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Using an Instructional Fluency Approach to Teach Addition Skills in a Pupil Referral Unit: A Pilot Study

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ABSTRACT

Pupil referral units (PRUs) in Wales accommodate children who present with a range of difficulties that cannot be managed within a mainstream setting. Many children attending PRUs in Wales do not develop the numeracy skills that they need to support their learning across the curriculum. In an effort to teach and assess addition skills, the authors assessed the effects of using a combination of direct instruction (DI) and precision teaching (PT) in a PRU. Over six school weeks, we worked with five children (aged 7 to 10 years) on a 1:1 basis through the *Corrective Mathematics* addition curriculum (Engelmann and Carnine, 2005). Following each lesson, the children completed an individualised fluency assessment, which we tailored to their needs using PT methods. We collected baseline and follow-up data using the Test of Early Mathematics Ability (TEMA-3), the Wide Range Achievement Test (WRAT-4) and the *Corrective Mathematics* placement test. We also interviewed the children post-intervention to gain insight into their experience of the approach. The results provide evidence to support the use of an instructional fluency approach in a PRU setting to help children develop early mathematics skills, particularly for children who engaged in the sessions regularly. Due to the small sample size, the results of this study have limited generalisability but may help shape future

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research investigating effective strategies for teaching mathematics in PRUs.

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Pupil referral units (PRUs) accommodate children with complex needs that cannot be managed within mainstream school settings. Children attend a PRU for a variety of reasons relating to behaviour or illness; with a large proportion holding a diagnosis for an additional learning need (Estyn, 2015; Welsh Government, 2018). In their report, Estyn (2017) indicated that there are currently twenty-five registered PRUs in Wales, educating approximately 665 children. Estyn judged the quality of provision to be adequate in 36 per cent of these PRUs and unsatisfactory in a further 14 per cent. This was mainly due to the wide variability of outcomes children achieve (including the narrow breadth of qualifications) and low levels of participation and engagement in learning. Their inspection report also revealed that four in ten PRUs taught numeracy skills targeted towards early development and that many of the children did not develop the skills they needed to support their learning across the curriculum. As a result, the curriculum these units offered lacked challenge, did not ensure that all children in the classroom achieved and that the pace of learning was too slow.

The term *mathematics* refers to an international discipline, which integrates concepts, rules and procedures involving quantities and symbols. The term *numeracy* refers to the application of mathematical knowledge to every-day life (Resnick and Ford, 2008; Welsh Government, 2019). Longitudinal studies provide evidence to suggest that acquisition of age-expected mathematics and numeracy skills during early childhood is predictive of later mathematical achievement (Desoete et al., 2010; Missall et al., 2012). Underachievement in mathematics and numeracy also has wider implications with regards to access to further education and employment prospects (Banerjee, 2016; Geary, 2011). This highlights the need to identify pedagogical approaches that can help improve the mathematics outcomes of children who attend PRUs.

The Foundation Phase Framework (Welsh Government, 2015) and the Mathematics Programme of Study in Wales (Welsh Government, 2016) describe the outcomes that children should be able to achieve at the end of each school year in Wales. By the end of the foundation phase (year 2, aged 7 years), children should be able to solve simple addition problems. Examples of these skills include being able to solve one-step word problems that involve addition (e.g. 7 + __ = 9); using known number facts when adding three single digit numbers; and being able to mentally add 10 to a given number up to 100 (Welsh Government, 2015). By the end of primary education (year 6, aged 11 years), the curriculum prescribes that children should be able to identify missing numbers in a sequence using their addition skills and simplify formulae involving the addition of variables (Welsh Government, 2016).

The new Curriculum for Wales (Donaldson, 2015) places an emphasis on making mathematics and numeracy experiences engaging and accessible for all children. Schools in Wales have the statutory duty to teach numeracy across the curriculum to help children to apply their skills and solve problems in real-world contexts (Welsh Government, 2013). The Curriculum for Wales 2022 guidance identifies that children need to be able to fluently use the four basic arithmetic operations (addition, subtraction, multiplication and division) and understand the relationship between them (Welsh Government, 2019). Jordan et al. (2003) demonstrated that children aged 7 to 9 years who were unable to perform arithmetic facts fluently (i.e. reach the correct answer in less than 3 seconds) performed significantly lower on mathematics tests compared to age-matched children who were more fluent. These tests included questions encompassing story problems, place value, forced retrieval of number facts, calculation principles and written computation. The Education Endowment Foundation (2017) recommend that schools support children to develop fluent recall of mathematics facts pertaining to the four arithmetic operations. They acknowledged that without this knowledge children are likely to encounter difficulty understanding and using the mathematical concepts taught later in the curriculum.

Binder et al. (2002) explained that schools traditionally view children achieving 100 per cent accuracy as ‘mastery’, but with additional practice children can recall facts both quickly and accurately (i.e. fluently). Without the ability to perform basic skills fluently children are likely to struggle to master complex skills (Johnson and Layng, 1996; Nelson et al. 2013). For example, if a child is not able to recognise numbers fluently, they will be unable to read single digit addition sums without hesitation. Binder (1996) coined this phenomenon 'cumulative dysfluency' and believed it can explain academic underachievement and failure within education. There are several evidence-based strategies that aim to remediate cumulative dysfluency, including direct instruction (DI; Kozioff et al. , 2001) and precision teaching (PT; Gallagher et al. , 2006).

The importance of teachers considering a range of teaching approaches, including more direct teaching, is recognised as one of the twelve pedagogical principles in the Curriculum for Wales (Donaldson, 2015). DI is an approach that aims to teach component skills to mastery. Using a combination of behavioural techniques (e.g. reinforcement principles and task analysis), DI programmes teach learners skills in a sequential, explicit and scaffolded order (Kinder and Carnine, 1991). DI lessons incorporate teacher demonstrations and guided practice to establish learner independence and skill acquisition (Archer and Hughes, 2011). Several studies have demonstrated that commercially available DI programmes can help children learn basic mathematics skills, including *Corrective Mathematics* (Glang et al., 1991; Parsons et al., 2004; Sommers, 1991). These programmes aim to supplement classroom teaching rather than replacing it by focusing on children's skill deficits. This makes DI programmes useful for both mainstream learners who have fallen behind age-expected norms and those with additional learning needs (Flores and Kaylor, 2007). In their meta-analysis, Stockard et al. (2018) demonstrated that DI programmes can significantly improve academic outcomes across the curriculum. Analysis of 328 studies across a sixty-year period revealed that DI approaches consistently yield positive effect sizes, with most estimates falling within the range of medium or large. This provides strong evidence to support the use of DI approaches in education, including its use to teach component mathematics skills to children.

A complementary approach to support skill development is precision teaching (PT). PT is a method of assessment that aids decisions about subsequent instruction. There is also a specific focus on building fluency to help children master skills to a level that promotes maintenance, endurance, stability, application and generalisability (Johnson and Layng, 1992; Johnson and Street, 2013). Within a PT approach, educational practitioners identify the skill(s) they want to help their learner develop, they provide opportunities to practise the skills, record data and try new/altered techniques to teach skills when necessary (White, 1986). These decisions are guided by learning pictures that emerge on a standard celeration chart

(SCC; see Lindsley, 1995). For example, if a child's data demonstrates that they are answering fewer questions correctly within one minute over several consecutive days, then a practitioner should consider changing the task that they have set. A feasible suggestion might be to assess if the child has mastered all of the associated prerequisite skills (Kerr et al. , 2003). Chiesa and Robertson (2000) demonstrated that employing PT methods can support children's mathematical fluency development. The results from their study suggest that PT driven fluency training (including daily practice, time probes and individually tailored materials) can help children to rapidly improve their mathematics outcomes.

Both PT and DI approaches focus on behavioural mastery and fluency. Desjardins and Slocum (1993) argued that integrating PT methods into DI programmes can help learners to establish mastery of key concepts. Combing these instructional technologies enables learners to fluently perform basic skills, provides efficient and effective practice opportunities, and ensures that learners are able to perform skills at a proficient level before progressing onto more difficult ones. An emerging evidence-base reports the benefits of using an instructional fluency approach to teach literacy skills to mastery (e.g. Adda Ragnarsdóttir, 2007; Hulson-Jones et al., 2013). The available data in this field suggest that the approach can support a variety of learners, including children who attend mainstream primary schools (Kubina et al., 2009) and children with additional learning needs (Morrell et al., 1995) to develop fundamental literacy skills. However, no known published research has investigated whether this approach can be applied to a manualised DI mathematics curriculum and elicit positive outcomes for children who attend a PRU.

The current small-scale exploratory study aimed to assess the effects of using an instructional fluency approach to teach and build fluency of addition skills. The authors used a commercially available DI programme – *Corrective Mathematics* (CM) – in a PRU situated in north Wales. At the end of each CM lesson, the children completed a one minute timing to assess their fluency of basic addition skills (adhering to PT methods). We assessed the children's numeracy gains over six weeks of instructional fluency sessions. This article also includes data relating to session attendance, the children's literacy skills, as well as the children's attitudes towards the instructional fluency intervention.

Method

Ethics

This study received full ethical approval from the School of Psychology's research ethics committee at Bangor University (reference number: 2018–16417). After approaching a PRU in north Wales to take part in this research, we sent opt-in consent forms to the head teacher and children's parents/guardians. Children provided assent to complete the assessments and intervention sessions. If at any point they did not want to participate they returned to their classroom. We later gave children the opportunity to return and complete the given task. Throughout this article we refer to each child by a pseudonym to protect their anonymity.

Sample

We received parental consent to assess ten of the children who attended the PRU. Following the baseline assessments, we identified five children (aged 7 to 10 years) to participate in the instructional fluency intervention. We selected these children on the basis that they attended the unit on the days that the first author was able to conduct the intervention sessions and they completed all baseline measures. This allowed us to ensure that the children had the appropriate prerequisite skills to participate in the intervention (e.g. they could recognise numbers, write numbers independently and were able to read basic addition problems independently). The assessment also enabled us to identify the children who would benefit from the CM

Table 1. Characteristics of the children who participated in the instructional fluency intervention at baseline.

Child	Diagnostic label	Chronological age
Tom	ASD, ADHD	7 years 8 months
Dean	ADHD	10 years 1 month
Will	–	10 years 1 month
Chris	ASD, ADHD	9 years 10 months
Leo	–	9 years 11 months

Note: ASD = Autism Spectrum Disorder, ADHD = Attention Deficit Hyperactivity Disorder.

addition programme – these children were all placed on a lesson within the addition module. Table 1 outlines the characteristics of the children who participated in the intervention.

Assessments

The researchers who administered the assessments were blind to the aims of the project and were not involved in delivering the intervention sessions; the purpose of this was to minimise administration bias. The children completed all of the assessments on a 1:1 basis with a researcher in a quiet room in the PRU they attended. In order to reduce the effects of fatigue and the likelihood of behaviours that challenge occurring as a result of demand, the researchers ensured that the children had sufficient breaks between assessments and sub-tests. None of the children completed all of the assessments in one sitting but did complete them within a one-week period. Following the completion of each assessment, the researchers rewarded each child with verbal praise and a sticker. The baseline assessments for this study took place in April 2019. Following seven weeks (inclusive of six weeks of intervention sessions and one week of half-term), the children completed the follow-up assessments in June 2019.

***Test of Early Mathematics Ability* (TEMA-3).** The TEMA-3 (Ginsburg and Baroody, 2003) identifies children who are likely to develop numeracy difficulties. Typically used with children aged between 3 years 0 months and 8 years 11 months, this assessment offers insight into children's ability to perform the mathematics skills that are typically taught during early schooling (e.g. reading numbers, counting forwards and backwards, using finger displays and using a number line).

The TEMA-3 offers an entry point for the assessment based on the child's age. We used this recommendation to limit administration time. A researcher worked forward through the test items from the age entry point until the child reached a ceiling (i.e. they answered five consecutive items incorrectly). The researcher also ensured that they had a basal measurement for each child (i.e. they had answered at least five consecutive items correctly); in some cases, this required the researcher and child to work backwards from the age entry point.

The researcher sat opposite the child across a table with the picture book and examiner record booklet. Each item on the TEMA-3 has a script for assessors to follow. The researcher read this out loud and waited for the children to respond. Some of the questions required the children to respond using their fingers, answer orally or provide a written response.

To account for repeated administration, the TEMA-3 offers two parallel test forms. Bliss (2006) reported that these forms have high levels of internal reliability (α); with previous research reporting reliability coefficients between .92 and .96. At baseline the children completed Form A and at follow-up they completed Form B.

***Wide Range Achievement Test* (WRAT-4).** The WRAT-4 (Wilkinson and Robertson, 2006) provides a battery of measures assessing reading, sentence comprehension and mathematical computation. The WRAT-4 assesses an individual's ability to decode letters and words, gain meaning from words, count, identify numbers, solve oral mathematics problems and calculate written mathematics problems (from basic arithmetic to advanced operations). This is a norm-referenced assessment which practitioners can use with individuals aged 5 years through to 94 years.

For this study, a researcher worked through the script that accompanies the administration of each sub-test (reading, comprehension and mathematics). The children responded either verbally or in written form on the corresponding test form. Some of the children were unable to read passages of text independently. The researcher read the comprehension passages and literacy-based mathematics questions out loud for these children.

The WRAT-4 offers two parallel test forms for repeated administration. Dell, Harrold and Dell (2008) indicated that the forms have high internal consistency, with reliability coefficients ranging from .92 to .98. Within this study, the children completed the Blue form at baseline and the Green form at follow-up.

***Corrective Mathematics* (CM) placement tests.** The CM programme is comprised of systematically sequenced lessons for key mathematics skills. In order to place children on a lesson that meets their needs, they can sit the CM placement test. This test is a paper-based assessment and requires children to write their responses on the test form. A researcher administered the CM placement test to identify if all of the children met the criteria for the addition module (i.e. they made more than one error on Part A of the assessment). The data from this assessment also enabled us to place the children on an appropriate lesson within the CM addition programme.

All of the children completed Parts A and B of the assessment. Part A assessed the children's ability to answer addition sums in columns: starting with single to single digit addition and progressing onto adding four multi-digit numbers together. Part B assessed the children's ability to answer subtraction calculations (including single to single digit and double to double digit calculations), as well as their ability to answer subtraction word problems. A researcher read the word questions to the children if they were unable to read independently.

The children had twenty minutes to answer as many questions collectively from Part A and B as they could. If they identified that they could not answer any more of the questions before the end of the timing period, the researcher stopped the assessment and scored their responses. If any of the children made one error, or less, on both Parts A and B, then they would have met the criteria to progress onto Parts C (multiplication) and D (division); however, none of the children met this threshold at baseline or follow-up.

Follow-up interviews. Following the intervention, the first author interviewed each of the children who participated in the instructional fluency intervention. The informal interviews also took place on a 1:1 basis in a quiet room within the PRU. The first author asked the children if they had enjoyed taking part in the intervention, what aspects about it they liked or disliked, and if they would like to continue taking part in the intervention sessions in the future. When appropriate, the first author asked the children to elaborate further on their answers and/or asked follow-up questions. Please refer to table 6 in the appendix for a list of the predetermined questions and prompts.

Materials

DI programme. CM (Engelmann and Carnine, 2005) is a commercially available DI programme that offers seven modules to build children's understanding of key mathematics skills. For this study, we focused on the addition module and aimed to complete one lesson per session. During each lesson, the first author used the presentation book, which contained a script for each exercise and for correcting children's errors. The exercises within each lesson required the children to respond either verbally, by pointing to an answer, or by writing the answer down. The lesson-specific worksheets provided the children with an opportunity to practise and review their skills throughout the programme. The children came out of class to complete the sessions in a separate room within the PRU.

Randomised practice sheets. To support the PT element of the sessions, the first author generated a collection of addition practice sheets. Each sheet contained thirty random column addition sums, tailored to the ability of each child (e.g. all single digit addition combinations containing digits 0 to 9; exclusively +0 and +1 sums). In order to complete the worksheet, the children had to write the correct answer underneath each column sum. They had one minute to answer as many questions as they could, working from left to right. They could skip questions if they did not know the answers fluently and return to them at the end if time permitted them to do so. The children completed one practice sheet following each CM lesson. The first author scored the answers based on the number of correct digits written within one minute.

The Standard Celeration Chart (SCC). The first author plotted the children's scores from the randomised practice sheet activity onto their individualised SCCs. This enabled the first author to make decisions about whether each child was making sufficient progress across sessions or if the activity needed to be altered. As a general rule, if a child did not make desired progress (i.e. their score decreased or maintained) over three consecutive sessions, the first author made a change to the practice sheet activity (e.g. altered the content of the worksheets or provided the children with some further instruction to help them answer specific questions).

Results

Attendance

The children had the opportunity to attend three intervention sessions a week, for six school weeks. Tom, Will and Leo started but did not complete one of their sessions due to refusal to comply. In these instances, the first author terminated the lesson and the children returned to their classroom. Tom and Will repeated the CM lesson that they did not complete in the following session. Despite given the opportunity, Leo refused to attend any more sessions for the remainder of the intervention period. Table 2 displays the total number of sessions each child attended.

Table 2. Progress through the CM curriculum over the intervention period.

Child	Starting CM lesson	End CM lesson	Total number of sessions attended
Tom	1	9	9.5
Dean	1	14	14
Will	1	16	16.5
Chris	1	11	11
Leo	23	26	3.5

Note: Some of the children started a lesson but did not complete them due to refusal; this is denoted by 0.5.

There are several reasons why some of the children did not attend all of the sessions. Reasons for non-attendance included: refusal to leave the classroom (i.e. lack of assent), school trip, illness or a competing activity within school that required the child's participation (e.g. another intervention). Some of the children also attended a mainstream school for half a day throughout the week, so they were not always in the unit to attend the sessions due to timetabling changes.

TEMA-3

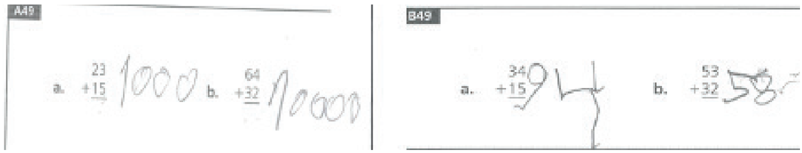
The raw scores from the TEMA-3 assessment can be found in table 3. A Wilcoxon signed-rank test revealed that the children's TEMA-3 raw scores did not vary significantly following the instructional fluency intervention ($Z = 1.63$, $p = .10$). It is worth noting that this analysis does not account for the variance of session attendance across the five children. However, the age equivalence data demonstrated that all of the children who attended the instructional fluency sessions, except Leo, made greater gains than would be typically expected over a seven-week period. Leo did not improve on this measure, but also engaged in the fewest number of sessions (completing only three CM lessons across two weeks).

Figure 1 shows evidence of Tom's development between baseline and follow-up. Despite reversing the numbers in his answer, his single digit addition computation skills improved. Although he did not demonstrate digit reversal during the intervention sessions, he made this mistake

Table 3. The children's baseline and follow-up outcomes on the TEMA-3.

	Raw score			Age equivalence		
Child	**Baseline**	**Follow-up**	**Difference**	**Baseline**	**Follow-up**	**Difference**
Tom	47	52	5	7 years 0 months	7 years 9 months	9 months
Dean	49	53	4	7 years 3 months	7 years 9 months	6 months
Will	33	48	15	6 years 0 months	7 years 3 months	1 year 3 months
Chris	45	46	1	6 years 9 months	7 years 0 months	3 months
Leo	64	63	-1	8 years 9 months	8 years 9 months	0 months

Figure 1. Tom's responses on the TEMA-3 at baseline (left) and follow-up (right).



consistently across the TEMA-3 follow-up assessment. This affected his overall raw score performance on the TEMA-3 and is reflected in his age-equivalence outcomes.

WRAT-4

A Wilcoxon signed-rank test revealed that the children's standard scores did not significantly improve on the mathematics sub-measure between baseline and follow-up (Z = 0.41, p = .69). Whilst the CM programme aims to help children master basic mathematics concepts, there is a level of literacy involved in reading and solving mathematics problems (e.g. reading word problems and deducing the corresponding calculation). Therefore, it is important to consider literacy skills in the wider context of numeracy development. We conducted signed-rank test on the children's standard reading (Z = 0.00, p = 1.00) and comprehension scores (Z = -0.27, p = .79). Neither analysis revealed a significant improvement between baseline and follow-up. Table 4 outlines the children's standard scores on the mathematics and literacy sub-tests.

CM placement test

The scoring system for the CM programme considers the number of errors children make whilst solving computation problems. Part A focuses on addition skills. A Wilcoxon signed-rank test revealed that the children's scores Part A did not significantly vary across the intervention period (Z = -0.16, p = .88). Part B assesses the children's ability to answer subtraction calculations. Our results suggest that as a group, the children did not significantly reduce the number of errors they made on Part B between baseline and follow-up (Z = -0.85, p = .40). Table 5 outlines the children's individual progress across Parts A and B.

Table 4. The children's baseline and follow-up standard scores on the WRAT-4.

Child	Word reading			Comprehension			Mathematics		
	Baseline [95% CI]	**Follow-up [95% CI]**	**Difference**	**Baseline [95% CI]**	**Follow-up [95% CI]**	**Difference**	**Baseline [95% CI]**	**Follow-up [95% CI]**	**Difference**
Tom	64	76	12	107	107	0	88	83	-5
	[57, 74]	[69, 85]		[99, 115]	[101, 112]		[79, 98]	[74, 95]	
Dean	79	80	1	84	78	-6	67	65	-2
	[72, 88]	[73, 89]		[77, 92]	[73, 84]		[59, 79]	[55, 76]	
Will	68	68	0	96	97	1	61	68	7
	[61, 78]	[61, 78]		[88, 104]	[91, 103]		[53, 73]	[61, 78]	
Chris	98	96	-2	100	87	-13	56	69	13
	[90, 106]	[89, 103]		[92, 108]	[82, 93]		[49, 68]	[60, 82]	
Leo	94	89	-5	102	111	9	87	84	-3
	[86, 103]	[81, 98]		[94, 110]	[105, 116]		[78, 97]	[74, 96]	

Table 5. The number of errors each child made on the CM placement tests.

Child	Part A			Part B			Addition lesson placement
	Baseline	Follow-up	Difference	Baseline	Follow-up	Difference	Baseline
Tom	8	8	0	11	4	-7	1
Dean	8	8	0	9	10	1	1
Will	8	4	-4	10	10	0	1
Chris	8	8	0	13	10	-3	1
Leo	3	4	1	9	9	0	23

SCC data

Here we present Dean's SCC as an example to illustrate its use by the first author across the intervention period (see figure 2). Refer to figures 3–6 in the appendix to see Tom's, Will's, Leo's and Chris's fluency progress.

Dean made limited progress on the single digit addition practice sheets over the first two weeks (celeration: x1.05, bounce: x1.3). It appeared that he was struggling to answer questions where the answer exceeded 10 and he needed to use his fingers to count. The first author altered the practice sheet activity to focus on building fluency on single digit addition sums where the answer did not exceed ten first, with the plan to reintegrate more difficult sums following their introduction in the CM programme. This alteration saw an improvement in Dean's progress (celeration: x1.1). After showing limited acceleration in his correct responses after half-term, the first author made the decision to simplify the activity even further, by focusing on adding 0, 1 and 2 to numbers 0–9. This saw an initial improvement in Dean's data before the intervention ended.

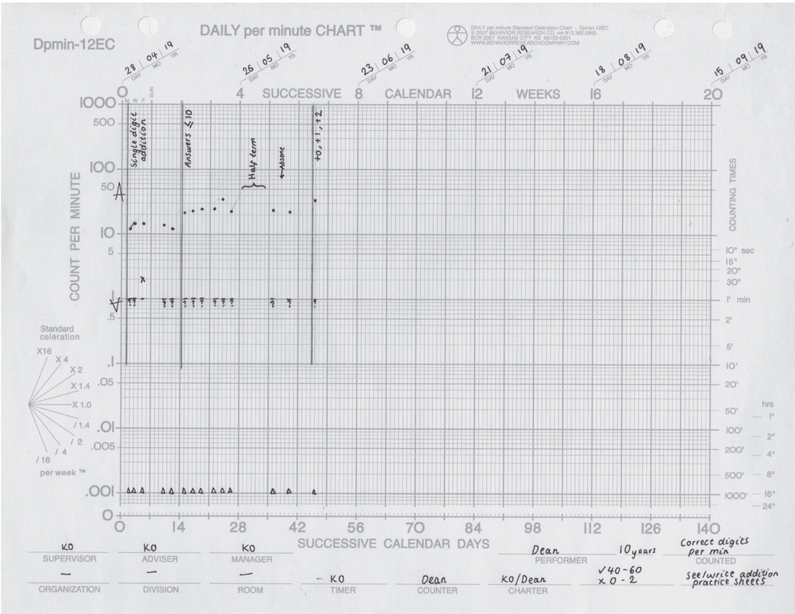
Follow-up interviews

When asked which aspects of the intervention they enjoyed, Tom reported that he liked seeing his progress reflected on the SCC (particularly in reference to the correct responses going up), doing the work well and receiving stickers for taking part. Will found the sessions fun and enjoyed learning new things and Leo enjoyed the mathematics content that we covered within the sessions.

With regards to the elements of the session that they enjoyed the least, Dean expressed that he did not like the one-minute fluency timings we completed after each lesson. Despite not enjoying this aspect, he acknowledged that it was a useful element of the session as he was now able to write down numbers faster. Chris claimed that he did not enjoy the sessions due to the repetitiveness of the content. Will did not enjoy answering sums containing big numbers.

All five of the children appreciated that the intervention was useful for them. Dean, Will and Tom claimed that the content they had learnt and practised within the sessions had helped them with their classwork. Will and Tom felt like taking part in the intervention helped them to get smarter. Dean and Chris both identified that the sessions were useful in the sense that they helped them to learn.

Figure 2. Dean's SCC. The dots represent the number of correct digits Dean wrote in 1 minute, the crosses and question marks refer to the number of incorrect digits in 1 minute (with the question marks denoting zero errors), and the triangles depict the number of timings Dean completed each session (i.e. one timing a day).



Dean, Will and Tom indicated that they would like to carry on using the instructional fluency approach to help them learn mathematics. Leo and Chris did not want to engage with the programme anymore, with Leo indicating that he felt that the sessions took too long. Leo and Tom both revealed that they did not always engage with the sessions due to a more appealing activity being available in their classroom (i.e. they chose to play a game with their peers rather than completing an instructional fluency session).

Discussion

Estyn (2017) identified that half of the PRUs in Wales are not supporting the children they accommodate to satisfactory standards. Many of the children who attend these units are unable to perform age-expected mathematics skills, they display a lack of interest towards learning and the pace of learning is too slow for them. Through this small-scale exploratory study, the authors aimed to investigate whether children attending a PRU would engage with an instructional fluency intervention targeting addition skills and whether it would help accelerate their mathematics outcomes. To the authors' knowledge this is the first study to investigate the use of this combined instructional technology to teach mathematics skills in a PRU.

The TEMA-3 results suggest that regular attendance at instructional fluency sessions can help children attending a PRU to learn some of the mathematics skills that they would typically be expected to have learnt and acquired during early childhood. However, the approach did not significantly affect the children's standard scores on the WRAT-4 assessment. This may suggest that six weeks of the intervention are not sufficient to help children access higher-level mathematics content or develop comprehension skills. Three of the five children who participated in the intervention indicated that they would like to continue using the instructional fluency approach to learn mathematics. Overall, the results from this study suggest that it is feasible to use an instructional fluency approach on a 1:1 basis in a PRU to improve mathematics outcomes. The outcomes appear to be contingent on the children complying to attend and showing willingness to engage with the intervention sessions.

DI programmes aim to identify and teach to children's skill deficits, using explicit and scaffolded teaching methods. That is, DI programmes teach skills in a sequential order based on the premise that children have to master certain prerequisite skills in order to access higher-order content (Kinder and Carnine, 1991). Previous research has demonstrated that DI is an effective remedial intervention for children in mainstream schools and children with additional learning needs (Flores and Kaylor, 2007). After six weeks of instructional fluency sessions, Will, Tom, Dean and Charlie increased their age equivalence scores on the TEMA-3 by at least three months. This suggests that the intervention helped the children attending the PRU to master some of the mathematics skills that children typically acquire during early schooling. This data supports the contention that using an instructional fluency approach can accelerate learning and remediate skill deficits (see, for example, Morrell et al., 1995). Moreover, this finding is in line with the wider DI and PT literature which provides strong evidence to support the benefits of using these technologies in education (see, for example, Stockard and Wood, 2018; Chiesa and Robertson, 2000).

Administering the WRAT-4 enabled us to assess the children's literacy and numeracy abilities comparative to age-expected norms. The data suggest that all of the children who participated in the intervention had low-level word reading skills for their age at baseline; with Dean and Will also displaying low-level comprehension skills. This may have wider implications with regards to the children's ability to access specific mathematics content on the curriculum. Passolunghi and Pazzaglia (2005) suggested that solving reading comprehension questions and mathematics word problems require children to use the same problem-solving skills. In order to answer these types of questions, a child's working memory needs to process the relevant text and ignore irrelevant information. This might explain why the children who participated in this study were unable to answer the mathematics word problems on both the WRAT-4 and CM placement test (Part B). The CM addition programme integrates word problems and associated comprehension strategies from lesson 19. We only ran this intervention for six school weeks, meaning that Tom, Dean, Will and Chris did not engage with these lessons within the CM addition curriculum. Future research could extend the intervention period and investigate the effects of the lessons on children's literacy and numeracy skills.

Estyn (2015) noted that a large proportion of children who attend PRUs in Wales have additional learning needs. In our sample, Tom, Chris and Dean held a diagnostic label for autism spectrum disorder (ASD) and/or attention deficit disorder (ADHD). Children with underlying developmental disorders often display attention difficulties which can make them more suspectable to poor academic outcomes and long-term behavioural problems (May et al., 2013). Jordan and Levine (2009) identified five mathematical competencies that children typically acquire during early childhood: the ability to rapidly recall small qualities up to four items; counting abilities; magnitude comparison; estimation, and arithmetic operations. These early numerical competencies provide the foundation for later mathematics skills to be built upon (Geary, 2000). Titeca et al., (2014) identified that pre-school children with high-functioning ASD perform at the same level as typically developing children on these numerical competencies. When it comes to the higher-order mathematics skills, children aged 6 to 7 years old with ASD perform significantly lower than their typically developing peers on questions pertaining to number fact retrieval and word problems. This theory might explain some of the disparity between the children's chronological age and age equivalence on the TEMA-3 assessment within the current study. The items on the TEMA-3 assessment increase in complexity from the age entry point, so without mastery of the foundation numerical competencies (e.g. counting objects) it is possible that the children who participated in our study were unable to tackle the questions that integrate the higher-order skills despite their chronological age.

All of the children who participated in this study also performed below average for their age (i.e. a standard score < 100) of the WRAT-4 mathematics sub-test. Without sufficient mastery of early mathematical skills the children who participated in this study might have been unable to understand the concepts and procedures underlying more complex mathematics problems. Both DI (Celik and Vuran, 2014; Rockwell et al., 2011; Thompson et al., 2012) and PT (Brady and Kubina, 2010) have documented benefits when researchers have used the strategies with children with developmental disabilities, including ASD and ADHD. Limited research in this field has demonstrated that educators can use DI and PT in conjunction with one another to help remediate mathematical skill deficits amongst populations with additional learning needs (see, for example, Delli Sante et al., 2001).

Of the five children who participated in the intervention, three indicated that they would like to continue using the approach to improve their mathematics skills. This finding suggests that the children associated some level of social validity with the intervention. Extensions of this research should consider collecting further data to assess common aspects of the intervention that children enjoy and elements that might need further refinement. Researchers could use these data to develop strategies to enhance children's engagement with instructional fluency sessions. This extension may also help identify some of the barriers in education that prevent children attending PRUs from engaging in similar intervention programmes (e.g. competing classroom activities).

Some of the children who participated in this study attended the PRU on a part-time basis, spending a percentage of their time in mainstream primary school. It was not possible to gather information on the strategies used to teach mathematics in each school and compare these to the instructional fluency approach described in this paper. We appreciate that the findings and conclusions drawn from the current study would have been enhanced by the inclusion of a control group: the incorporation of which would have helped established whether the instructional fluency approach has any additional advantages for children attending PRUs, compared to the typical classroom teaching that they are exposed to. Due to the variability in the children's ages, diagnostic labels, the percentage of time they attend the PRU and their baseline mathematical abilities, we were unable to match the children who returned their parental consent forms. In addition to this, we had a small dataset as a result of some of the children being unable to complete all of the necessary assessments: this was due to lack of assent and/or non-attendance. Due to the time constraints surrounding this project we were also unable to explore the possibility of the children acting as their own controls and measuring their performance growth before and after exposure to the intervention. Future replications of this research should consider the recruitment of control data to establish the differences in performance gains between children in PRUs who attend instructional fluency sessions regularly compared to the effects of their typical exposure to mathematics instruction.

The researchers who conducted the assessments for this study were unaware of the aims of this project and which children had been selected to participate in the intervention. We hoped that this would reduce any confounds surrounding administration bias. However, it is important to consider that the children might have altered their behaviour as a result of their participation in this study. McCarney et al. (2007) explained that it is important for researchers to consider the impact of the Hawthorne effect in relation to the generalisability of research to day-to-day life. In the context of the current study, it is possible that the children's performance is an under-representation or over-representation of their performance in the classroom due to their reactivity to the testing conditions.

The data presented in this paper suggest that the instructional fluency approach can support children in a PRU to improve their basic mathematics skills. There is a strong evidence-base supporting the use of DI to teach children literacy (Przychodzin-Havis et al., 2005; Simonsen and Gunter, 2001) and numeracy skills (Przychodzin et al., 2004). Moreover, practitioners have used PT methods to record and monitor performance for many academic skills, such as mathematics (Chiesa and Robertson, 2000), reading (Hughes et al., 2007), content-specific terminology

(Beverley et al., 2009; Stockwell and Eshelman, 2010) and second-language acquisition (Beverley et al. 2016). Whilst researchers have put forward the argument that DI and PT can complement each other to create a superior instructional technology (Binder and Watkins, 1990; Desjardins and Slocum, 1993), further research is necessary to show the generalisability of an instructional fluency approach across different curriculum subjects to remediate children's skill deficits. Investigations in this area could validate the use of an instructional fluency approach in PRU settings to help children improve their academic performance.

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Appendix

Table 6. A list of the questions and prompts the first author asked during the follow-up interviews.

Main question	**Prompts**
Do you enjoy coming out of class and doing maths with me?	• Why do you/don't you enjoy it? • What parts of the session do you/don't you enjoy? • What's your favourite part of the sessions? • What is your least favourite part of the sessions?
Do you think that coming out to do these sessions is useful?	• Why do you think that the sessions are/aren't useful?
Would you like to carry on doing these maths sessions?	• Why/why not?

Figure 3. Tom's SCC.

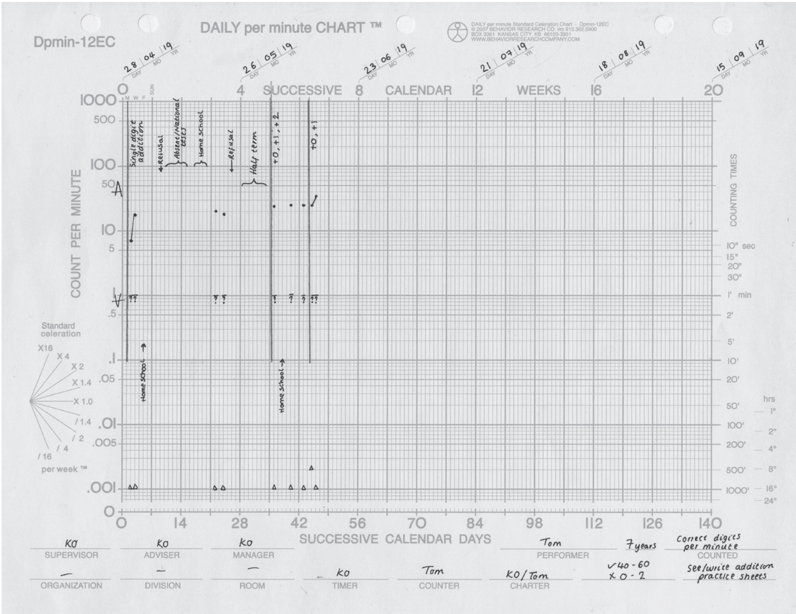

Figure 4. Will's SCC.

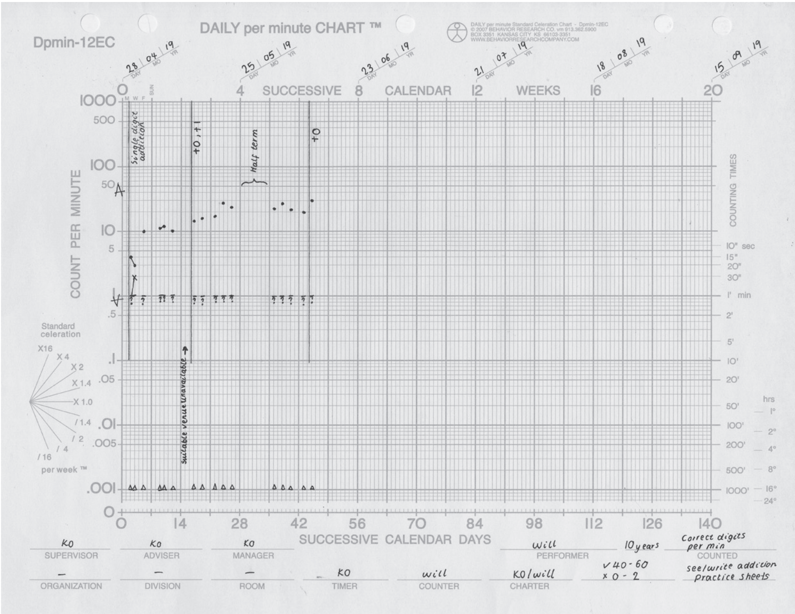

Figure 5. Leo's SCC.

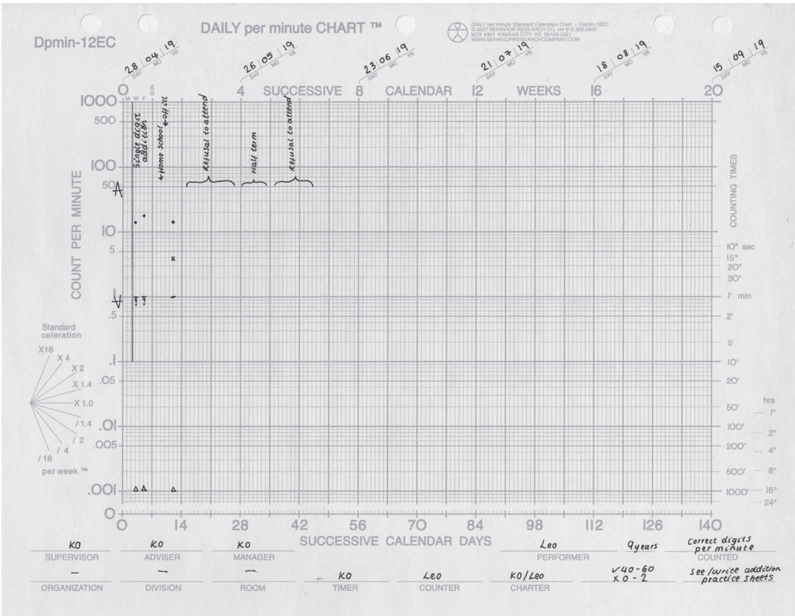

Figure 6. Chris's SCC.