consumers are price-takers. That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))} >/< 0,$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) > \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))}, \text{ and}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) < \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} \tag{25}$$

(ii) The equilibria given below are an improvement over the results of ap Gwilym and Ebrahim (2013).

Equilibrium (**P')$_{\mathbf{1}(t+1)}$** **(Solution 7)**:

Here, the futures pricing is determined solely by the Producers, while: (i) the Speculators are price-takers; and (ii) the Consumers are priced out of the market ($q_{C_{t+1}} = 0$). That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} < 0, \text{ and}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) < \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))}. \tag{26}$$

Equilibrium (**C')$_{\mathbf{1}(t+1)}$** **(Solution 8)**

Here, the futures pricing is determined solely by the Consumers, while: (i) the Speculators are price-takers; and (ii) the Producers are priced out of the market ($q_{P_{t+1}} = 0$). That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} >/< 0, \text{ and}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) > \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))} \tag{27}$$

Equilibrium (**S')$_{\mathbf{1}(t+1)}$** **(Solution 12)**

Here, the futures pricing is determined solely by the Speculators, while: (i) the Consumers are price-takers; and (ii) the Producers are priced out of the market ($q_{P_{t+1}} = 0$). That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))} >/< 0, \text{ and}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) < \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))}. \qquad (28)$$

(iii) Equilibrium (**P'**)$_{2(t+1)}$ **(Solution 10)**:

Here, the futures pricing is determined solely by the Producers, while: (i) the Consumers are price-takers; and (ii) the Speculators are priced out of the market ($q_{S_{t+1}} = 0$). That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} < 0, \text{ and}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) < \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} \qquad (29)$$

Equilibrium (**C'**)$_{2(t+1)}$ **(Solution 14)**

Here, the futures pricing is determined solely by the Consumers, while: (i) the Producers are price-takers; and (ii) the Speculators are priced out of the market ($q_{S_{t+1}} = 0$). That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) = \frac{Cov_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}), \tilde{p}_{t+1})}{E_t(U'^{C}_{C}(\tilde{c}_{C_{t+1}}, \tilde{\rho}_{c_{t+1}}))} >/< 0, \text{ and}$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) > \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))} \qquad (30)$$

Equilibrium (**S'**)$_{2(t+1)}$ **(Solution 15)**

Here, the futures pricing is determined solely by the Speculators, while: (i) the Producers are price-takers; and (ii) the Consumers are priced out of the market ($q_{C_{t+1}} = 0$). That is,

$$f_{t+1} - E_t(\tilde{p}_{t+1}) \;=\; \frac{Cov_t(U_S'(\tilde{c}_{S_{t+1}}), \tilde{p}_{t+1})}{E_t(U_S'(\tilde{c}_{S_{t+1}}))} \;>/<\; 0,$$

$$f_{t+1} - E_t(\tilde{p}_{t+1}) \;>\; \frac{Cov_t(U_P'(\tilde{c}_{P_{t+1}}), \tilde{p}_{t+1})}{E_t(U_P'(\tilde{c}_{P_{t+1}}))}\,. \tag{31}$$

Thus, the presence of the three tier equilibria in a dynamic setting can lead to the market moving between multiple equilibria, alternating between normal backwardation and contango, thereby increasing volatility and making the empirical analysis of time series data unintelligible, leading to a puzzling behavior of futures prices.

Q.E.D.

**Table 1 [Interior Solution for the case of perishable commodity]**

| Solution No. | Producer's Hedging Constraint ($\mu_{t+1}$) | Consumer's Hedging Constraint ($\Phi'_{t+1}$) | Speculator's Risk Mgt. Constraint ($\Phi''_{t+1}$) | Producer's K-T Condition ($q_{P_{t+1}}$) | Consumer's K-T Condition ($q_{C_{t+1}}$) | Speculator's K-T Condition ($q_{S_{t+1}}$) | Equilibrium Nomenclature (Yellow highlight indicates double equilibria) |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | > 0 | > 0 | > /< 0 | $PCS_{t+1}$ with Prod., Con. and Spec. setting price of futures |
| 2 | 0 | 0 | 0 | > 0 | > 0 | 0 | $P'C'_{t+1}$ with Prod. and Con. setting price and Spec. not participating ($q_{S_{t+1}} = 0$) |
| 3 | 0 | 0 | 0 | > 0 | 0 | > /< 0 | $P'S'_{t+1}$ with Prod. and Spec. setting price and Con. not participating ($q_{C_{t+1}} = 0$) |
| 4 | 0 | 0 | 0 | 0 | > 0 | > /< 0 | $C'S'_{t+1}$ with Con. and Spec. setting price and Prod. not participating ($q_{P_{t+1}} = 0$) |
| 5 | 0 | 0 | > 0 | > 0 | > 0 | > /< 0 | $PC_{t+1}$ with Prod. and Con. setting price, while spec. is a price taker (extricating economic surplus) |
| 6 | 0 | 0 | > 0 | > 0 | > 0 | 0 | This equilibrium does not make sense as constraint and K-T condition contradict each other. |
| 7 | 0 | 0 | > 0 | > 0 | 0 | > / < 0 | $P'_{1(t+1)}$ with Prod. setting price, Spec. being a price-taker and Con. not participating ($q_{C_{t+1}} = 0$) |
| 8 | 0 | 0 | > 0 | 0 | > 0 | > / < 0 | $C'_{1(t+1)}$ with Con. setting price, Spec. being a price-taker and Prod. not participating ($q_{P_{t+1}} = 0$) |
| 9 | 0 | > 0 | 0 | > 0 | > 0 | > / < 0 | $PS_{t+1}$ with Prod. and Spec. setting price, while consumer is price-taker (extricating economic surplus) |
| 10 | 0 | > 0 | 0 | > 0 | > 0 | 0 | $P'_{2(t+1)}$ with Prod. setting price, Con. being a price-taker, and Spec. not participating ($q_{S_{t+1}} = 0$) |
| 11 | 0 | > 0 | 0 | > 0 | 0 | > / < 0 | This equil. does not make sense as constraint and K-T condition contradict each other. |
| 12 | 0 | > 0 | 0 | 0 | > 0 | > / < 0 | $S'_{1(t+1)}$ with Spec. setting price, Con. being a price taker, and Prod. not participating ($q_{P_{t+1}} = 0$) |
| 13 | > 0 | 0 | 0 | > 0 | > 0 | > / < 0 | $CS_{t+1}$ with Con. and Spec. setting price, while Prod. is a price taker (extricating |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | economic surplus) |
| 14 | > 0 | 0 | 0 | > 0 | > 0 | 0 | $C'_{2(t+1)}$ with Con. setting price, Prod. being a price-taker and Spec. not participating ($q_{S_{t+1}} = 0$) |
| 15 | > 0 | 0 | 0 | > 0 | 0 | > / < 0 | $S'_{2(t+1)}$ with Spec. setting price, Prod. being a price-taker and Con. not participating ($q_{C_{t+1}} = 0$) |
| 16 | > 0 | 0 | 0 | 0 | > 0 | > / < 0 | This equil. does not make sense as constraint and K-T condition contradict each other. |
| 17 | 0 | > 0 | > 0 | > 0 | > 0 | > / < 0 | $P_{t+1}$ with Prod. setting price, while both Con. and Spec. are price-takers. |
| 18 | 0 | > 0 | > 0 | > 0 | > 0 | 0 | This equil. does not make sense as constraint and K-T condition contradict each other. |
| 19 | 0 | > 0 | > 0 | > 0 | 0 | > / < | This equil. does not make sense as constraint and K-T condition contradict each other. |
| 20 | 0 | > 0 | > 0 | 0 | > 0 | > / < 0 | This equil also does not make sense as Prod. is supposed to set prices while the other two agents are price-takers. However, $q_{P_{t+1}} = 0$. |
| 21 | > 0 | 0 | > 0 | > 0 | > 0 | > / < 0 | $C_{t+1}$ with Con. setting price, while Prod. and Spec. are price-takers (extricating economic surplus). |
| 22 | > 0 | 0 | > 0 | > 0 | > 0 | 0 | This equil. does not make sense as constraint and K-T condition contradict each other. |
| 23 | > 0 | 0 | > 0 | > 0 | 0 | > / < 0 | This equil also does not make sense as Con. is supposed to set prices while the other two agents are price-takers. However, $q_{C_{t+1}} = 0$. |
| 24 | > 0 | 0 | > 0 | 0 | > 0 | > / < 0 | This equil. does not make sense as constraint and K-T condition contradict each other. |
| 25 | > 0 | > 0 | 0 | > 0 | > 0 | > / < 0 | $S_{t+1}$ with Spec. setting price, while Con. and Prod. are price-takers. |
| 26 | > 0 | > 0 | 0 | > 0 | > 0 | 0 | This equil also does not make sense as Spec. is supposed to set prices while the other two agents are price-takers. However, $q_{S_{t+1}} = 0$. |
| 27 | > 0 | > 0 | 0 | > 0 | 0 | > / < 0 | This equil. does not make sense as constraint and K-T condition contradict each other. |
| 28 | > 0 | > 0 | 0 | 0 | > 0 | > / < 0 | This equil. does not make sense as constraint and K-T condition contradict each other. |

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