Bangor University

DOCTOR OF PHILOSOPHY

The psychology of information selection and reasoning

Morris, M. Frances G.

Award date:
1997

Awarding institution:
University of Wales, Bangor

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal ?

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Download date: 02. May. 2024

THE PSYCHOLOGY OF INFORMATION SELECTION AND REASONING

M. Frances G. Morris

I'W DDEFNYDDIO YN Y
LLYFRGELL YN UNIG

TO BE CONSULTED IN THE
LIBRARY ONLY

Ph. D.
University of Wales, Bangor

1997

U.C.N.W. LIBRARY C.P.G.C. LLYFRGELL

THE PSYCHOLOGY OF INFORMATION SELECTION AND REASONING

Testing the Predictions of the Oaksford and Chater (1994) Bayesian Model of Optimal Data Selection

CORRECTION SHEET

ABSTRACT

This thesis is about the psychology of information selection and reasoning. It investigates the way in which the probability of information influences the selection of information. Information which is expected to reduce uncertainty the most in a given probabilistic context is assumed to be the most relevant information to select.

Several computer-designed studies, two of which vary the probability of information in a learning phase, test the precise predictions and assumptions of the Oaksford and Chater (1994) model of optimal data selection, which specifically explains card selections in Wason's (1966) four card problem or selection task in terms of probability-dependent optimal data selection. This way of explaining reasoning in the selection task contrasts with traditional reasoning theories and explanations which assume that a reasoner's goal in the selection task is to falsify, and that truth-preserving rules of inferences, for example, logical deduction, underlie the inferential processing component of behaviour in this reasoning task.

In order to compare and contrast the O&C optimality approach with other theories in the psychology of reasoning, major theories of reasoning, as well as specific explanations of reasoning in the selection task, are reviewed. General optimality approaches to cognition are also reviewed (Stephens and Krebs, 1986) in order to place in proper theoretical context the O&C (1994) model of optimal data selection and how it explains, in particular, affirmative abstract versions of the selection task.

It is concluded that, at a psychological level, the O&C model of optimal data selection contributes significantly to the theoretical understanding of human reasoning because it proposes that a simple, adaptive, optimality-preserving decision rule (i.e. to select optimal data) governs the selection of information and what is perceived as optimal will change in different probabilistic contexts. Experimental results support an optimal data approach and demonstrate that it is not necessary for counterexamples to be represented in order to produce apparent "falsificationist" behaviour, as simply manipulating the probability of p and q in a learning phase prior to a selection phase can change, in accordance with precise predictions, selection task performance. At an optimality model level, because it adopts certain classical optimality assumptions regarding the optimality-preserving decision rule governing selection behaviour, the O&C model has similar strengths and weaknesses as simple optimality approaches, and these are discussed.

CONTENTS

Chapter 4 - Experiments

ACKNOWLEDGEMENTS

I thank Mike Oaksford for giving me the opportunity to work on and learn about the Oaksford and Chater (1994) model of optimal data selection. Carrying out research in this area of psychology has been an enlightening challenge, and I have been fortunate enough to have had the time to develop a true appreciation of the clarity of thought needed to author an optimal data selection approach to reasoning.

I acknowledge Martin Keyes, Guy Monson and Ann Thurgarland who helped me run four experiments when they were undergraduate students at University of Wales, Bangor.

Dr Pauline Horne, Dr Nick Ellis and Professor Fergus Lowe of the School of Psychology, and Dr John Perkins, at the University of Wales, Bangor, have, in different ways, made it possible for me to finish the writing of my thesis in Bangor, and I thank them for their support.

I shall always remember the encouragement of my close friends and family, here and abroad, present and absent: thank you.

My research was made possible by a University of Wales postgraduate studentship award.

Chapter 1
INTRODUCTION

This thesis investigates the psychology of information selection and reasoning and has both a theoretical and empirical component. Theoretically, and in the light of recent trends towards optimality approaches to cognition (Anderson, 1991, and Oaksford and Chater (1994) ("O&C"), I review and re-assess assumptions underlying the psychology of reasoning. In particular, I compare the O&C model of optimal data selection with other rationalist interpretations of reasoning behaviour and to this end several definitions of what it means to be rational are introduced in this chapter 1.

In order to compare the O&C model with other models of reasoning, the logic-based decision rules or inferential procedures traditionally assumed to underlie reasoning are reviewed in chapter 2, as well as the decision rules and inferential procedures of recent, specific theories and explanations of reasoning, and Wason's (1966) selection task in particular[1]. For example, mental logics (Rips, 1983; O'Brien, 1993, 1995); mental models (Johnson-Laird, 1983; Johnson-Laird and Byrne, 1991, 1993a); pragmatic reasoning schema (Cheng and Holyoak, 1985, 1989); linguistically relevant biases and mental models (Evans, 1984, 1989, 1995); optimal relevance, Sperber, Caro and Girotto, 1994); subjective expected utility and mental models (Manktelow and Over, 1990, 1991); subjective expected utility, signal detection theory and mental models (Kirby, 1994); and cost-benefited social contracts and optimally adaptive "cheater detection" algorithms (Cosmides, 1989).

In contrast to theories of reasoning which assume that inferences in the selection task apply some form of truth-preserving or logic-based decision rules, the O&C model of optimal data selection views reasoning in the selection task as optimal data selection. Optimal data selection is an example of a "design" or behaviour which has evolved and adapted from a cognitive environment in which information competes for attention and processing. It is an optimal design simply because an adaptive behaviour or bias which governs the way in which relevant information is selected minimises constraints on attention and memory, which then enables cognition to be optimally focused in different contexts (for example, enabling different prioritisation of events and/or information in different contexts). The O&C (1994) model of optimal data selection assumes that, rather than truth-preserving logical principles in the form of truth tables, or rules of derivation, or mental models, being the ways in

1 The selection task is detailed in section 1.2 of this chapter 1.

which inferences are made in the selection task, the principle of maximising or optimising one set of properties in the environment (i.e. optimising expected information gain) is the decision rule governing selection task performance. The theoretical assumptions (including the maximisation principle) underlying optimality approaches generally and the O&C model in particular are reviewed in chapter 3.

The empirical component of my thesis involves testing the assumptions and predictions of the O&C (1994) model of optimal data selection in a series of studies which use novel experimental procedures. In chapter 4, studies in which modified versions of Wason's (1966) affirmative abstract selection tasks are reported. Three experiments obtain judgements about perceived card informativeness, two of which studies obtain a direct rating of information gain perceived to be provided by each card, and in the other study an indirect scaling of the informativeness of cards is obtained. In Section B of chapter 4, studies in which the probabilities of cards are systematically varied in a learning phase prior to the selection task phase are reported.

In Part I of chapter 5, I consider and discuss model-specific and general theoretical implications of experimental results for the O&C model of optimal data selection and reasoning generally. At a psychological and philosophical level, I conclude that the O&C model makes a significant contribution to the understanding of reasoning because it offers a simple, optimality-preserving decision rule which governs which cards are optimally relevant to select in different probabilistic contexts. At a general optimality model level, because it adopts the same decision rule assumptions as a classical optimality model (Stephens and Krebs, 1986) the O&C model has the same strengths and weaknesses. For example, in classical, optimal foraging theory the probabilities and consequent optimality of only one property in an environment, i.e. energy gain (or "expected information gain" in the O&C model) are varied. In Part II of chapter 5, I consider the ways in which the O&C model of optimal data selection may be refined, for example, to include the calculation of decrements in information gain over time, as well as the possibility of calculating probabilities of other properties in the cognitive environment besides information gain. Finally, I outline ways in which an optimality approach to cognition is useful in explaining behaviour generally.

Having outlined the basic structure of this thesis, I now introduce a number of other interpretations of what it means to be rational, besides the rationality of behaviour being optimally adapted to an environment.

1.1. What does it mean to be rational?

The Concise Oxford Dictionary defines *rational* as "being endowed with reason", and *reason* is defined as "the intellectual faculty characteristic especially of human beings by which conclusions are drawn from premisses". This definition is concerned with the ability of human beings to reason *deductively* and is the type of reasoning on which research in cognitive psychology has traditionally focused. Psychological research has involved experiments which seem to provide evidence that human beings are not very good at deductive inferences. However, even though most people do not perform reasoning tasks such as Wason's selection task successfully, it appears that logical and therefore rational reasoning performance can be facilitated in various ways (Johnson-Laird and Wason, 1970a; Wason and Shapiro, 1971; Griggs and Cox, 1982; Cheng and Holyoak, 1985, 1989; Cosmides, 1989; Manktelow and Over, 1991; Kirby, 1994). These and other studies, and their respective theoretical motivations, are reviewed in chapter 2. This first chapter introduces the selection task, and general concepts and assumptions on which the selection task and psychological theories of reasoning and cognition are traditionally based.

One way that arguments for and against the deductive logicality or rationality of human behaviour is framed has been to distinguish between underlying reasoning *competence* and reasoning *performance*. This distinction was first used by the linguist, Noam Chomsky, when he argued that language was a competence or ability which is seen in its impure form in an individual's actual performance or output. He felt that linguists should focus on competence and disregard performance errors, pauses, and lapses of memory, and that psychologists should study the latter (Gardner, 1985).

Applying this competence/performance distinction, Cohen (1981) argued that human beings have rational ability or competence and reasoning errors reflect the difficult nature of a reasoning task (for example, the unfamiliarity and artificiality of the selection task where abstract materials are used), or a person may not have been taught logic and so cannot be expected to perform competently. For these reasons, Cohen concludes that the irrationality of human behaviour, or performance, cannot be demonstrated.

Not all approaches to cognition rely solely on the competence/performance distinctions to decide if behaviour is rational or not. For example and in response to Cohen, the view of Evans (1984, 1989) is that a "biases approach" is essential to understanding human inferential performance and why rational reasoning exhibited under one set of circumstances is absent in others. The assumptions of bias research in psychology traditionally derive from *both* decision theory and theories of logical deduction. Evans (1993a) incorporates these different foundations and distinguishes two further definitions of rationality, rather than relying on the competence/performance distinction, when debating rationality. The distinction which Evans called *rationality$_1$* is considered a *personal* or subjective rationality, and is about the ability to be purposeful and adaptive and make choices based on personal *beliefs and goals which will produce the best outcome for an individual.* This decision-theoretic definition of rationality is defined by Evans and Over (1996) as meaning "reasoning or acting in such a way as to achieve one's goals". Evans' second rationality definition is called *rationality$_2$* and it is considered a more objective or "*impersonal* rationality" as it is concerned with the *processes* of rationality, rather than beliefs, biases and goals, and its foundations are in the psychology of deductive reasoning. Evans and Over's (1996, p. 2) definition of rationality$_2$ is "reasoning or acting in conformity with a relevant normative system such as formal logic or probability theory". It is an implicit assumption that rationality$_2$ serves rationality$_1$ and that logical reasoning will lead to the achievement of goals (Evans, Over and Manktelow, 1993 at p. 168). Evans argues against rationality$_2$ as the standard against which rational behaviour should be measured and proposes that rationality$_1$ should be the yardstick against which human rationality should be assessed.

The above two definitions of rationality are in some respects similar to the conceptualisations of rationality made by Simon (1986 p. 26). Simon writes that in economics rationality is viewed in terms of the choices it produces and is a "substantive rationality" because a rational person always reaches the decision that is objectively, or substantively, best because utility of consequences is assumed to have been maximised[2]. Rationality$_1$ is also

2 The technical notion of utility and maximisation is detailed in chapter 3. But, briefly, maximising utility is about the way in which people have preferences, and preferences are assumed to be distinguished on the basis of the amount of benefit, or worth or value which they give an individual. Maximising utility is therefore about the decision rule or "bias" to consistently choose preferences which provides the most value or benefit.

concerned with the goal of maximising utility, although it differs from Simon's *objective* or "substantive rationality" as it is concerned with *subjective* utility and *subjective* probability. In addition, and in contrast to Evans' view that rationality$_1$ rather than rationality$_2$ should be the way in which behaviour is investigated, Simon argues that because a utility maximising approach has not sufficiently explained and predicted behaviour, in consumer economics at least, the emphasis of psychological studies should be on the *processes* of rationality rather than goals (Simon ,1986, p. 39).

Notwithstanding the recent trend to incorporate a rationality$_1$ component when studying the psychology of reasoning (Cosmides, 1989; Manktelow and Over, 1991; Kirby, 1994; Evans and Over, 1996), theories which focus on the processes (rationality$_2$) assumed to be implicated in reasoning (e.g. propositional logic, mental logic, mental models or schema theories) have dominated philosophy, psychology and cognitive science and they often use some form of competence/performance distinction in order to explain irrational or illogical reasoning behaviour. For example, mental modellers Johnson-Laird and Byrne (1993b) modify the Chomskian competence-performance distinction, and their competence or process component of reasoning assumes that rationality is based on impeccable rules of inference but assume impeccable competence is not always demonstrated in reasoning performance. Johnson-Laird and Byrne propose that belief in an impeccable rationality fails and they assume that a reasoner's goal is to apply a *rational meta-principle* which seeks to maintain truth: i.e. an inference is valid provided there is no model of the premises in which its conclusion is false. To argue that errors arise as a result of performance is therefore, they argue, misleading "because it suggests a failure to put into practice correct rules, whereas there are no correct rules to put into practice, only higher-order meta-principles" (Johnson-Laird and Byrne, 1993b p. 194). Johnson-Laird and Byrne assume that this meta-principle is part of universal human deductive competence and rationality.

While Johnson-Laird's theory of mental models and his views on rationality focus on a meta-competence, a formal or mental logics approach to rationality assumes that when people are irrational, it is not because they lack logical competence, but irrational performance is a result of the demands of the situation which exceed their logic skills or because inferences from nonlogical sources are made, or because they are reasoning from irrational assumptions (O'Brien, 1993, p. 110). Cheng and Holyoak's (1985) theory of pragmatic reasoning schema focuses on *performance errors* and rationalises errors in terms of the lack of facilitating deontic information[3]. Part II of chapter 2 reviews these theories of reasoning in depth.

The purpose of mentioning the above theories now is to show that there are several views and definitions of what is means to be rational. Most theories of reasoning assume that human beings have an ability to be rational (using processes or inferential procedures such as logic, or mental models or pragmatic reasoning schema) but different theories give different reasons why reasoning performance does not always appear to reflect underlying rational competence. Before introducing the specific rationality assumptions of the O&C model of reasoning as optimal data selection, it is appropriate to describe the selection task, to review the initial explanations for illogical performance observed on this task, and then to discuss the underlying assumptions motivating such explanations.

1.2 What is the selection task?

The selection task or four card problem is the most widely used task in the psychology of reasoning. All theories mentioned above have attempted to explain "illogical" reasoning performance on this task. However, few theories explicitly challenge assumptions underlying the goals and decision rules of the selection task and this has perpetuated studies which seek to improve or facilitate logicality in various ways. In order to understand why these explanations of performance on the selection task are problematic, the task's assumptions have to be made clear and then re-evaluated.

The abstract selection task (see Figure 1.1 below) has become an accepted tool used to measure deductive reasoning. The correct solution of the four card problem is assumed to involve applying principles of propositional logic concerned with conditional statements of the form *if p then q* (i.e. material

3 Deontic reasoning provides information about what *ought* to be the case, and it includes terms such as "must", which type of information is apparently lacking in standard conditional (if *p* then *q*) sentences. For example, the conditional sentence "if you go shopping (*p*), then you must fill the car with petrol afterwards (*q*)", because it explicitly includes the deontic term "must", is assumed to further facilitate rational reasoning. A deontic task thus involves decisions being made about whether the conditional rule is obeyed or violated (Manktelow and Over, 1991), whereas in standard versions of the selection task the task is to evaluate the truth or falsity of the selection task rule.

implication). The four card problem was specifically designed in 1966 by Peter Wason to show how logical implication is understood, or rather not understood, by reasoners. The selection task is furthermore assumed to model an hypothesis testing setting in which falsification or refutation of an hypothesis is a necessary condition of good science, and therefore necessary for correct, or logical, task performance.

Figure 1.1: example of Wason's (1966) Selection Task

A K 2 7

There is a letter on one side of the card and a number on the other side.
Rule: If a card has a vowel on one side then there is an even number on the other side.

The selection task instructions require people to name those cards, and only those cards, which need to be turned over in order to determine whether the rule, typed above in bold, is true or false. The *logically correct* solution is to turn over the A (p) and the 7 ($-q$) cards. This logically valid inferential procedure for testing an hypothesis is assumed to be derived by the application of the reasoning principle *modus tollendo tollens* (on which falsificationism is based) which principle allows a logically correct deduction to be made. The logical rules and procedures which permit this deduction to be made are detailed in chapter 2, when the methods and assumptions of propositional logic are reviewed.

Notwithstanding the logical appropriateness of applying the principle of *modus tollendo tollens* ("MTT"), the observed mean proportions of cards selected when attempting to solve the four card problem are: for the p or the "A" card .89, for the q or the "2" card .62, for the $-q$ or the "7" card .25, and for the $-p$ or the "K" card .16; i.e. the consistent ordering of card selections is $p>q>-q>-p$ (see meta-analysis of affirmative abstract versions of the selection carried out by Oaksford and Chater, 1994, p. 613, where 13 studies reported 34 selection tasks involving 845 participants). These card selection results are assumed to demonstrate that human beings are illogical, because they do not apply the appropriate logical MTT principle and select the logically correct p and $-q$ cards, and must therefore be irrational reasoners and hypothesis testers.

1.3 How Peter Wason explained illogical performance on the selection task

In an endeavour to explain selection task *errors*, i.e. why *p* and *-q* were not selected, Wason initially made two assumptions:

(i) He assumed that reasoners do not apply and are therefore not constrained by the rules of propositional logic where a conditional sentence or rule has two outcomes or values: true or false. Instead, Wason argued that reasoners must think that a conditional sentence has three outcomes or truth values: where the outcome or contingency (or card selections) "PQ" made the rule *true*; the contingency "P-Q" made the rule *false*; and the contingencies "-PQ" or "-P-Q" were *irrelevant* to the rule. Given the above, Wason argued that reasoners must believe that the *q* card is a plausible (but not logical) selection, in order to see if it is associated with the *p* card making the conditional rule true. In other words, reasoners must apply "defective truth tables" (details of truth tables used in proposition logic are detailed in part II of chapter 2)

(ii) Performance on the selection task also motivated Wason to argue that reasoners are biased to expect a relation of truth correspondence or match to hold between sentences and states of affairs. This expectation he assumed is learned, because true instances are more frequently encountered than false instances. The high frequency of truth correspondence in the world is thus why reasoners seldom propose a false correspondence in order to reach a conclusion or make a decision (Wason, 1968, p. 274). In more technical terms, Wason argued that infrequent experience of contrapositive inferences explained why *-p* is seldom deduced from *-q*, which derivation is permitted by the logic principle MTT, details of which are in chapter 2.

Applying these logic-based theoretical assumptions, Wason sought to correct illogical and therefore irrational performance errors by moving behaviour away from truth correspondence. To this end, he designed *therapy* or therapeutic experiments which would facilitate the contrapositive inference, as *insight* into the principle of falsification would be given.

In the first therapy experiment, Wason (1968) attempted two kinds of therapeutic manipulations, both designed to break what he believed was a mental set for expecting a relation of truth. The first therapy experiment used an abstract letters/numbers, binary selection task[4], and involved the *projection of falsity* therapy, where participants were asked to verbalise what could be on the unexposed reverse of cards they selected. They were then asked to say whether those chosen but unseen reverse value/s would make the conditional rule *false*. After "projecting falsity" and if participants wished, initial card selection were then revised. Finally each card was turned over to see the actual reverse values and participants then said whether these actual reverse values made the rule true or false. Falsity projection therapy was not successful and in a second experiment Wason (1968) (using a less arbitrary shapes/coloured squiggles, binary selection task) applied another therapy: *the restricted contingency programme.* This involved participants being given examples of four outcomes or contingencies (PQ, -PQ, -P-Q, and P-Q) and then being told that only one contingency falsified the rule. This therapy did not facilitate logical reasoning either.

In order to remove the "deep fixation" causing reasoning errors, Wason (1969) devised another therapy experiment, using another less abstract, coloured triangles/coloured circles, binary selection task, to induce *contradictions* between current information and previous choice. It was hypothesised that card selection would be corrected to produce logically correct selections if contradiction was successfully induced. To implement this hypothesis, different strengths of contradiction of a participant's card selection were verbalised by the experimenter. For example, on the *initial choice* of card, the contradiction of the experimenter was made "in a casual tone of voice". A *weak hypothetical contradiction* was the next level of contradiction, made by the experimenter by pointing to the *p* card and asking "What could be on the other side of that card?": if the response was *q*, the participant was reminded that the task instructions were to test whether sentences were true or false. A direct and *strong hypothetical contradiction* was made when the experimenter pointed to the *-q* card. A *concrete contradiction* involved the participant physically (not hypothetically) turning over the cards selected and then saying

4 "Binary" meaning that a value was actually printed on both sides of a card, and reverse card values could actually be seen (rather than hypothesised) if the card were turned over.

whether the card made the sentence true or false, after which the experimenter turned over the *-q* card if this had not already been selected and asked if the sentence was still true. The *final judgement* was when the participant had consistently failed to select the *-q* card and was told that previous card selections were incorrect and asked to think again.

The results of the contradiction therapy showed that the frequency of choosing *-q* cards cumulated as the different degrees of contradiction were confronted. However, the logically valid "P-Q" contingency only increased after strong hypothetical contradictions, concrete contradictions and at the final judgement. From these results, Wason (1969) concluded that *insight* into the logical structure of the task more often than not facilitated logical reasoning performance when contradictory information is encountered successfully. The few participants for whom contradictory evidence did not provide insight into the principle of falsification were speculated as having temporarily regressed to Piagetian pre-formal operational thought.

In another therapy experiment, Wason and Johnson-Laird (1970) sought to induce insight by rectifying any misconstrual of the words “other side of the card” as referring to the side which was face downwards . All information was presented on one side of the card and information which had been on the other side of the cards in previous experiments was masked. Reasoners were required to evaluate (or passively judge prior to card selection) the truth or falsity of the contingencies PQ and -QP. The rule is this experiment was "every card which has a circle on it has two borders round it". Logically valid performance was not improved in these experiments.

Finally, experiments using concrete stimuli which had to be placed in boxes, investigated the influence of a reduced array selection task ("RAST") (Johnson-Laird and Wason, 1970a experiment 1). It is called the RAST because only the *q* and *-q* are used, i.e. given a rule "all the triangles are blue" choices or selections are made from a blue shape and a red shape (i.e. only a single consequent stimuli , as opposed to conjoined antecedent/consequent stimuli of the form "a D on the reverse of a 3", has to be considered. Logical facilitation was eventually gained in this RAST experiment and it was concluded that "familiarity with a simple task enables insight into the logical structure of implication to be permanently gained" (Johnson-Laird and Wason, 1970a, p. 59).

Based on the above therapy experiments and conclusions, Johnson Laird and Wason (1970b) formalised a theoretical model of the stages of insight into the falsificatory principle. Details of this insight model, and how it explains performance on the selection task, are in Part II of chapter 2.

In addition to attention being given to the elimination of logical errors in selection task performance, consistent *biases* were also investigated, as discussed in the next section.

1.4 Biases in the selection task

Wason's (1966, 1968) explanation of poor deductive reasoning performance on the selection task assumed an inherent tendency to expect a truth relation to hold between sentences and states of affairs. In other words, a *verification* bias was being displayed, as evidence which verified the rule was selected instead of evidence to disconfirm it.

Evans and Lynch (1973) also proposed that errors on the selection task are a form of verification bias, which they termed *confirmation bias,* and this bias, in turn, was assumed to be a by-product of *matching bias.* Matching rather than confirmation bias was experimentally observed by Evans and Lynch when a negative, rather than affirmative, version of the rule in the selection task was used. For example, given a selection task rule in the conditional form $p \rightarrow -q$[5], reasoners chose the p and $-q$ cards, the logically correct falsification selection. In order to ascertain whether the conditional rule was being confirmed or whether cards in the rule were being matched, when the antecedent (p) was negated rather than the consequent (q), i.e. when the selection task rule was in the form $-p \rightarrow q$, the "matching" $-p$ and $-q$ cards were selected, whereas selection of p and $-q$ cards is the "correct" falsificatory or logical response.

Evans and Lynch (1973) argued that *matching* reflected cognitive failure at an early stage of processing. This dual-process view of selection task

5 The condition statement or rule "if p then $-q$" is represented by the notation $p \rightarrow -q$. Details of this way of formalising statements are in given in chapter 2 when the methods and assumptions of proposition logic are reviewed.

performance was theorised by Wason and Evans (1975) as comprising a pre-attentive non-verbal stage determined by relevance biases, and a verbal or analytic stage. They further proposed that matching was a more fundamental manifestation of an unconscious bias to be indifferent to logical negation. More specifically that matching bias reflects a complex, linguistically determined relevance judgement rather than a simple availability of information effect (Evans, 1977). Evans (1984, p, 31) writes that matching bias occurs because reasoners prefer to choose the cards actually named in the rules and find it hard to see the relevance of instances which match neither the antecedent nor consequent values named[6]. Matching bias is therefore considered to be a good example of selective processing of problem information and is deemed to support Evans (1984, 1989, 1995) dual-component, heuristic-analytic approach to selection task performance, a full review of which is in Part II, chapter 2.

In order to eliminate reasoning errors and/or biases as described above, and as it was assumed appropriate to facilitate logical reasoning, many experiments have investigated the role of content, and the familiarity and availability of information, and how these factors influence performance on the selection task, as discussed below.

1.5 Content, familiarity and availability of information

As therapy experiments did not always successfully induce insight into the falsification principle, attention was given to the role of content and context. Investigating the role of selection task content can involve experimental changes to materials, procedures and instructions, as well as making rules more contentful and framed within realistic or *thematic* contexts. For example, Wason and Shapiro (1971) designed a thematic version of the selection task where cards represented either towns, or transport which could be used to get there. For example, the *p, q, -q* and *-p* cards were: "Manchester" (*p*) and "Leeds" (*-p*) and the other two cards represented two different modes of transport "train" (*q*) and "car" (*-q*). The conditional rule in this more realistic version of the selection task was: "Every time I go to

6 In support of the matching biases hypothesis, Reich & Ruth (1982) propose that a reasoner's tendency is to match, rather than to verify. Pollard (1985) supports this matching bias stance rather than the verificationist view . However, Oaksford and Stenning (1992) argue that matching only occurs when insufficient or ambiguous information prevents negations being interpreted (discussed further in Part I of chapter 5)

Manchester [*p*], I get the train [*q*]". A control experiment used a standard abstract letters/numbers selection task where the rule was "every card which has a D on one side has a 3 on the other side" and the cards representing *p, -p, q* and *-q* were D, K, 3 and 7, respectively. Facilitation was reported in the realistic or thematic study and it was concluded that it is not so much the logical structure which makes the abstract selection task difficult, but the arbitrariness of the material which "seems to defy the reasoning process" (Wason and Shapiro, 1971, p. 70).

The role of familiarity was also investigated, although without successful facilitation, by Johnson-Laird, Legrenzi and Legrenzi (1972). In this study of the selection task, envelopes instead of cards were used, and there were two *-q* cards, an envelope with a 40 lire stamp on it and an unstamped envelope. Participants were told to imagine they were postal workers checking the rule "if the letter is sealed then it has a 50 lire stamp on it". There was an 81% facilitation of the falsificatory response but this was found to be culture-dependent and not facilitatory in contexts where this rule was unfamiliar. This study was criticised as not reflecting reasoning at all but rather the retrieval of real situations from memory.

Also investigating the familiarity of material, Manktelow and Evans (1979) replaced the abstract materials of the original letters/numbers selection task with more realistic materials and the rule "If I eat haddock then I drink gin" but logical facilitation was not achieved and it was concluded that materials remained arbitrary. Facilitation was, however, successful when Griggs & Cox (1982) investigated content and the role of memory cueing, and where the context was policing or enforcing a familiar and available "drinking age rule" in the form "If a person is drinking beer then that person must be over 19 years of age". The cards represented a person drinking in a bar with the beverage they were drinking on one side and their age on the other. This experiment also demonstrated the role of transfer effects, where a selection task in which the "drinking age rule" preceded an abstract version of the selection task produced logical facilitation in the abstract task. Griggs and Cox concluded that facilitation was a result of memory-cueing and not facilitation of logical reasoning.

Other facilitatory thematic selection task experiments include those carried out by Cheng and Holyoak (1985, 1989), Cosmides (1989), Manktelow and Over (1991), Sperber, Caro and Girotto (1994) and Kirby (1994). The theoretical motivations these experiments are reviewed in Part II of chapter 2.

Finally, Pollard and Evans (1981) studied the effects of prior beliefs in reasoning performance. And Pollard & Evans (1983) investigated the effects of "contrived experience" (1983) by introducing a probability learning task prior to an affirmative abstract version of the selection task, as well prior to a second experiment in which the "negations paradigm" selection task was used[7]. In both the 1983 studies the frequency of certain card presentations was varied prior to the selection task, and the differential learning or availability of the falsificatory response was assumed to facilitate correct logical selections[8].

Having introduced the ways in which facilitation of logical performance on the selection task has been investigated and explained, the next section makes explicit the assumptions underlying the selection task, as it is the acceptance of these assumptions which constrains understanding and clear explanation of the selection task, reasoning and rationality.

1.6 Assumptions underlying the selection task

By accepting that performance on the selection task is illogical but that logicality can be facilitated in some form or another, the necessity to question assumptions underlying the selection task, human reasoning ability and performance, and rationality generally are not fully debated. If reasoning and the psychology of information selection is to be properly understood, assumptions underlying this task and the psychology of reasoning must be re-assessed.

The main assumption of the selection task is that falsifying an hypothesis is the reasoner's goal. The assumption that looking for a falsifying instance is the correct way of ascertaining the truth of hypotheses also forms the basis of

---

7 Using four forms of rule: $p \rightarrow q$, $p \rightarrow -q$, $-p \rightarrow q$ *and* $-p \rightarrow -q$ is known as "the negations paradigm" selections task as antecedents and consequents of rules contain negated constituents.

8 Pollard and Evans (1981) and (1983) studies are considered in detail in chapter 4, section B, and in Part I of chapter 5.

Karl Popper's theory that falsificationism should be the criterion to demarcate scientific from non-scientific knowledge (Popper, 1959). Popper argued that science could only progress by systematically attempting to falsify previously advanced hypotheses. He maintained that seeking to confirm an hypothesis does not determine whether an hypothesis is true but only lends it temporary support until negative evidence overthrows it. In similar terms, Wason assumed that the correct, logical rule of inference in conditional reasoning must be the application of the logical principle *modus tollendo tollens*, so if falsifying evidence is found it means that there is logical certainty that the rule is false. However, it is debatable whether falsification or the application of logical principles such as *modus tollendo tollens* are appropriate explanations of the way in which human beings make inferences about conditional rules in the selection task or generally.

Another assumption to reconsider is that the selection task is a tool for measuring deductive reasoning ability and that the psychology of reasoning should be confined to investigating the context of justification or hypothesis testing, rather than induction or construction of a theory, hypothesis or rule. The selection task may well have been designed by Wason to demonstrate how logical principles should form the basis of deductive reasoning but, notwithstanding Wason's intention, there is confusion as to what selection task performance is reflecting as illustrated by inconsistencies in its classification. Gilhooly (1988, p,113), for example, writes about the selection task in a chapter entitled “Inductive Reasoning”. John Anderson, in the 1990 edition of his book “Cognitive Psychology and its Implication”, writes about the selection task in a section on deductive reasoning entitled “Conditional Reasoning: Failure to apply Modus Tollens” (p. 295). A way of resolving inconsistency must involve the redefinition of rationality, and a re-examination of the decision rules and procedures assumed to comprise the selection task itself.

1.7 Rationality redefined

As regards a re-definition of rationality, I introduced earlier in this chapter Evans' distinctions between rationality$_1$ and rationality$_2$ which definitions are essentially distinctions between personal, goals, beliefs and actions and their utility *versus* impersonal processes underlying inferential behaviour. Evans' definitions assumes that a clear distinction can be made between personal utility of beliefs/goals and processes, and in this way he is able to propose that pre-attentive relevance-biased beliefs influence the way in which inferences using mental model procedures are made. A full review of Evans' theoretical approach is detailed in chapter 2, but its relevance at this stage is that it is an example of the way in which many theories of reasoning assume both an *inferential* component in which deductive inferences using truth-preserving rules in some form or another are made, but that a pre-inferential or *interpretational* component of cognition creates the context (or interprets premises) on which inferential rules then operate.

The O&C model of reasoning as optimal data selection, however, assumes that an optimality-preserving decision rule governs selection behaviour. More specifically and rather than assuming that the decision rule of the selection task is to falsify or apply MTT, O&C assume that the decision rule is to *select optimal data,* i.e. to select the cards expected to provide the most information as such cards reduce uncertainty the most about the truth of the selection task rule. The assumption that optimal data selection is the decision rule governing selection behaviour is based on the consistently observed ordering for certain cards selection in the four card problem, i.e. $p>q>-q>-p$. An optimality approach to cognition assumes that such a consistently observed transitive ordering reflect that a maximizing principle is being applied to preferences (O&C assume that these preferences are about which data or cards are the most optimal because they are expected to provide the most gain in information).

The O&C model is also concerned with the role of context discovery and construction but this is not conceptualised as a separate, interpretational component on which logical decision rules in an inferential stage of reasoning then operate. An optimality approach to cognition makes it difficult to distinguish between interpretational and inferential components. Instead the selection task is assumed by O&C simply to reflect the adaptive behaviour of optimal data selection, where card selections reflect each card's perceived information gain which is probability- or context-dependent. The O&C's optimality approach to rationality and cognition is similar to that of John R. Anderson (1990a, 1991, 1993), whose research into the adaptive character of thought is reviewed in chapter 3.

1.8 Optimality calculated

To formalise the intuition that reasoners are good information optimizers rather than poor logical falsifiers, the expected amount of information to be gained if a given card were turned over in order to see what is on its reverse side is first calculated by O&C. This involves calculating the amount by which each card reduces uncertainty *before* and *after* evidence is collected. The difference between prior and posterior rule- or model-uncertainty is the measure of each card's optimality. Having calculated each card's expected informativeness, and as O&C assume that information gain is probability dependent, card informativeness is then varied by systematically increasing or decreasing the probability of each card, which in turn, increases and decreases the optimality of each card. Different probabilistic context are predicted to change the informativeness of cards and so produce different card selections[9]. In other words, card selections are governed by a simple optimality-preserving decision rule to select the most informative data or, in optimality terms, to optimise expected information gain. Full details of the O&C model of optimal data selection, its assumptions, constraints, predictions, and the way in which expected information gain of each card is calculated using Bayesian methods, are fully detailed in Part II of chapter 3.

1.9 The structure of this thesis

Having outlined the motivations of this thesis and introduced the assumptions underlying the O&C model of optimal data selection, the structure and contents of the ensuing chapters are outlined below.

Chapter 2, Part I includes a short introduction to the philosophy of logic and cognitive science. Part II reviews major theories of reasoning processes, including propositional logic, insight models of reasoning, formal rules or mental logics, the theory of mental models and the theory of pragmatic reasoning schema. Two theories which focus on what makes information relevant are then reviewed: the heuristic-analytic approach and relevance theory. Theories of relevance then pave the way for reviewing theories and

---

9 Optimal data selection assume that p and q card are optimal selections when the probability of these cards in comparison to the $-p$ and $-q$ cards is low. In this specific probabilistic context, p and q cards are predicted to be frequently selected because they reduce uncertainty the most about whether the selection task rule or another "foil" hypothesis holds, and the $-q$ card and the $-p$ card are the least informative cards to select in this probabilistic context. When the probability of p and q are high, card informativeness ordering is predicted to change to the ordering: $p>-q>q>-p$.

explanations concerned with the maximisation of utility, as well as the optimization of items in a social contract and "fitness"[10].

Chapter 3, Part I outlines the assumptions underlying the principle of maximization, and a rational analysis of consumer behaviour in economic decision making is used to illustrate this approach more concretely. The way in which decision rules in utility theory have evolved is then reviewed, whereafter the theoretical assumptions of classical, optimal foraging and optimal diet selection theories are reviewed. Part II outlines the approach of John Anderson's (1990a, 1991, 1993) adaptive character of thought, and the O&C model of optimal data selection is explained within the framework of Anderson's six steps in the development of a rational analysis of behaviour.

Chapter 4 reports seven studies. In Section A, the "Four Cards" study and the "Single Card" study control for computerised, multiple (four card) selection task presentation, and computerised, multiple (single card) selection task presentation, respectively[11]. Three "card informativeness" studies are then reported, the "Pilot Ratings" (four card presentation) study, the "Single Ratings" study, and the "Binary" (two cards presentation) study, in all three of which different forms of data about the perceived informativeness of cards are collected. In Section B, two probability learning studies are reported, in each of which there are two experimental conditions where the probabilities of the p and q cards are varied in a probability learning phase prior to the selection task phase. In one experimental condition the probabilities of p and q are high and in the other condition, the probabilities of these cards are low.

Chapter 5 has two parts. In Part I, I discuss the experimental results and learning procedures used in the Section B studies and their implications for the O&C model specifically. A number of ways in which the O&C model may be refined are considered, and the results of Pollard and Evans (1983) studies are explained within the framework of Bayesian optimal data selection. In Part II, I discuss optimality approaches to cognition generally, and also discuss the way in which different decision rules may be more or less appropriate in

10 The concepts of "fitness" is fully defined in chapter 3.

11 Computerised, multiple, single card, selection task presentation are design features of probability manipulation studies reported in Section B and these first two studies control for these novel procedures.

different contexts. Finally, the implications of this way of explaining selections task behaviour for the psychology of reasoning is discussed.

The next chapter 2 reviews the philosophical assumptions underlying cognitive psychology and science, and the major theories in reasoning psychology.

Chapter 2
THEORIES OF REASONING

In the first chapter, I introduced the notion of rationality and showed how performance on the selection task impels the redefinition of what it means to be rational. In chapter 2, I introduce philosophical and psychological views of what is means to reason and how it is done. There are two sections to this chapter: Part I provides a brief history of logic and the philosophical foundations of cognitive science and the psychology of deductive reasoning. Part II reviews the major psychological theories of reasoning: formal rules or mental logics; mental models; schema theories; and the biases and heuristics approach. The way in which a theory of optimal relevance explains the selection task is also reviewed, and two explanations of the selection task in which subjective expected utility is combined with a mental models approach are also reviewed, as is an evolutionary approach to cognition which explains the selection task in terms of cost-benefited social contracts and optimally adapted decision rules.

Part I - The philosophy of reasoning psychology

2.1.1 The history of logic

The relation between logic and thinking has been a philosophical issue in Western thought since the time of Plato and Aristotle. The ideas of these philosophers came from Socrates who invented what is known as the Socratic Method of "cross-examination". He used this technique with his students as he sought to make them aware of "the conceit of knowledge without the reality" (Lewes, 1857 p. 129). By this Socrates meant that the use of loose conceptions should be substituted by the application of rigorous and distinct concepts. He achieved this method of reasoning by formulating *Definitions*, as to know the essence of a thing it must be considered as distinct from everything else, therefore it must be defined. The philosophical method devised by Socrates was the first instrument by means of which Knowledge was assumed to become Science, as it enabled the thinker to separate a particular thought to be expressed from other thoughts which clouded it.

Plato was taught by Socrates and he preserved the Socratic Method which relied on Definitions. Plato enlarged on these principles, adding the processes of generalisation and classification. Plato achieved this by introducing the art of discoursing, or the art of thinking using general propositions or general terms and universals, or logic. Aristotle was a student of Plato and he also sought to analyse the processes of mind in order to exhibit the art of thinking in all its detail. Aristotle proposed that logic, being the science of seeking truth or falsehood, was the only instrument of thought, and that human beings alone had this art or skill.

These ancient philosophers were the first to study what it means to reason logically, and their claim that logic is the mechanism which enables this has been the foundation stone of the psychology of reasoning. The philosophy of science and cognitive science (of which cognitive psychology and the study of artificial intelligence are sub-disciplines) were also informed by past ontologies of the mind and epistemological issues. Those of particular relevance are introduced in the next section.

2.1.2 The assumptions of cognitive science

Cognitive science is concerned with the issues of ontology and epistemology. *Ontology* is the study of the form in which something exists or "what there is". It was debated in the seventeenth century by René Descartes, who formulated an ontological theory of mind known as Cartesian dualism. This theory proposed that human beings were made from two substances: one physical or material in nature, from which the body was composed; and another non-physical, non-spatial substance, from which thinking was composed. Descartes claimed that these two substances interacted in the brain of human beings, but how such interaction took place was not clearly explained.

Descartes was also concerned with the nature, structure and origins of knowledge, the study of which is known as *epistemology*. He argued that all our knowledge of the external world is mediated by representations, which stand for things. Consistent with his dualistic ontological stance, he inferred that there is no necessary connection between representations and the things they represent. But, as representations have a relatively constant relation to the things they represent, he proposed that they are able to guide our activity in the outside world. Using this notion of "representational scepticism", Descartes was able to argue that the mind could be studied without paying attention to the reality it represents. In other words, it is only necessary to study the interrelation of symbols and processes going on inside a mind (a concept at the centre of traditional cognitive psychology and termed "methodological solipsism" (Fodor, 1975, 1983)). Hobbes added to Descartes' theorising on the nature and structure of knowledge by saying "all reasoning is but reckoning", by which is meant that thought can be understood as a kind of calculation, perhaps often unconscious, using formal operations on symbols stored in the mind (Stillings, Feinstein, Garfield, Rissland, Rosenbaum, Weisler, Baker-Ward, 1987, p. 307).

Descartes' epistemological and ontological theories and their implications are mirrored in the way in which cognitive psychologists have studied cognition, and reasoning in particular. The questions have remained the same as in Descartes' day: what is the nature of psychological processes; and what is their relation to physical states? Different assumptions about these issues have lead to differences in, for example, behaviourist and cognitive approaches to psychology. Contemporary approaches to cognition which derive from these earlier philosophical views and which bear directly on recent theorising in the psychology of reasoning are outlined below.

2.1.3 Ontology: Modularity of Mind

The philosopher and psychologist, Jerry Fodor a major proponent of methodological solipsism, sees merit in Cartesian dualism as he believes it recognises the existence of rational mental states, and mechanisms which operate on these states. However, Fodor rejects the view that his separation of mentalism from mechanism is equivalent to dualism as envisaged by Descartes, as he does not believe that there are two substances, one for mind and one for matter. Fodor is a "dualist" in another way as he separates *perception* from *cognition*[1]. He argues that only perception or perceptual inputs are amenable to psychological investigation as they are, amongst other things, isolated from the semantic world and "informationally encapsulated" or domain specific. Cognition, on the other hand, is not a modular system but a central, symbol processing system or network from which our knowledge of the world and expectations derive[2]. Fodor argues that as a centralised symbol

---

1 The psychological systems Fodor classifies are, in fact, threefold: transducers (which transform energy patterns into neural events), input systems and central processes.

2 Fodor assumed that perceptual inputs are "fast" (as a reflex response is fast) and "stupid" as no cognitive processing is required. The central cognitive system, on the other hand, is deemed to be slow and intelligent, and rational.

manipulating system, cognition cannot be psychologically studied, whereas modular input systems can be investigated.

Fodor's (1983) Modularity of Mind thesis relies on the assumption that perception is a collection of many independent modules. For example, in order to study language its components are classified as being part of perception in Fodor's terms and there is a module for word recognition system, a face recognition system, in fact, there is an input system which identifies any object in the world. Fodor developed his thesis regarding the ontology of the mind while a student of Noam Chomsky at the Massachusetts Institute of Technology in the 1950s. Chomsky believed that language had it own unique mental domain whose sole function was language acquisition, processing and production. In order to study the domain of language, as I briefly outlined in the introductory chapter, Chomsky made a distinction between linguistic *competence* (a speaker-hearer's knowledge of their language) and *performance* (a speaker-hearer's actual use of language in the world). In this way, Chomsky was able to account for language competence and he argued that linguistic performance did not reflect the ideal level of a speaker-hearer's linguistic competence. This was because memory limitations, distractions, shifts of attention and errors in knowledge application interfered with innate linguistic ability. Chomsky viewed the development of a theory of competence as the task of linguists, whereas psychology's task was to formulate a theory of performance and when studying linguistic performance, or any other complex cognition, competence must be seen as only one factor to consider. Chomsky also argued that language acquisition was genetically determined, i.e. human beings are wired with some sort of innate "Universal Grammar". This argument that there were universal grammars allowed Fodor to address epistemological questions and formulate his language of thought hypothesis, as outlined below.

2.1.4 Epistemology - Language of Thought

Fodor's thesis on the Modularity of Mind is concerned with ontology or the structure of the mind. Epistemological questions regarding knowledge, how it is acquired and the standards by which it can be judged as reliable or true knowledge, were addressed by Fodor in his earlier thesis on the "Language of Thought" (1975). Fodor's co-influence in this regard was Hilary Putnam, a mathematically trained philosopher who believed that the invention of computing machines was an important event in the philosophy of mind because many processes termed "thinking" (meaning symbol manipulation as considered more fully below) could be realised by machines constituted of entirely different components to human minds.

As well as the ontological issue that thinking computers brought into question, they also raised the epistemological issues of what is knowledge and where does it come from. Adopting a functionalist approach[3], Fodor argued that a system depends not on its hardware or physical realisation to acquire knowledge, but on its software where symbols or mental representations are manipulated according to innate rules. His thesis regarding the "Language of Thought" allowed Fodor to propose that the truth of knowledge is not *semantically* assessed, but it is derived purely in terms of *syntax* in the form of proof theory which requires no recourse to the external world.

Fodor's Modularity of Mind and Language of Thought are important to the psychology of reasoning because logic (which is symbol manipulation) is traditionally assumed to govern the way in which human beings make logical inferences. Because of the above assumption made by Fodor and others, the way in which the external world or context affects reasoning has not been considered an appropriate topic for cognitive psychology. Having reference to the external world (i.e. context) is important, however, and in order to maintain a distinction between the process of deduction and context, the competence and performance distinctions are invoked by psychological theories of reasoning to explain any mismatch between assumed logical competence and observed illogical performance. David Marr provides three different distinctions or levels of analysis which have informed more recent theories in the psychology of reasoning, as outlined in the section below.

2.1.5 At what level should reasoning be analysed?

David Marr characterises three, loosely associated, levels at which an information processing device or "any machine carrying out an information-processing task" (Marr 1982, in Anderson 1990a, p. 5) must be understood.

---

3 "Functionalist" because (i) mental representations are assumed to be autonomous from actual realisations in the world and (ii) there are assumed to be inferential relations among mental states; and (iii) there is multiple realisability of mental states, i.e. computers can infer therefore they have mental states.

(i) The top level is concerned with stating the *computational* hypothesis or theory of "what" a system is doing (i.e. its decision rule) and "why" it is doing it. For example, normative theories of reasoning assume that "what" a reasoner is or should be doing in the selection task is logical inference and looking for falsifying instances in particular. And the reason "why" this is what a reasoner is or should be doing is because this is the method by which an hypothesis is logically proved to be false, or "scientifically" refuted in Popper's falsificationism terms.[4]

(ii) Marr's *algorithmic* level specifies "how" falsification is performed (for example, by means of logic, or mental models, or pragmatic reasoning schema). Evans' rationality$_2$ definition is concerned with this level of analysis.

(iii) The *implementational* levels relates to how these processes are actually specified in the mind/brain/computer.

The levels of analysis formulated by Marr are of relevance to the O&C model of optimal data selection as it is designed to be a computational explanation of the selection task. I outlined in the introductory chapter that the O&C model of optimal data selection is similar to John Anderson's (1990a, 1991, 1993) theory of the adaptive character of thought. Anderson equates Marr's computational level of analysis with his *rational* level of analysis where rationality is defined as being optimally adaptive behaviour. I review the O&C model of optimal data selection in detailed in chapter 3 within the framework of Anderson's six steps in the development of a rational analysis of cognition.

An important point is therefore that different theoretical and philosophical assumptions about the level at which cognition should be analysed produce different explanations of reasoning performance and reasoning competence.

---

4 See Oaksford and Chater (1995a, p. 133) : "what" a reasoner is doing in their terms is "probabilistic optimal data selection". "Why" this is done is because it is an adaptive behaviour which reduces uncertainty.

In summary, Fodor's assumption that truth can be assessed without recourse to semantics or the world, were also implicitly accepted by Wason when he designed the selection task. Most recent theories of reasoning make similar distinctions between performance and competence and assume that reasoning in the selection task must involve both an "inferential component" (at which stage deductive processes or mechanisms in some form operate on premises or information) and an "interpretational component" (at which stage the selection and induction of information, and/or the interpretation of premises, takes place). Making such distinctions has enabled theories of the psychology of reasoning to maintain that truth-preserving principles or decision rules govern reasoning at an inferential or deductive stage of reasoning, whilst acknowledging that interpretation of premises or context, prior to the stage of deductive reasoning, influences what inferences are made, i.e. what conclusions are reached.

In the second part of this chapter, I review the way in which propositional logic has influenced general theories of reasoning and thinking. I then review specific psychological theories of reasoning and how they explain the selection task.

Part II -The psychology of reasoning

In Part I of this chapter, I introduced the philosophical assumptions underlying logic, cognitive science, the psychology of reasoning, and deductive inference. In Part II, I review the decision rules, procedures and assumptions of *propositional logic* from which initial theoretical explanations of the selection task derive. The *insight models* of logical errors in selection task performance, and three major psychological theories of reasoning competence or processes are then reviewed: *formal* or *mental logic, mental models* and *pragmatic reasoning schema.* The *heuristic-analytic* approach, which identifies relevance biases in deductive reasoning tasks rather than proffering a theory of process, and another *Relevance Theory* are thereafter reviewed. Finally, I review three general theoretical approaches within which explanatory accounts of reasoning performance on the selection task are framed. These general approaches are: *decision theory*, *signal detection theory* and *evolutionary theory.*

Truth-preserving models of reasoning

In philosophy, logic has traditionally been assumed to be the process underlying reasoning and how knowledge and truth are acquired. A number of psychologists continue to take the view that propositional logic, in particular, is implicated in reasoning and that it is the means by which knowledge and truth are acquired. Psychologists taking this approach include Peter Wason, Rips (1983, 1990), and David O'Brien (1993, 1995).

Before reviewing formal rule theory, or mental logic as it is also known, the inferential components or decision rules in standard logic, which are assumed to underlying human thought, are described. The two *methods* by which truth or validity is proved in propositional logic, i.e. truth tables and rules of derivation, are first described.

2.2.1 Propositional logic

Arguments in logic are comprised of propositions and operators (or connectives). A proposition is a simple assertion that cannot be added to and it can either be true or false. For example, the sentence "it is raining" is one proposition, and it can be represented with the symbol "P". Another proposition is "it is snowing", which can be represented with the symbol "Q". Propositions or propositional variables can be premises or conclusions. If there was another propositions besides P and Q, it could be represented with "R", and if there were another it could be represented by "S". A new variable could also be represented as P', and another as P", and another as P'", and so on, there is no theoretical limit to the number of propositional variables there can be (Lemmon, 1965).

Propositions are connected together using logical operators[1] which represent "and, "or" "if ... then", and "it is not the case that". The logical operators for these four connectives are "**&**", "**v**", "→" and "-", respectively. Thus a combination of the two sentences above into "it is raining **and** it is snowing" is represented by the symbols P**&**Q. And the symbols P→Q mean that there is a conditional relationship between propositions, i.e. **if** it is raining **then** it is snowing[2].

This symbolic language is known as propositional logic or propositional calculus, and its exact rules of grammar or syntax allow the validity of arguments to be proved. There are two methods to do this: truth tables, or rules of derivation. I shall explain the method of proving the validity or truth of arguments using truth tables first.

2.2.1.1 Truth Tables

Propositional variables must be reduced to all the possible conditions under which they are true ("T") or false ("F"). For example, the four conditions under which the variables P and Q are logically true are set out below:

PQ	TT
PQ	TF
PQ	FT
PQ	FF

---

1 Operators are symbols which have a specific "meaning" or value attached to them.
2 The rule in the selection task follows this conditional "if *p* then *q*" format.

For example, in the second line above, P is true (T) but Q is false (F). Three propositional variables, e.g. PQR, would require eight lines of truth conditions, and four variables, e.g. PQRS, would require 16 lines of truth conditions. Performing a truth table is purely mechanical, however, and it becomes increasing difficult as the number of variables increases[3].

Having made the truth values of the variables explicit, truth conditions need to be written for all the propositions in an argument. For example, given the sentence or argument (PvQ)→P, P |- -Q, the truth values for all the **operators**, i.e. the conditions under which these operators are true (given two variables P and Q), are specified below - reading *downwards* see standard format of truth tables in the footnote below[4], where the table values for "v" are TTTF, the truth values for the "→" are TFTT, and the truth values for "-" (relating to the negation of Q, the truth values for Q being TFTF) are FTFT:

P Q	(PvQ)	→P,	P |- -Q
TT	**T**	**T**	**F**
TF	**T**	**F**	**T**
FT	**T**	**T**	**F**
FF	**F**	**T**	**T**

Reading across the lines of the above truth table, as there are two true premises (T) and a false conclusion (F) in the first line this is a logically invalid argument.

Truth tables are, in the language of logic, a "semantic" way of testing for validity (or logical truth) and invalidity of arguments as they are concerned with the "meaning" (or truth function) of the operators and the contribution operators make to the truth value of arguments. The second method of testing for the validity (but not invalidity) of arguments uses rules of derivation, rather than truth tables, to prove an argument.

---

3 Computational intractability (Oaksford and Chater, 1993) is one of the main arguments against logic being the mechanism underlying reasoning.

4 Each operator has a specific grammatical or syntactic "meaning", which is defined in term of the conditions under which the operator is logically valid.

PQ	**P & Q**	**P v Q**	**P → Q**	**P ↔ Q**
TT	T	T	T	T
TF	F	T	F	F
FT	F	T	T	F
FF	F	F	T	T

Negation: Q:	**not Q**
T	F
F	T

2.2.1.2 Rules of Truth Derivation

The "syntactic" method of proving an argument uses rules of truth derivation as set out below.

RULES OF TRUTH DERIVATION IN NATURAL LOGIC

Rule	Derivation	
Assumption	A	**(see A below)**
MPP[5]	$P \rightarrow Q, P \therefore Q$	**(see MPP below)**
MTT	$P \rightarrow Q, -Q \therefore -P$	**(see MTT below)**
Double negations	$- -P \therefore P$	**(see DN below)**
Conditional Proof	Assume P, derive q	
	$\therefore P \rightarrow Q$	**(see CP below)**
Conjunctions:	$P, Q \therefore P \& Q$	**(see &-Introduction below)**
	$P\&Q \therefore P$	**(see &-Elimination below)**
Disjunction	$P \therefore P \vee Q$	**(see vIntroduction below)**
	$P \vee Q$, not $P \therefore Q$	**(see vElimination below)**
Reductio ad absurdum	Assume P, derive $Q\&-Q$	
	$\therefore -P$	**(see RAA and * below)**

A This is the first rule of derivation and it allows, at any stage of an argument, for an assumption to be introduced. An assumption is different from a premise as it is not a result of any deduction or reasoning. Whereas a premise is used to draw a conclusion.

MPP **modus ponendo ponens** (MPP) is a principle of conditional reasoning which concerns the operator "$\rightarrow$". MPP permits the consequent of the conditional to be drawn as a conclusion. For example: given $P \rightarrow Q$ and P, deduce Q.

MTT **modus tollendo tollens** (MTT) is another principle in conditional reasoning which is concerned with the operator "$\rightarrow$". MTT permits the negation of the antecedent of the conditional to be drawn as a conclusion. For example, given $P \rightarrow Q$ and -Q, deduce -P. This MTT reasoning principle is the basis of falsification, on which successful solution of the four card problem is assumed to rely.

DN The fourth rule of derivation permits a reasoner, given a double negation of a proposition, to draw the proposition itself as a conclusion.

CP This fifth rule allows a reasoner to derive a conditional conclusion. For example, if by assumption P then Q can be deduced, CP permits that the implication $P \rightarrow Q$ must be true. Then only If P needs to be shown true, and Q will follow.

&I Given two propositions as premises, this rule permits us to their conjunction i.e. P&Q, as a conclusion

&E Given a conjunction as a premise, this rule permits either P or Q as a conclusion.

vI Exclusive conjunction means either P or Q (but not both)

vE Inclusive disjunction means either P or Q (or both)

RAA This last rule rests on the principle that, if a contradiction can be deduced from a proposition P, P cannot be true, so its negation -P can be affirmed.

* **|- - means that a deduction from P to Q is possible**

For example, using the MPP rule of derivation and given the propositions $P \rightarrow Q$, P, a reasoner can validly deduce Q. In formal logic the derivation of this proof is as follows (read footnotes, 6, 7, 8 and 9 first):

---

5 The asymmetrical conditional relationship between $P \rightarrow Q$ is known as "material implication". A symmetrical or bi-directional relationship between $P \leftrightarrow Q$ is known as "material equivalence".

1[6]	(1)[7]	$P \rightarrow Q$[8]	A[9]
2	(2)	P	A
1,2	(3)	Q	1, 2 MPP.

Having outlined the methods used in propositional logic to assess whether an argument is valid, "insight" models of selection task performance, which are based on the assumption that reasoners apply defective truth tables and procedures, are reviewed next.

2.2.2 Insight Model

I review insight models at this stage, directly after discussing methods used in propositional logic, because, as introduced in chapter 1, insight models assumed that errors in the selection task were based on defective truth tables. This theoretical assumption was formalised by Johnson-Laird and Wason (1970b) when they specified the way in which a computer would make deductions. They assumed that computer-based deductions could provide the appropriate standard against which "ideal" human reasoning performance should be compared and contrasted. The steps which Johnson-Laird and Wason assumed a computer program would follow are set out below.

Step 1: A computer is programmed to retrieve the appropriate truth table. For example, and as the *operator* connecting the sentences in the selection task is of the form **$P \rightarrow Q$**, the computer selects the truth table for the "$\rightarrow$" conditional connective, which is:

PQ	$P \rightarrow Q$
TT	T
TF	F
FT	T
FF	T

Step 2 Having retrieved the $P \rightarrow Q$ truth table, the computer scans each card to see how it relates to the four combinations of the truth table. As the above truth table shows, $P \rightarrow Q$ is only falsified then when the proposition P is true and the proposition Q is false (i.e. -Q in second line). The computer selects the P card

6 The number in this column reflects the *assumption/s* on which a conclusion rests.
7 Bracketed numbers reflect the next step in the proof as a whole.
8 *Premises* are in this column.
9 This column specifies the *rule of derivation* used and the *premises* to which the rule applies.

as the rule is falsified when this card is associated with the -Q card, and this latter card is also selected.

On the basis of poor reasoning performance on Wason's (1966) selection task, Johnson-Laird and Wason (1970a and 1970b) argued that people do not perform in accordance with the above computer-based truth table procedure and scan the four ways the values of P and Q can be combined. Instead a "defective" truth table, as below, is applied which assumes that any combination associated with -P is *irrelevant* to truth or falsity of the rule[10]. Furthermore, this "defective" truth table is not always applied correctly.

PQ	P→Q
TT	T
TF	F
FT	Irrelevant
FF	Irrelevant

As a result of Wason's (1968) additional unsuccessful "therapy" experiments involving the "projection of falsity" and the "restricted contingency programme", and in order to make the principles of falsification more evident, Johnson-Laird and Wason (1970b, p. 138) argued that reasoning errors were made because most people have different degrees of insight into the importance of falsification as opposed to verification. On this assumption, a two-tiered information processing model of reasoning performance was formulated where two kinds of insight were postulated: possessing "insight (a)" leads to checking only the *p* card, while possessing "insight (b)" leads to checking the *p*, *q* and -*q* cards. Possessing both insight (a) and insight (b) leads to choosing both the *p* and -*q* cards.

However, further non-facilitatory effects of therapy experiments (Wason, 1969) prompted the two-tier insight model to be revised to incorporate three levels of insight: no insight, partial insight and complete insight. This revised model predicted that in a state of *no insight* reasoners focus on cards mentioned in the rule and verify (this was not an explicit assumption in the two-tier insight model). If the rule was interpreted as being biconditional (P↔Q), then both *p* and *q* cards are chosen. Otherwise, if the rule was interpreted as being asymmetrical (P→Q), only the *p* card is selected. Having *partial insight* meant that cards which verify and falsify are selected, i.e. *p*, *q*

10 See chapter 1 at section 1.3 for Wason's (1966, 1968) explanation of original selection task errors in terms of three outcomes: true, false or irrelevant.

> and *-q* cards, but the *-p* card is irrelevant as it cannot verify or falsify. The preliminary insight model assumed that this level of insight was independent from complete insight. Having *compete insight* meant that only cards which falsify are selected. Therefore the *p* and *-q* are selected and the *q* card rejected as it cannot falsify the rule. Finally, if a biconditional interpretation is rejected (i.e. P→Q is accepted), then complete insight may be gained after partial insight when the *q* card is tested for the first time. If a biconditional interpretation (i.e. P↔Q) is accepted, it is less likely that complete insight is achieved as there is no reason to retest the *q* card and develop insight. The greater the insight, the greater the number of routines.
>
> The Johnson-Laird and Wason (1970b) insight model of selection task performance supported the view that, in Piaget's terms, some individuals are not logical as they have not reached the stage of formal operations, or may have regressed to a pre-formal operations stage of illogical reasoning (Wason 1969).

I have included this insight model in this literature review as it was the first theoretical explanations of reasoning behaviour in the selection task and inducing degrees of insight was the experimental procedure used to facilitate logical reasoning. Inducing insight appears to be an appropriate course of action *if* falsification is what a reasoner's goal in the selection task should be. O&C (1994) assume that the goal of a reasoner in the selection task is to select optimal data, not look for falsifying instances of a rule. They propose that the various unsuccessful attempts to induce insight using therapy experiments reflect the application of a simple optimality-preserving decision rule: to select cards on the basis of their informativeness. Furthermore, it is only when probabilistic context is varied that card selection behaviour changes, as the informativeness of cards is probability or context-dependent. Therefore, without the probability of cards being varied, participants in the insight studies had to try to override a basic optimality decision rule and, instead, select cards which do not provide the most information gain.

In Bayesian optimal data terms, therefore, inducing degrees of insight into truth-preserving decision rules is not appropriate, theoretically or experimentally, if selection behaviour is to change. However, facilitating falsification is the goal of most selection task experiments in the psychology of reasoning, although not all theories assume that using defective truth tables is the way in which we reason.

In the next section of Part II, I review the formal rules or mental logic theory of reasoning. This theoretical approach assumes that constructing mental proofs similar to the rules of derivation used in propositional logic, rather than truth tables, is the method used to explain reasoning behaviour.

2.2.3 Theory of mental-logic or formal rules

In a formal rules or mental logic approach to reasoning, the truth-preserving rules of derivation method of validation used in propositional logic is used rather than truth tables. More specifically, general purposes inferences or schema whose function is prove the validity of conclusions are applied to premises. The first step a reasoner is assumed to make is to uncover the logical form of the premises; a reasoner's repertoire of inference schema is then accessed; and a proof is then constructed using a particular inference schema showing that the conclusion is a valid one.

Mental logic differs from propositional logic in that, in propositional logic, any argument even if it has contradictory premises is valid if the conclusions are also true. In mental logic this cannot be the case and nothing can be inferred in these conditions, except that a premise is wrong. Inferential schema are only applied to premises if premises are accepted and not contradictory (O'Brien page 198)[11].

Mental logic is also different to propositional logic in that it assumes that some inferences or schemas are more available than others, and that it why some forms of reasoning are more difficult than others. In fact, some sets of inference schemas are completely unavailable as most theories of mental logic do not incorporate a set of inference schema for *modus tollendo tollens* ("MTT") although formal theories do have inferential schemas for *modus ponendo ponens* and conditional proof[12]. Therefore, and in order to make the MTT deduction in the selection task, in mental logic it is usually necessary to make a series of complex inferences. A deductive problem requiring the application of the reasoning principle of MTT is thus harder, mental logicians

---

11 A problem with this mental logic assumption is that decisions first have to be made about which premises are acceptable and not contradictory. In other words, mental logic does not consider what decision rule is being applied to make premises appropriate to select in the first place.

12 Osherson (1976) does include MTT (see Table 2.2 in Johnson-Laird and Byrne (1991, p 30)

argue, than one requiring the application of MPP which is part of the mental logic repertoire.

For example, in order to falsify an argument, *modus tollendo tollens* needs to be applied to the premise $P \rightarrow Q$, but as there is no principle of *modus tollendo tollens* in mental logic, no simple proof in the form $P \rightarrow Q$, $-Q \therefore -P$ can be derived as is the case in propositional logic, and a longer proof is required. One possible proof of validity that mental logicians can apply is as follows:

given premises of the form: if *p* then *q*, *not q*, reasoners can hypothesise *p*, from which they can derive *q* (by MPP from hypothesis and first premise). This conclusion, together with the second premise, yields a self contradiction *q* and *not q* (by conjunction). The reductio ad absurdum rule entitles the negation of any hypothesis that leads to a self contradiction to be derived, i.e. *not p* (by reductio). (Johnson-Laird and Byrne, 1991 p. 42)[13]

As far as computer instantiations of theories are concerned, a formal or mental logic approach assumes a *reasoning program* controls the way in which proof is constructed and selected at different stages. In addition, a *comprehension component* is assumed to decode the premises into logical form prior to reasoning and then recodes the conclusion subsequent to reasoning. For example, a reasoning program could have direct and indirect reasoning routes. Direct reasoning routines start with the premises and then inferential rules or schema matching the form of the argument are accessed. Decision rules or inferential schema are then applied and each inference is added to the premise set until a conclusion is reached, or a proposition that is incompatible with the given conclusion reached. When the direct reasoning routine fails, indirect reasoning strategies are applied. For example, the derivation of the consequent in conditional proof is not assumed to be the result of logic or direct reasoning and it could be the result of pragmatic or indirect inferences (O'Brien, 1995 p. 197).

In similar terms to the above computer instantiations of mental logic assumptions, and after a reasoner has abstracted the form of the premises, mental logic predicts that easily validated inferences are made by a reasoner accessing a corresponding elementary inference or decision rule. It is also predicted that errors will occur because deduction depends on whether there is a corresponding inference or truth-preserving decision rule to access, e.g. there

13 See also Table 9.1 in O'Brien (1995) at page 202, where there is a sample argument of a conditional proof for Wason's selection task.

may be no MTT; or errors may also occur because, although there may be a rule, it is inaccessible; or errors may occur because reasoners may choose to use pragmatic heuristics. Mental logic also predicts that the length of a derivation can cause errors and proofs will take longer.

Notwithstanding the above predictions about reasoning generally, mental logic is not easily able to explain selection task performance because MTT is not part of the mental logic repertoire. On this basis alone, it could be argued that mental logics is not a complete theory of reasoning. However, in optimal data terms, the application of truth-preserving decision rules (or inability to apply such rules because they are not available), either in the form of truth tables or rules of derivation, is an inappropriate way to explain reasoning performance in the selection task, because cards selections are assumed to reflect that there is a relationship between the expected information gain of each card and which card are optimal selections in a given context.

It is clear that the methodological solipsism advocated by Fodor is adopted by mental logicians as they are only concerned with establishing truth-preserving relationships between propositions, and are not concerned with what makes premises relevant in the first instance. In the next section, I review the mental models approach to reasoning, which argues against the methodology used in mental logic.

Semantic mental models

2.2.4 **Mental Models**

A mental models approach argues that mental logic is not able to explain why the *content* of a problem affects the conclusions drawn. In logic, any set of premises can support many valid (but not necessarily true) conclusions and appropriate conclusions are not isolated from trivial ones. Reasoners do not ignore semantic information, but they discriminate between appropriate and trivial conclusions. Given these problems with mental logic, the assumption that formal inference schema or mental logic is the underlying mechanism allowing deductive ability or competence is rejected by Johnson-Laird (1983). He also argues that people do not think using truth tables as reasoners do not naturally know the meaning that logic gives to connectives: a truth functional approach (i.e. using truth tables) is rather only a possible way of thinking, rather than an habitual one.

Rather than propositional and/or mental logics governing the way in which we make correct inferences in daily life, Johnson-Laird formulates a theory of mental models in which new semantic information can be added and where the deductive mechanism is not deemed to be syntactic in its operation but semantic[14]. To achieve a semantic method of proof, Johnson-Laird turned to the "search of problem space" framework used in the field of artificial intelligence. Marr's (1982) levels of analysis also influenced the development of the mental models approach, which Johnson-Laird assumes characterises deduction at Marr's computational level. At this level, the questions asked are: What is being computed? Why is it being computed? What constraints may assist the process[15]?

Deduction is what is computed, is the mental model response to the first question posed by a computational level of analysis. Why it is being computed is so that inferences can be drawn. Johnson Laird proposes that *everyday inferences* depend on the ability to interpret sentences by constructing mental models of the states of affairs they describe, and *deliberate, logical deductions* depend on the further ability to search exhaustively and systematically for alternative models that violate putative conclusions. Constraints which assist the process of building mental models, the third of Marr's questions, are specified as:

(i) not throwing semantic information away, as logical form is not all that governs good reasoning;

(ii) conclusions must be more parsimonious than premises (not, for example, Ann is clever. Snow is white. Therefore, Ann is clever and snow is white); and

(iii) a conclusion should, if possible, assert something not explicitly stated in the premises (for example: Mark is over six feet tall and Karl is taller than him. Therefore Karl is over six feet tall).

---

14 The use of "semantic" by Johnson-Laird is not limited to the way in which operators in truth tables in propositional logic have meaning and are thus "semantic".

15 Mental models is also assumed to be a theory about the process of deduction and therefore falls within Marr's algorithmic ("how") level of analysis (Johnson-Laird and Byrne, 1991) - see chapter 2 (part I), section 2.1.5.

When there is no conclusion that meets these three constraints or assumptions then "nothing follows" is all that can be concluded. In formal logic, this response would be wrong. In summary, "to deduce is to maintain semantic information, to simplify, and to reach a new conclusion." (Johnson-Laird and Byrne 1991, p. 22).

But what exactly are mental models? Mental models are not perceptual images which represent objects. Nor are they strings of symbols that correspond to natural language, i.e. propositional representations. Johnson-Laird (1983) differentiates mental models from propositional representations by using the example of a maze, the routes through which are mapped and remembered on the basis of mental models - there is no representation of verbal propositions involved. Mental models are rather "structural analogues of the world" (Johnson-Laird, 1983, p. 165) comprised of premises in the form of tokens which are arranged in a particular way to represent a state of affairs in the world which can be a true situation, a possible situation or an imagined situation.

A mental model may or may not include imagery, and can take many forms and serve many purposes and contents of mental models are varied. They can represent spatial, temporal or causal relations and their structure corresponds to the perceived or conceived structure of the world. A mental model (or models) does not have arbitrarily chosen syntactic structure like propositional logic, but its structure plays a direct representational role as it is perceived as an analogue of the real world.

For these reasons, Johnson-Laird proposes that mental models provide a more realistic way of representing premises, and their manipulation makes it possible to reason without logic. In particular, he argues that there is no need to translate premises into unwieldy truth tables of the true and false conditions of *p*s and *q*s as used in propositional logic. However, the process involved in assessing the validity of mental models is similar to the truth table method of proof. I shall compare mental model "truth tables" and truth tables used in propositional logic after outlining two stages implicated in mental model construction, and a third stage which implicates deduction.

The mental models approach assumes that the process of mental model construction and deduction is comprised of three stages. In the first stage, reasoners use their knowledge of language to *comprehend premises* and then *construct a mental model* which describes the state of affairs in the world[16]. To the first model of a premise is added a model for a second premise, and so on. The second stage involves formulating a *description of the models of premises* constructed in the first stage. This description[17] asserts something not stated in the models of premises. For example, if all artists are beekeepers is a model of one premise, and all beekeepers are chemists is a model of another premise then, at this stage, a model x, model y, or model z is formulated. If no descriptive model about the state of affairs represented by the premise models can be generated then nothing can be concluded from the premise models. Specifically, when there is indeterminacy, and as soon as this is detected, all that can be said is that "nothing follows" from the models of the premises.

In the third stage, a reasoner *searches for models of the premises where the conclusion is false*. If there are no such models then the conclusion is valid. But if a counterexample or falsificatory model is found, then the reasoner should return to the second stage and discover whether a true conclusion can be derived from the models of premises already constructed. If a valid conclusion is found then the search for counterexamples of that valid conclusion will be necessary, until all sets of possible models has been exhausted and there is a final conclusion. Only this third stage involves deductive reasoning, the other two are assumed to be the processes of comprehension and description (Johnson-Laird and Byrne 1991, p. 36).

Having explained the stages involved in mental model construction and deduction, I now detail how truth is derived using mental models as outlined in stage three above. As I described in section 2.2.1 above, propositional logic uses truth tables in order to derive the conditions under which statements containing operators or connectives are valid or invalid. Mental models also has models for propositional connectives or operators and below are some models (see Johnson-Laird and Byrne, 1991 p. 51, Table 3.1):

16 As was the case in mental logic, mental modelling does not consider what decision rule is being applied to makes premises, or a model of a premise, relevant to focus on in the first place.

17 Formulating a description is similar to drawing a conclusion from premises.

MENTAL MODELS OF CONNECTIVES

p and q	implicit model	p	q		
	explicit model	[p]	[q]		
p or q	implicit models	p			
			q		
	explicit model	Inclusive		Exclusive	
		[p]	[-q]	[p]	[-q]
		[-p]	[q]	[-p]	[q]
		[p]	[q]		
if p then q	implicit models:	p	q		
		...			
	explicit models	conditional		biconditional	
		[p]	[q]	[p]	[q]
		[-p]	[q]	[-p]	[-q]
		[-p]	[-q]		
p only if q	implicit models	[p]	q		
		-p	[-q]		
		...			
	explicit models	conditional		biconditional	
		[p]	[q]	[p]	[q]
		[-p]	[-q]	[]p]	[-q]
		[p]	[q]		

For example, by comparing the propositional logic truth table for an inclusive disjunction, and the mental models for inclusive disjunction, the similarity between mental models' and truth tables' method of assessing validity becomes clearer:

TRUTH TABLE for p v q (or both):

	p	q	p v q	
1	T	T	T	*
2	T	F	T	*
3	F	T	T	*
4	F	F	F	

MENTAL MODELS of p v q (or both)

p or q	implicit models	p		
			q	
	explicit model	Inclusive		
		[p]	[-q]	(see 2 above)
		[-p]	[q]	(see 3 above)
		[p]	[q]	(see 1 above)

It is proposed that mental models differs from truth tables in propositional logic because a mental model represents only true contingencies (marked "*" on the truth table above). Mental models is assumed to show that reasoners try to be as cognitively efficient as possible by using models that make explicit as little information as possible. In this way reasoners reduce the computations associated with the truth tables used in propositional logic[18]. In

18 See section 2.2.2, where I describe how Johnson-Laird and Wason (1970), having based insight models on truth tables in proposition logic, specify the way in which a computer should retrieve and scan *all* values, not only true values, in truth tables.

other words, mental models theory assumes that implicit models are preferred by reasoners because they minimise the cognitive load on working memory. Within these terms, mental modellers argue that a model "fleshed out" to include falsification examples will improve MTT conditional reasoning performance, because explicit models of falsification will reduce memory load. Mental modellers assume that this is why experimental manipulations which flesh out models by way of content and context enhancement facilitate logical reasoning performance. Mental models theory predicts that any manipulation that reduces memory load, such as content or context-enhancing selection task experiments, will be effective in improving deductive reasoning performance (Johnson-Laird, 1995, p. 132). For example, mental models theory argues that it should be easier to deduce from a diagram which makes explicit alternative possibilities, than from logically equivalent verbal premises. With a diagram, reasoners construct an internal description or model from which they can extract a logical form. In support of this assumption, Johnson-Laird (1995, p. 144) quotes a study which he and a colleague have carried out[19] which shows that when premises are in the form of diagrams rather than sentences, reasoning is reliably faster and more accurate.

Finally, and as regards deductive competence and how it is acquired, Johnson-Laird and Byrne (1993, p 332) write "what has to be acquired is a capacity to build models of the world, either directly by perception or indirectly by understanding language, and a capacity to search for alternative models. The acquisition of these abilities is less problematic than the acquisition of formal rules". This quotation contrasts with Fodor's view that in principle all concepts are innate and that inferential ability cannot be learned. A mental models approach assumes that children learn about the contributions of connectives, but only when they have acquired language do they make inferences which involve connectives and deduce valid conclusions.

A mental models approach thus argues against methodological solipsism as proposed by Fodor, as it assumes that the goal of a reasoner is to establish the truth between mental models and states in the world, rather than truth-preserving relationships between propositions and propositions. For this reason, mental models deems itself to be a "semantic" theory of reasoning.

19 (Bauer and Johnson-Laird, 1993)

I conclude that a mental models approach to reasoning essentially remains a truth-preserving model of reasoning as it assumes that a reasoner's goal, in the deductive stage of reasoning, is to search for models in which premises are falsified. In addition, in terms of optimal data selection, making falsifying instances of a rule explicit in the selection task, as mental models assumes is necessary to facilitate logical reasoning in the selection task, is not theoretically or experimentally necessary in order to change selection behaviour at a fundamental level. If experiments using diagrams appear to facilitate reasoning, such facilitation need not due be due to diagrams making alternative possibilities explicit so logical form can be more readily extracted. Rather, diagrams simply provide more information about the probability of events in a given environment and so change the optimality of events (cards) in that environment. When probabilistic context is varied by varying the amount of information people have about events, optimal data selection assumes that selection behaviour will also change to reflect that an optimality-preserving decision rule (to select optimal data) governs selection behaviour.

Having described the mental logics and mental models theories of reasoning, I now review a theory of reasoning in the selection task in particular, rather than reasoning generally.

Meaning-based reasoning schema

2.2.5 Pragmatic Reasoning Schema

Cheng and Holyoak's (1985) pragmatic reasoning schema is an account of selection behaviour in *thematic* versions of the selection task. This thesis focuses on affirmative abstract versions of the four card problem, but I include this explanation of thematic selection tasks because it is an example of the methods used in the psychology of reasoning to facilitate a falsificationist response.

Cheng and Holyoak (1985) argue that mental models does not introduce a novel knowledge structure, as the validity or truth of an argument is still assessed using methods applied in logic. They propose that when reasoning in thematic versions of the selection task, people use knowledge structures acquired as a result of ordinary life experiences. Such knowledge structures they term *pragmatic reasoning schema.* They were influenced in their choice of knowledge structure by the general notion of "schemas", evidence for which was first obtained by in 1932 by F.C. Bartlett when he was researching mechanisms for elaborating information[20]. In the 1970s, research into schema and meaning-based knowledge representations was in full flourish, and distinctions were made between "propositions" (which were assumed to represent the atomic meaning of sentences) and "sets of propositions" which cohere and become "schema" (Anderson, 1990b, p. 143). The view that pragmatic reasoning schema are dynamic knowledge structures, comprising generalised abstracted knowledge and rules learned from experience, derives from this era and area of academic interest[21].

As well as believing it necessary to reject the processes which mental logic and mental models implicate in reasoning and introduce a novel mechanism, the approach formulated by Cheng and Holyoak was motivated by the need to explain the role of context on selection task performance. They particularly sought to show the inadequacy of "availability" explanations of reasoning performance, where available information and the retrieval of associated available information from memory were deemed to facilitate reasoning on the selection task (rather than simply making falsifying instances explicit as mental models assumes is necessary).

A number of selection task studies appear to support the weakness of a simple availability explanation. For example, in the "Sears Problem" (D'Andrade in Evans, 1989), falsifying behaviour was adopted although reasoners had no previous, personal experience of the specific context of the selection task rule available in memory. Participants in this study had to imagine they were store managers checking receipts and the rule was *if any purchase exceeds $30, then the receipt must be approved by the departmental manager.* About 70% of reasoners in this study chose the falsificatory response, even though they could not all have had direct, personal experience of the role of store manager available in memory. In section 1.5 in chapter 1, I introduced a number of other "thematic" studies which manipulated context and the availability of

20 See Anderson (1990b, p. 195) for reproduction of "The War of the Ghosts" story Bartlett used in his research.

21 The research of Schank and Abelson (1977) best exemplifies this era and area of reasoning, for example their event script or schema of dining at a restaurant (see Anderson, 1990b, p. 141 for a summary).

information. For example, Wason and Shapiro's (1971) thematic "towns and transport" task facilitated logical reasoning performance; but results in the "postal rule" selection task of Johnson-Laird, Legrenzi and Legrenzi (1972) were found to be culture-dependent; and the "drinking age" rule selection task of Griggs and Cox (1982) showed a facilitating transfer effect when the drinking age rule study preceded an abstract version of the selection task.

Because of inconsistent facilitation effects, Cheng and Holyoak reject a simple availability explanation of the selection task. They also reinterpret the "drinking age" rule as expressing deontic relations, i.e. statements about what *ought* to be the case. They propose that a deontic rule, unlike the abstract rule in Wason's (1966) selection task, provides sufficient cues for a facilitating reasoning schema to be retrieved from memory (see chapter 1, section 1.1., footnote 3). The rules associated with schema, then activate appropriate inferences. Cheng and Holyoak thus extend a simple "availability of memory cues" explanation into a theory of pragmatic reasoning schema, where knowledge structure is assumed to be qualitatively different to theories previously advanced.

Having outlined the theoretical and experimental motivations of Cheng and Holyoak's approach, I now explain the details of pragmatic reasoning schema. Cheng and Holyoak assume that abstract knowledge structures or pragmatic reasoning schema are acquired from life experience. Furthermore, as there can be scripts and schema for different events and circumstances each with different goals, different pragmatic reasoning schema also have different goals. For example, they envisage facilitating pragmatic reasoning schemas for *conditional permissions* (where satisfaction of the precondition bestows a *right* to take a regulated action as conveyed by the word "may"), and facilitating pragmatic schemas for *conditional obligation* (where satisfaction of the precondition imposes a *duty* to take the relevant action as conveyed by the word "must").

Whether or not a precondition is satisfied, and whether or not action is to be taken, can be defined in four ways for both permissions and schemas. Using the "drinking age" rule as an example, Cheng and Holyoak assume this will evoke a permission schema, which can be represented by the following rules:

P1	If the action is to be taken, then the precondition *must* be satisfied
P2	If the action is not to be taken, the precondition need not be satisfied
P3	If the precondition is satisfied, then the action *may* be taken
P4	If the precondition is not satisfied, then the action must not be taken.

The corresponding rules for obligation schema are:

O1	If the precondition is satisfied, then the action must be taken
O2	If the precondition is not satisfied, then the action need not be taken
O3	If the action is to be taken, then the precondition may have been satisfied
O4	If the action is not to be taken, then the precondition must not have been satisfied.

Cheng and Holyoak's basic thesis is that people make inferences using pragmatic reasoning schema where premises are mapped onto a particular set of context-sensitive rules attached to relevant schema. Such schema vary in their degree of correspondence with the conditional $P \rightarrow Q$ (or the material conditional or material implication). This idea is more easily understood when the four permission rules are compared to the four *possible* inferences of the material conditional, i.e. *modus ponendo ponens, modus tollendo tollens*, denial of the antecedent and affirmation of the consequent, (the latter two being logically invalid inferences) see their derivations below:

MPP	$P \rightarrow Q, P \therefore Q$
MTT	$P \rightarrow Q, -Q \therefore -P$
DA	$P \rightarrow Q, -P \therefore -Q$
AC	$P \rightarrow Q, Q \therefore P$

Cheng and Holyoak argue that when a situation evokes a permission schema, the entire set of rules comprising the schema becomes available. When a conditional statement is in the form of P1 (and using the "if a person is drinking beer then that person must be over 19 years of age" example), a facilitative permission schema is evoked. Pragmatic reasoning schema theory argues that the P1 rule has the same result as MPP, although this does not imply that MPP and P1 are equivalent. Pragmatic reasoning schema are thus used as production or decision rules by reasoners to deduce valid conclusions.

As well as allowing valid inferences to be made, Cheng and Holyoak propose that whether inferences are useful can also be assessed using pragmatic reasoning schema. This is because pragmatic goals guide the process of conditional inference. To illustrate this argument Cheng and Holyoak (1985, p. 396) use the propositions: if I have a headache (P), then I should take some aspirin (Q). MTT inference: if I don't need to take some aspirin (-Q), then I don't have a headache (-P). Because reasoners do not make this type of inference, Cheng and Holyoak argue that pragmatic goal-directed rules must guide the process of inference.

Cheng and Holyoak carried out a number of experiments to test their predictions, and I shall outline their "cholera problem" or "immigration task". This thematic version of the selection task was carried out in two conditions: in one, participants were given a reason or rationale and in the other condition no rationale for their task was given. For example, all participants were asked to imagine they were immigration officers and they had to check each incoming passenger's travelling documents in particular a certain form H. On one side of the form whether the passenger was entering the country or in transit was indicated. On the other side a list of diseases, including cholera was listed. Four cards or forms were shown: on one form the word "ENTERING" was printed (representing *p*); "TRANSIT" was printed on another form (representing -*p*), a third form listed the diseases "CHOLERA, TYPHOID, HEPATITIS" (representing *q*) and on the fourth form the diseases "TYPHOID, HEPATITIS" were printed (representing -*q*). The task was to ensure that "if the form says ENTERING on one side, then the other side includes cholera among the list of diseases" and to this end participants had to indicate which of the cards (or forms) they would have to turn over in order to check this rule. The "rationale version" of the task was identical to this except that instead of saying that the form listed names of tropical diseases, initial instructions were that the form listed inoculations passengers had had in the past six months. In addition, the condition rule "if the form says ENTERING on one side, then the other side includes cholera among the list of diseases" was followed by the rationale "this is to ensure that entering passengers are protected against the disease". Both conditions facilitated logical responses, but the rationale version produced significantly more falsifying card selection. Cheng and Holyoak (1985) concluded that pragmatic goal-directed rules guide the process of inference.

In conclusion, and as was the case when I reviewed mental logic and mental models, pragmatic reasoning schema theory assumes that in the selection task a reasoners goal is to falsify. In addition, experiments are specifically designed to facilitate falsification by using thematic materials which versions of the selection task are assumed to show that context-dependent pragmatic inferential rules govern correct selection performance. However, Cheng and

Holyoak's theory does not generalise to selection behaviour in abstract versions of the task, i.e. pragmatic reasoning schema does not explain *what* it is that reasoners *are* consistently doing in the selection task. Instead attention is focused on what reasoners ought to be doing, i.e. falsifying the selection task rule.

I have now reviewed three theories of reasoning *processes*. In the next two sections of this chapter, I review theories of relevance and the role relevance plays in reasoning *performance*. The first relevance account is the heuristic-analytic theory of Evans (1984, 1989) which explains errors and biases in reasoning performance. I then review the way in which Sperber, Caro and Girotto (1995) apply another Relevance Theory (Sperber & Wilson, 1986) in order to explain reasoning performance in the selection task.

Models of relevance

2.2.6 Heuristic-analytic theory

I introduced Evans' (1984, 1989) heuristic-analytic theory in chapter 1 and proposed it was an example of theories of reasoning which assume that truth-preserving decision rules underlie an inferential component of reasoning behaviour, while another interpretational component, or other stages of reasoning, permits context to be taken into account[22]. Evans' approach, however, unlike mental logics, mental models and pragmatic reasoning schema, considers what makes selections relevant in the first place, and biased reasoning performance is assumed to be caused by language mechanisms which select relevant information in a linguistically determined way. Selection of relevant information is assumed to occur at a pre-attentive *heuristic* stage of reasoning rather than at an *analytic* stage where inferences operate on information selected as relevant. The main assumption of this approach to reasoning is that "relevance" influences at the heuristic stage cause inferential performance carried out at an analytic stage of reasoning to appear illogical.

22 For example, mental models is divided into three stages, the first two of which are concerned with the comprehension of premises and description of conclusions, and the last of which is concerned with the inferential process. This third stage, may be compared to Evans' analytic stage.

This two-tier view of reasoning is rooted in a dual process hypothesis first formulated by Wason and Evans (1975). This earlier theory was based on studies (carried out by Goodwin and Wason, 1972) in order to corroborate Johnson-Laird and Wason's (1970b) insight model which required self-reports or introspections about selection task performance to be completed by reasoners after they had carried out the task. On analysis of the protocols or self-reports completed by reasoners, a "matching bias" was observed by Wason and Evans, although matching was not reported by reasoners as a method used by them to solve the selection task problem[23].

In order to explain this biased reasoning performance, matching was assumed by Wason and Evans (1975) to be a non-verbal process determined by relevance which reasoners were not explicitly aware of (Evans, 1989 p. 32). What reasoners were aware of and self-reported in the 1975 studies were termed *post hoc* rationalisations. The inconsistency between, on the one hand, matching performance and, on the other hand, reasoners' self-reports of their performance which did not include that they matched cards with the rule, prompted Wason and Evans to theorise that reasoning reflects two different processes. Inconsistency between card selections and subsequent rationales for card selections were also reported by Wason and Golding (1974).

Evans' research has focused on formulating an explanatory account of biases in reasoning performance. His basic theoretical explanation of biases assumes that pre-conscious processes or heuristics (such as matching bias) lead to a limited representation of a problem space. Therefore, if a problem has been selectively represented when inferences are made, valid deductions cannot be expected. In developing his heuristic-analytic explanation of reasoning performance, Evans draws on several distinctions and theoretical frameworks and these are now considered. For example, Chomsky's competence/performance distinction has informed Evans in his choice to focus on reasoning performance rather than the formulation of a theory of the processes implicated in reasoning. While he acknowledges that reasoners possess an underlying logical competence (as logical principles are understood

23 When "matching" is performed in the selection task it means that cards named in the selection task rule are also the cards selected by reasoners to solve the task's problem. For example, given the *negative* selection task rule "if there is not an A on one side, then there is not a 2 on the other", a reasoners' selection of the A card and the 2 card is called matching, as their card selections match elements of the rule.

in one case but not in others)[24], Evans is of the view that progress in the psychology of reasoning is better made by first explaining reasoning performance.

Notwithstanding the boundaries which limit a bias-focused performance approach from being a complete theory of reasoning, and in order to specify what should be required if psychological theories of reasoning are to be complete theories, Evans (1991) argues that three issues need to be addressed. The first issue a complete theory of the psychology of reasoning needs to address is that of *competence*, and how it is that logical tasks can be solved with above chance accuracy. The second issue needing to be addressed is concerned with *biases*, and why logically relevant features of a reasoning task are ignored and attention deflected away from logical competence. In other words, what decision rule is governing selection behaviour at the interpretational stage of reasoning? The third issue is about the role of *content and context* in reasoning tasks, and why they are attended to if they are meant to be irrelevant to logical or formal solution. Evans classifies mental logics and mental models as the two major theories of reasoning processes addressing the issue of competence or inferential ability. Theories which he proposes primarily address the issue of content and which use the selection task as their sole experimental tool, are Cheng and Holyoak's (1985) pragmatic reasoning schema, and Cosmides (1989) theory of social exchange and social contracts[25]. Evans proposes that his heuristic-analytic theory addresses the issue of biases.

As well as being influenced by Chomsky's competence and performance distinctions, theories of statistical inference have also been informative in Evans' explanation of reasoning performance, especially research undertaken in the 1970s by Daniel Kahneman and Amos Tversky regarding judgement under uncertainty. Their heuristic and biases approach to decision making is concerned with the way in which frequency and objective probability judgements are made using short-cut heuristics or rules of thumb, such as "representativeness" and "availability", but using such heuristics to make judgements or inferences, although useful, can lead to errors and biases. The

---

24 Logical competence meaning the ability to infer rather than the underlying mechanism or process that allows such logical competence or inference.

25 Cosmides' (1989) theory will be considered later in this chapter.

availability heuristic used by Kahneman and Tversky has a specific definition: it is a cognitive strategy or method used when frequency probabilities are estimated. It is argued that an availability heuristic leads to contradictions of basic laws of probability, for example, the "extension rule": if the extension of a set A includes the extension of a set B, then the probability of A must be greater than or equal to the probability of B. A study illustrates this more clearly: when asked to rate whether words with the end "----ing" or "------n" would most frequently occur in a novel, "----ing" words were rated to be more frequent, although the set of "----ing" words are part of the set of "------n" words, and this latter set is the most frequent set. Differential availability of information is deemed to explain biased judgements in this study (Gilhooly, 1988 p. 143).

Using an availability heuristic assumes that decision making and problem solving is biased because decisions are based on selecting readily available information. In other words, the decision rule is "select the most available information". Pollard (1982) extended the definition of availability to the psychology of reasoning and proposed that there is a direct relationship between available information accessed from memory and inferences subsequently made. More specifically while therapy and thematic versions of the selection task make information more "available", these modified tasks do not guarantee that enhanced context will be perceived as relevant to a reasoner's inferences. "Simply concretising the terms is not sufficient; the materials must directly stimulate the recollection of information *relevant* to the solution" (Evans, 1984 p. 458).

To better explain reasoning performance and when formulating his heuristic-analytic approach, the notion of relevance is therefore emphasised by Evans. He offers an idea of what is meant by *relevance* using an analogy of the way in which expert chess players automatically and "immediately 'see' that only a few moves are 'relevant'" (Evans, 1984 p. 452). This rapid, perceptual-like identification of relevance is assumed to be part of the nature of a reasoner. Such a bias is assumed to resolve the problem of complexity or combinatorial explosion[26] in long term and short-term or working memory by reducing

26 Oaksford and Chater (1992, 1993) make this point when they talk about "bounded rationality" and the computational intractability of reasoning if it is assumed to be based on truth-preserving decision rules.

search space as irrelevant information is not selected for further processing (Evans, 1995).

In order to provide a more specific answer to the question "what makes something *relevant*", Evans (1977, 1983) proposes that relevance effects or biases are linguistically determined. In order to show how biases are linguistically determined, a distinction is made between *topic* and *comment* and it is assumed that the propositions included in a statement form the topic, while modifiers such as negation affect only the comment. For example, the statements "I am writing a book" and "I am not writing a book" have the same topic, but make a different comment. He argues that linguistically, the relevant focus of attention in a sentence of the form -P→-Q is always on P and Q regardless of the presence of negations. Logically or syntactically, however, the relevant focus of attention in a sentence is on its operators, of which negations are one.

Having made the above distinction, Evans assumes that matching reflects a complex, *linguistically* determined, bias or relevance judgement. For example, matching in the abstract selection task reflects a "*not*-heuristic", and in thematic versions of this task, an "*if*-heuristic". These heuristics are embodied in the mechanisms which allow language to be understood and they have specific functions. The "not-heuristic" has the function of treating a proposition with a negation in it as irrelevant. The "if-heuristic" has the function of treating false antecedents as irrelevant.

Matching is thus considered a relevance effect at the heuristic, not analytic, stage of reasoning. At a superficial level this relevance effect manifests itself as a bias or tendency to prefer to select or evaluate as relevant cards which match a rule. Matching also superficially reflects a general, pre-conscious "positivity bias" to select and evaluate positive rather than negative information. At a more fundamental level, however, matching is a assumed to be a linguistic bias as, for example, the "not-heuristic" makes it hard for reasoners to see the relevance of instances which neither match the antecedent nor consequent values named[27].

27 Oaksford and Stenning (1992) argue against the existence of these heuristics and O&C (1994) characterise and formalise "relevance" as optimal information. The way in which Bayesian optimal data selection explains matching in the "negations paradigm" of the selection task

In general theoretical terms then, the function of a *pre-attentive* relevance heuristics is to determine what information a reasoner selects and such heuristics will automatically "select 'relevant' information for analytic processing" and information deemed irrelevant will not be further analysed. In other words, the decision rule at the heuristic stage of reasoning is: select "relevant" information, and at the analytic stage the goal of a reasoner in the selection task is to falsify.

As a pre-attentive heuristic or process, therefore, relevance is not a deliberative process of reasoning but selecting relevance is an "immediate, given and non-introspectible" process (Evans, 1995 p.149). This definition of heuristics as a process, contrasts with Kahneman and Tversky's definition of heuristics as conscious, short-cut decision making strategies for making judgements or inferences, which Evans regards as being part of the second, analytic stage of reasoning as I describe below.

The second *analytic stage* of the heuristic-analytic explanation of reasoning is when explicit, *conscious* analytic reasoning is applied to relevant information and inferences then drawn. If information about a problem is selected and represented at the heuristic stage of reasoning in a biased way, then this will produce biased reasoning in the analytic stage of the inferential process. As regards the *mechanism* that underlies analytic processes used to derive inferences from 'relevant' data or information, Evans rejects the view that mental logics underlies this reasoning and analytic process. He aligns himself with mental models which is deemed "an ingenious account of how people reason without logic" (Evans, 1984, p. 466). As I have already described, mental models stresses that people reason by manipulation of mental models representing premises of possible states in the world. Evans argues that this way of theorising about reasoning is compatible with the role of relevance-heuristic processes whose function is to select linguistically determined, relevant information or premises on which the representation of a problem or argument is based.

---

used in Pollard and Evans (1983) probabilities learning studies is fully discussed in Part II of chapter 5.

In the most recent development of his work, Evans (1995) links his theory and mental models by equating the concept of relevance with the notion of explicit representation in a mental model. To achieve a strong linkage, Evans elaborates his notion of relevance and argues that is identical to the notion of "focusing effects"[28] now incorporated into mental models. He believes that by theorising in these terms, mental modellers endorse his view that relevance corresponds to focusing on explicit mental models, and both arise "from selective or biased attention to that which is explicitly represented in a model" (Evans 1995 p. 153).

By merging two approaches to reasoning, mental models and the heuristic-analytic approach, the scope of an heuristic-based biases explanation is widened, and he essentially proposes that *relevant mental models* underlie deductive reasoning *competence* and they also bias deductive inferential *performance*. More specifically, biases and errors are assumed to occur because at the heuristic stage (a) logically relevant (i.e. negations) information is not selected as being relevant and represented; or (b) logically irrelevant (i.e. linguistically-determined relevant) information is included (Evans, 1985 at 148).

Evans' heuristic-analytic approach has evolved since the dual process hypothesis was first formulated by Wason and Evans in 1975 to become an account of how reasoning or analytic problems arise by selective representation at the heuristic stage. Theoretical integration with mental models now enables Evans to address the issue of competence or *how* people reason at the analytic stage, whereas, previously, Evans' approach has been limited to explaining *what* information people reason about. As well as falling within Evans' own two definitions of rationality, a relevant mental models approach to reasoning strives to be a complete theory of reasoning because it addresses the issues of competence, content and biases.

In support of Evans' relevance-heuristic-analytic approach, many studies have been carried out and I deal in depth with the probability learning studies carried out by Pollard and Evans (1983), in chapters 4 and 5. These studies, one of which uses the "negations paradigm" of the selection task, reflect

---

28 Focusing effects are when individuals restrict their thoughts to what is explicitly represented in their models (Legrenzi, Girotto and Johnson-Laird, 1993 in Evans , 1995 at page 152).

Evans' view that a reasoner's beliefs at the heuristic stage influence reasoning in the analytic stage. This reflects "availability" and mental models assumptions about making instances explicit, and experimental manipulations in Pollard and Evans' (1983) work include making falsifying instances explicit in order to facilitate logical reasoning performance.

Evans' two-stage theory is interesting because it seeks to explain the way in which our rationality is constrained by decision rules which govern the way reasoners select relevant information. However, in terms of optimal data selection, it is not necessary for there to be two (or more) components in reasoning, as selection of optimally relevant cards is *all* reasoners are doing in the affirmative abstract versions of the selection task. The assumption that other truth-preserving decision rules, at an analytic stage, need to be applied to relevant information in order to ascertain the truth of falsity of the selection task rule is not an appropriate issue in optimality approaches to cognition. In Part I of chapter 3, I review general optimality assumptions about selection behaviour and the decision rules which govern such behaviour.

Having considered the role of relevance in Evans' heuristic-analytic theory of reasoning, Relevance Theory as formulated by Sperber and Wilson (1986), and as applied to the selection task by Sperber, Caro and Girotto (1995), is reviewed in the next section.

2.2.7 Relevance Theory

Sperber and Wilson's (1986) Relevance Theory is different to Evan's (1984, 1989, 1995) relevant mental models approach as no distinction between successive heuristic and analytic processes is envisaged. Rather, all reasoning has both heuristic and analytic components: the heuristic component guides the analytic as well as receiving feedback from the analytic component (Sperber, Caro and Girotto, 1995, p. 80).

Sperber and Wilson (1986) also characterise "relevance" differently to Evans (1984, 1989, 1995), although Sperber, Caro and Girotto's (1995) explanation of the selection task also has a linguistic component. Relevant information in this relevance theory comes about firstly because of the assimilation of new information with previously available beliefs and conjectures. Assimilated information then becomes relevant information *if* it brings about *cognitive* *effects* or changes in belief which could not have occurred as a result of the new information by itself, or as a result of prior beliefs by themselves. Relevant information therefore yields cognitive effects which lead to (a) the abandonment of old beliefs as they are contradicted by assimilated information, and (b) the formation of new beliefs.

Changes in belief or cognitive effects involve a cognitive cost in the form of processing or *cognitive effort*. The greater the effort involved in achieving cognitive effects, the less relevant information is. How there can be degrees of relevance is illustrated by Sperber, Caro and Girotto (1995, p. 49) using an example set out below, where information about train schedules to Manchester is given with different amounts of information which require more or less processing effort.

The next train to Manchester is:
(a) at 5.30 pm;
(b) some time after 4.00 pm;
(c) is scheduled to leave 7500 seconds after 3.25 pm.

The first bit of information is the most relevant as from it can be inferred the earliest time to travel to Manchester, it also includes the inferences contained in (b) and (c), as well as any other inferences which an individual may regard as following on from the train leaving at 5.30 pm. The second bit of information is less relevant and yields fewer inferences (only (c) above). The third bit of information is the least relevant, not only because inferences are difficult to generate, but the information given in (c) is cognitively cost-inefficient to process as it is complex information. The information contained in (a) above is therefore the most relevant, as it requires the least cognitive effort and it also provides the greatest cognitive effects or change in belief.

Sperber and Wilson (1986) in formulating Relevance Theory make a number of assumptions as summarised below:

(i) The most relevant information at any given time is always attended to and is a basic tenet of Relevance Theory. This notion is formally termed The First (Cognitive) Principle of Relevance and it assumes that a reasoner is governed by a decision rule whose function is to maximize relevance, i.e. process "the most relevant information available in the most relevant way" (Sperber , Caro and Girotto, 1995, p. 48/49)[29].

(ii) The motivation to process the most relevant information in the most relevant way is guided or driven by what can be termed a reasoner's *expected cognitive **effect*** as well as *expected **effort***.

(iii) The expectation of effect and the consideration of cognitively efficient effort cause attention to be directed to items in the environment which are expected to provide the desired information.

(iv) In the absence of expected cognitive effect, *expected effort* plays a direct role in directing attention and memory retrieval, as follows:

(a) the most easily representable information (being the most accessible *consequent* in a conditional statement), at a given time is the most relevant information at that time.

(b) the most easily accessed contexts at a given time are those in which relevance is likely to be maximised, although relevance may not always be maximised as a rapidly changing environment and culture may work against this mechanism[30].

In summary, only if information is salient and easily representable in the first place will it be attended to in the hope of gaining cognitive effects with the least cognitive effort.

Besides the First (Cognitive Principle of Relevance defined in (i) above, another basic tenet of Relevance Theory is The Second (Communicative) Principle of Relevance where "every utterance conveys a presumption of its own relevance". This principle is concerned with communicated information being different to environmental information. It is different because a communicator wants a receiver of the communication to presume that the information being communicated is relevant (i.e. provides some information

---

29 This theory of relevance therefore adopts an optimality approach to cognition similar to O&C (1994).

30 I discuss the way in which an optimality approach debates the issue of the maximisation principle not always being applied in Part I of chapter 3 and Part II of chapter 5.

gain in optimal data terms). This second principle of Sperber and Wilson's relevance theory is the means by which performance on the selection task is explained.

In order to decipher whether a communication is relevant or not, Sperber, Caro and Girotto (1995) assume that on its receipt a specific "comprehension strategy" is spontaneously applied by receivers in order to identify the cognitive effects intended to be relevant by the communicator[31]. Furthermore, a receiver's perception or assessment of relevance is assumed to depend on *expected cognitive effects* which should be the most easily accessible information (and the most easily accessible consequents in conditional statements), and *expected effort* which should be minimal effort. When the expected, adequate level of relevance comprehension is achieved (if it is achievable), then it is rational for the application of a relevance comprehension strategy to stop.

To illustrate when the expected level of relevance is achieved, Sperber et al give the following example:

Peter: Do you want to go to the party at the Smiths?
Mary: They came to our party.

The issue of relevance to Peter is receipt of an answer to his question about whether or not to go to the Smith's party. The reply given by Mary is not a direct one and it also carries a presumption of relevance, as Peter has to ascertain how the reply is relevant to him. To do this, an implicit premise (about a social rule of reciprocation) needs to be retrieved from memory by Peter and then assimilated with the explicit information contained in Mary's reply and Peter's own explicit prior beliefs. The assimilated information (comprising new information assimilated with prior beliefs) will then provide expected information (or cognitive effects) from which an implicit conclusion can be derived.

Relating the above theoretical interpretation of relevance to laboratory settings of reasoning problems, Sperber et al argue that information given to reasoners, in the selection task discourages expectation of relevance. What drives the

31 Comprehension strategies include "disambiguating, assigning reference, narrowing down or loosening literal meaning, and identifying cognitive effects".

comprehension of relevance in the abstract selection task is economical effort, rather than expected cognitive effects. This is because information, about consequents in a conditional rule, are not easily accessible, although thematic versions do raise expectations of cognitive effects as information is more easily accessible.

In order to construct a "facilitating" or easy selection task, Sperber et al argue that the task must be constructed so that the selection task is interpreted as a denial of the antecedent, i.e. "there are no P and (-Q)" cases, even though this is more effortful (involving two negations) than representing "there are P and Q cases". Relevance Theory (1986) predicts, because of the psycholinguistics of negation, a negatively constructed selection task and the subsequent denial thereof causes logically correct card selections. In other words, in order to cause the selection of the P and -Q cards, the consequences of the selection rule (the consequences being "there are no P and (-Q) cases") should be made more accessible and richer in cognitive effects than the consequences of "there are P and Q cases" (Sperber et al, p. 89).

The explanation of the selection task by Sperber, Caro and Girotto (1995) in terms of Sperber and Wilson's (1986) Relevance Theory is compatible with O&C (1994) model of optimal data selection because it assumes that the decision rule governing the selection of information is the optimisation of relevance, and information which provides the most cognitive effects (or gains) with the least effort is the most relevant or optimal information to select. Oaksford and Chater, (1995b) propose that their model of optimal data selection formalises Sperber et al's characterisation of relevance. Optimal data selection, also has a more parsimonious account of the way in which negations facilitate reasoning based on the probabilities attached to negations and how this affects the optimality of negated constituents. I detail the way in which optimal data selection explains negations in Part I of chapter 5 when Pollard and Evans (1983) negations paradigm selection task study is discussed.

So far in Part II, I have reviewed three theories which describe processes which may underlie reasoning (mental logics, mental models and pragmatic reasoning schema) and two theories which emphasise the influence of relevance on reasoning performance. In the next two sections, I review the decision-theoretic motivations underlying the experimental research of Manktelow and Over (1991), and Kirby

(1994) where selection task behaviour is assumed to be influenced by utilities and probabilities.

Models of utility and probability

2.2.8 Subjective utilities, subjective probabilities and extended mental models

It is because of Manktelow and Over's (1990, 1991, 1993) theoretical investigation of *subjective utilities*[32] and *subjective probabilities*, which notions they incorporate into explanation of *deontic* reasoning, that their decision theoretic approach to reasoning is reviewed in this section. In abstract or indicative versions of the selection task, the notion of *subjective epistemic utility* (or variations in the value of knowledge or information to an individual depending on an individual's goal) is deemed to be more appropriate. Manktelow et al argue that mental models and pragmatic reasoning schema theory do not provide a way of representing and assessing subjective utilities of possible choices or preferences in thematic versions of the selection task. Evans and Over (1996) similarly criticise the O&C model of abstract versions of the selection task because subjective epistemic utility is not included[33].

The theoretical motivation underlying research into the benefits, costs and probable outcomes of choices derives from normative decision theory, where taking account of preferences is a basic notion. When subjective utilities and subjective probabilities of an outcome are combined in normative decision theory, this combination is called *subjective expected utility* ("SEU")[34]. SEU can be used to formalise or predict which choices are made and when they can be expected to be made. Manktelow and colleagues do not use SEU to mathematically formalise their predictions. Rather, the general principles of a decision-theoretic approach are adopted. This is because it is assumed that judgements of utility and probability cannot be represented by precise numbers as classical decision theory requires. Neither is it assumed that utility and probability judgements always conform to probability calculus (Manktelow and Over, 1990, p. 125). For example, if the probability of A is

32 "Subjective utility" is a term used in decision theory for the value of an individual's preferences, i.e. it is a subjective measure of the benefits or costs of one choice or course of action over the benefits and costs of another choice or course of action.

33 The role of subjective epistemic utility in the O&C model is discussed in chapter 5.

34 Formally this can be annotated as $SEU = \sum s_i U_i$, where s is subjective probability and U is its subjective utility (Evans, Over and Manktelow, 1993 p. 166)

.5, and the probability of -A is .5, then in formal terms they are equally likely to be true or false. But the degree of belief in p, may be greater than the degree of belief in $-p$, as reasoners are usually concerned with what follows from their *own* subjective relevant beliefs of premises, which may or may not have a certainty value of 1, and therefore the conclusion inferred from these uncertain premises may also not be highly probable[35].

Furthermore, it is not assumed that the principle of maximization will always be the decision rule used to influence choice. Neither is it assumed that preferences will always be consistent, although having consistent preferences is a basic assumption of normative decision theory. It is argued that preferences are not always consistent because preferences are not always a consequence of basic drives and immediate desires. Rather, ideal preferences are influenced by social or moral decision rules and the ability to make inferences from them is constrained by a form of bounded rationality (Evans, Over and Manktelow, 1993 p. 181).

By adopting a non-normative or "informal" decision-theoretic approach to reasoning problems, Manktelow and Over (1991) propose that a modified mental models theory where subjective utilities are represented in mental models could provide a complete account of what is involved in "fleshing out" mental models. To achieve an enhanced theory of mental models, what ought to be represented is not just possible states of affairs and possible consequences or outcomes of various actions; a reasoner's preferences for these different possible outcomes must also be represented. The role of subjective probability in mental models theory is also emphasised because there can be more or less confidence that an action will lead to a certain outcome. To reflect the influence of subjective probabilities, Manktelow and Over propose that mental models could assign weights to subjective probabilities, as well as utilities. It is not assumed that precise values for utilities and probabilities need to be assigned to mental models and the states they represent. Instead, values could be given to mental models without using

35 An assumption underlying probability calculus is that the probability of the conclusion of a valid inferences given its premises be assigned the value of 1 (i.e. in terms of formal logic which underlies probability theory there is a 100% certainty of the conclusion being true, given that the premises are true).

precise numbers by simply indicating that some mental models were more preferable or probable than others[36].

In conclusion, the general, decision-theoretic approach to reasoning adopted by Manktelow and colleagues is of importance to the psychology of reasoning because it draws attention to the need to incorporate the role of utility and probability judgements in models of reasoning. However, because an extended mental models approach to reasoning essentially assumes separate interpretational and inferential components of cognition, and deduction in the form of falsifying or looking for violations is assumed to be the goal of a reasoner in the selection task, an extended mental models explanation is not compatible with the O&C (1994) optimality approach to cognition. I review decision theory and how it relates to optimality approaches in more detail in Part I of chapter 3.

In the next section, I review the theoretical approach of Kris Kirby (1994) who combines SEU principles with signal detection theory in order to distinguish between confounding "content" variables, and utility and probability variables in *both* abstract and deontic selection task performance.

2.2.9 Probabilities and utilities of fictional outcomes

Kirby (1994) argues that theories of reasoning which examine how cards selections change as problem content changes have arisen because analysis of inferential performance on the selection task is believed to be confounded by non-inferential variables. In similar terms to Manktelow and Over (1991), Kirby argues that important variables influencing card selections are probabilities and utilities. His theoretical approach to reasoning and selection task performance is summarised below:

(i) How problem content is understood in the first place is important, and these prior beliefs in problem content, in turn, influence a reasoner's own assessment of *probabilities* or the likelihood of an event producing a desired outcome, i.e. the likelihood of finding certain features on the back of a card.

---

36 Manktelow, Sutherland and Over's (1995) study on the role of probabilities in deontic tasks is discussed in Section B of chapter 4.

(ii) In addition, how likely a particular choice is to yield a desired outcome is, in turned, weighed against a reasoner's assessment of the *utilities* or value or benefits /costs of making that choice.

Kirby proposes that *subjective expected utility* (which is the combination of subjective utility and subjective probability) is the criterion (or decision rule) reasoners use when deciding whether to select, or not select, a card. For example, the subjective expected utility ("SEU") of, say, making Choice A is the sum of the utilities of the possible outcomes of that Choice A, with each outcome being weighted by the reasoner's own subjective estimate of the probable likelihood of that outcome.

Kirby specifies four possible outcomes (or "fictional" outcomes of card selections[37]) which he identifies in signal detection terms, as follows:

(i) a *hit* is when a card which yields an outcome "inconsistent with the conditional", i.e. falsification of an hypothesis in abstract selection tasks and violation of a rule in thematic tasks, is selected;

(ii) a *miss* is when a card which yields an outcome inconsistent with the conditional rule is not selected;

(iii) a *false alarm* is when a card that yields a consistent outcome is selected; and finally

(iv) a *correct rejection* is when a card which yields a consistent outcome is not selected.

Each of the above possible outcome has a positive or negative utility for a reasoner.

SEU theory predicts that selections, say, Choice A will be made when the value or SEU of choosing A is *greater* that the value or SEU of not choosing A, i.e.:

SEU(choosing A)>SEU (not choosing A).

---

37 Because only the face of a card is seen in the selection task and what is actually on the reverse sides of each card is not known as a fact, Kirby includes the adjective "fictional".

SEU theory thus assumes that the more highly valued preference or SEU will *always* be selected.

Kirby applies SEU theory to the selection task in order to determine what makes one choice or card selection preferable to another. He formalises the SEU of **selecting a card** as being:

(i) equal to the utility of a *hit* ,

(ii) multiplied by the probability that an inconsistent outcome is present on the back of card C (C = visible card face, as opposed to hidden or fictional reverse of card);

(iii) plus the utility of a *false alarm* ,

(iv) multiplied by the probability that an inconsistent outcome is absent on the back of card C.

The SEU of ***not* selecting a card** (say, the -*q* card) is formalised as being:

(i) equal to the utility of a *miss*,

(ii) multiplied by the probability that an inconsistent outcome is present on the back of card C,

(iii) plus utility of a *correct rejection,*

(iv) multiplied by the probability that an inconsistent outcome is absent on the back of card C.

As the likelihood of an outcome increases, selections should increase.

Given the above formalised criteria which underlie how choices, i.e. card selections, are made, the main hypothesis tested by Kirby is that logically correct card selections are influenced by the probabilities of making a hit. In other words, he predicts that a card is **more** likely to be selected when the probable likelihood of *p* cards increases (from small to medium to large probabilities) thereby increasing that card's availability or likelihood of being a hit (i.e. having the outcome -*q* on the reverse of *p* will increase). This prediction was substantiated in the experimental results[38]. Kirby also predicts that a card is **less** likely to be selected as the utility or benefit of a correct rejection increases, or the cost of a false alarm increases. However, when the probability of *p* and -*p* cards decreased, experimental results (which Kirby

38 Section B in chapter 4 describes Kirby's studies in more detail.

could not explain in terms of his theoretical approach) showed that the proportion of selections of *-p* cards also increased[39].

Kirby concludes that his research supports the assumption that decisions are influenced by the probability of finding a desired outcome. More generally, he proposes, in similar terms to the heuristic-analytic approach of Evans (1984, 1989), that reasoning should be viewed as comprising both inferential processes (of which he believes mental models gives a plausible account) and preference or choice processes which are independent of the inferential process. By separating reasoning and choice processes, Kirby is thus able to argue that inferential reasoning is not influenced by content.

In conclusion, Kirby's work is noteworthy because it is one of the few studies in the psychology of reasoning in which the probability of information is manipulated in abstract versions of the selection task. However, his experimental results are fully explainable in terms of optimal data selection, and distinctions between interpretational and inferential components of reasoning do not need to be invoked when explaining a reasoner's behaviour in the selection task. I discuss this fully in chapter 5.

In the final section of Part II, I review the work of Leda Cosmides (1989) whose theoretical assumptions include mechanisms which maximise the utilities of exchange items in social contracts, as well as decision rules whose long-term fitness maximisation is assumed to underlie reasoning about social exchange generally.

2.2.10 **Darwinian algorithms and social contracts**

In order to explain selection task performance, Manktelow and Over (1991) and Kirby (1994) adopt a decision-theoretic approach where it is assumed that a reasoner's goal or decision rule in an interpretational stage of reasoning is to maximize subjective expected utility ("SEU"). The maximisation of SEU is not the main issue in the evolutionary approach of Cosmides (1989) social contracts theory. This is because the main goal in an evolutionary analysis of behaviour is to determine which decision rules are selected over generations to promote their own inclusive fitness. On a broad level, therefore, Cosmides' social contracts theory investigates the role of maximization of long-term

---

39 The O&C model of optimal data selection detailed in Part II of Chater 3 predicts that when the P(*p*) and P(*q*) are high, selection of *-q* and *-p* cards will increase.

cognitive "fitness" as well the short term maximisation of items in a social contract[40].

For example, as far as the selection task (which is assumed to be a form of social contract) is concerned, reasoning in this task is assumed to be governed by decision rules to "detect cheaters". And whether there is a social contract in the first place, depends on whether the costs and benefits of items in a social contract have been negotiated. The "cheater detection" decision rules and the cost/benefits mechanisms assessing social contracts make up what Cosmides terms "Darwinian algorithms". These algorithms, or cognitive programs, are the components of Cosmides' research given most attention in psychology as her experiments test the hypothesis that "cheater detection" decision rules or algorithms (rather than logical procedures) determine selection task performance.

However, Cosmides' theory of social contracts also raises the general theoretical issue of maximisation of local and global and long-term and short-term preferences. For example, she assumes that different social contracts will be preferred depending whether short-term or long-term benefits are being maximised. As regards local and global preferences, Cosmides (1987 p. 279) refers to the usefulness of natural selection theory to psychology as it predicts that behaviour will vary enormously and an individual's behaviour will often appear far from optimal when optimality is defined in isolation from the environment as a whole. For example, if two individuals are behaving in order to maximize their own individual fitness, there may be a conflict in "fitness" interests. In these circumstances, an outcome can only be optimal for one party but not both parties or it can be non-optimal for both parties, but optimal for a larger group of individuals (perhaps altruistic behaviour). Patterns, or consistent preferences, in behavioural variations should thus be used as clues to the nature of the psychological mechanisms that produce behaviour.

It is because of these general issues that I review Cosmides' social contracts theory at this stage, as her approach to cognition provides an apposite way in

40 Chapter 3 provides a full review of the assumptions underlying optimality approaches to cognition, including maximisation of overall fitness in contrast to the maximisation of a set of properties in the environment (e.g. SEU or O&C's expected information gain).

which to introduce different maximization assumptions prior to my review of general optimality approaches to selection behaviour in Chapter 3.

The theoretical assumptions underlying Cosmides' theory of social contracts derive from both cognitive psychology and evolutionary theory. For example, she assumes that the cognitive architecture of the mind does not consist of a few powerful general mechanisms, but instead comprises specialised, domain-dependent, cognitive programs or Darwinian algorithms whose function is to be adaptive, reliable and efficient information-extractors, as well regulators of how extracted information is processed. More specifically, information is assumed to be extracted efficiently and reliably by way of domain-dependent algorithms which evaluate and represent the benefits and costs, and future expectations of situations (such as expected social contracts). What is done to this extracted information is governed by domain-dependent procedures or algorithms (such as cheater detection rules). Darwinian algorithms, as well as constraining and structuring the way in which information is interpreted and processed, have thus also allowed optimal human reasoning generally to evolve from the "domain of activity" concerned with mutual co-operation and social exchange[41]. The environment does not therefore provide all information needed for social interaction, as utility maximizing algorithms also constrain which information is extracted from the environment.

Social contracts theory thus assumes that, because the mind is modular, each cognitive specialisation can contain domain-specialised algorithms or design features which are not activated by other domains. Cognitive processes which appear to be domain-independent are therefore fundamentally a consequence of prior, long-term environmental and cultural shaping and maximisation, and the design of the human mind reflects these richly textured domain-specialised psychological adaptations (Cosmides and Tooby, 1992 p. 165)[42].

---

41 Besides the domain of social exchange a Darwinian algorithm for which domain comprises the "look for cheaters" decision rule or procedure, other domains of human activity that should have Darwinian algorithms are aggressive threat, mate choice, sexual behaviour, pair-bonding, parenting, parent-offspring conflict, friendship, kinship, resource accrual, resource distribution, disease avoidance, predator avoidance, to name but a few (Cosmides and Tooby, 1987 p. 286).

42 In Cosmides' terms therefore maximization cannot be a general principle underlying all cognition. A number of domain-specific maximising decision rules must be triggered by domain specific schema. This contrasts with the O&C, general, optimality-preserving decision rule to optimise information gain.

Criteria for recognising evolved design include cognitive economy, efficiency, complexity, precision, specialisation and reliability. For example, the design of reasoning has evolved because social exchange based on mutual co-operation has given rise to the negotiation of social contracts where individuals pay costs or meet a requirement in order to receive benefits. Procedures that make human beings good at detecting violations of social contracts result in large fitness benefits, whereas the failure to detect violations results in large fitness costs. Cosmides argues that the ability to engage in social exchange in the first place could not have developed without violation detection procedures being directly programmed into Darwinian algorithms.

I now summarise how Cosmides uses social contracts theory to explain performance in the selection task. As already mentioned, she firstly assumes that the selection task is a form of social contract which is regulated by specialised, social exchange algorithms. The Darwinian algorithms for the domain of social exchange comprise:

(i) social exchange-specific algorithms which evaluate and represent benefits/costs and future expectations of social contracts; and

(ii) rather than general, logical inferential procedures, social-exchange domain-dependent "cheater detection" decision rules or algorithms (where cheating is defined as a failure to pay costs even though benefits may have been receive) regulate social contracts (Cosmides, 1989 p. 196, Cosmides and Tooby, 1987 p. 299 and Cosmides and Tooby, 1992 p. 220). This cheater detection algorithm focuses attention specifically on situations where benefits have been accepted but costs have not been paid. The definition of cheating is "directly programmed into social contract algorithms, and the problem of learning the definition by trial and error does not arise (Cosmides, 1989 p. 259).

The perceived costs and benefits of *items* exchanged in a social contract are thus assumed to be *the* diagnostic cues which permit a situation to be interpreted and then represented as a social exchange/social contract situation

(Cosmides, 1989 p. 230)[43]. Social exchanges in which perceived costs exceed the perceived benefit are avoided. It is the inclusion of "utilities" in this sense that distinguishes social contracts theory from pragmatic reasoning schema which represent the world in the terms of "actions to be taken" and preconditions (obligations and permissions) to be satisfied. Cosmides argues that social contracts embody permission and obligation rules, whereas the more general level representation adopted by pragmatic schema does not accommodate the lower level representations of costs/benefits of those rules. Cost/benefited social contract representations are thus assumed to be more plausible than Cheng and Holyoak's (1985) pragmatic reasoning schema representations which have no utility evaluation.

Costs and benefits evaluation and schema production represent the "interpretative component" of social contract algorithms (Cosmides, 1987 p. 229). The other necessary component of social contract algorithms is the "look for cheaters" decision rules and procedures (the "inferential" component), which Cosmides assumes only operate when a rule has the cost/benefit structure of a social contract. When a social contract has been translated into cost-benefits terms, the cheater detection procedures *operate* on that particular social contract representation. However, whether violation of a cost-benefited social contract is perceived as cheating in the first place, depends on a reasoner's view or perspective of whether a social contract exists or not. For example, a schema representation of a person enforcing an acknowledged social contract comprising expected costs and benefits of the social exchange items triggers detection of cheaters algorithms, whereas a schema representations of a person enforcing a social purposes schema (as envisaged by pragmatic reasoning schema) cannot trigger such decision rules.

I now outline a study in which Cosmides tests the hypothesis that social contracts are not driven by MTT or other logical principles but that a "looking for cheaters" algorithm is the cause of facilitatory effects in the selection task. The rule in Cosmides' thematic version of the selection task is "if a man eats cassava roots, then he must have a tattoo on his face". This rule has come about because in the area in which "the Kaluame tribe" live, cassava root is a

43 When using the terms "benefit" and "costs", Cosmides does not prejudge the values assigned to items by parties entering into social contracts (Cosmides and Tooby, 1992 p. 199). In addition, algorithms are assumed to be item-independent and handle a wide variety of items of exchange (Cosmides, 1989, p. 196).

scarce but powerful aphrodisiac. In addition, among the Kaluame tribe only married men have tattoos on their faces. Elders of the tribe have established the cassava rule because they strongly disapprove of sexual relations between unmarried people. However, many unmarried men are tempted to violate the rule, i.e. cheat and eat cassava roots. Given this contextualised selection task, participants were presented with four cards: one side of the card told which food a man was eating, and the other side told whether or not the man had a tattoo on his face. The cards were "eats cassava root" (p) , "tattoo" (q), "eats molo nuts" ($-p$) and " no tattoo" ($-q$). There were two version of this task. One condition involved taking the perspective of an elder enforcing a social law. As an elder of the tribe, the task was to indicate only the cards definitely needed to turn over to catch any Kaluame men who violate the social contract or rule. The other condition involved taking the perspective of an anthropologist observing a social law. The anthropologist's task was to study the Kaluame and to indicate only the cards definitely needed to turn over to investigate the claim about the social contract or rule. The perspective adopted by participants was shown to influence responses and when the law had to be *enforced*, 75% of participants selected the p and $-q$ cards. Cosmides concluded that her experiments support the view that the selection task is a form of social contract governed by optimally adapted decision rules which have evolved to detect cheaters of social contracts.

In conclusion, Cosmides' theory of social contracts is of note because it assumes decision rules other than truth-preserving decision rules may govern behaviour in the selection task. The theory of social contracts also incorporates the role of utilities of items in expected social contracts in an interpretational component of cognition. This aspect of her work is interesting as theoretically (and notwithstanding that social exchange algorithms are assumed to be domain-specific) it provides a *general* way in which to envisage the variation of the utility and probability of items or events in abstract versions of the selection tasks. The way in which perspective influences selection behaviour is also highlighted by Cosmides. In broader terms, Cosmides draws attention to the role of long-term fitness maximisation, which rational mechanism has enabled optimal decision rules such as "cheater detection" decision rules to evolve. This and other general optimality issues are reviewed in detail in the next chapter 3.

Chapter 3
A RATIONAL ANALYSIS OF SELECTION BEHAVIOUR

In Chapter 2, I introduced the philosophical and psychological assumptions underlying the traditional interpretation of rationality in the psychology of reasoning. The main theoretical approaches to reasoning were also reviewed to show how performance on the selection task is explained by each approach. Chapter 3 details the assumptions underlying an approach which assumes that selection behaviour is governed by principles of optimization or maximization. Part I reviews various optimality approaches, and Part II introduces John Anderson's rational analysis of cognition, and describes the O&C (1994) model of optimal data selection.

Part I - Optimality Approaches

3.1.1 General optimality assumptions and the O&C model

O&C do not consider "logicality" (as measured by either logical competence, or performance of falsification of an hypothesis) to be equivalent to "rationality". They are, however, rationalists because problems of variability in performance on the selection task are deemed explainable if the sources of variability are found. This broad notion of rationality is made specific by adopting an approach to cognition in which behaviour is assumed to be rational because it is optimally adapted to the environment. Furthermore, and by using formal mathematical modelling, the O&C hypothesis that reasoning and selection task behaviour in particular is designed to be optimized to the environment can be evaluated. Evaluation is possible because optimality modelling requires that:

(i) unambiguous *assumptions* (i.e. decision assumptions, currency assumptions and constraint assumptions) about the behaviour being analysed should be made ; and

(ii) precise *predictions* should also be made about what behaviour ought to occur in given circumstances.

An optimality model is substantiated if the experimental studies carried out to test it produce the same behaviour as the model predicts.[1]

The assumptions of optimality modelling as set out in (i) above are specified by O&C's model as follows. The O&C model is testing the hypothesis that a reasoner's goal in the selection task is to select the "best" or most informative data. "Optimal data selection" is thus the *decision assumption* or variable being analysed by the O&C model. The *currency* O&C use to compare alternative values of the decision variable is "optimizing[2] the expected amount of information to be gained by turning each card" (Oaksford and Chater, 1994, p. 609). The maximizing or optimizing principle is explained in more detail in section 3.3. *Constraint assumptions*, and *precise predictions* as required in (ii) above, are detailed in Part II of this chapter, where the O&C model of optimal data selection is fully specified.

3.1.2 Other optimality approaches in psychology

An optimality approach is concerned with behaviour, and whether one behaviour is better than another. It is behaviourist in practice if not in a strict theoretical sense, as it is not concerned with investigating the *processes* of behaviour which topic has been the emphasis of some of the previously discussed theories in the psychology of reasoning.

Within cognitive psychology and the psychology of reasoning, O&C's optimality model and assumptions are most similar to those of John Anderson's model of the "Adaptive Character of Thought" (1990a, 1991, 1993). The assumptions underlying the O&C model of optimal data selection and Anderson's adaptive approach to cognition are akin to models of optimal foraging (McFarland, 1993) and optimal diet selection (Krebs and Kacelnik, 1991).

The assumptions underlying the optimality approach adopted by O&C and Anderson have different foundations to the rational analyses traditionally used in the psychology of reasoning and decision making, where Utility Theory, whose roots are in game theory, is used. This latter form of rational analysis of behaviour is the basis of Evans' rationality$_1$ definition.

---

1 See Stephens and Krebs (1986) (p. 5) for full details of components of optimality models.

2 Another choice principle, although not a currency of the O&C optimality model, is "minimisation".

Both game-theory derived Utility Theory and evolutionary derived Optimality Theory, shall be explained in the following sections of this chapter. When the distinctions and assumptions of these two forms of rational analysis have been made, the assumptions underlying Anderson's model of rational cognition will be explained in Part II before detailing the O&C model of optimal data selection.

3.1.3 Assumptions underlying the principle of optimization

All optimality approaches to the study of behaviour makes several basic assumptions in order to ensure that the principle of maximization or optimization is possible to implement.

Firstly, they assumes that information or alternatives in an environment, for example, A and B and C, are *discriminated* and then *selected* from the whole array of information in an environment[3].

The second assumption is that there is a consistent and preferential ordering of selections, i.e. there is consistent *transitivity* (which presupposes reflexivity and symmetry) of choice. For example, if A is preferred to B, and B is preferred to C, then A will be preferred to C, and this is always the case.

Thirdly, an optimality approach assumes that, if it can be shown that discriminations and selections have been made from a comprehensive array and that there are also consistent transitive preferences amongst these selections, then it can be inferred that *a maximizing principle is being used.* For example, if A and B and C are the selections made, and these selections are assigned preferential values where A has a higher value than B and C (A>B>C>), then *the maximization principle of selecting the larger value* can be inferred if A is always preferentially selected rather than B and C.

In economics, the word *utility* is given to the quantity or value that is maximised and economists use the currency terminology "the consumer maximises utility". It should be noted that utility is a notional measure of the psychological value of things as it cannot be known whether or not utility does come into the decision making process. The equivalent to utility in animal

---

3 Whether the whole array of is information is available will be debated in the chapter 5.

behaviour is known as *benefit*, and in the O&C model the equivalent is *information gain*.

Finally, an optimality approach assumes that while *utility increases* each time an alternative is selected, there is a *decrease in the incrementation in the utility* of that alternative. This assumption is based on the concept of "satiation" which is considered in more detailed in section 3.1.4 below.

It is important to note that, if the preferential ordering or transitivity assumption (A>B>C) is not satisfied, then a "rational analysis" is not possible as there is no consistently more valued preference whose utility can be maximised.

How economists and animal behaviourists take an optimality approach, the important distinctions and additions they make, and the use made thereof in the psychology of reasoning, are illustrated in the subsequent sections of Part I of this chapter.

3.1.4 Rational analysis in economic decision making

In order to model individual consumer behaviour and make predictions about the distribution of income, an optimality approach in microeconomics assumes that a consumer will distribute income in a way that *maximises subjective utility* (i.e. the commodities or purchases a consumer values the most will be chosen). The "maximization principle" is the basis of Utility Theory, and optimality approaches in general

But how exactly can Utility Theory predict that, say, buying a pair of shoes will maximize subjective satisfaction or utility more than giving to charity? The role of Utility Theory in economics and the maximizing principle in general are illustrated with a simple example.

If, as an avid book reader you are given a token to buy ten books, would you select only your most favourite kind of books (novels) or would you also buy some of your next preferred books (autobiographies)[4]?

To make any predictions about choice of books to purchase, Utility Theory (and optimization generally, as briefly outlined in the previous section)

---

4 See Mazur (1994) (p. 199)

assumes first of all that one type of book is preferred to another type of book which is preferred to another type of book, and so on. It also assumes a value can be assigned to this book-type preference.

In addition, the more items of type X one obtains, the more the utility or value decreases for each *additional* type X item obtained. For example, the first novel purchased will be highly valued, but the second novel will be less valued, and the tenth novel, if ten novels were purchased, would have a very small utility or value in comparison to the first novel purchased. Similarly, the first autobiography purchased will be more valued than the tenth autobiography purchased, and so on. In other words, while *subjective utility increases* each time an item is selected, there is a *decrease in the incrementation in the subjective utility* of that item.

Given the above assumptions, what books would be selected if ten books could be purchased, and if novels were preferred to autobiographies?

It is predicted that a person with precisely these preferences for these two types of books would, first of all, maximize *overall subjective utility* by choosing a combination of the two kinds of books. Secondly, and more specifically, it is predicted that six novels and four autobiographies will be chosen. This is because the first autobiography purchased, (even though it is not the most favourite type of book, novels are), would have more subjective utility than a tenth novel, if it were purchased. And the purchase of a second autobiography would have more utility than if a ninth novel were purchased, and so on. In other words, books that have the maximum subjective value will be chosen, and the best selection of book would be comprised of books with the "best" or maximum or optimal overall subjective utility.

The above example is concerned with a simple environment or set of choices/alternatives. It also does not consider the role of subjective probabilities and how they affect subjective utility. Notwithstanding these limitations, an optimality approach holds that behaviour is essentially the same in more complex environments and the consequences of behaviour are assumed always to be directed to maximise subjective utility.

How the role of subjective probabilities have been incorporated into Utility Theory will now be discussed.

3.1.5 Assumptions of Utility Theories

The study of decision making under risk goes back to the 18th century when French noblemen asked their court mathematicians to advise them how best to gamble (Coombs, Dawes and Tversky, 1970, p. 117). One theory of decision making with incomplete knowledge is known as "Expected Utility Theory" ("EUT") and an example of its basic approach was illustrated in the previous section. A value or expected utility based account of behaviour was formulated in 1944 by John von Neumann and O. Morgenstern and was part of their "Theory of Games" which sought to provide a rational theory of preferences among gambles. The von Neumann and Morgenstern expected utility theory "revived maximization of expected value as a prime criterion of economic rationality" (Gigerenzer, Swijtink, Porter, Daston, Beatty and Kruger, 1989 p. 269)[5].

A criticism of EUT revolves around the concept of probability used which was formulated in terms of gambles whose numerical probabilities were assumed to be known in advance. Probabilities in EUT were therefore assumed to be objective probabilities.

Why probabilities are important can be illustrated as follows. Even though A and B are selected from an array of alternatives, and even if A is preferred to B, this is not sufficient to ensure that the alternative with the greatest utility, i.e. A, will be chosen. This is because the probabilities associated with the likelihood of B may be more than the probabilities associated with the likelihood of A. In other words, even though A is preferred to B, B may be a common occurrence and for this reason the selection of B may be more likely that the selection of A.

To remedy this flaw in EUT, in 1954 L. J. Savage developed a theory leading to the simultaneous measurement of subjective utility and subjective probability. Savage replaced the objective probabilities of EUT with subjective probabilities and so extended the classical economic utility approach, making it a more general and subjective model of decision making.

5 von Neumann and Morgenstern's EUT improved on a previous utility theory in which the utility used was "objective utility" and probabilities were "objective probabilities" .

Savage's revision of EUT is known as "Subjective Expected Utility Theory" ("SEUT" or "SEU") (Coombs et al, 1970, p. 129).

The assumptions of SEU and how they differ or add to EUT are recapitulated as follows. EUT assumes that the decision maker has a well defined set of alternatives or decisions from which to choose; SEU also assumes this. SEU further assumes that each alternative can be associated with one or more consequences, which has a certain *utility function* (which measures a person's liking or preference) as well as a given *subjective probability* (also known as a *joint probability function*) which is it assumed the decision maker can assign to all future sets of alternatives or decisions (Simon 1983, p. 13). The alternatives with the highest *subjective expected utility* (the combination of subjective utility and subjective probability is known as subjective expected utility) is selected. Finally SEU assumes that a principle of maximizing subjective expected utility will be used and influence choice.

Utility Theory approaches to cognition as described above are derived from game theory assumptions of what is means to be rational. Optimality modelling assumptions, derived from evolutionary biology and behavioural ecology rather than game theory, will now be reviewed.

3.1.6 Modelling animal behaviour

The idea of the "survival of the fittest" is the core of Darwinian thinking. It simply means that "the best" design ultimately survives and reproduces. In optimality terms, when an organism's behaviour is optimally adapted to its environment, its chances of surviving and reproducing offspring with similar behavioural tendencies to reproduce should increase.

Studying the adaptation of an animal to its environment in order to see whether it survives is not, however, where attention should be focused as far as an *evolutionary* optimality approach is concerned[6]. For example, crypsis is the most common adaptation used by animals and, by means of changing colour and/or patterns, animals can masquerade as specific items in the environment which are not normally eaten, or they can resemble a random sample of backgrounds (Shettleworth, Reid and Plowright, 1993). When taking a long-term or evolutionary approach to studying behaviour, the important question to focus on is how design (e.g. crypsis in this instance)

6 Stephens and Krebs (1986, preface)

relates to an animal's expected "fitness" or *overall* chances of survival and reproduction. The answer is that crypsis is related to overall fitness because such a design feature allows an animal to defend itself against predator detection or identification and therefore it ultimately survives. Cryptic insects, like praying mantis which can mimic a dead leaf, and colour changing animals like chameleons, are examples of "the best" or optimal design and how design relates to (maximises) overall fitness[7].

How to assess whether one design or behaviour is better than another is illustrated in the next section.

3.1.7 How is optimal design assessed?

The previous section in this chapter has shown that crypsis can be the "best" overall behaviour or optimal design in a long-term survival situation. As already mentioned, the currency used to assess whether one design (or value or utility or preference) is better than another is to say that an organism will act in a way that maximises either:

(a) overall fitness; or
(b) in classical optimality terms, a set of properties in the environment.

For example, in the case of crypsis, an organism behaving in a way that "*minimises the cost of predation*" is also maximising overall survival and fitness. As far as foraging and diet selection is concerned, an organism behaving in a way that "*maximises rate of energy intake*", is maximising one particular set of properties, energy, in the environment.

A study showing how optimization is used when studying foraging behaviour is quoted below as the technical terms necessarily adopted to make optimality theory precise are less confusing when placed in a "down to earth" context.

> "Goss-Custard (1977) recorded the foraging behaviour of redshank *Trigo totanus*, a shorebird. One of the redshank's main prey was the polychaete worm *Nereis diversicolor*. These worms occur in different sizes, and redshank are sometimes seen to feed exclusively on large worms and sometimes to take both large and small ones. This is not simply because of variation in availability of small worms: they are always plentiful in the mud. What determines the redshank's decision to eat or reject small worms? An intuitive argument is that large worms are more "profitable", that is to say they yield a higher amount of energy per unit of pursuit and handling time. When large worms are

---

7 Overall or long-term fitness and how it relates to shorter term maximization will be consider in section 3.1.9.

> sufficiently common, it never pays to miss opportunities of catching them while pursuing or handling less profitable small worms. However, when large worms are rare, the small, unprofitable, worms are worth eating when they are encountered because the alternative is to spend a long time searching for the occasional large item. (Krebs and Kacelnik, 1991 p. 106)

To be certain that selective foraging does reflect a strategy of maximizing rate of energy procurement, and to increase certainty that the correct cost-benefit relationships underlie foraging behaviour, more precise predictions can be made.

> "For example, ... where there are two classes of worm, large and small, it is possible to derive a precise quantitative prediction about when the redshank should and should not take small worms. The crucial variable, namely the energy values of the large and small prey types and their handling times, are all measurable, so in principle it is possible to test a prediction."

The advantages of this kind of formal mathematical modelling have already been discussed (unambiguous assumptions and precise predictions) and the above study simply illustrates that optimality modelling can expect an answer to the question "what behaviour is expected in particular circumstances", rather than simply ask "what behaviour is expected?".

The next section will look at Optimal Diet Theory as it is an example of classical optimality theories all of which are concerned with the maximization of some properties in the environment (energy or nutrition or information gain), rather than optimizing overall fitness.

3.1.8 Classical optimal diet theory

The "prey model" [8] of the redshank's diet selection is an example of classical foraging theory as it uses the principle of maximizing in order to evaluate rate of energy or nutrition gain, and it makes predictions about what behaviour is expected in given circumstances. Most tests of diet selection models have used organisms whose diet selection is usually a compromise between small prey that can be rapidly captured and consumed and large prey that require longer handling times but provide more energy per item.

A classical optimal diet model makes the following specific predictions:

---

8 A "prey model" is about the decision variable "whether to eat or not eat prey" (Stephens and Krebs, 1986 , p. 13). There can also be "patch models" where the decision variable is "how long to stay in a patch or environment."

(i) Foragers should select *more profitable* prey. Other considerations or trade-offs such as predation are not accounted for in classical optimal diet theory.

(ii) The tendency to specialise on more profitable prey should increase with an increase in encounter rates with (i.e. increased probability of) more profitable prey. Therefore, as encounter rates with more profitable prey increase, less profitable prey should be dropped from the diet.

(iii) Attack probabilities on less profitable prey should not be affected by their own encounter rate (i.e. even if the probability of less profitable prey is high, selection of these non-optimal prey will not increase).

Classical maximizing models like the optimal diet theory above outlined have three major disadvantages. They do not encompass trade-offs, state, or stochasticity (Krebs and Kacelnik, 1991, p. 117). These disadvantages are also limitations in utility maximizing models used in psychology and economics, and are outlined below.

(a) Classical maximizing models do not consider the role of trade-offs or substituting alternative activities. For example, animal studies show that minimising risk of predation is *traded-off* with maximizing food intake, and animals will feed in less profitable (in terms of food energy intake) patches in order to avoid the risk of predation.

(b) When there is a trade-off between, for example, feeding and danger, the choice of patch or behaviour in that patch depends on an animal's *state*. Classical maximizing models do not analyse how behaviour might change with state changes. For example, Krebs and Kacelnik (19971) cite a study by Milinski and Heller (1978) in which the state of satiation of sticklebacks (either hungry or well-fed) showed that hungry fish were more likely to feed in a patch where the rate of food intake was higher but danger of predation was also higher.

(c) Finally, classical maximizing models assume that the environment is *deterministic*, while in reality encountering food and being attacked by predators occur in a stochastic or unpredictable way.

In addition to the three limitations above outlined, optimal diet theory and classical optimality approaches assume that organisms have complete information about the relevant parameters or characteristics of the environment[9]. In particular optimal diet theory assumes that types of information are unambiguous. For example, that a forager will know encounter rates (i.e. probabilities), profitabilities and gain functions. However, information is often ambiguous and whereas a forager may be able to recognise types of things to select, e.g. conifer trees, it may not be able to distinguish sub-types, for example, cones with no insects in them versus cones with lots of insects in them. (Stephens and Krebs, 1986, p. 75).

Finally, as a simple model of behaviour, optimal diet theory is only concerned about evaluating "energy maximizing", rather than general "fitness maximizing" which would take into account other than energy factors.

The above factors limit the predictive accuracy of classical maximizing modelling in animal behaviour studies and a number of people in the field of animal behaviour have sought to extend classical modelling in a number of ways. For example, David McFarland (1989, 1993) extends classical optimality models by incorporating the role of state-space. The O&C model of optimal data selection has also sought to reduce the limitations of classical optimality modelling by incorporating changes in probabilistic state. It is this inclusion of state-dependent optimising behaviour which distinguishes the O&C optimization approach from classical maximizing theories of utility traditionally used in psychology. As the O&C model is to some extend based on McFarland's work, his approach (and other ways in which classical optimality has been extended) will be introduced in the next section.

3.1.9 The importance of state and its determination

In the previous section it was shown that classical maximizing models assume that the environment is *deterministic* and specifiable. This assumption raises an important point and brings into debate two opposing views regarding the causes of variations in behaviour and how and if they can be determined.

9 See section 3.1.3, where the first assumption of optimization is that selections are assumed to be made from the "whole array of information in the environment".

One view, which David McFarland takes, is that behavioural variations occur because of failures to control for them in the environment. Sibly and McFarland (1974) writes that "behavioural systems are basically deterministic and that the problems of variability should be met by attempts to identify the source of variation and then specify its cause". McFarland believes the source of variation is the *state* of an organism. The implication is therefore that if state can be determined, variability in behaviour can be controlled.

McFarland's inclusion of state derives from the role of behaviour in the regulation and maintenance of homeostasis. Sibly and McFarland (1974, p. 214) write "provided that an animal is acclimatised to its environment, and the structure of the environment is known, the animal is motivationally stable, and its overall motivational state, being a combination of physiological state and perception of environmental stimuli, can be specified directly in terms of the structure of the environment". McFarland represents motivational state as a point in space and, as a consequence of behaviour, state changes and so the point describing a trajectory in space. The state-space concept links design optimality and the consequences of behaviour to mechanisms or processes (homeostasis, in this instance) of decision making.

O&C refer to McFarland's work and incorporate his basic argument: "that state at any particular time determines which behaviour occurs" (McFarland, 1989) p. 28) into their model of optimal data selection. The idea of state was first developed in system theory where the state of a system is thought of as the information needed if the system's behaviour is to be predicted. It is the incorporation of state in this sense that differentiates the O&C model from the optimality approaches described in previous sections of this chapter. The O&C model is also different to previous optimality approaches in psychology as it predicts that changes in probabilistic state will change what is maximised.

3.1.10 The importance of state and stochasticity

A different view to McFarland's deterministic approach to defining state and how variability thereof can be controlled, is taken by Alasdair Houston (1993) who assumes that variability in behaviour will persist even when careful environment controls are incorporated in the experimental design. This is because behavioural systems are by their very nature stochastic, unlike McFarland's assumption that there is a deterministic relationship between behaviour and change in state. Houston argues that more realistic modelling should involve probabilistic relationships between the action chosen and the consequences for state. For example, if the probability of finding food is uncertain then a stochastic or indeterminable relationship between behaviour and changes in state results (Houston, in Hughes, 1993 p. 11).

Houston uses a stochastic dynamic programming technique which analyses dynamic overall fitness optimization (rather than maximization of one property in the environment) and therefore incorporates trade-offs, state *and* stochasticity or the unpredictable nature of events. Such a technique tries to evaluate how short-term behavioural decisions contribute to long-term fitness. For example, whether a meal is eaten at lunch time depends on whether one is hungry at lunch time as well as whether it is known if a meal will be eaten later that evening. If there is going to be an evening meal, then there is less need to eat lunch[10]. A whole sequence of present and future events therefore need consideration rather than one action taken at a particular time. When an action is considered in isolation the contribution it makes to fitness in the future cannot be ascertained.

In order to consider sequences of states in time, a stochastic dynamic programming model works backwards from the final time through successive time periods computing at each stage the behaviour that will maximize survival in the long-term. The end result is a matrix of choices that will maximize survival probability for each state and each time interval. This is known as optimal policy (Krebs and Kacelnik, 1991, pp 121 and 122). For example, in "patch 1" there is a probability of 0.5 of finding food in a given time interval and probability of 0.5 of finding no food and a probability of 0.0 of being killed by a predator. In "patch 2" the corresponding parameters are 0.75, 0.10 and 0.15. Patch 1 thus has a lower rate of food intake but it is safer.

Dynamic modelling can represent and analyse the state-dependent trade-off between, for example, foraging and predation. It assumes that searching for food costs one unit of energy whichever patch is chosen and that finding an item of food yields two units of energy. Given these values it is possible to compute the *expected terminal reward* resulting from different courses of action at different times. In contrast, classical maximization assumes that a unit of energy will always make the same contribution to fitness, i.e. that expected terminal reward is always the same. However, when an animal's

10 This example is used by Krebs and Kacelnik (1991, p. 121)

food reserves at T1 are high, dynamic programming shows that it pays to choose the safe but poor patch. If reserves are low it pays to choose the dangerous patch with a higher probability of food. At T2 where results are neither high nor low, it pays to choose the safe patch followed by either the safe or the dangerous patch at T1 (see table below from Krebs and Kacelnik, 1991, p. 122, where "x" = energy reserves, and "TR" = terminal reward)

At T-1

		Find food		No food		Death by predation
(a)	x = 3 (i.e. energy reserves are low)					
	Choose patch 1: TR	0.5 x 4.0	+	0.5 x 2.0 = 3.0		
	Choose patch 2: TR	0.75x4.0	+	0.1x 2.0	+	0.15x0 = 3.2
	TR (x=3) is greater if patch 2 is chosen at T-1.					
(b)	x=5 (i.e. energy reserves are high)					
	Choose patch 1: TR	0.5 x 6.0	+	0.5 x 4.0= 5.0		
	Choose patch 2: TR	0.75x6.0	+	0.1x4.0	+	0.15x0 = 4.9
	TR (x=5) is greater if patch 1 is chosen at T-1.					

(c) Summary of similar calculations for other states

x(T-1)	Optimal choice	X(T) = TR
5	1	5.00
4	2	4.05
3	2	3.2
2	2	2.34
1	2	1.50

At T-2

It is assumed that the animal will make the optimal choice at T-1, so pay offs are calculated substituting x(t) for x(T-1), e.g. if x=4 at T-2

Choose patch 1: TR	0.5 x 5.0	+	0.5 x 3.2= 4.10		
Choose patch 2: TR	0.75x5.0	+	0.1x3.2	+	0.1x3.2 = 4.07

If x=4 at T-2 it pays to choose patch 1. The optimal choice at T-1 depends on the outcome of T-2. If food is found, x(T-1)=5 and the optimal choice is patch 1. If no food is found, x(T-1)=3 and patch 2 should be chosen. On average it pays to choose patch 2 at T-1.

Maximization as used in classical optimality approaches ignores the fact that food may have consequences for an animals other than energy balance. Houston's view is that a general relationship between energy gain *and* other fitness can be found.

When comparing maximization models and stochastic dynamic programming which incorporates dynamic feedback between states, Krebs and Kacelnik (1991 p. 126) conclude that there is a role for both approaches: "maximizing models may give a simple intuitive feel of what is going on and often generate successful predictions. SDP models with their greater realism and detail may be able to explain more of the complexity of decision making".

In summary, the O&C model of optimal data selection incorporates *state*. In addition, by working backwards from a given state, and using a similar technique to Houston, the probabilities of events which comprise *another state in time* are calculated. The O&C model also includes *learning* about the actual probabilities which comprise a given state. However, the O&C model maintains one element of the classical maximization approach: it is concerned only with the optimization of a particular property in the environment, specifically, expected information gain. It is not therefore a general optimization model of *overall fitness* as envisaged by Houston.

3.1.11 The Role of Optimality Modelling

In Part I of this chapter I show that optimality modelling is useful in a number of ways. Firstly, it provides general organising principles for predicting and describing behaviour and designing experiments. Secondly, formal modelling provides a way of investigating and evaluating hypotheses or models of behaviour.

In Part II, I introduce John Anderson's (1990a, 1991) model of the Adaptive Character of Thought and then detail the O&C (1994) formal model of optimal data selection, its assumptions, constraints and predictions.

Part II
The O&C (1994) Model of Optimal Data Selection

In the first part of this chapter, I reviewed various optimality approaches to cognition in order place the O&C (1994) model of optimal data selection in theoretical context. I also detailed the way in which an optimality approach to cognition requires that unambiguous decision assumptions, currency assumptions and constraint assumptions about the behaviour being analysed be made, as well as precise predictions about what behaviour ought to occur in a given environment. In Part II, I outline John Anderson's (1990a, 1991, 1993) assumptions about the way in which a rational analysis of behaviour are achieved and, within this framework I detail the way in which Oaksford and Chater (1994) achieve a rational analysis of the selection task in terms of optimal data selection.

John Anderson's (1991) work on the adaptive character of thought adopts assumptions basic to optimal foraging theory (Stephens and Krebs, 1986) in order to understand and explain human cognition. He specifies six steps in the development of a rational analysis of behaviour, the aim of which analysis is to show that human cognitive behaviour is also optimally adapted to an environment. The six assumptions or steps and the way in which O&C adopt these optimality assumptions are explained below.

3.2.1 Anderson's (and optimality approaches generally) first step in the development of a rational analysis of behaviour is *the precise specification of the goals of the cognitive system.* As detailed in Part I, a simple goal or decision rule governing animal behaviour is assumed to be the maximisation of energy intake, i.e. the most nutritious food is the best food selection to make. O&C assume that a reasoner's goal in the selection task behaviour is to reduce uncertainty by selecting relevant data, and relevant data is specified as being that which is expected to provide the most gain in information in a given context, i.e. the decision rule to optimise expected information gain is assumed by O&C to determine selection task behaviour[1].

Reasoning or acting in a purposeful away so as to achieve a goal is the way Evans and Over (1996) define rationality$_1$. The O&C model of optimal data

1 The O&C model of optimal data selection is thus a simple or classical optimality model of behaviour as its decision rule involves the optimisation of one set of properties in the environment, i.e. to optimise expected information gain. The O&C model also assumes that the most optimal information will always be selected in a given context and that it is only optimality or information gain (not the decision rule) which changes when state changes. This issue is discussed in chapter 5.

selection thus seems to provide a rationality$_1$ explanation of the selection task (see chapter 1 p. 4).

3.2.2 When the goals of a system have been clearly specified, Anderson's second step in the development of a rational analysis of behaviour requires that a *formal model be specified of the (basically probabilistic) environmental structure to which the behaviour is designed to be optimised.* Anderson acknowledges problems in specifying probabilistic context, for example, only a portion of an environment may be specified and modelling of behaviour in this limited environment may not generalise to other parts of the environment. Also there is the problem of behaviour being different over time, as past behaviour shapes our future behaviour which is continually evolving over time. In other words, Anderson is aware of the role played by short-term and long-term goals (the maximisation or optimisation of either of which may be an optimal decision rule in a given environment).

In order to specify the environment, Anderson uses Bayesian terminology and methods in which *prior probabilities* (i.e. prior uncertainties) about events or contingencies, i.e. the way in which contingencies or events in an environment are distributed prior to any collection of evidence, is computed. *Conditional probabilities* are the outcomes or behaviours produced given (conditional upon) certain contingencies and these are computed using Bayes Theorem. *Posterior probabilities* are the inferences made given data or events/contingencies and they are also computed using Bayesian Methods. When prior probabilities or uncertainty are ignored, this is when non-optimal or irrational behaviour is assumed to be produced. In other words, ignoring priors assumes that an "organism should have the same model of the situation as the experimenter" (Anderson, 1991 p. 473).

Conceptualising the selection task in Bayesian optimal data terms requires that two hypotheses be explicitly specified so that hypotheses or models are not tested in isolation but are compared to one another. Evidence or data collected will then weigh in favour of *either* the first specified hypothesis (formalised as "H1") *or* an alternative (foil) hypothesis (formalised as "H2"), as illustrated in below:

H1<--------------------------->H2

where: H1 is specified as being, say, that x hypothesis is true;
H2 is specified as being, say, that y hypothesis is true.

This Bayesian hypothesis-*constructing* methodology contrasts with hypothesis-*testing* methodology on which theoretical assumptions Peter Wason based the design of the selection task, i.e. only one hypothesis is tested and there is no account of the way in which the amount and kind of information collected constructs context and belief in an hypothesis. The way in which the O&C optimality model specifies the structure of the selection task environment and formally calculates changes in selection behaviour as a function of changes in card optimality which is probability dependent is described below.

O&C assume that the structure of the selection task environment is defined by the frequency or probability of properties or events occurring in that environment. They assume that data collected will change the structure of the selection task environment as data weigh in favour of either a model of dependence between certain events or properties in the environment ("MD"), or a model of independence about certain events or properties in the environment ("MI"), as illustrated below:

uncertainty

Certainty/MD<--------------|-------------->MI/Certainty

where: MD is specified as being a model or context in which if *p* holds then *q* must hold
MI is specified as being a context or model in which *p* and *q* are completely independent events[2].

In addition to beliefs (or uncertainty) about which of two models is true, O&C assume that the events which define the structure of the selection task are in the form of expectations about what is on the reverse of a card and how useful or informative cards are perceived to be in reducing uncertainty about which model holds. For example, optimal data selection assumes that the more certain a reasoner is about either MI or MD being true, the less optimal or

2 MI is the simplest possible alternative to MD. Laming (1996) proposes that other (more general) models can be used as an alternative to MD. A Bayesian approach requires that both hypotheses or models must be specified so that they are both true and that there must be a specific alternative hypothesis. For example, if H^1 = the population mean is equal to 34, it is not sufficient for the alternative hypothesis to be inexact or general such as H^2 = the population mean is not equal to 34. Furthermore, when two true hypothesis are specified, say, H^1 is about "Fair coins" being the case and H^2 is about "tail bias coins" being the case, when data are obtained which are *unlikely* given the truth of H^1, then the data are relatively *likely* under the truth of the *alternative* hypothesis. Appropriate model specification is therefore important, for example, when it is known that a coin, if it is biased, favours tails not heads, then 83 heads or more out of 100 is unlikely for a fair coin, but it is even more unlikely for a tail-biased coin. So if the two true hypotheses are about "Fair coins" and "tail bias coins," the data favour the fair coin hypothesis (Phillips (1973 p. 329), although in this example, it can be inferred from the data that neither the fair coin nor tail-biased coin hypotheses are true, and that a head-bias coin hypothesis is more likely. In other words, the frequency of data will substantiate or not the appropriateness of MI and MD and as O&C (1996 p. 386) indicate their "simple model accurately captures empirical results".

informative p and q cards will become and so their selection should decrease. A card's optimality or informativeness (which are assumed to be probability-dependent) is thus specified as the set of properties or events which change the structure of the selection task environment. A formal measure of the probability-dependent informativeness of cards is therefore necessary. Informally, O&C conceptualise card informativeness as the *difference* between model uncertainty *before* receiving data and model uncertainty *after* receiving data. This is formalised by comparing prior probabilities (or prior model uncertainty) with posterior probabilities (revised model uncertainty). Bayes Theorem is the method used to calculate conditional and posterior probabilities.

The three main parameters in O&C's formal account of the selection task as optimal data selection are, therefore, the expected frequency or probability of p and q, i.e. the P(p) and the P(q); and prior beliefs in the probable likelihood or prior probabilities of the rule "if p then q" formalised as (P(MI).

As already mentioned, a necessary requirement if Bayesian methods are used is the specification of two, true hypothesis, O&C specify MD and MI as described above. O&C set the *prior probabilities* of MD and MI (the likelihood that these models are true) as being equal, where the P(MI) = .5 and (MD) = 1 - P(MI)[3]. The prior probability of p (P(p) is the same in both MD and MI[4]. The prior probability of q in the absence of p (P(q|-p) (represented as parameter b in Appendix 3.2) is also the same in both models[5]. The prior probability of q (P(q)) is not the same in both models because the P(q) depends on whether the selection task rule is true (if the rule is true, i.e. MD holds, O&C assume that there is inequality between p and q, and there should be more q cards than p cards).

Having specified two models, O&C specify the set of properties which comprise each model, i.e. each card's informativeness or the degree to which each card reduces uncertainty about which model holds. They measure uncertainty by calculating the prior probabilities or model uncertainty *before*

3 In other terms, there is maximum entropy or uncertainty (non-information) for each model.

4 P(p) is also represented as parameter a - see Appendix 3.2 which is described in subsequent paragraphs below.

5 O&C (1994 p. 610) propose that equating b in MD and MI reflects both psychological and logic-based assumptions that the -p card is irrelevant to the truth or falsity of an if p then q conditional statement.

data using the Shannon-Weiner measure of information $I(H_i)$ (see equation (1) in Appendix 3.1).

The *posterior probabilities* are then calculated, as model uncertainty after the collection of information by turning a card will produce a revised or new model uncertainty, formalised in Bayesian terms as *P(MI)* and *P(MD)*. See equation (2) in Appendix 3.1 where the new measure of uncertainty is indicated as *I(H|D)*. Calculation of posterior probabilities involves terms of *conditional probabilities* such as *P(MI|D) and P(MD|D)*, i.e. the probability of a model given some data. Bayes Theorem (see equation (3) in Appendix 3.1) is used to calculate the conditional probability for *each* whole card given both MD and MI[6]. In other words Bayes Theorem is the method used to specify formally the posterior probability of a model given data in terms of the prior probabilities of each model, as well as the likelihood of (or degree of belief in) all possible outcomes given the model.

The difference between prior and posterior degree of belief (for each card given each model) is referred to by O&C as the "difference in information gain" produced by a particular datum, i.e. the degree to which a card reduces uncertainty about which model, MI or MD, holds. This measure of reduction in uncertainty or I_g (information gain, which produces a negative value) is formalised as set out in equation (4) in Appendix 3.1. However, because it is not known what will be on the reverse of a card, i.e. the *actual* outcome is not known only possible outcomes are known, O&C calculate the *expected* information gain for each card $E(I_g)$. For example, if q is the face card, there are assumed to be two possible reverse sides or outcomes p or $-p$, therefore the prior probabilities or $E(I_g)$s of these outcomes are calculated, as set out in equation (5) in Appendix 3.1[7].

For example, O&C calculate the $E(I_g)$ of the q card from the prior probabilities or uncertainties about MD and MI, and the probabilities of the

6 For example, given MD, the probability of finding a p card with a q on its reverse *P(p|q|MD)*; a *p, -q* card *P(p)|q|MD*; a *-p, q* card *P(-p|q|MD)*; and a *-p,-q* card *P(-p|-q|MD)*, and similarly for MI (see O&C, 1994 p. 610).

7 The fact that I_g produces a negative value is criticised by Evans and Over (1996) and they believe that it is an inappropriate measure of information gain. O&C (1996 p. 381) point out that $E(I_g)$ or expected informativeness which is the basis of their analysis is non-negative. However, taking into account the negative feature of Ig, O&C (1996) formalise reduction in uncertainty by measuring the difference between new and old degrees of belief using the Kullback-Liebler distance *D*, where *D* is always non-negative. O&C (1996) found, using this measure of uncertainty reduction or information gain, that expected informativeness values remained the same as in their original (1994) analysis, which used the Shannon-Weiner measure of information gain, as predictions are based on expected information gain.

possible outcomes (i.e. p or $-p$) if the q card were turned over. $E(I_g|q)$ is thus the uncertainty after selecting the q card, weighted by the probability of (i.e. degree of belief in) each possible outcome, less the prior uncertainty. The probabilities of p and $-p$ given q are therefore *conditional* upon both the visible face of the card as well as which model is thought to be true: MD ($P p|q$, MD) and $P(-p|q$, MD) or the equivalent for MI (see equation (6) in Appendix 3.1)[8]. Finally, the probability of q in the absence of p $P(q|-p)$ (represented by parameter b in Appendix 3.2) is calculated using the formula set out in equation (7) in Appendix 3.1)

Expected information gain is calculated in this way for each card given both MI and MD. The greater the $E(I_g)$ of a card, the more useful it will be perceived in deciding whether MI or MD holds. In other words, cards which reduce uncertainty the most, by providing the most gain in information about which of two hypotheses or models holds, will be perceived as optimal data to select.

The probability modelling of the two hypotheses, MD and MI, thus involves three parameters:

(a) the frequency of p in the population or the $P(p)$;

(b) the frequency of q in the population or the $P(q)$;

(O&C assume that in standard abstract versions of the selection task these above events are rare in comparison to $-p$ and $-q$);

(c) and prior beliefs about the probable truth of the selection task rule "if p then q" (P(MI).

O&C assume that all quantities needed to calculate expected information gain can be defined in terms of these three factors or parameters (see Appendix 3.2). To see if the O&C optimality model of the selection task would produce predicted behaviour and produce orderings which reflect expected information gain, $E(I_g)$ was varied by systematically varying one of the three model parameters at a time, and setting constant values for the other two parameters.

8 In contingency tables which O&C use to model the rule "if p then q" (see Appendix 3.2), the equivalent of the term($P p|q$, MD) is a.

When the **parameter P(MI)** was varied (using four probability values: 0.2, 0.4, 0.6 and 0.8) and a and b parameters were set at .1, the $E(I_g(cards))$ ordering (p>q>-q>-p) did not vary when P(MI) was varied although $E(I_g)$ were scaled, (i.e. the range of $E(I_g)$s was between 0.2 to just over 0.6 (in comparison to a range from 0.2 to over 0.8). In addition, when uncertainty about MD and MI was at its highest (0.5) the frequency of p was at its highest. The $E(I_g)$ value for **Parameter *a*** was then varied (using the same four probability values: 0.2, 0.4, 0.6 and 0.8) and the other two parameters were fixed where P(MI) was set at .5 and parameter b was set at .1. The expected information gain for each card varied as a function of varying parameter a, so that when the parameter a or P(p) increased so did the expected information gain of -q, and when P(p) was small the expected information gain of -q was also small. When **parameter *b*** or P(q|-p) was varied and the remaining two parameters set, $E(I_g(cards))$ also varied as a function of parameter b so that when the P(q|p) was low the $E(I_g)$s for p and q were high but the $E(I_g)$s for -p and -q were low.

From these $E(I_g)$ or card informativeness results O&C concluded that reasoners in the selection task base their card selections on the expected informativeness of cards, and that varying the probability of p and q are important factors in changing selection task behaviour. Specifically, O&C found that the consistently produced selection task ordering (p>q>-q>-p) depended on probabilities being for the p card (i.e. for parameter a) less than .35, and for the q card (i.e. parameter b) less then .25. In other words, the consistently observed card ordering is conditional upon p and q being perceived as rare events in comparison to -p and -q; when this rarity assumption no longer holds, i.e. when the probability of p and q increases beyond the above probability values, the expected informativeness of cards changes which, in turn, affects card selections[9].

Finally, and in order to specifically model the Pollard (1985) finding that p and q are associated card selections in the selection task and -p and -q are associated selections, i.e. similarly valenced cards are associated with each other, O&C (1994) calculated the scaled expected information gain of each

---

9 Almor and Sloman (1996 p. 378) raise the question of "when should we assume rarity and when should we not?" In other words, there may be situations were rarity is not violated but p and -q responses are produced. O&C (1996 p. 387) acknowledge that certain experiments (O&C cite Green and Larking, 1995 and Platt and Griggs, 1993 and 1995) may suggest that rarity violations may not be a necessary condition to produce p and -q behaviour, and proposes that rarity is a sufficient rather than necessary condition.

card. The $SE(I_g)$ for each card is derived by adding .1 (to add noise in signal detection terms) to the information gain for each card. O&C make this transformation to $E(I_g)$ card values as they assume that all cards have some probability of being selected. By scaling $E(I_g)$ therefore the *-p* card can be more informative than other cards (if the $SE(I_g)$ of other cards is less distinguishable than the $SE(I_g)$ for *-p*), as well as being less informative than other cards because the $SE(I_g)$ of other cards is usually greater than the $SE(I_g)$ of *-p*. The $E(I_g)$ +.1 for each card was then divided by the mean value for all four cards (see O&C, 1994 p. 614). When the $SE(I_g)$s for all four cards were computed, parameters *a* and *b* were again varied using five different probability values (.04, .08, .12, .16 and .20, i.e. all respecting rarity or low probability of *p* and *q*). O&C predicted that $SE(I_g)$ of *p* and *q* would increase in step with each other, and *-p* and *-q* would increase in step with each other across these five probability values, which they did and dissimilarly valenced cards were negatively correlated in terms of $SE(I_g)$s.

3.2.3 Given that the environment has been fully specified and formalised, Anderson's third step in the development of a rational analysis of behaviour requires that *minimal constraints be imposed on what can or cannot be optimised.* For example, basic cognitive constraints include limited cognitive processing capacity or computational difficulties involved in processing alternative models, and human short-term memory limitations impose a "bounded rationality" on cognition (Oaksford & Chater, 1992, 1993). O&C assume that the basic constraints of their model of selection task behaviour are that there is little benefit incurred (i.e. costs are incurred) if irrelevant or non-optimal information is selected.

3.2.4 Having specified the goals or decision rules governing the behaviour to be modelled, as well as having specified the probabilistic environment with minimal assumptions about computational limitations, *optimal behaviour given those precise circumstances must be computed* is Anderson's fourth step in the development of a rational analysis of behaviour. The precise predictions of the O&C optimality model are as follows:

(i) Generally, the probability of *p* and *q* will influence the selection of *-q*.

(ii) There will be an increase or decrease in other card selections besides the *-q* card. When cards are informative because they reduce uncertainty the most about which model MI or MD holds, selections are predicted to increase in comparison with other cards, and when cards are uninformative because they provide little information gain about which model holds, selection is predicted to decrease in comparison to other cards. Specifically, when rarity is not violated, selection of *p* cards will increase, -*p* card selections will decrease, *q* card selections will increase and -*q* card selections will decrease. When rarity is violated, selection of *p* cards will decrease, -*p* card selections will increase, *q* card selections will decrease and -*q* card selections will increase. These increases and decreases in card selections are assumed by optimal data selection to be a function of the variations in the expected informativeness of cards, which are dependent on the P(*p*) and P(*q*).

(iii) As regard card selection ordering, which is assumed to reflect preferences about which card provides the most gain in information, when the P(*p*) and P(*q*) are low, i.e. assuming rarity, card selection ordering is predicted to replicate the consistently observed *p*> *q*>-*q*>-*p* affirmative abstract selection task card ordering. When the P(*p*) and P(*q*) are high, i.e. when the rarity assumption has been violated, card selection ordering is predicted to change to reflect the card informativeness preferences *p*>-*q*>*q*>-*p*. Analysis of each card's informativeness or utility is relevant in optimality models of cognition because when a consistent transitive ordering such as *p*>*q*>-*q*>-*p* is observed it is assumed to reflect that an optimality-preserving maximising principle is being applied to a set of properties in an environment i.e. cards, rather than logic-based decision rules such as falsification.

3.2.5 *Examination of evidence in order to either substantiate or refute a model, its assumptions and predictions* is Anderson's fifth step in the development or a rational analysis of behaviour stipulates. An optimality model is substantiated if the experimental studies carried out to test it produce the same behaviour as the model predicts.

In order to test O&C's optimal data selection assumptions and predictions in an experimental setting, the P(*p*) and P(*q*) are experimentally varied in studies reported in Section B of chapter 4. Probabilities are varied by systematically increasing or decreasing the frequency of cards in a probability learning phase prior to the selection task phase. In order to specify the frequencies of cards, O&C work backwards from MD to MI[10], the probable likelihood of each card is varied where in one experimental condition the P(*p*) and P(*q*) is calculated to be low (.2) (i.e. the rarity assumption holds) and a second experimental condition the probabilities of these two cards is calculated to be high (.8) i.e. the rarity assumption is violated (see Appendix 3.3 for the way in which frequencies for high and low P(*p*) were calculated). These two different probabilistic contexts are predicted to produce specific increases and decreases in card selections as detailed in 3.2.4 (ii) above, as well as specific card ordering behaviour as predicted in 3.2.4 (iii) above.

3.2.6 The sixth step in Anderson's (and optimality approaches general) development of a rational analysis of behaviour, and if the model has not been substantiated, involves the *refinement of the model*, its assumptions, its constraints and possibly its predictions[11].

In summary, in order to develop a rational analysis of behaviour in the selection task, O&C have specified the precise goals of the reasoner in the selection task. They assume that behaviour on the selection task is rational as it reflects a reasoner's goal of selecting optimal data in a given probabilistic environment. O&C have specified and formalised the environment of the selection task using Bayesian methods to formalise the measure of informativeness of each card, which probability-dependent informativeness values define the structure of the selection task environment. Constraint assumptions have also been specified and so have precise predictions about selection task behaviour in different probabilistic contexts. Existing data substantiate predictions made by the O&C optimality model, and the experiments which I report in chapter 4 test the precise predictions of the O&C model of optimal data selection for the affirmative abstract selection task.

As regards the way in which the O&C model relates to optimality approaches of cognition generally and as discussed in Part I of this chapter, the O&C (1994) optimality model assumes that the decision rule to optimise expected information gain governs selection task behaviour, which decision rule assumption is the basis of classical optimality modelling (i.e. optimal foraging theory) in which one set of

---

10 This method of modelling behaviour, which words from final (or true state or state of certainty in terms of reasoning about the selection task rule through to a state of uncertainty) is used by Krebs and Kacelnik as described in Part I section 3.10 of this chapter.

11 Oaksford, Chater, Grainger and Larking (in press) consider ways in which O&C (1994) model of optimal data selection could be refined. For instance, the O&C (1994) model assumes that there will always be the same amount of information gain , i.e. that there is no decrement in information gain. The role decrements in information gain may play in causing changes in focal attention and selection task behaviour is considered in the chapter 5.

properties in the environment (for example, energy intake) are maximised. In addition, and like crypsis (the common adaptation animals use to survive in a competitive environment), optimal data selection can be viewed as an example of the best or optimal design in a cognitive environment in which information competes for attention and processing. It is an optimal design simply because an adaptive behaviour or bias which governs the way in which relevant information is selected minimises the constraints on attention and memory which then enables attention to be optimally focused in different contexts (for example, to prioritise events differently in different contexts).

In conclusion, the optimality approach to cognition taken by O&C (1994) does not assume that logicality (either logical competence or logical performance) as measured by falsifying an hypothesis is the criterion by which rationality should be measured. An optimality approach assumes that the problems of variability in performance on the selection task can be explained if the sources of variability are found. In order to test the optimality assumptions and predictions of the O&C model of optimal data selection, experiments on the abstract version of the selection task were carried out and are reported next in Chapter 4 as outlined below.

Seven studies (plus one replication) are reported in total, five studies in Section A and two probability learning studies in Section B. The first two studies in Section A control for computerised multiple selection task presentation, and computerised multiple single card presentation. The next three studies in Section A, called the card informativeness studies, require judgements to be made about the informativeness of cards. In two of the card informativeness studies, card informativeness judgements are obtained using informativeness ratings scales (which provide a direct measure of informativeness). In a third card informativeness study, the task requires that preferences regarding the informativeness of cards be made and two cards at a time are presented on a computer screen. The two studies in Section B both manipulate the probability of p and q, i.e. in each study there are two conditions. In one experimental condition $P(p)$ and $P(q)$ are high and in the other experimental condition $P(p)$ and $P(q)$ are low.

The experimental design and rationale for each study prefaces the next chapter 4.

Chapter 4
EXPERIMENTS

Experimental Rationale

In chapter 3, I reviewed general optimality assumptions about selection behaviour, and then detailed the way in which O&C (1994) apply these assumption to their optimality model for affirmative abstract versions of the selection task. In this chapter, I report the experiments carried out to test this model of optimal data selection. Five affirmative abstract selection task studies are reported in Section A, Parts I to V as summarised below, and two affirmative abstract selection task studies are reported in Section B, Parts VI and VII.

The five studies reported in Section A are:

I **Four Cards study:** this study comprises multiple affirmative abstract selection tasks where four cards are simultaneously presented on computer screen;

II **Single Card study**: this study also comprises multiple affirmative abstract selection tasks but one card at a time is presented on computer screen;

III **Pilot Ratings**: this study comprises one affirmative abstract card informativeness ratings task where four card are simultaneously presented on sheet of paper;

IV **Single Ratings:** this study comprises multiple affirmative abstract card informativeness ratings tasks where one card at a time is presented on computer screen[1];

V **Binary study**: this study comprises multiple affirmative abstract card and/or card pairings informativeness judgement tasks where two cards at a time are presented on computer screen.

In Section B, Parts VI and VII, I report two **probability learning studies**.

---

1 The Single Ratings study was replicated and the results of the "Ratings Replication" study are also reported in Part IV.

In the **Four Cards** study, I used a one-factor within-participants design. The independent variable or factor is **type of card** which has four levels: *p*, *q*, *-p* and *-q* cards. This is a factor in all studies. The dependent variable in the Four Cards study is frequency of **selection of *p*, *q*, *-p* and *-q* cards**. The main purpose of the Four Cards study is to control for **multiple computerised task presentation** rather than standard presentation of one, paper and pencil version of the selection task. More specifically, this study involves performing on a computer 12 selection tasks each with a different rule and cards in order to investigate whether computerised multiple task presentation changes card selection behaviour.

As regard card selection behaviour, in a meta-analysis of studies reporting affirmative abstract versions of the four card problem compiled by Oaksford and Chater (1994 p. 613), a $p>q>-q>-p$ card selection ordering was consistently observed across 13 studies comprising 34 standard abstract selection tasks involving 845 participants. This consistently observed card selection ordering is assumed by optimal data selection to reflect the application of a decision rule or maximising principle to optimise expected information gain about which model (MI or MD) holds[2]. More specifically it is assumed that people select the *p* card most frequently because it is expected to provide the most gain in information and therefore reduce uncertainty the most about which model (MI or MD) holds. The *-p* card is the least informative card to select because it provides the least reduction in uncertainty about which model holds.

The multiple computerised task presentation component of the Four Cards study is not expected to produce card selection ordering behaviour different to other affirmative abstract versions of the selection task.

The **Single Card study** is also a one-factor within-participants design. As in the Four Cards study, **type of card**, i.e. *p*, *q*, *-p* or *-q*, is the independent variable and the dependent variable is frequency of **selection of *p*, *q*, *-p* and *-q* cards**. The main purpose of the Singles study is to control for **computerised single card presentation** in preparation for the probability learning studies detailed in Section B where cards are also presented one at a time on a computer screen. The Single Card study therefore acts at a "control" for the probability learning studies and it is important that the Single Card study produces the same card selection behaviour as the Four Cards study and other affirmative abstract versions of the selection task. As this study is the

2 "MI" is a model of independence between *p* and *q* (i.e. there is independence between *p* and *q*) and "MD" is a model of dependency between *p* and q (i.e. if *p* holds then *q* holds).

same as the Four Cards study in all respects except for the number of cards presented on the screen at any one time, *p*, *q*, *-p* and *-q* card selections in the Four Cards and Single Card studies are compared as if they were two different conditions (i.e. four or single card presentation) of the same study.

The **card informativeness studies**, i.e. the Pilot Ratings, Singles Ratings and Binary studies reported in Parts III, IV and V, respectively, of Section A, require participants to judge card informativeness, which task is very different to that of selecting necessary cards in order to evaluate whether the selection task rule is true or false. However, as the O&C (1994) model of optimal data selection assumes that people choose the cards which are most informative in a given context, results of the informativeness studies are not expected to differ from the Four Cards and Single Card studies[3]. Consequently, any deviation from standard performance using the informativeness ratings tasks could constitute strong falsifying evidence against optimal data selection.

The results of the card informativeness studies are expected to reflect the above optimality assumptions, and cards which reduce uncertainty the most about whether MI or MD holds, i.e. *p* and *q*, are expected to be rated more informative than the *-p* and *-q*. However, *p* and *q* are optimal or informative only if the *p* and *q* rarity assumption is not violated, i.e. if the P(*p*) and P(*q*) remain low in comparison to the P(*-p*) and P(*-q*). In other words, it is because *p* and *q* are rare that they are most informative, therefore, if this specific probabilistic context changes, card selection ordering is predicted to change to reflect changes in expected card informativeness.

As the probability of cards has not been manipulated in any way in the card informativeness studies (or in other studies detailed in Section A), card selection ordering is predicted to remain the same as in standard affirmative, abstract versions of the selection task. Specifically, the consistently observed and context-dependent $p>q>-q>-p$ card selection ordering is expected to be replicated in the judgement of card informativeness studies because card selections are related to perceived card informativeness.

The main purpose of the informativeness studies is to provide data to test the hypothesis that the decision rule governing card selection behaviour is that of optimal data selection, rather than the logic-based decision rule of falsification. To this end,

3 As regard context, the probabilistic context of affirmative abstract versions of the selection task is assumed by optimal data selection to be one of uncertainty about whether a model of independence (MI) holds or whether the model is one of dependency (MD)

the informativeness studies provide two different forms of data both of which are expected to provide converging evidence for the appropriateness of an optimal data selection explanation of selection task performance. For example, the Pilot Ratings and Singles Ratings studies produce ordinal data (a ratings scale is the methodology used to investigate judgements about expected card informativeness, or the amount of information expected to be gained by turning a card over). The Binary study produces nominal data (pair comparison is the methodology used to judge card informativeness: for example, which of two cards is the most useful card to turn over?).

The way in which the informativeness of cards is judged is therefore investigated using a *direct rating* of expected card informativeness as well as an *indirect scaling* of expected card informativeness. The direct rating of expected card informativeness is the task performed in the **Pilot Ratings** study where four cards are simultaneously presented as in the Four Cards study and standard versions of the selection task. The experimental design of this pilot informativeness study is a one-factor (being **type of card**: *p*, *q*, *-p* or *-q*) within-participants design. The dependent variable is the **informativeness rating given to *p*, *q*, *-p* and *-q* cards.**

The direct rating of expected card informativeness is also the task performed in the **Single Ratings** study[4], a one-factor within-participants experimental design with **type of card** (*p*, *q*, *-p* or *-q*) being the independent variable. Card presentation is one card at a time in this study which, as well as providing more evidence about whether multiple computerised single card presentation changes behaviour, permits an absolute rather than a comparative judgement of the informativeness of a card to be made. The dependent variable is the **informativeness rating given to *p*, *q*, *-p* and *-q* cards.**

Indirect scaling of expected card informativeness is the task performed in the **Binary** study, where cards are presented in pairs in order that comparative card informativeness judgements may be made when presented with every possible pair of *p*, *q*, *-p* and *-q* cards. The Binary study, like the Four Cards, Single Card and Single Ratings studies, has multiple computerised task presentation. This informativeness study is a one-factor within-participants design, the independent variable being **type of card** and/or **type of card pairing**. The dependent variable is the **scaled informativeness of *p*, *q*, *-p* and *-q* cards and/or card pairings.**

4 (and Ratings Replication study)

Having designed experiments to control for computerised multiple task presentation and computerised single card presentation, as well as to obtain different forms of data about judgements about *p, q, -p* and *-q* informativeness, in Section B of Chapter 4, I detail experiments designed to test whether manipulation of probabilistic context - by systematically varying the **probability of *p, q, -q* and - *p* cards** and consequent **optimality or expected information gain of cards** - makes a difference to card selection behaviour.

The first experiment described in Part I of Section A is the Four Cards study where the *p*, *q*, *-p* and *-q* cards are simultaneously presented on the computer screen and selection task wording is the same as in standard affirmative abstract versions of the selection task. Data from this study are compared with those from the Single Card study in Part II.

Chapter 4
EXPERIMENTS - SECTION A

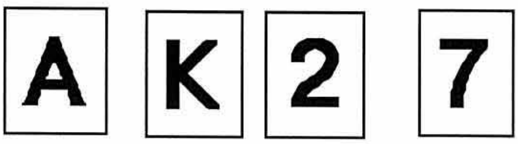
Part I - Four Cards Study

Computerised Four Card Presentation
Multiple Selection Task Presentation
Original Selection Task Wording

Introduction

As detailed in chapters 1 and 2, the connection between logic and thinking has been an issue in Western thought since at least the time of Socrates and continues to the present day, where the implementation of logical rules and procedures is assumed to be the mechanism which allows people to draw inferences and reason rationally. These assumptions were accepted by Peter Wason when he designed the four card problem or selection task. An example of the selection task is set out in Figure 4.1 below:

Figure 4.1 - example of Wason's Selection Task



There is a letter on one side of the card and a number on the other side.
Rule: If a card has a vowel on one side then there is an even number on the other side.

The original selection task instructions required people to "*name those cards, and only those cards, which need to be turned over in order to determine whether the rule* (typed in Figure 4.1 above in bold) *is true or false*". The logically correct solution is presumed to involve applying principles used in propositional logic to ascertain whether statements in the conditional form "if p then q" are valid statements or hypotheses. If these principles are applied, the logically correct solution would be to turn over the A(p) card and the 7($-q$) card.

The logically correct solutions thus requires the falsification of an hypothesis or conditional statement. However, in the O&C (1994) meta-analysis of studies reporting affirmative abstract versions of the selection task, *p* and *-q* are not the cards which most people select when performing Wason's four card problem. Meta-analysis total card selection frequencies were: p = 754, q = 522, $-q$ = 215 and $-p$ = 137.

From these and other selection task results, and as reviewed in chapter 1, it is often argued that people are irrational reasoners because logical principles are not applied in order to make correct, falsificationist selections and/or inferences about the truth or falsity of the hypothesis in the selection task. Such "irrational" performance has motivated the formulation of several psychological theories in order to explain illogical errors and biases in reasoning. For example, and as detailed in chapter 2, proponents of a theory of "mental logics" assume that people reason by applying mental rules of inference to information or premises, and irrational inferences in the form of reasoning errors are assumed to be made mainly because the correct logical rules and truth-preserving procedures are not available, or not accessed and applied to information (O'Brien, 1995). Johnson-Laird and Byrne (1993) propose that "mental models" are constructed which represent possible states of the world consistent with premises or information given. The validity or truth of a mental model is ascertained by the search for and attempted construction of counter-examples where the premises are true but the conclusions are false. If no such counter-examples are found, then a mental models approach to reasoning assumes that inferences are then valid.

Another psychological approach focuses on apparent biases, as opposed to errors, in reasoning, where it is assumed that inferences only appear to be illogical and irrational because, at a pre-conscious stage of information processing, there is a bias to select only linguistically relevant information (Evans, 1977, 1984, 1989). For example, a "positivity" bias ignores negative information as it is not considered relevant and only positive information is selected and processed. In these terms reasoning is deemed to be rational because conclusions inferred are consistent with premises.

The Oaksford and Chater (1994) model of optimal data selection is also concerned with "biased" selection of information, but it does not assume that applying logical principles to selectively biased information is what constitutes rational reasoning in the selection task. Rather, it is argued that reasoning performance reflects that a maximizing principle is being applied in order to ensure that the most beneficial or useful information in a given context is always chosen. Beneficial or optimal information is assumed to be information which increases certainty or expected information gain and reduces uncertainty the most about which of two hypotheses holds.

As I detailed in chapter 3, a similar cost-benefit approach or "rational analysis" of decision making behaviour is used in economics to study the way in which consumers make decisions, which are assumed to be rational decisions because they reflect a consistent selection of preferences about what consumable goods provide the most value or utility to an individual. Consistent card selection is also the reason why the O&C model of optimal data selection argues that reasoning in the selection task is rational, and card selections are assumed to reflect the way in which people perceive the informativeness of each card. Specifically, the consistently observed *p>q>-q>-p* card selection ordering is assumed to reflect that a maximising principle is being applied as a result of which the *p* card is always preferred, because in standard affirmative abstract selection tasks it is always perceived to provide more gain in information about which model holds than the *q* card; the *q* card is the next preferred card selection, as it is perceived to provide more information gain in comparison to the *-q* card; and the *-q* card and-*p* card provide the least gain in information.

The main purpose of the Four Cards study is to replicate the consistently observed *p>q>-q>-p* card ordering when there is multiple and computerised selection task presentation rather than standard presentation of one, paper and pencil version of the affirmative abstract selection task. The Four Cards study thus involves performing on a computer 12 affirmative abstract selection tasks each with a different rule and it has been designed to control for multiple computerised task presentation, which is a design feature of the probability learning studies reported in Section B of this chapter. It is important therefore that card selection behaviour in this study does not differ from other affirmative abstract versions of the selection task.

The methodology of the Four Cards study is detailed below.

Method

Participants

There were 20 participants, 11 males and nine females, all of whom were undergraduate students at the University of Wales, Bangor aged between 18 and 50. No participants had prior knowledge of logical principles or the rationale for this study.

Design

This was a one-factor within-participants design. The dependent variable was the **frequency of *p*, *q*, -*p* and -*q* card selections**. The independent variable or factor was **type of card** which had four levels for *p*, *q*, -*p* and -*q* cards. It was hypothesised that there would be a difference in card selection behaviour depending on whether the card was a *p*, *q*, -*p* or -*q*, and that card selection behaviour would not be affected by **multiple computerised (four card) selection task presentation**. In other words, performing on a computer 12 consecutive selection tasks, each task with a different rule and cards and where four cards at a time are displayed on a computer screen, would produce the same card selection behaviour (for example a $p>q>-q>-p$ card preference ordering) as found in standard (four card) affirmative abstract versions of the selection task. Table. 4.1.1.1 below illustrates the experimental design of this Four Cards study.

Table 4.1.1.1: Packs/rules and *p*, *q* -*p* and -*q* cards simultaneously displayed on computer screen

PACK	PACK RULE	CARDS SIMULTANEOUSLY AND RANDOMLY DISPLAYED ACROSS BOTTOM OF SCREEN			
		p	*q*	-*p*	-*q*
	LETTERS				
	***p* as vowel**				
1	if A then 2	A	2	K	7
2	if U then 8	U	8	A	6
3	if E then 5	E	5	A	2
	***p* as consonant**				
4	if J then 6	J	6	W	9
5	if X then 9	X	9	U	2
6	if D then 3	D	3	F	1
	NUMBERS				
	***p* as even number**				
7	if 4 then Q	4	Q	6	L
8	if 8 then E	8	E	4	C
9	if 8 then Z	8	Z	1	P
	***p* as odd number**				
10	if 1 then A	1	A	5	U
11	if 3 then B.	3	B	1	A
12	if 9 then G	9	G	7	W

For example, there were 12 different packs of cards. Each pack had a different rule as well as two different letter cards and two different number cards representing *p, q, -p* or *-q*. In addition the *p* card in the rule was either a vowel, consonant, odd or even number. More specifically, there were three vowel *p* cards and three consonant *p* cards, and three even-numbered *p* cards and three odd-numbered *p* cards.

Randomisation and counterbalancing for order effects, i.e. random selection of the multiple (12) selections tasks or packs (each with different rules and cards), was programmed in PsyScope (see Apparatus section) for each participant. In addition, each of the *p, q, -p* and *-q* cards in each of the twelve packs/trials was programmed to appear randomly in either the first second, third or fourth position across the bottom of the computer screen.

For example, pack 4 in Table 4.1.1.1 above, where the *p* card in the rule was a consonant: "if there is a J on one side of a card, there is a 6 on the other side", may have been randomly selected for trial/selection task 1. The pack 4 cards may then have been displayed on the computer screen in the following random sequence: 6 (*q*), J (*p*), 9 (*-q*) and W(*-p*). In other words, for one participant a *q* card with "6" on its face may have appeared first, in the far left-hand port, a *p* card with a "J" may have appeared on its right, a *-q* card with a "9" on its face may have appeared in the third port, and the last or fourth card on the far right of the screen may have been a *-p* card with a "W" on its face. Table 4.1.1.2 below details the 24 possible permutations for card screen position of pack 4 *p, q, -p* and *-q* cards.

Table 4.1.1.2: 24 permutations of card screen positions (1st, 2nd, 3rd or 4th port or position) using pack 4 cards as an example

Card Permutations	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Port																								
1st	6	6	6	6	6	6	J	J	J	J	J	J	W	W	W	W	W	W	9	9	9	9	9	9
2nd	J	J	W	W	9	9	6	6	W	W	9	9	J	J	6	6	9	9	6	6	J	J	W	W
3rd	W	9	J	9	W	J	W	9	9	6	W	6	9	6	J	9	J	6	J	W	W	6	J	6
4th	9	W	9	J	J	W	9	W	6	9	6	W	6	9	9	J	6	J	W	J	6	W	6	J

The 24 permutations for the different *p*, *q*, *-p* and *-q* cards in each of the 12 selection tasks and/or packs were also calculated and randomised.

Apparatus

PsyScope, a graphic interface experimental design application, was used to program and run this experiment and record responses. All instructions and stimuli were presented on a monitor attached to a Macintosh LCIII computer. A Macintosh keyboard and mouse recorded responses.

Procedure

Participants were tested individually and sat facing a Macintosh LCIII computer monitor. It was explained that instructions about the task and the task itself would be displayed on the computer screen. Below the computer and on the same desk was a Macintosh keyboard and mouse and it was explained that using the keyboard and mouse would record responses.

Before the experimenter left the research room, the participant was handed a sheet of paper on which *Participant's Rights* as set out below (see Appendix 4.1 for precise format) were typed.

> Although you have agreed to participate in this study, you are at liberty to withdraw at any time. Any data which you produce will remain confidential and your anonymity is guaranteed. We are NOT interested in whether your responses are right or wrong. This is NOT a test of intelligence or ability, and you will not be timed. CLICK MOUSE ONCE FOR FURTHER DETAILS ABOUT THIS STUDY.

This statement was also on the computer screen. When Participant's Rights had been read and the mouse had been clicked to proceed with the experiment, the experimenter left the room.

The next screen instructions as set out below (see Appendix 4.2 for precise screen format) gave *general instructions* about the study:

> This study uses several packs of cards. All the cards in these packs have a LETTER on one side and a NUMBER on the other side. There are rules about what letters and numbers can go together. For example: "If a card has a 2 on one side then it has a T on the other side. CLICK MOUSE FOR MORE INSTRUCTIONS ABOUT 'YOUR TASK'...

When these general instructions had been read and assimilated, instructions were to click the mouse in order to obtain further instructions about the task.

The next screen as set out below (see Appendix 4.1.1 for precise screen format) gave study and *task specific instructions* :

> FOUR cards at a time will be dealt from one of several packs used in this study. Only one side of each card will be displayed on the screen. A rule will also be shown. Your task will be to name those cards, and only those cards, which need to be turned over in order to determine whether a rule is true or false of the pack then being used. You will be prompted to press one, or more keys on the keyboard in order to record your card selection. Your task will continue until you have made card selections from several packs. You will be prompted when you have reached the end. If you would like to review these task instructions, PRESS THE "A" KEY. If you are clear about what to do CLICK THE MOUSE ONCE TO BEGIN.

The next screen displayed the statement "*New Pack! New Rule...*" (see Appendix 4.3 for precise screen format) in order to make the participant aware that a new pack of cards with a new rule rather than the rule given as an example in the general instruction (see Appendix 4.2) was now being used.

The next screen of *selection task instructions* as set out below (see Appendix 4.1.2 for precise screen format) was automatically displayed after "*New Pack! New Rule...*" had been displayed on the screen for five seconds:

> Rule: If a card has a [*p*] on one side then it has a [*q*] on the other side.
>
> Your task is to name those cards and only those cards which need to be turned over in order to determine whether the above rule is true or false of the pack now in use. To record your selection, PRESS one, or more, or all of the equivalent keys on the keyboard.
>
> CLICK MOUSE ONCE when you have finished your card selection.
>
> [CARD] [CARD] [CARD] [CARD].

For example, at the beginning of each trial, the rule on the screen randomly changed and the cards belonging to the pack then in use were randomly displayed in the first, second, third or fourth positions across the bottom of the screen. Card selection responses were recorded by pressing a card-equivalent key or keys on the keyboard[1]. When the participant had finished making selections for that trial/pack of cards and in order to proceed with the task, instructions were to click the mouse once. The "*New Pack! New Rule...*" instructions were again immediately displayed for five second, after which the next trial/selection task with a new pack of cards commenced.

The above screen instructions (change of pack and rule instructions before each new trial) continued for 12 selection task trials/packs of cards, at the end of which the *final screen instruction* as set out below (see Appendix 4.4 for precise screen format) were displayed:

> You have now completed this task. If you would like to know more about this study, we shall be happy to answer any questions. Thank you for your time. CLICK THE MOUSE ONCE TO END THIS SESSION.

The Four Cards study took each participant between 10 to 15 minutes to complete.

See Appendix 4.1.3 for an example of one participant's raw datasheet and how responses were recorded in this Four Cards study.

Results

Regarding the hypothesis that there would be a difference within this Four Cards study in *p*, *q*, *-p* and *-q* card selections depending on card type (*p*, *q* *-p* or *-q* cards), and whether multiple computerised task presentation would alter card selections when compared with standard one task versions of the selection task, mean cards selections are summarised in Means Table 4.1.1 below[2].

---

1 In all studies in this thesis, only keys which were relevant to a particular pack of cards (or appropriate at a particular time) were active keys and able to record responses or progress the study. For example, only the 6 J W 9 keys would have been active when pack 4 was the selection task/trial being performed. In other words, all non-relevant keys on the keyboard, including the mouse if this was not an appropriate response to make at a given time, were disabled.

2 As each of the 20 participants in this study performed 12 selection tasks, each participant's mean selections for *p*, *q*, *-p* and *-q* cards were calculated and form the basis of this analysis.

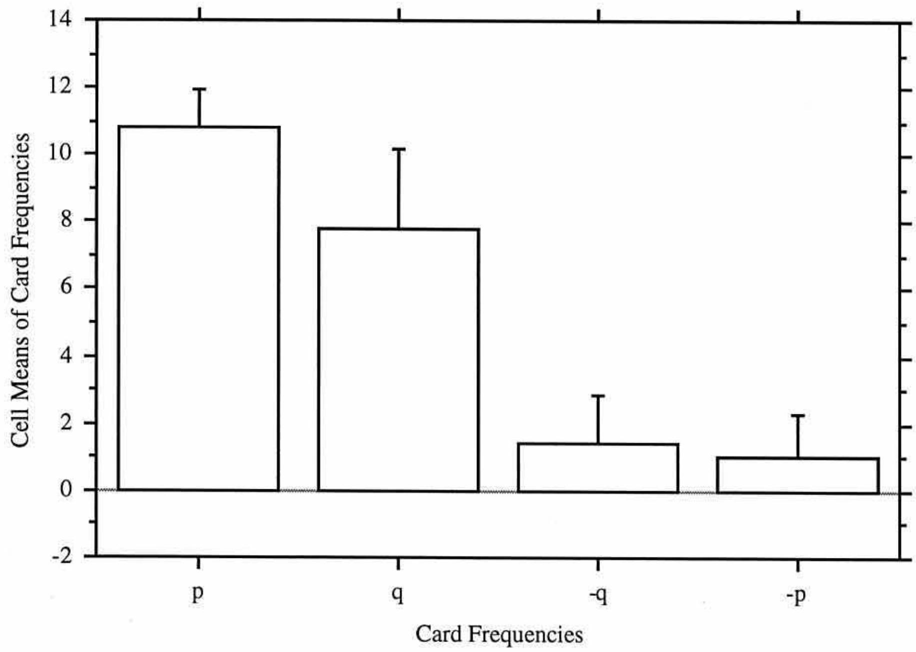
Means Table 4.1.1: *p, q, -p* and *-q* card selections in the Four Cards study

	Count	Mean	Std. Dev.	Std. Error
p	20	10.850	2.254	.504
q	20	7.800	5.012	1.121
-q	20	1.400	3.102	.694
-p	20	1.050	2.625	.587

A one-way ANOVA was performed which showed a significant effect between card selections depending on card type (F (3, 19) = 40.516, MSE = 468.817, $p < .0001$)[3].

Pairwise comparisons of means were carried out in order to investigate which card selections were significantly different from each other. This analysis revealed that at the .05 level there were significant effects between *q* and *p* (p = .0063); *-p* and *q*, *-p* and *p*, *-q* and *q*, and *-q* and *p* were all significantly different from each other (p = .0001 for these four pairwise comparisons); but there was no significant effect between *-p* and, *-q* card selections (p = .7461). Figure 4.1.1 below illustrates the card selections as well as predicted *p>q>-q>-p* card preference ordering produced in this Four Cards study.

Figure 4.1.1 : card preference ordering of *p, q, -p* and *-q* card in Four Cards study



A linear contrast was performed on the selection ordering of *p, q, -q* and *-p* cards in this study, which was significant (F (1,19) = 110.763, MSE = 1281.640, $p < .0001$).

3 All ANOVA summaries are in Appendix 4.8.

The main results of this Four Cards study have now been reported.

Before discussing their implications, the design of this study makes it possible to carry out further analysis to see if there was a difference in card selections depending on whether the *p* **card in the selection task rule was a consonant, vowel, odd number or even number**. The rationale for this analysis is that the usage of vowels is more frequent in the English language in comparison to the usage of consonants, and it would be interesting to see whether *-q* card selections increase when the *p* card is a vowel, i.e. when P(*p*) is "higher", in comparison to card selections when the *p* card is a consonant, i.e. when P(*p*) is "lower"[4]. The mean of all card selections depending on whether *p* cards were consonants, vowels, odd numbers or even numbers are summarised in Means Table 4.1.2 below:

Means Table 4.1.2: *p*, *q*, *-p* and *-q* card selections depending on whether *p* cards were consonants, vowels, odd numbers or even numbers

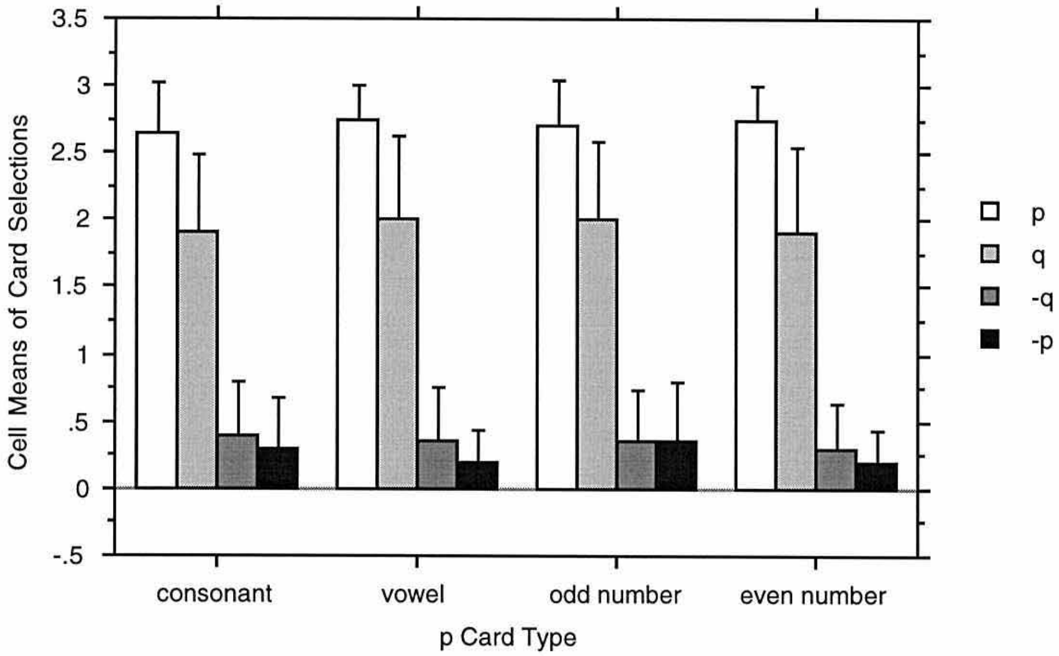
	Count	Mean	Std. Dev.	Std. Error
p, consonant	20	2.650	.813	.182
p, vowel	20	2.750	.550	.123
p, odd number	20	2.700	.733	.164
p, even number	20	2.750	.550	.123
q, consonant	20	1.900	1.252	.280
q, vowel	20	2.000	1.338	.299
q, odd number	20	2.000	1.257	.281
q, even number	20	1.900	1.373	.307
-q, consonant	20	.400	.821	.184
-q, vowel	20	.350	.875	.196
-q, odd number	20	.350	.813	.182
-q, even number	20	.300	.733	.164
-p, consonant	20	.300	.801	.179
-p, vowel	20	.200	.523	.117
-p, odd number	20	.350	.933	.209
-p, even number	20	.200	.523	.117

A 4 x 4 within-participants ANOVA was performed which showed that there was no difference in *overall* card selections based on *p* card type (i.e. when *p* card was a consonant, vowel, odd or even number) ($F(3, 76) = .067$, $MSE = .054$, $p = .9772$).

4 Kirby (1994) proposes that the probability of vowels is low as there are only five vowels whereas there as 21 consonants. However, when placed in the context in which letters are used in the English language, the five vowel letters of the alphabet are more frequently used than the 21 consonant letters of the alphabet.

Neither was there any significant *interaction* between *p* card type and *p*, *q* *-p* and *-q* cards selections ($F(9, 76) = .081$, $MSE = .068$, $p = .9998$). Figure 4.1.2 below illustrates card selections depending on *p* card type. It can be seen that the card preference ordering when *p* cards were consonants, vowels and even numbers was the predicted *p>q>-q>-p* , but when *p* cards were odd numbers, the *-p* and *-q* cards were equally selected (means for *-p* and *-q* card selections = .350).

Figure 4.1.2: preferential ordering for *p*, *q*, *-p* and *-q* cards when *p* cards were in either consonants, vowels, odd numbers or even numbers.



In order to see which, if any, card selection were significantly different from each other depending on card type, pairwise comparisons were carried for each card: there were no significant effects at this level of analysis. In other words, the selection frequency of *p* when *p* cards were consonants was no different to the selection frequency of *p* when *p* cards were vowels, odd or even numbers. This was the case for the selection frequency of *q*, *-q* and *-p* cards.

Discussion

The Four Cards study has replicated the consistently observed card selection ordering as reported by Oaksford and Chater (1994) in the meta-analysis of affirmative abstract selection tasks, where a *p>q>-q>-p* card selection ordering was found to exist across 13 studies comprising 34 standard affirmative abstract selection tasks involving 845 participants.

In terms of optimal data selection this precise card ordering would be expected in a probabilistic context where *p* and *q* are rare events (i.e. when the probability of these cards is low in comparison to *-p* and *-q*). In such a probabilistic context, *p* and *q* cards are predicted to be the most frequently selected cards because they provide the most information gain and optimally reduce uncertainty about which model (MI or MD) holds.

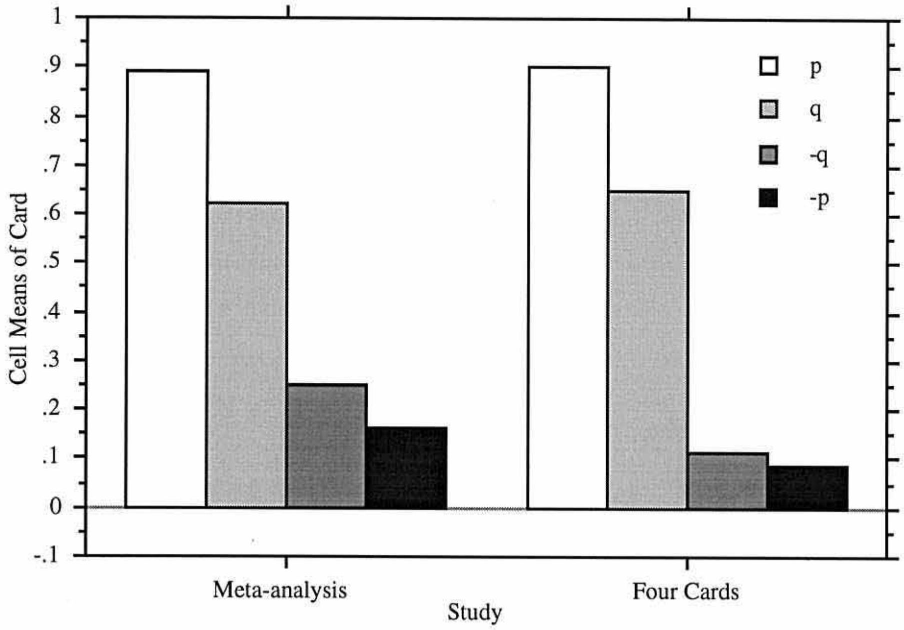
To investigate the way in which the frequencies of card selections in this study may differ from studies comprising the meta-analysis, and for information purposes only, further post hoc analysis was carried out to compare cards selections made in the Four Cards study with card selections from meta-analysis studies. Table 4.1.4 below gives the frequencies and mean proportions of card selections in these studies.

Table 4.1.3: comparison of Four Cards and O&C (1994) meta-analysis card selections

Study	Data	*p*	*q*	*-q*	*-p*
Meta-analysis	Frequency	754	522	215	137
N = 845	*M* proportions	.89	.62	.25	.16
Four Cards	Frequency	217	156	28	21
N = 240	*M* Proportions	.904	.65	.116	.0875

Table 4.1.3 above shows that, although card preference ordering is the same in both studies, i.e. *p>q>-q>-p*, the Four Cards study produces different proportions of card selections to the meta-analysis studies. For example, in the Four Cards study the proportion of *p* and *q* cards selections have *increased* slightly and the proportion of *-p* and *-q* card selections have *decreased* in comparison to the meta-analysis. Figure 4.1.3 below illustrates this more clearly.

Figure 4.1.3: comparative card selections (in proportions) between O&C (1994) meta-analysis and Four Cards study



A chi-square "goodness of fit" statistical test was performed on the meta-analysis and Four Cards frequency data, and it showed that *-q* and *-p* card selections in the Four Cards study were significantly different (lower) to meta-analysis *-q* and *-p* card selections: *-q* card χ^2 (1) = 18.62 p <.0001 and the *-p* card χ^2 (1) = 7.29 p < .01. Whereas there was no significant divergence between the meta-analysis and Four Cards study for the *p* and *q* card selections: *p* card χ^2 (1) = 1.11 p = .29, *q* card χ^2 (1) = 1.29 (p = .26). Specifically, when df = 1 at the .5 significant level one-tailed, the expected table value of χ^2 = 3.841. As observed χ^2 did not exceed this value in *p* and *q* card instances there is no evidence for a divergence between the proportion of *p* and *q* cards selections in the meta-analysis and Four Cards study. However, observed χ^2 did exceed the critical value for *-q* and *-p* card selections. Thus there is a significant divergence in the proportion of *-q* and *-p* cards selection in the meta-analysis and Four Cards study[5].

Notwithstanding decreased *-q* and *-p* card selections, which may be a task-specific interaction between computerisation of the four card task, multiple presentation of this task, as well as MI/MD model uncertainty, the hypothesis tested in this first study (that multiple computerised task presentation would not alter card selection ordering

5 In thinking about "fit", results which are not significantly different to each other are "good" results. This analysis therefore shows that *p* and *q* selections were the same in both the meta–analysis and Four Cards study, which is good, but the *-q* and *-p* selections were different from the meta-analysis.

in comparison to standard selection task studies) is supported as cards are selected as predicted and produce the predicted p>q>-q>-p card preference ordering. However, whether observed decreases in -q and -p cards selections is unique to this Four Cards study and reflects a task specific interaction of factors which affects overall cognitive performance is considered further in the Singles study in Part II of this chapter. The main purpose of the Singles study is to control for single card presentation in preparation for the probability learning studies, but, as multiple selection tasks are also performed on a computer in the Singles study, it is possible to compare its results with the Four Cards study as if they were two conditions of the same study.

Part II- Single Card Study

Computerised Single Card Presentation
Multiple Selection Task Presentation
Original Selection Task Wording

Introduction

The theoretical motivations of the Single Card study are the same as those detailed in the Introduction to the Four Cards study, i.e. to investigate whether the decision rule to optimise expected information gain governs selection task behaviour, rather than the logic-based decision rule to falsify. However, and unlike the Four Cards study, the main purpose of the Single Card study is to control for single card presentation of the selection task, in preparation for the probability learning studies detailed in Section B where cards are presented one at a time on a computer screen.

The Single Card study therefore acts as a "control" for the probability learning studies and it is important that it produces the same card selection behaviour (i.e. the consistently observed $p>q>-q>-p$ card ordering) found in affirmative abstract versions of the selection task and the Four Cards study reported in Part I.

Method

Participants

There were 20 participants, seven males and 13 females, all of whom were undergraduate students at the University of Wales, Bangor aged between 18 and 50. No participants had prior knowledge of logical principles or the rationale for this study.

Design

This Single Card study was a one-factor within-participants design. As in the Four Cards study, the dependent variable was the **frequency of p, q, $-p$ and $-q$ card selections** and the independent variable or factor was **type of card** which had four levels for p, q, $-p$ and $-q$ cards.

It was hypothesised that there would be a difference in card selection behaviour depending on whether the card was a *p*, *q*, *-p* or *-q* and that card selection behaviour would not be affected by **multiple computerised (single card) selection task presentation**. In other words, performing 12 multiple selection tasks, where one card at a time was presented on a computer screen, would produce the same card selection behaviour (for example a *p*>*q*>*-q*>*-p* card preference ordering) as found in the Four Cards study and standard affirmative abstract versions of the selection task. Table 4.2.1.1 below illustrates the experimental design of this Single Card study.

TABLE 4.2.1.1: Packs/rules and *p, q -p* and *-q* cards displayed one card at a time on computer screen

PACK	PACK RULE	SINGLE CARD PRESENTATION RANDOMLY DISPLAYED IN CENTRE OF COMPUTER SCREEN			
		p	*q*	*-p*	*-q*
	LETTERS				
	***p* as vowel**				
1	if A then 2	A	2	K	7
2	if U then 8	U	8	A	6
3	if E then 5	E	5	A	2
	***p* as consonant**				
4	if J then 6	J	6	W	9
5	if X then 9	X	9	U	2
6	if D then 3	D	3	F	1
	NUMBERS				
	***p* as even number**				
7	if 4 then Q	4	Q	6	L
8	if 8 then E	8	E	4	C
9	if 8 then Z	8	Z	1	P
	***p* as odd number**				
10	if 1 then A	1	A	5	U
11	if 3 then B.	3	B	1	A
12	if 9 then G	9	G	7	W

For example and as in the Four Cards study, there were 12 different packs of cards. Each pack had a different rule as well as two different letter cards and two different number cards representing *p, q, -p* or *-q*. In addition there were three vowel *p* cards and three consonant *p* cards, and three even-numbered *p* cards and three odd-numbered *p* cards.

As in the Four Cards study, randomisation and counterbalancing for order effects, i.e. random selection of the multiple selection tasks, was programmed in PsyScope for each participant. In addition, each of the *p*, *q*, *-p* and *-q* cards in each of the twelve packs/trials was programmed to appear randomly one card at a time at the bottom centre of the computer screen.

For example, pack 4 in Table 4.2.1.1 above (where the *p* card in the rule was a consonant: "if there is a J on one side of a card, there is a 6 on the other side") may have been randomly selected for trial/selection task 1. The pack 4 cards may then have been displayed one card at a time on the computer screen in the following random sequence: 6 (*q*), J (*p*), 9 (-*q*) and W(-*p*). In other words, for one participant a *q* card with "6" on its face may have appeared as the first single card displayed on the screen, a *p* card with a "J" may have replaced the *q* card and appeared as the second single card displayed, a -*q* card with a "9" on its face may have replaced the *p* card and appeared as the third single card displayed, and the last singly displayed card may have been a -*p* card with a "W" on its face. Table 4.2.1.2 below details the 24 possible permutations for card display sequence, using pack 4 *p*, *q*, -*p* and -*q* cards as an example.

Table 4.2.1.2: 24 permutations of single card display sequence (1st, 2nd, 3rd or 4th) using pack 4 cards as an example

Card Permutations																								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Card																								
1st	6	6	6	6	6	6	J	J	J	J	J	J	W	W	W	W	W	W	9	9	9	9	9	9
2nd	J	J	W	W	9	9	6	6	W	W	9	9	J	J	6	6	9	9	6	6	J	J	W	W
3rd	W	9	J	9	W	J	W	9	9	6	W	6	9	6	J	9	J	6	J	W	W	6	J	6
4th	9	W	9	J	J	W	9	W	6	9	6	W	6	9	9	J	6	J	W	J	6	W	6	J

The 24 permutations for the *p*, *q*, -*p* and -*q* cards in each of the 12 selection tasks and/or packs were also calculated and randomised.

Apparatus

As in the Four Cards study, PsyScope, a graphic interface experimental design application, was used to program and run this experiment and record responses. All instructions and stimuli were presented on a monitor attached to a Macintosh LCIII computer. A Macintosh keyboard and mouse recorded responses.

Procedure

As in the Four Cards study, participants were tested individually and sat facing a Macintosh LCIII computer monitor. It was explained that instructions about the task and the task itself would be displayed on the computer screen. Below the computer and on the same desk was a Macintosh keyboard and mouse and it was explained that using the keyboard and mouse would record responses.

Before the experimenter left the research room, and as in the Four Cards study, the participant was handed a sheet of paper on which *Participant's Rights* as set out below (see Appendix 4.1 for precise format) were typed.

> Although you have agreed to participate in this study, you are at liberty to withdraw at any time. Any data which you produce will remain confidential and your anonymity is guaranteed. We are NOT interested in whether your responses are right or wrong. This is NOT a test of intelligence or ability, and you will not be timed. CLICK MOUSE ONCE FOR FURTHER DETAILS ABOUT THIS STUDY.

This statement was also on the computer screen. When Participant's Rights had been read and the mouse had been clicked to proceed with the experiment, the experimenter left the room.

The next screen instructions as set out below (see Appendix 4.2 for precise screen format), and as in the Four Cards study, gave *general instructions*:

> This study uses several packs of cards. All the cards in these packs have a LETTER on one side and a NUMBER on the other side. There are rules about what letters and numbers can go together. For example: "If a card has a 2 on one side then it has a T on the other side. CLICK MOUSE FOR MORE INSTRUCTIONS ABOUT 'YOUR TASK'...

When these general instructions had been read and assimilated, instructions were to click the mouse in order to obtain further instructions about the task.

The next screen as set out below (see Appendix 4.2.1 for precise screen format) gave Single Card study *task specific instructions* :

> ONE card will be dealt from a pack. Only one side of this card will be displayed on the screen. A rule will also be shown. Your task will be to decide if the card needs to be turned over in order to determine if a rule is true or false of the pack then in use. You will be prompted to press an appropriate key on the keyboard in order to record your decision. The task will continue until you have made decisions about cards in several packs. You will be prompted when you have reached

> the end. If you would like to review these instructions, PRESS THE "A" KEY. If you are clear about what to do CLICK THE MOUSE ONCE TO BEGIN."

As was the case in the Four Cards study, the next screen displayed the statement "*New Pack! New Rule...*" (see Appendix 4.3 for precise screen format) in order to make the participant aware that a new pack of cards with a new rule rather than the rule given as an example in the general instruction (see Appendix 4.2) was now being used.

The next screen of *selection task instructions* as set out below (see Appendix 4.2.2 for precise screen format) was automatically displayed after "*New Pack! New Rule...*" had been displayed on the screen for five seconds:

> Rule: If a card has a [*p*] on one side then it has a [*q*] on the other side. Must the card below be turned over in order to determine whether the above rule is true or false of the pack now in use? If it is necessary to turn the card below over, press the "Y" key. If it is not necessary to turn this card over, press the "N" key.
>
> [CARD].

For example, at the beginning of each trial (one trial or selection task comprising the display of four singly-displayed cards from the same pack) the rule presented at the top of the screen randomly changed and one pack card was randomly displayed in the bottom centre of the screen. The participant was instructed to respond to the card displayed on the screen by pressing either the "Y" key (representing a "Yes, it is necessary to turn the card over" response) or the "N" key (representing a "No, it is not necessary to turn the card over" response") on the keyboard[1].

When the participant had made either a "yes" or "no" keyboard response, this first pack card was then replaced at the centre of the screen by a second card from the same pack of cards. When the participant responded either "yes" or "no" to this second card, a third card selected from the same pack replaced it at the centre of the screen. When a response regarding this third card was made, the last and fourth card for that pack/trial was displayed at the centre of the screen. When a response was

---

1 In all studies in this thesis, only keys which were relevant to a particular pack of cards (or appropriate at a particular time) were active keys and able to record responses or progress the study. In other words, all non-relevant keys on the keyboard, including the mouse if this was not an appropriate response to make at a given time, were disabled.

made for this last card in the first trial by pressing either the "Y" or "N" key on the keyboard, the screen automatically displayed instructions advising that a new pack of cards (i.e. a new trial) with a new rule was now being used (see Appendix 4.3). Five seconds after this message was displayed on the screen, the next trial of the selection task (again comprising four single card presentations in one trial) automatically commenced.

The above screen instructions (change of pack and rule instructions before each new trial) continued for 12 selection tasks (48 single card presentations in total), at the end of which the *final screen instruction* as set out below (see Appendix 4.4 for precise screen format) were displayed as in the Four Cards study:

> You have now completed this task. If you would like to know more about this study, we shall be happy to answer any questions. Thank you for your time. CLICK THE MOUSE ONCE TO END THIS SESSION.

The Single Card study took each participant between five and ten minutes to complete.

See Appendix 4.2.3 for example of one participant's raw data and how responses were recorded in this single card presentation study.

Results

Regarding the hypothesis that there would be a difference within this Single Card study in *p*, *q*, *-p* and *-q* card selections depending on card type (*p*, *q* *-p* or *-q* cards), the mean cards selections for this study are summarised in Table 4.2.1 below[2].

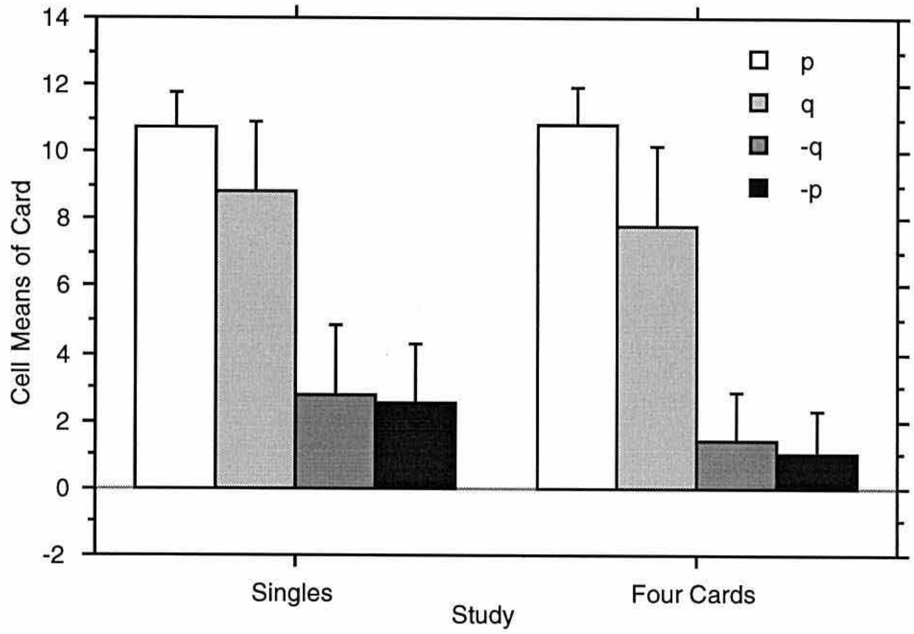
Table 4.2.1: mean *p*, *q*, *-p* and *-q* card selections in the Single Card study

	Count	Mean	Std. Dev.	Std. Error
p	20	10.700	2.319	.519
q	20	8.800	4.432	.991
-q	20	2.800	4.348	.972
-p	20	2.550	3.734	.835

2 As in the Four Cards study and as each of the 20 participants in this study performed 12 selection tasks, each participant's mean selections for *p*, *q*, *-p* and *-q* cards were calculated and form the basis of this analysis.

A one-way ANOVA was performed and revealed a significant effect between card selections depending on card type (F (3, 19) = 28.113, MSE = 345.946 p = .0001)[3]. Pairwise comparisons were carried out in order to investigate which card selections were significantly different from each other in the Single Card study. This analysis revealed that at the .05 level there were nearly significant effects between q and p (p = .0922); $-p$ and q, $-p$ and p, $-q$ and q, and $-q$ and p were all significantly different from each other (p = .0001 for these four pairwise comparisons); but there was no significant effect between $-p$ and $-q$ card selections (p = .8225). Figure 4.2.1 below illustrates the above card selections as well as the p>q>$-q$>$-p$ card preference ordering produced in the Single Card and Four Cards studies.

Figure 4.2.1: card selections and p>q>$-q$>$-p$ ordering in Singles and Four Cards studies



For information purposes only, the total frequencies of card selections in the Single Card and Four Card studies are summarised in table 4.2.2 below.

Table 4.2.2: p, q $-q$ and $-p$ card frequencies in Singles and Four Card Studies

Study	*p*	*q*	*-q*	*-p*
Singles	214	176	56	51
Four Cards	217	156	28	21

A linear contrast was performed on the Singles study card p>q>$-q$>$-p$ preference ordering which was significant (F (1, 19) = 75.349, MSE = 927.202 p = .0001).

3 ANOVA summaries are in Appendix 4.8.

In order to see if there were significant differences in card selections depending on the number of cards presented on the computer screen at any one time, i.e. depending on study (Four Cards or Single Card studies), a 4 x 2 ANOVA was performed. There was no significant interaction between card selections depending on study (F (3, 38) = .479, MSE = 5.723 p = .6973). Pairwise comparisons were performed on each card to see which, if any cards were significantly different from each, but no significant effects were found at this level of analysis (p selections = 8368, q selection: p = .5079; $-q$ selections: p = .2484; and $-p$ selections: p = .1499). In other words, there was no statistical difference in the selection of the p card in the Single Card study and the selection of the p card in the Four Cards study, and this was the case for the other three cards.

The main results of this Single Card study have now been reported.

Before discussing their implications, and as was the case in the Four Cards study, analysis was carried out to see if there was a difference in card selections depending on whether the ***p* card in the selection task rule was a consonant, vowel, odd number or even number**. The means of all card selections depending on whether p cards were consonants, vowels, odd numbers or even numbers are summarised in Means Table 4.2.3 below:

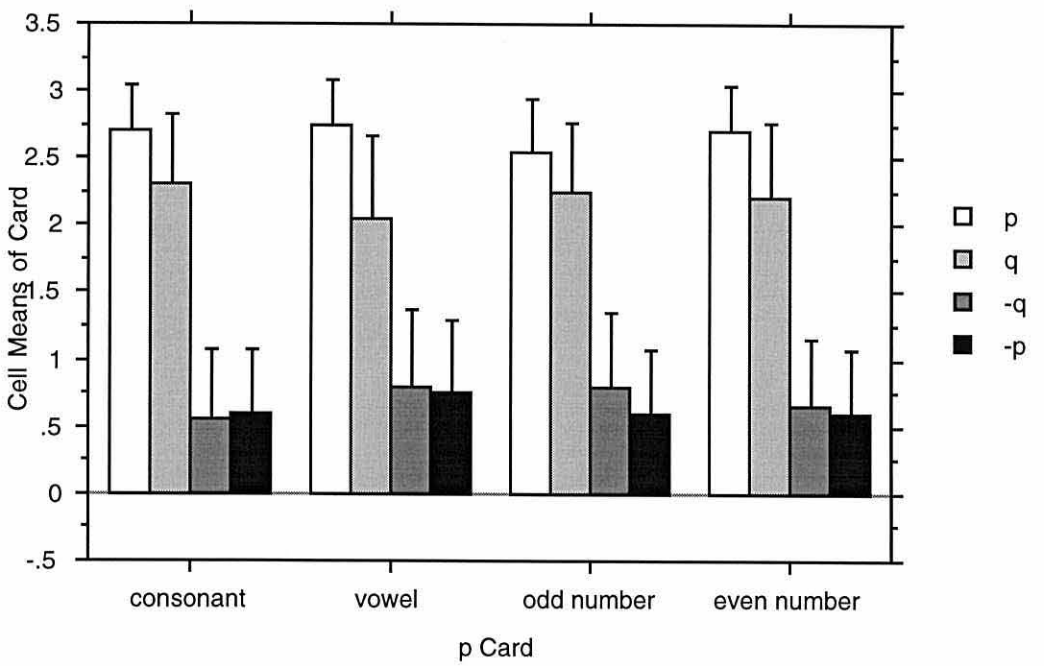
Means Table 4.2.3: p, q, $-p$ and $-q$ card selections in the Singles study depending on whether p cards were consonants, vowels, odd numbers or even numbers

	Count	Mean	Std. Dev.	Std. Error
p, consonant	20	2.700	.733	.164
p, vowel	20	2.750	.716	.160
p, odd number	20	2.550	.826	.185
p, even number	20	2.700	.733	.164
q, consonant	20	2.300	1.129	.252
q, vowel	20	2.050	1.317	.294
q, odd number	20	2.250	1.118	.250
q, even number	20	2.200	1.196	.268
-q, consonant	20	.550	1.099	.246
-q, vowel	20	.800	1.240	.277
-q, odd number	20	.800	1.196	.268
-q, even number	20	.650	1.089	.244
-p, consonant	20	.600	.995	.222
-p, vowel	20	.750	1.164	.260
-p, odd number	20	.600	.995	.222
-p, even number	20	.600	.995	.222

A 4 x 4 within-participants ANOVA was performed which showed that there was no difference in *overall* card selections based on *p* card type (i.e. if *p* card was a consonant, vowels, odd or even number) ($F(3, 76) = .030$, $MSE = .045$, $p = .9928$). Neither was there any significant *interaction* between *p* card type and *p*, *q* -*p* and -*q* card selections ($F(9, 76) = .256$, $MSE = .250$, $p = .9852$).

Figure 4.2.2 below illustrates the above card selections as well as card preference ordering depending on *p* card type more clearly. It can be seen that when *p* cards were vowels, odd numbers or even numbers the predicted *p*>*q*>-*q*>-*p* ordering was produced, but when *p* cards were consonants the card preference ordering produced was *p*>*q*>-*p*>-*q* (-*p* mean = .600 and -*q* mean = .550)[4].

Figure 4.2.2: card ordering of *p*>*q*>-*q*>-*p* cards in Singles study



A one-way ANOVA was performed on each of the four cards in the Single Card study and pairwise comparisons were carried to see which, if any, card selections were significantly different from each other depending on *p* card type: there were no significant effects in the Single Card study at this level of analysis. In other words, there was no significant differences in the selection frequency of *p* when *p* cards were consonants, vowels, odd or even numbers. There were also no significant differences

4 A *p*>*q*>-*p*>-*q* ordering was produced in the Four Cards study when *p* cards were odd numbers, the -*p* and -*q* were equally selected (means for both were .350).

in the selection frequency of *q*, -*q* and -*p* cards when *p* cards were consonants, vowels, odd or even numbers.

Further analysis was carried out to investigate whether there were any significant effects between card presentation (single or four cards), *p* card type (*p* card as either a consonant, vowel, odd or even number) and the selection of *p*, *q*, -*p* and -*q* cards. A 4 x 4 x 2 mixed ANOVA was performed and no significant interaction was found (F (9, 152) = .232, MSE = .211, p = .9898). Pairwise comparisons were carried on each card to see which, if any, card selections were significantly different from each other depending on card presentation (i.e. Single or Four cards studies) and *p* card type. There were no significant effects at this level of analysis.

Discussion

Analysis of the Single Card study data shows that there is a within-participants difference in the selection of the four cards and that the predicted card selection ordering (p>q>-q>-p) was produced. Furthermore, comparative analysis of the Single Card and Four Cards studies showed that there is no significant difference across studies in card selection ordering depending on card presentation, as both the Single Card and Four Cards studies produced the predicted ordering p>q>-q>-p. Given these results, it is concluded that computerised single card presentation, as well as multiple selection task presentations, have no significant effect on card selection behaviour. These factors were important to control for as computerised multiple task presentation with single card presentation is the design of the probability studies reported in Section B of this chapter.

It is interesting to note that selection of -*q* cards (mean selections = 2.800) in the Single Card study has increased in comparison to the Four Cards study (mean selections 1.400), and so has the selection of -*p* cards, 2.550 in the Singles study and 1.050 in the Four Cards study. These cards selections were lower in the Four Cards study in comparison to the meta-analysis studies. The Single Card study may therefore have produced a context in which cognitive resources were less overloaded and so there was less general and task uncertainty, as may have been the case in the Four Cards study as a results of a possible interaction between multiple and computerised selection tasks as well as four card presentation.

Regarding whether the type of p card presented makes a difference to card selections, Figure 4.2.2 in the results section of this Singles study shows that comparative increases and decreases in card selections when p is a vowel and p is a consonant, although not statistically significant, are consistent with optimal data selection predictions. For example, when the P(p) is "high" (i.e. when p is a vowel), $-q$ card selections should increase, which they did. The P(p) is comparatively lower when the p card is a consonant and in this comparatively "low" probabilistic context $-q$ card selections should decrease, which they did when the p card was a consonant.

Having controlled for computerised single card presentation (the main purpose of this Single Card study), as well as multiple selection task presentation, and having briefly discussed selection task results in terms of an optimality approach to reasoning and cognition, the next studies investigate whether asking participants to judge the informativeness of cards changes selection behaviour.

Part III - Pilot Ratings

Four Card Presentation
Card Informativeness Ratings Task

Introduction

The Pilot Ratings study requires participants to judge the informativeness of *p, q, -p* and *-q* cards. This task is very different to the selection tasks performed in the Four Cards and Single Card studies reported in Parts I and II of this chapter, where instructions were to select the cards necessary to turn over in order to evaluate whether a rule is true or false. Consequently, any deviation from standard performance using the informativeness ratings would constitute strong falsifying evidence against optimal data selection. However, the results of this Pilot Ratings study are not expected to differ from the Four Cards and Single Card studies because the O&C (1994) model of optimal data selection assumes that there is a relationship between card selections and card informativeness: people choose cards which they perceive as being the most informative card selections in a given context.

As detailed in chapter 3 and as outlined in the experimental rationale which prefaces this chapter 4, in affirmative abstract selection tasks, cards which reduce uncertainty the most about whether a model of independence ("MI") or a model of independence ("MD") holds[1] are predicted to be *p* and *q*, and these two cards are consequently predicted to be rated more informative than *-p* and *-q*. However, *p* and *q* are optimal or the most informative cards only if the *p* and *q* rarity assumption is not violated, i.e. if the P(*p*) and P(*q*) remain low in comparison to the P(*-p*) and P(*-q*). In other words, it is because *p* and *q* are rare that they are most informative, and if probabilistic context changes, card informativeness ordering is predicted to change.

As context has not been manipulated in this Pilot Study (or in the other studies detailed in Section A of this chapter), card selection ordering is expected to remain the same as in standard affirmative, abstract versions of the selection task, and as replicated in the Four Cards and Single Card studies. Specifically, the consistently

---

1 " "MI" is a model of independence between *p* and *q* (i.e. there is no relationship between *p* and *q*) and "MD" is a model of dependency between *p* and q (i.e. if *p* holds then *q* holds).

observed and context-dependent *p>q>-q>-p* card selection ordering is expected to be replicated in the Pilot Ratings study because card selections are related to perceived card informativeness.

The purpose of the Pilot Ratings Study is therefore to produce data to test the hypothesis that the decision rule governing card selection behaviour is that of optimal data selection, rather than the logic-based decision rule of falsification. To this end, the Pilot Ratings study uses a ratings scale in order to investigate the way in which people judge the informativeness of cards. This ratings methodology produces ordinal data, as the informativeness of cards is rated on a scale of 0 to 8 (where 0 indicates that the card is not useful and an 8 rating indicates that a card is extremely useful when testing a rule). A *direct rating* of expected card informativeness is achieved using this ratings scale methodology. The Method of this Pilot Ratings study is detailed below.

Method

Participants

There were 33 participants aged between 18 and 60 in this Pilot Ratings study. All participants were first year undergraduate students from the Department of Sociology and Social Policy of the University of Wales, Bangor. No participants had prior knowledge of logical principles or the rationale for this study.

Design

This study was a one-factor within-participants design. The dependent variable, unlike the Four Cards and Singles studies, was ***p*, *q*, *-p* and *-q* card informativeness ratings.** The independent variable was the same as previous studies: **type of card** (i.e. *p*, *q*, *-p* or *-q* card). The experimental hypothesis was that there would be a difference in informativeness ratings depending on card type (i.e. *p*, *q*, *-p* or *-q*). It was predicted that a *p>q>-q>-p* card preference ordering would be produced as was the case in the Four Card and Single Card studies, and standard affirmative abstract versions of the selection task.

Table 4.3.1 illustrates the way in which the ordering of cards used in this study, *p* (U), *q* (8), *-p* (X) and *-q* (9) (which were presented four cards at a time on an A4 sheet of paper), was randomised and counterbalanced in order to control for order effects.

Table 4.3.1: 24 card ordering permutations of the four cards, *p* (U), *q* (8), -*p* (X) and -*q* (9) used in the Pilot Ratings study

Permutations of cards U 8 X 9	1st card position	2nd card position	3rd card position	4th card position	Participants (3 Withdrew)
1.	8	9	X	U	5
2.	8	9	U	X	4
3.	8	U	9	X	0[W]
4.	8	U	X	9	0 [W], 21
5.	8	X	U	9	7
6.	8	X	9	U	6
7.	9	X	U	8	24
8.	9	X	8	U	22
9.	9	U	8	X	8
10.	9	U	X	8	1
11.	9	8	U	X	3
12.	9	8	X	U	2
13.	U	X	8	9	9, 23
14.	U	8	9	X	10, 27
15.	U	X	9	8	11, 25
16.	U	9	8	X	13, 26
17.	U	9	X	8	12, 28
18.	U	8	X	9	15, 29
19.	X	U	8	9	14, 30
20.	X	U	9	8	16,
21.	X	8	U	9	18 , 20
22.	X	8	9	U	19
23.	X	9	8	U	0[W]
24.	X	9	U	8	17

Materials

An A4 sheet of paper on which task instructions were typed.

Procedure

Participants carried out this pilot study as a group. They were advised that the study would take no more than four minutes to complete and that they could withdraw from the study at any time. Consenting students were handed a task sheet as set out below (see Appendix 4.3.1 for precise format):

> Good afternoon! This study involves four cards, depicted below.
>
> [CARD] [CARD] [CARD] [CARD]
>
> Each card has a letter on one side and a number on the other side.
>
> Your task is to rate how useful or informative each card would be if it were turned over to test a rule.

W means that participants withdrew and did not complete task.

The rule is: if there is a vowel on one side of the card then there is an even number on the other side.

Please circle the informativeness rating you choose for each card. For example: an 8 rating means the card is extremely useful; a 6 rating means the card is very useful; a 4 rating means the card is quite useful; a 2 rating means the card is not so useful; a 0 rating means the card is no use at all.

Thank you for participating.

The above task sheet was placed before each participant face-down and the experimenter advised when the task sheets could be turned over in order to begin. There was no set time to complete the task. When it was evident that all participants had completed the study, all task sheets were collected. The experimenter thanked everyone for their time and for agreeing to participate.

This Pilot Ratings study took 10-12 minutes to complete from the time the experimenter entered the lecture room to the time the experimenter left the lecture room.

Results

Regarding the hypothesis that there would be a difference in this Pilot Ratings study in ***p*, *q*, -*p* and -*q* card informativeness ratings**, the mean informativeness ratings of all *p*, *q*, -*p* and -*q* are summarised in Means Table 4.3.1 below.

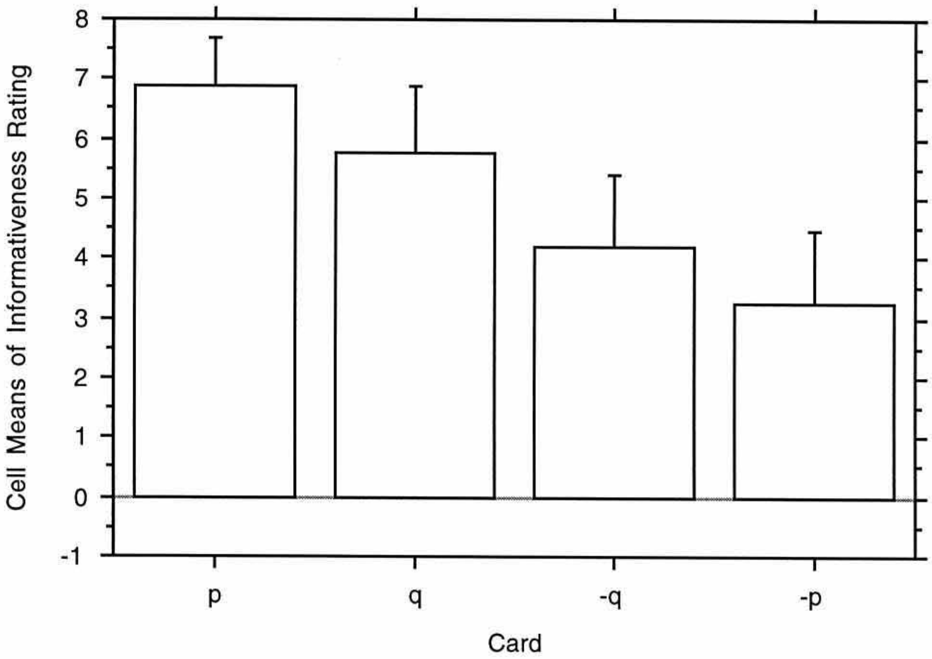
Means Table 4.3.1: *p*, *q*, -*p* and -*q* card informativeness ratings in Pilot Ratings study

	Count	Mean	Std. Dev.	Std. Error
p	30	6.900	2.139	.391
q	30	5.767	2.967	.542
-q	30	4.200	3.263	.596
-p	30	3.267	3.205	.585

A one-way ANOVA was performed and a significant difference at the .05 significant level was found in informativeness ratings depending on card type (i.e. whether a card was a *p*, *q*, *-p* or *-q* card) ($F(3, 29) = 12.245$, $MSE = 78.378$, p = .0001)[2].

Pairwise comparisons were carried out in order to investigate which card informativeness ratings were significantly different from each other. This analysis revealed that at the .05 level there were nearly significant effects between *p* and *q* (p = .0863); *p* and *-q*, *p* and *-p* were significantly different from each other (p = .0001 for these two pairwise comparisons) and so were *q* and *-p* (p = .0002) and *q* and *-q* (p = .0186); but there was no significant effect between *-q* and *-p* card selections (p = .1566). Figure 4.3.1 below illustrates the above card informativeness ratings and shows more clearly that informativeness preference ordering for the *p*, *q*, *-p* and *-q* cards was the predicted *p*>*q*>*-q*>*-p*.

Figure 4.3.1 : informativeness rating preference ordering of *p*, *q*, *-p* and *-q* card in the Pilot Ratings study



A linear contrast was performed on the above informativeness ordering of *p*, *q*, *-q* and *-p* cards, which was significant ($F(1, 29) = 36.422$, $MSE = 233.127$, $p = .0001$).

---

2 ANOVA summaries are in Appendix 4.8.

Discussion

The purpose of this study was to ascertain whether perceived informativeness of cards related to cards which participants in the Four Cards and Singles studies selected as necessary cards to turn over in order to test whether a given rule was true or false. Notwithstanding the way in which this informativeness ratings task differs from standard version of the selection task, this Pilot Study replicates the preferential ordering produced in affirmative abstract versions of the selection task, as well as the Four Cards and Single Card studies. Therefore, it may be concluded that there is a relationship between card selections and their perceived informativeness.

More specifically, the informativeness ratings task was quite different to the selection task, however, the consistently observed preferential ordering $p>q>-q>-q$ was produced in this pilot card informativeness ratings study. In terms of an optimality approach to cognition, this preferential ordering simply reflects that the p card was perceived by participants to be more useful or informative than the q card (an informativeness rating of 6.900 in comparison to 5.767), the q card was rated more informative than the $-q$ card (4.200), and the $-q$ card was rated more informativeness than the $-p$ card (3.267).

Given this $p>q>-q>-q$ ordering, it can be inferred that p and q are dominant selections because they are rare events in comparison to $-p$ and $-q$. In other words, the P(p) and the P(q) is perceived to be low in this ratings study. In this specific probabilistic context it is assumed that there will be uncertainty about whether the model is MI (there is independence between p and q) or MD (if p holds then q holds). Therefore it should be expected that p and q will be the most informative cards or optimal data as they will be perceived as reducing uncertainty the most about whether MI or MD holds.

Having replicated the $p>q>-q>-p$ card ordering behaviour in this card informativeness ratings study, and having inferred that this preferential ordering reflects that there is a relationship between card selections and card informativeness, the next study to be reported is the Single Ratings study. In this second card informativeness study, participants were asked to rate the informativeness of cards but the task was presented on a computer screen, one card at a time was displayed during the selection task, and there were multiple task presentations as in the Four Cards and Single Card study.

Part IV - Single Ratings Study

Computerised Single Card Presentation
Multiple Informativeness Ratings Tasks

Introduction

The Single Ratings study was designed to provide further data to test the hypothesis that the decision rule governing card selection behaviour is that of optimal data selection, rather than logic-based decision rules such as falsification. To this end, and as was the case in the Pilot Ratings study, direct rating of expected card informativeness is the task performed in the Single Ratings study (and Single Ratings Replication study). This ratings methodology produces ordinal data but, unlike the Pilot Ratings study, the informativeness of cards in this study is rated on a scale of 1 to 5 (where 1 indicates that the card is uninformative and 5 indicates that a card is informative when testing a rule).

The mode of card presentation in this card informativeness study is the same as detailed in Part II where the Single Cards study was reported: i.e. one card at a time is presented on a computer screen. As well as providing more evidence about whether multiple computerised single card presentation changes behaviour, this mode of presentation also permits an absolute rather than a comparative judgement about the informativeness of a card to be made. (Comparative informativeness judgements were made in the Pilot Ratings study as mode of presentation was four cards at a time, and in the Binary study reported in Part V of this chapter where two cards at a time are presented on a computer screen and pair informativeness preference judgements are made.)

As was the case in the Four Cards, Single Card and Pilot Ratings studies, the results of this Single Ratings study are not expected to differ from standard affirmative, abstract versions of the selection task because the O&C (1994) model of optimal data selection assumes that there is a relationship between card selections and card informativeness. More specifically, cards which reduce uncertainty the most about whether a model of independence ("MI") or a model of independence ("MD") holds[1]

1 "MI" is a model of independence between p and q and "MD" is a model of dependency between p and q (i.e. if p holds then q holds).

are predicted by optimal data selection to be *p* and *q*. These two cards are consequently predicted to be rated more informative than *-p* and *-q*. This prediction is conditional upon the *p* and *q* rarity assumption not being violated, i.e. if the P(*p*) and P(*q*) remain low in comparison to the P(*-p*) and P(*-q*).

As context has not been manipulated in this Single Ratings study (or any studies reported in Section A of this chapter), the consistently observed and context-dependent *p*>*q*>*-q*>*-p* card selection ordering produced in affirmative, abstract versions of the selection task, as well as in the Four Cards, Single Card and Pilot Ratings studies, is expected to be reproduced in this Single Ratings studies because card selections are related to perceived card informativeness.

The Method section of this Single Ratings study is detailed below.

Method

Participants

There were 20 participants, nine males and 11 females aged between 18 and 50 in this Single Ratings study. All participants were undergraduate students at the University of Wales, Bangor. No participants had prior knowledge of logical principles or the rationale for this study.

Design

This study was a one-factor within-participants design. As in the Pilot Ratings study, the dependent variable was the **informativeness rating of *p*, *q*, *-p* and *-q* card**. The independent variable or factor was **type of card** which had four levels for *p*, *q*, *-p* and *-q* cards.

It was hypothesised that there would be a difference in card informativeness ratings depending on whether the card was a *p*, *q*, *-p* or *-q* , and that card informativeness ratings would not be affected by multiple computerised (single card) task presentation. In other words, performing 12 multiple ratings tasks, where one card at a time was presented on a computer screen, would produce the same card ordering behaviour (for example *p*>*q*>*-q*>*-p*) as standard affirmative abstract versions of the selection task, and the Four Cards, Single Card and Pilot Ratings studies.

Table 4.4.1.1 below illustrates the experimental design of this Single Ratings study.

TABLE 4.4.1.1: Packs/rules and *p, q -p* and *-q* cards displayed one card at a time on computer screen in Single Ratings study

PACK	PACK RULE	SINGLE CARD PRESENTATION RANDOMLY DISPLAYED IN CENTRE OF COMPUTER SCREEN			
		p	*q*	*-p*	*-q*
	LETTERS				
	***p* as vowel**				
1	if A then 2	A	2	K	7
2	if U then 8	U	8	A	6
3	if E then 5	E	5	A	2
	***p* as consonant**				
4	if J then 6	J	6	W	9
5	if X then 9	X	9	U	2
6	if D then 3	D	3	F	1
	NUMBERS				
	***p* as even number**				
7	if 4 then Q	4	Q	6	L
8	if 8 then E	8	E	4	C
9	if 8 then Z	8	Z	1	P
	***p* as odd number**				
10	if 1 then A	1	A	5	U
11	if 3 then B.	3	B	1	A
12	if 9 then G	9	G	7	W

For example and as in the Four Cards and Single Card studies reported in Parts I and II of this chapter, respectively, there were 12 different packs of cards. Each pack had a different rule as well as two different letter cards and two different number cards representing *p, q, -p* or *-q*. In addition there were three vowel *p* cards and three consonant *p* cards, and three even-numbered *p* cards and three odd-numbered *p* cards.

Randomisation and counterbalancing for order effects, i.e. random selection of the multiple selection tasks, was programmed in PsyScope for each participant. In addition, each of the *p*, *q*, *-p* and *-q* cards in each of the twelve packs/trials was programmed to appear randomly one card at a time at the bottom centre of the computer screen. For example, Pack 5 in Table 4.4.1.1 above where the *p* card in its rule was a consonant: "if there is an X on one side of a card, then there is a 2 on the other side" may have been randomly selected as the first trial for one participant. Pack 5 cards would then have been randomly displayed one card at a time in the centre of the screen as fully detailed in the Single Card study (Part II of this chapter).

As in the Single Card study, there were 24 possible permutations for card display sequence, and these were calculated for each of the 12 selection tasks and/or packs and randomised, as detailed in the Single Card study.

Apparatus

PsyScope, a graphic interface experimental design application, was used to program and run this experiment and record responses. All instructions and stimuli were presented on a monitor attached to a Macintosh LCIII computer. A Macintosh keyboard and mouse recorded participants' responses.

Procedure

As in the Four Cards and Single Card studies, participants were tested individually and sat facing a Macintosh LCIII computer monitor. It was explained that instructions about the task and the task itself would be displayed on the computer screen. Below the computer and on the same desk was a Macintosh keyboard and mouse and it was explained that using the keyboard and mouse would record responses.

Before the experimenter left the research room, the participant was handed a sheet of paper on which *Participant's Rights* as set out below (see Appendix 4.1 for precise format) were typed.

> Although you have agreed to participate in this study, you are at liberty to withdraw at any time. Any data which you produce will remain confidential and your anonymity is guaranteed. We are NOT interested in whether your responses are right or wrong. This is NOT a test of intelligence or ability, and you will not be timed. CLICK MOUSE ONCE FOR FURTHER DETAILS ABOUT THIS STUDY.

This statement was also on the computer screen. When Participant's Rights had been read and the mouse had been clicked to proceed with the experiment, the experimenter left the room.

As was the case in the Four Cards and Single Card studies, the next screen instructions as set out below (see Appendix 4.2 for precise screen format) gave *general instructions.*

> This study uses several packs of cards. All the cards in these packs have a LETTER on one side and a NUMBER on the other side. There are rules about what letters and numbers can go together. For example: "If a card has a 2 on one side then it has a T on the other

side. CLICK MOUSE FOR MORE INSTRUCTIONS ABOUT 'YOUR TASK'...

When these general instructions had been read and assimilated, instructions were to click the mouse in order to obtain further instructions about the task.

The next screen as set out below (see Appendix 4.4.1 for precise screen format) gave Single Ratings study *task specific instructions* :

"ONE card at a time will be been dealt from one of several packs used in this study. Only one side of a card will be displayed on the screen. A rule will also be shown. Your task will be to rate (on a scale from 1 to 5) how much information a card, if turned over, provides about whether a rule is true or false of the pack then being used. You will be prompted to press an appropriate key on the keyboard in order to record your rating of each card. When you have assessed the informativeness of a small sample of cards in several packs, the task will end. If you would like to review these instructions, PRESS THE "A" KEY. If you are clear about what to do CLICK THE MOUSE ONCE TO BEGIN.

As was the case in the Four Cards and Single Card study, the next screen displayed the statement "*New Pack! New Rule...*" (see Appendix 4.3 for precise screen format) in order to make the participant aware that a new pack of cards with a new rule rather than the rule given as an example in the general instruction (see Appendix 4.2) was now being used.

The next screen of *selection task instructions* as set out below (see Appendix 4.4.2 for precise screen format) was automatically displayed after "*New Pack! New Rule...*" had been displayed on the screen for five seconds:

> Rule: If a card has a [*p*] on one side then it has a [*q*] on the other side. Please rate how much information the card below, if turned over, provides about whether the above rule is true or false of the pack now in use. Select either "1", "2', "3", "4" or "5" where:
>
> 1= card is USELESS (provides NO INFORMATION about truth/falsity of rule.)
> 2 = card is INADEQUATE (provides LITTLE INFORMATION...)
> 3 = card is HELPFUL(provides SOME INFORMATION ...)
> 4 = card s IMPORTANT(provides LOTS OF INFORMATION..)
> 5 = card is VITAL(provides MOST INFORMATION ...)
>
> [CARD]

For example, at the beginning of each trial, the rule presented at the top of the screen randomly changed and one pack card was randomly displayed in the centre of the screen. Instructions were to respond by rating the informativeness of the card displayed on the screen by pressing either the "1" key, the "2" key, the "3" key, the "4" key or the "5" key on the keyboard[2]. When the participant had rated the informativeness of the first card, this first card was then replaced at the centre of the screen by a second card from the same pack of cards. When the participant rated the informativeness of this second card, a third card selected from the same pack replaced it at the centre of the screen. When the informativeness rating for this third card was made, the last and fourth card for that pack/trial was displayed at the centre of the screen. When the informativeness rating response for this last card in the first trial was made, the screen automatically displayed instructions advising that a new pack of cards (i.e. a new trial) with a new rule was now being used (see Appendix 4.3). Five seconds after this message was displayed on the screen, the next trial of the selection task (again comprising four single card presentations in one trial) automatically commenced.

The above screen instructions (change of pack and rule instructions before each new trial) continued for 12 informativeness ratings tasks (48 single card presentations in total), at the end of which the *final screen instruction* as set out below (see Appendix 4.4 for precise screen format) were displayed as in the Four Cards and Single Card studies :

> You have now completed this task. If you would like to know more about this study, we shall be happy to answer any questions. Thank

2 In all studies in this thesis, only keys which were relevant to a particular pack of cards (or appropriate at a particular time) were active keys and able to record responses or progress the study. In other words, all non-relevant keys on the keyboard, including the mouse if this was not an appropriate response to make at a given time, were disabled.

you for your time. CLICK THE MOUSE ONCE TO END THIS SESSION.

See Appendix 4.4.3 for example of one participant's raw data and how responses were recorded in this Single Ratings study.

This study took each participant between eight and 12 minutes to complete.

Results

Regarding the hypothesis that there would be a difference within this Single Ratings study in ***p*, *q*, -*p* and -*q* card ratings**, the mean informativeness ratings of *p*, *q*, -*p* and -*q* cards are summarised in the Means Table 4.4.1 below[3].

Means Table 4.4.1: *p*, *q*, -*p* and -*q* card informativeness ratings in the Single Ratings study

	Count	Mean	Std. Dev.	Std. Error
p	20	4.539	.546	.122
q	20	4.113	.591	.132
-q	20	1.842	.581	.130
-p	20	1.640	.664	.148

A one-way ANOVA was performed and there was a significant effect in card informativeness ratings depending on card type (F (3, 19) = 1.71E2, MSE = 45.298 p = .0001)[4].

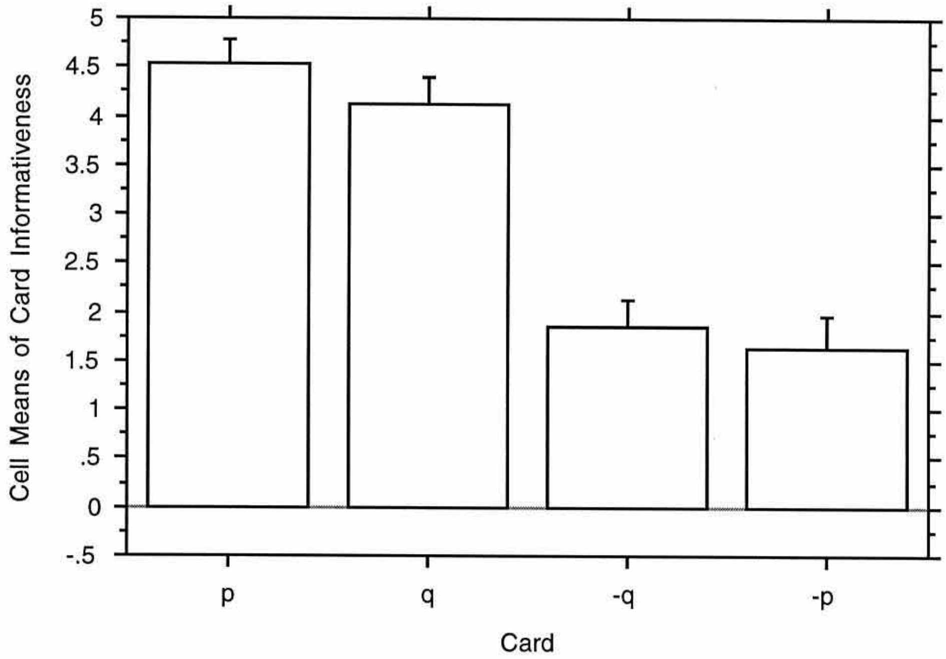
Pairwise comparisons were carried out in order to investigate which card informativeness ratings were significantly different from each other in the Single Ratings study. This analysis showed that at the .05 level there were significant effects between *q* and *p* (p = .0113); -*p* and *q*, -*p* and *p*, -*q* and *q*, and -*q* and *p* were all significantly different from each other (p = .0001 for these four pairwise comparisons); but there was no significant effect between -*p* and-*q* card informativeness ratings (p = .2220).

3 As in the Four Cards and Single Card studies, and as each of the 20 participants in this Single Ratings study performed 12 tasks, each participant's mean informativeness ratings for *p*, *q*, -*p* and -*q* cards were calculated and form the basis of this analysis.

4 All ANOVA summaries are in Appendix 4.8.

Figure 4.4.1 below illustrates more clearly the above card informativeness ratings as well as the predicted *p>q>-q>-p* card informativeness ordering produced in this Single Ratings study.

Figure 4.4.1: informativeness ratings and ordering for *p*, *q*, *-p* and *-q* cards in the Single Ratings study



A linear contrast was performed on the Single Ratings study card preference ordering which was significant (F (1, 19) = 453.285, MSE = 120.330, p = .0001).

The main results of this Single Ratings study have now been reported.

Before discussing these main results and as was the case in the Four Cards and Single Card studies, analysis was carried out to see if there was a difference in card selections, i.e. informativeness ratings, depending on whether the ***p* card in the selection task rule was a consonant, vowel, odd number or even number**. The means of all card ratings depending on whether *p* cards were consonants, vowels, odd numbers or even numbers are summarised in Means Table 4.4.2 below:

Means Table 4.4.2: *p*, *q*, *-p* and *-q* card informativeness ratings depending on whether *p* cards were consonants, vowels, odd numbers or even numbers

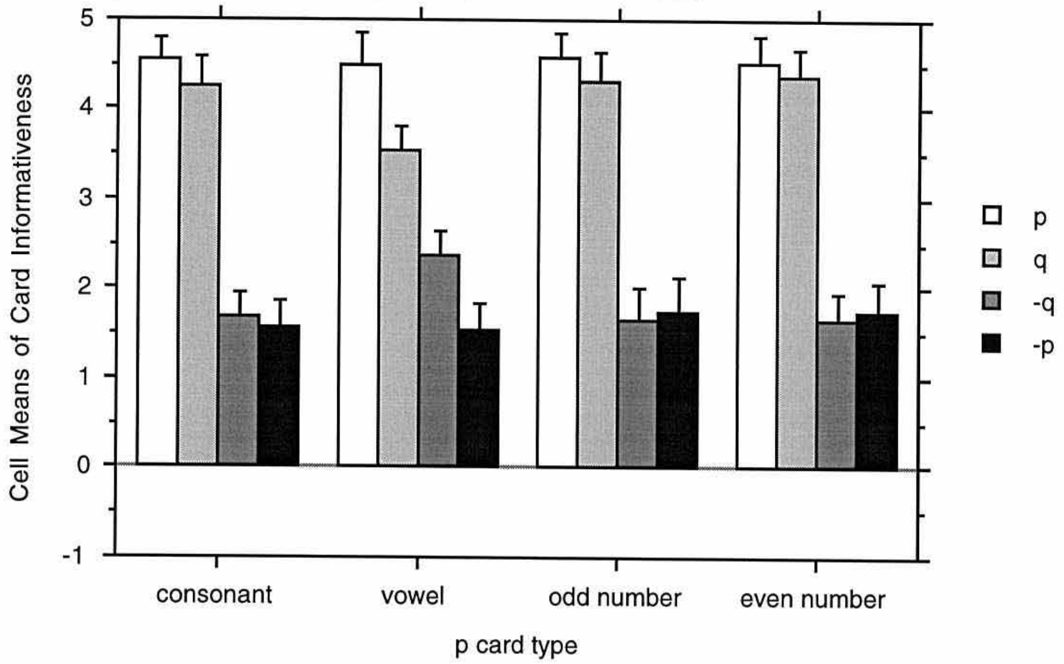
	Count	Mean	Std. Dev.	Std. Error
p, consonant	20	4.550	.499	.112
p, vowel	20	4.500	.737	.165
p, odd number	20	4.583	.601	.134
p, even number	20	4.533	.606	.136
q, consonant	20	4.250	.732	.164
q, vowel	20	3.533	.596	.133
q, odd number	20	4.300	.700	.157
q, even number	20	4.367	.648	.145
-q, consonant	20	1.683	.607	.136
-q, vowel	20	2.383	.554	.124
-q, odd number	20	1.650	.791	.177
-q, even number	20	1.650	.662	.148
-p, consonant	20	1.567	.650	.145
-p, vowel	20	1.533	.643	.144
-p, odd number	20	1.750	.830	.186
-p, even number	20	1.733	.697	.156

A 4 x 4 within-participants ANOVA was performed which showed that there was no difference in *overall* card ratings based on *p* card type (i.e. if *p* cards were consonants, vowels, odd or even numbers) ($F(3, 76) = .189$, $MSE = .142$, $p = .9038$). However, there was a significant *interaction* between *p* card type and *p*, *q* *-p* and *-q* card ratings ($F\ (9, 76) = 5.686$ $MSE = 1.924$, $p = .0001$).

Pairwise comparisons were carried out on each card to see which, if any, card informativeness ratings were significantly different from each other depending on *p* card type. As far as the *-q* card was concerned there were significantly different informativeness ratings between consonants and vowels (p = .0012), vowels and odd numbers (p = .0007) and vowels and even numbers (p = .0007). As far as the *q* card was concerned there were significantly different informativeness ratings between consonants and vowels (.0012), vowels and odd numbers (p = .0005) and vowels and even numbers (p = .0002). There were no significant effects for the *p* and *-p* cards.

Figure 4.4.2 below illustrates the above relative increases and decreases in card informativeness ratings, as well as the card informativeness ordering produced depending on *p* card type. For example, it can be seen that when *p* cards were vowels and consonants the predicted *p*>*q*>*-q*>*-p* informativeness ordering was produced, but when *p* cards were odd or even numbers the card informativeness ordering produced *p>q>-p>-q*.

Figure 4.4.2: informativeness ratings and ordering for *p*, *q*, *-p* and *-q* cards in the Single Ratings study when *p* cards were consonants, vowels, odd or even numbers



Discussion

In the Pilot Ratings study participants were asked to rate the informativeness of cards on a ratings scale of 0 to 8. In the Single Ratings study participants were instructed to rate card informativeness on a scale from 1 to 5. Notwithstanding the way in which the ratings tasks differ from standard affirmative abstract versions of the selection task, both card informativeness ratings studies produce the consistently found *p>q>-q>-p* ordering, but the Single Ratings study has produced results which reflect a marked contrast in the informativeness ratings of *p* and *q* card in comparison with *-q* and *-p* cards which were rated as being the least informative cards[5]. The Single Ratings study results therefore further substantiate the optimal data selection assumption that, if the probability of *p* and *q* are low, *-q* and *-p* will be rated as uninformative cards.

When Single Ratings data were analysed to see if there was a difference in card informativeness ratings depending on whether *p* cards were consonants, vowels, odd or even numbers, the O&C (1994) model of optimal data selection was again

5 Compare the Pilot Study's card informativeness preference ordering (Figure 4.3.1) with the Single Ratings study card informativeness preference ordering (Figure 4.4.1))

substantiated. For example and referring to Figure 4.4.2 above, when *p* cards were vowels (i.e. the P(*p*/vowel), *q* card informativeness ratings were significantly less and the informativeness ratings for the *-q* card significantly increased, as optimal data selections predicts should be the case in a "high" P(*p*) context. However, *p* card type made no significant difference to *p* and *-p* informativeness ratings, i.e. informativeness ratings for the *p* card and the *-p* card were substantially the same when *p* cards were consonants, vowels, odd or even numbers.

A replication of the Single Ratings study was carried out in order to be sure that the above Single Ratings study results (especially the *p* card type i.e. consonant, vowel, odd and even number increases and decreases) were reliable: the same results were produced as described below.

Single Ratings Replication

There were 35 participants, aged between 18 and 50, in the replication of the Single Ratings study. Participants were drawn from the subject panel of the Department of Psychology, University of Wales Bangor and were paid £2.50 for taking part and, if appropriate, a contribution of £1.50 towards travelling expenses was made[6]. Exactly the same computer generated experiment as detailed in the Method section of the first Single Ratings study was used.

The results of the Ratings Replication are detailed below.

Regarding the hypothesis that there would be a difference within this Ratings Replication in ***p*, *q*, *-p* and *-q* card ratings**, the mean selection of *p*, *q*, *-p* and *-q* cards are summarised in the Means Table 4.4.3 below[7].

Means Table 4.4.3: *p*, *q*, *-p* and *-q* card informativeness ratings in the Rating Replication

	Count	Mean	Std. Dev.	Std. Error
p	35	4.260	.789	.133
q	35	3.876	.891	.151
-q	35	2.069	.831	.140
-p	35	1.743	.885	.150

6 Participants who took part in the First Probability study, reported in Section B, Part VI of this chapter, performed the Ratings Replication study after completing the First Probability study.

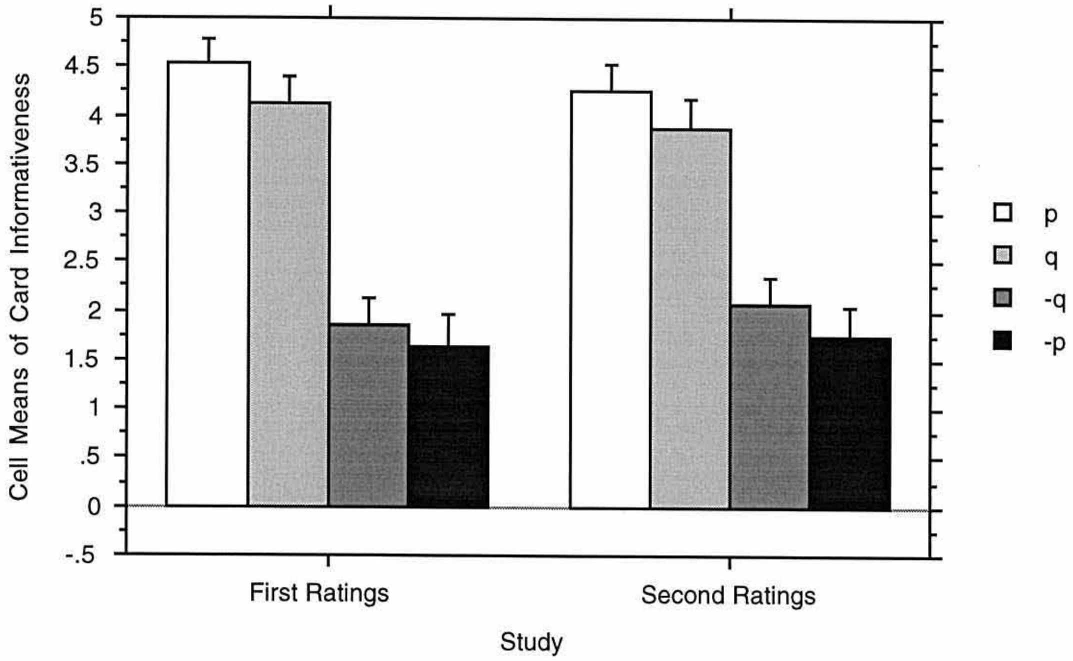
7 As in the first Single Ratings study, and as each of the 35 participants in the Ratings Replication performed 12 tasks, each participant's mean informativeness ratings for *p*, *q*, *-p* and *-q* cards were calculated and form the basis of this analysis.

A one-way ANOVA was performed and there was a significant effect in card informativeness ratings depending on card type (F (3, 34) = 1.06, MSE = 55.997, p = .0001).

Plairwise comparisons were carried out in order to investigate which card informativeness ratings were significantly different from each other in the Ratings Replication. This analysis showed that at the .05 level there were significant effects between *q* and *p* (p = .0300); *-p* and *q*, *-p* and *p*, *-q* and *q*, and *-q* and *p* were all significantly different from each other (p = .0001 for these four pairwise comparisons); and there was a nearly significant effect between *-p* and*-q* card selections (p = .0640). A linear contrast was performed on the Ratings Replication *p>q>-q>-p* card preference ordering which was significant (F (1, 4) = 288.753, MSE = 153.196, p = .0001).

Figure 4.4.3 below illustrates the card informativeness ratings and the *p>q>-q>-p* informativeness ordering produced in the (first) Single Ratings study and (second) Ratings Replication.

Figure 4.4.3: informativeness ratings for *p*, *q*, *-p* and *-q* cards in the (first) Single Ratings and (second) Ratings Replication studies



A 4 x 2 mixed ANOVA was performed in order to see if there was any statistical difference in *p, q, -q* and *-p* card informativeness ratings depending on the ratings study (i.e. the first Single Ratings study, or the second (Single) Ratings Replication). There was no significant difference in card informativeness ratings depending on ratings study (F (3, 53) = 1.831, MSE = .797, p = .1437). Pairwise comparisons were carried out on each card in order to see which, if any, card informativeness ratings were significantly different from each depending on study. There were no significant effects at this level of analysis, i.e. the informativeness ratings for *p* cards in the Ratings Replication were not significantly different from the informativeness ratings for *p* cards in the Single Ratings study (p = .1668), and this was the case for *q* cards (p = .2950),-*q* cards (p = .2845) and -*p* cards (p = .6515).

Analysis was also carried out on the Ratings Replication data to see if there was a difference in card informativeness ratings depending on whether the ***p* card in the selection task rule was a consonant, vowel, odd number or even number**. The means of all card ratings depending on whether *p* cards were consonants, vowels, odd numbers or even numbers are summarised in Means Table 4.4.4 below:

Means Table 4.4.4: *p*, *q*, -*p* and -*q* card informativeness ratings depending on whether *p* cards were consonants, vowels, odd numbers or even numbers

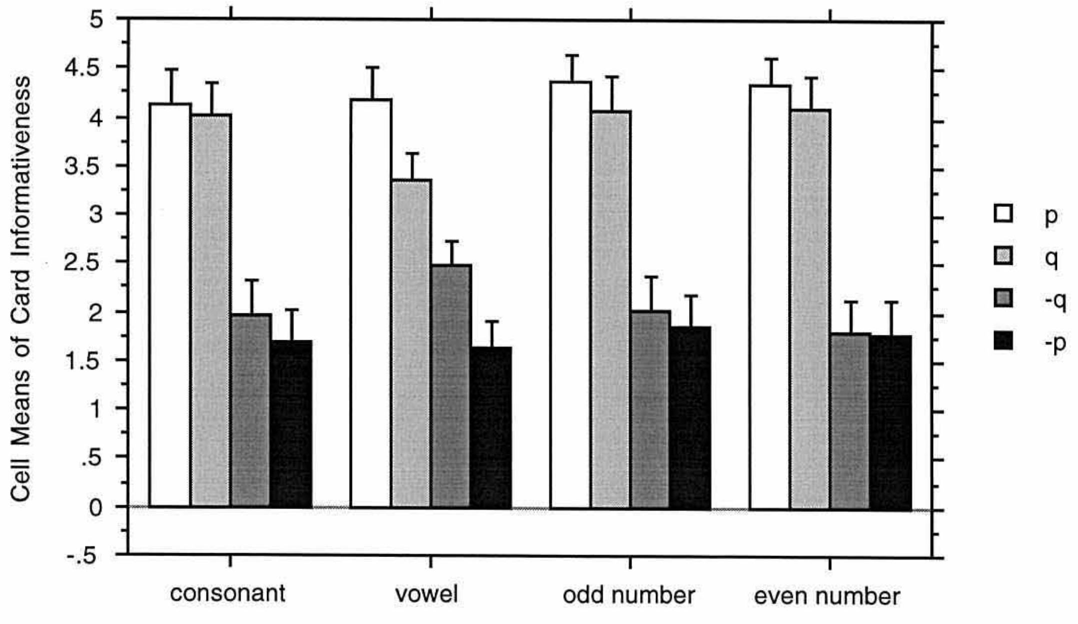
	Count	Mean	Std. Dev.	Std. Error
p, consonant	35	4.133	.971	.164
p, vowel	35	4.171	.958	.162
p, odd number	35	4.381	.797	.135
p, even number	35	4.352	.788	.133
q, consonant	35	4.009	1.002	.169
q, vowel	35	3.352	.828	.140
q, odd number	35	4.057	1.107	.187
q, even number	35	4.086	1.024	.173
-q, consonant	35	1.962	1.038	.175
-q, vowel	35	2.495	.678	.115
-q, odd number	35	2.019	1.054	.178
-q, even number	35	1.800	.957	.162
-p, consonant	35	1.676	1.011	.171
-p, vowel	35	1.648	.779	.132
-p, odd number	35	1.857	.968	.164
-p, even number	35	1.762	1.059	.179

A 4 x 4 within-participants ANOVA was performed which showed that there was no difference in *overall* card ratings based on *p* card type (i.e. if *p* cards were consonants vowels, odd or even numbers) (F(3, 136) = .491, MSE = .711, p = .6892). However, as was the case in the Single Ratings study, there was a significant *interaction* between *p* card type and *p*, *q* -*p* and -*q* card ratings (F (9, 136) = 3.552 MSE = 2.526, p = .0003).

A one-way ANOVA was performed on each of the four cards and pairwise comparisons of means were carried out in order to see which, if any, card informativeness ratings were significantly different from each other depending on card type. The results were similar to those found in the Single Ratings study. For example, and as far as the -*q* card is concerned there were significantly different informativeness ratings between consonants and vowels (p = .0195), vowels and odd numbers (p = .0367) and vowels and even numbers (p = .0025). As far as the *q* card was concerned there were significantly different informativeness ratings between consonants and vowels (.0066), vowels and odd numbers (p = .0036) and vowels and even numbers (p = .0025). As was the case in the Single Ratings study, there were no significant effects for the *p* and -*p* cards, i.e. informativeness ratings for the *p* card and the -*p* card in the Ratings Replication were substantially the same when *p* cards were consonants, vowels, odd or even numbers..

Figure 4.4.4 below illustrates the above relative increases and decreases in card informativeness ratings, as well as card informativeness ordering depending on *p* card type in the Ratings Replication study.

Figure 4.4.4: informativeness ratings and ordering for *p*, *q*, -*p* and -*q* cards in the Ratings Replication when *p* cards were consonants, vowels, odd or even numbers



It can be seen from Figure 4.4.4 above that the predicted $p>q>-q>-p$ informativeness ordering was produced for all four p card types in the Ratings Replication. (The Single Ratings study produced this ordering only when p cards were vowels and consonants, and the odd and even numbered p cards produced a $p>q>-p>-q$ card informativeness ordering - see Figure 4.4.2.)

A 4 x 4 x 2 mixed ANOVA was performed on the Ratings Replication and Single Ratings study consonant, vowel, odd/even number data which showed that there were no significant differences in card informativeness ratings depending on study (first Single Ratings study or second Ratings Replication study), card type (*p, q, -q* and *-q*) and *p* card type (consonants, vowels, odd or even numbers) ($F(9, 212) = .174$ $MSE = .101$ $p = 9966$). Pairwise comparisons were carried out on each card in order to see which, if any, informativeness ratings were significantly different from each other depending on study, i.e. the Single Ratings study and Ratings Replication. There were no significant differences at this level of analysis, i.e. there was no significant difference in the first Single Ratings study's informativeness ratings of, say, *-q* when the *p* cards were consonants when compared to the Rating Replication study's informativeness ratings of *-q* when the *p* cards were consonants, and this was the case for all cards.

Discussion of Single Ratings and Ratings Replication studies

The Ratings Replication has produced the same results as the Single Ratings study as far as card informativeness ordering is concerned: the predicted $p>q>-q>-p$ ordering was produced in both studies. Both studies also produced results which reflect a contrast in the informativeness ratings of *p* and *q* card in comparison with *-q* and *-p* cards which were rated as being the least informative cards, although this contrast is less marked in the Ratings Replication (see Figure 4.4.3).

When Ratings Replication data were analysed to see if there was a difference in card informativeness ratings depending on whether *p* cards were consonants, vowels, odd or even numbers, results similar to the Single Ratings study were again produced. For example, it was shown that the "P(*p*/vowel) condition" produced significantly different informativeness ratings as far as the *q* and *-q* cards are concerned compared to *q* and *-q* cards in the "P(*p*/consonant) condition" (see Figure 4.4.4). The Single Rating and Ratings Replication results therefore both substantiate the optimal data selection assumption that, if the probability of *p* and *q* are low, *-q* and *-p* will be rated as uninformative cards as they provide the least gain in information about which model, MI or MD, holds.

It is concluded from the above analysis of the Single Ratings and Ratings Replication studies that further support for an optimality explanation of the selection task has been provided. More specifically, increases and decreases in card informativeness ratings and informativeness ordering are as expected by the O&C model of optimal data selection when the P(p) and ((q) are low. Cards which were perceived to reduce this uncertainty the most (p and q) have been selected as the most informative cards in both the Single Ratings and Ratings Replication studies. In addition, a most interesting result is that, in both studies when the p card was a vowel, q card ratings decreased and -q ratings increased as would be expected when P(p) is increasing.

The Binary Choice Ratings study is detailed next in Part V of this Chapter 4, and it was designed to provide another form of card informativeness data in order to investigate further the relationship between card selections and card informativeness.

Part V - Binary Study

Two Card Presentation
Multiple Informativeness Comparison Tasks

Introduction

The theoretical motivation of the Binary study is as detailed in the Introduction to the Pilot Ratings study, i.e. to test the hypothesis that there is a relationship between card selections and card informativeness and that the decision rule governing card selection behaviour is that of optimal data selection rather than logic-based decision rules such as falsification.

The mode of card presentation in this card informativeness study differs from previously reported studies, however, and involves one pair of cards being displayed on the screen at a time and task instructions are to select the card which is perceived to provide the most information gain about whether the selection task rule is true or false. This pair comparison methodology, where card informativeness preferences judgement are made, produces data from which an indirect scaling of card informativeness can be computed[1]. The previous two informativeness studies have produced direct measures of card informativeness.

The Method of this Binary study is detailed below.

Method

Participants

There were twenty participants, six males and 14 females aged between 18 and 50, in this Binary study. All participants were undergraduate students at the University of Wales, Bangor. No participants had prior knowledge of logical principles or the rationale for this study.

1 In Part II of chapter 3, I detailed the way in which O&C *scaled* the expected information gain of each card in order to model the selection of similarly valenced cards (Pollard, 1985). The scaling of each participant's pair preferences should provide a similar measure of the way in which similarly valenced cards are associated selections.

Design

This Binary study was a one-factor within-participants design. The dependent variable was the **comparative informativeness of pairs of cards**. The independent variable or factor was **type of card and/or card pair**. The hypothesis was that there would be a difference in card informativeness judgements depending on whether the card was a *p*, *q*, -*p* or -*q* and depending on card pairing. The same card ordering as produced in the Four Cards study, Singles Study, standard affirmative abstract versions of the selection task, as well as the Pilot Ratings and Singles Ratings studies, i.e. *p*>*q*>-*q*>-*p*, was expected to be produced.

Table 4.5.1.1. below sets out the way in which cards may have been randomly paired together for one participant across two trials.

Table 4.5.1.1: possible pack/rule and card pairing and randomisation in Binary study

Trial/Pack/Pair	Left-hand side of card pairing	Right-hand side of card pairing
1.7.1	Q (*q*)	L(-*q*)
1.7.2	L (-*q*)	6(-*p*)
1.7.3	4 (*p*)	L (-*q*)
1.7.4	6 (-*p*)	L (-*q*)
1.7.5	6 (-*p*)	Q (*q*)
1.7.6	6 (-*p*)	4 (p)
1.7.7	L (-*q*)	4 (*p*)
1.7.8	4 (*p*)	Q (*q*)
1.7.9	4 (*p*)	6 (-*p*)
1.7.10	Q (*q*)	6 (-*p*)
1.7.11	Q (*q*)	4 (*p*)
1.7.12	L (-*q*)	Q (*q*)
	New Pack!	New Rule...
2.5.1	2 (-*q*)	U (-*p*)
2.5.2	X (*p*)	2 (-*q*)
2.5.3	9 (*q*)	2 (-*q*)
2.5.4	9 (*q*)	U (-*p*)
2.5.5	9 (*q*)	X (*p*)
2.5.6	X (*p*)	9 (*q*)
2.5.7	U (-*p*)	2 (-*q*)
2.5.8	2 (-*q*)	X (*p*)
2.5.9	U (-*p*)	9 (*q*)
2.5.10	X (*p*)	U (-*p*)
2.5.11	U (-*p*)	X (*p*)
2.5.12	2 (-*q*)	9 (*q*)
	New Pack!	New Rule...

For example, for one participant, a *q* card with "Q" on its face may have appeared on the left of a computer screen and been paired with the -*q* card for that pack which had an "L" on its face and which card appeared on the right of the screen . Then a -*q* card with an "L" on its face may have replaced the first card on the left-hand side of the screen. This second -*q* card was then paired with the -*p* card with a "6" on its face and which replaced the first card on the right hand side of the screen.

The task in the Binary study was therefore a card informativeness preference task, where one card from a pair of cards was selected as the preferred card as it was perceived to provide the most gain in information and therefore reduce uncertainty, about whether MI or MD (i.e. the selection task rule) holds, the most.

As in the Four Cards, Single Card and Single Ratings studies, there were multiple tasks, i.e. 12 card informativeness comparison tasks (see Table 4.5.1.2 below for the different cards and rules used in each task). As there were 12 tasks each with 12 card pairings, this study thus required participants to complete a total of 144 pair comparisons and/or card informativeness preference judgements.

TABLE 4.5.1.2: packs/rules and *p, q -p* and *-q* cards in Binary study

PACK	PACK RULE	CARDS RANDOMLY DISPLAYED *IN PAIRS* ON COMPUTER SCREEN			
		p	*q*	*-p*	*-q*
	LETTERS				
	***p* as vowel**				
1	if A then 2	A	2	K	7
2	if U then 8	U	8	A	6
3	if E then 5	E	5	A	2
	***p* as consonant**				
4	if J then 6	J	6	W	9
5	if X then 9	X	9	U	2
6	if D then 3	D	3	F	1
	NUMBERS				
	***p* as even number**				
7	if 4 then Q	4	Q	6	L
8	if 8 then E	8	E	4	C
9	if 8 then Z	8	Z	1	P
	***p* as odd number**				
10	if 1 then A	1	A	5	U
11	if 3 then B.	3	B	1	A
12	if 9 then G	9	G	7	W

Apparatus

As was the case in the Four Cards, Single Card and Single Ratings and Replication studies, PsyScope, a graphic interface experimental design application, was used to program and run this experiment and record responses. All instructions and stimuli were presented on a monitor attached to a Macintosh LCIII computer. A Macintosh keyboard and mouse recorded participants' responses.

Procedure

As in the Four Cards, Single Card and Singles Ratings studies, participants were tested individually and sat facing a Macintosh LCIII computer monitor. It was explained that instructions about the task and the task itself would be displayed on the computer screen. Below the computer and on the same desk was a Macintosh keyboard and mouse and it was explained that using the keyboard and mouse would record responses.

Before the experimenter left the research room, the participant was handed a sheet of paper on which *Participant's Rights* were typed as set out below (see Appendix 4.1 for precise format):

> Although you have agreed to participate in this study, you are at liberty to withdraw at any time. Any data which you produce will remain confidential and your anonymity is guaranteed. We are NOT interested in whether your responses are right or wrong. This is NOT a test of intelligence or ability, and you will not be timed. CLICK MOUSE ONCE FOR FURTHER DETAILS ABOUT THIS STUDY.

This statement was also on the computer screen. When Participant's Rights had been read and the mouse had been clicked to proceed with the experiment, the experimenter left the room.

The next screen instructions as set out below (see Appendix 4.2 for precise screen format), and as in the Four Cards, Single Card and Singles Ratings studies, gave *general instructions*.

> This study uses several packs of cards. All the cards in these packs have a LETTER on one side and a NUMBER on the other side. There are rules about what letters and numbers can go together. For example: "If a card has a 2 on one side then it has a T on the other side. CLICK MOUSE FOR MORE INSTRUCTIONS ABOUT 'YOUR TASK'...

When these general instructions had been read and assimilated, instructions were to click the mouse in order to obtain further instructions about the task.

The next screen as set out below (see Appendix 4.5.1 for precise screen format) gave Binary study *task specific instructions* :

> You will see TWO cards on the screen, both dealt from the same pack. Only one side of each card will be displayed on the screen. A rule will also be shown. Your task is to decide which of the two displayed cards, if turned over, provides more information about whether a rule is true or false of the pack then being used. You will be prompted to press an appropriate key on the keyboard in order to record your decision. The task will continue until you have compared a number of pairs of cards from several packs. You will be prompted when you have reached the end. If you would like to review these instructions, PRESS THE "A" KEY. If you are clear about what to do CLICK THE MOUSE ONCE TO BEGIN.

As was the case in the Four Cards, Single Card and Singles Ratings studies, the next screen displayed the statement "*New Pack! New Rule...*" (see Appendix 4.3 for precise screen format) in order to make the participant aware that a new pack of cards with a new rule rather than the rule given as an example in the general instruction (see Appendix 4.2) was now being used.

The next screen of *selection task instructions* as set out below (see Appendix 4.5.2 for precise screen format) was automatically displayed after "*New Pack! New Rule...*" had been displayed on the screen for five seconds:

> Rule: If a card has a [*p*] on one side then it has a [*q*] on the other side. Which of the two cards below, if turned over, is more helpful in establishing whether the above rule is true or false of the pack now in use? Press "1" if the card on the LEFT of the screen provides more information about whether the above rule is true or false of the current pack. Press "*" if the card on the RIGHT of the screen provides more information about whether the above rule is true or false of the current pack.
>
> [CARD] OR [CARD]

For example, at the beginning of each trial comprising 12 card pairings, a rule for the pack of cards then being used was randomly displayed at the top of the screen. At the same time as the rule being displayed, two cards from the pack in use were randomly presented on either the left or right hand side of the screen. Instructions were to respond by pressing either the "1" key (which was appropriately marked on the far left-hand side of the keyboard) or the "*" (which was appropriately marked on the far right-hand side of the keyboard). Pressing the "1" key represented that the participant wished to turn over the card on the left-hand side of the screen as it was perceived to be more helpful in deciding whether the rule was true or false of the pack then in use. Pressing the "*" key represented that the participant wished to turn over the card on the right-hand side of the screen as it was perceived to be more helpful in deciding whether the rule was true or false of the pack then in use.

When the participant's choice of cards from the first card pairing had been recorded by pressing the appropriate key on either the left or right hand side of the keyboard, the first pair of cards displayed on the screen was immediately replaced by a second pair of cards from the same pack. For example, see Table 4.5.1.1 above where each card in Pack 7 (i.e. card 4 (*p*), card Q (*q*), card 6 (*-p*) and card L (*-q*)) was randomly displayed six times, three times on the left hand side of the screen and three times on the right hand side of the screen. The 12 pair combinations which were randomly presently were: *p/q, p/-p, p/-q, q/p, q/-p, q/-q, -p/p, -p/q, -p/-q, -q/p, -q/q* and *-q/-p.*

When the participant's preferential card response was made for the last or twelfth card pairing in the first trial, the screen automatically displayed instructions advising that a new pack of cards (i.e. a new trial) with a new rule would be used in the next trial (see Appendix 4.3). Five second after this message was displayed on the screen, the next trial (comprising a further 12 card pairings) automatically commenced.

The above procedure for one trial or pack of cards was the same for all 12 packs of cards, at the end of which the *final screen instruction* as set out below (see Appendix 4.4 for precise screen format) were displayed as in the Four Cards, Single Card and Singles Ratings studies :

> You have now completed this task. If you would like to know more about this study, we shall be happy to answer any questions. Thank you for your time. CLICK THE MOUSE ONCE TO END THIS SESSION.

See Appendix 4.5.3 for example of one participant's raw data and how responses were recorded in this Binary study.

This study took each participant between 15 and 20 minutes to complete.

Results[2]

Regarding **card pair informative preferences**, the mean frequencies for all pairs of cards are summarised in Means Table 4.5.1 below:[3]

Means Table 4.5.1: frequencies of card pair comparisons in Binary study

	Count	Mean	Std. Dev.	Std. Error
p/q	20	19.050	5.256	1.175
p/-p	20	21.350	5.334	1.193
p/-q	20	20.600	5.538	1.238
q/p	20	4.950	5.256	1.175
q/-p	20	19.800	6.014	1.345
q/-q	20	19.450	6.992	1.564
-p/p	20	2.650	5.334	1.193
-p/q	20	4.200	6.014	1.345
-p/-q	20	14.000	5.731	1.281
-q/p	20	3.400	5.538	1.238
-q/q	20	4.550	6.992	1.564
-q/-p	20	10.000	5.731	1.281

For example, the *p* card was the preferred card as regards informativeness when compared to *q* (*p* card mean informativeness preference selections were 19.050), *-p* (*p* card mean informativeness preference selections was 21.350) and *-q* (*p* card mean informativeness preference selections were 20.600).

The *q* card was the preferred selection as regards informativeness when compared to *-p* (*q* card mean informativeness preference selections were 19.800) and *-q* (*q* card mean informativeness preference selections were 19.450), but the *q* card was less frequently selected as regards informative when compared to the *p* card (*q* card mean informativeness were 4.950).

---

2 All ANOVA summaries are in Appendix 4.8.

3 As each of the 20 participants performed 12 selection tasks each with 12 pair comparisons, the means for each pair comparison was computed for each participant and provided the basis of this analysis. For information purposes only, one-way ANOVA was performed on the card pairs data and there was a significant difference in card informativeness preferences depending on card pairs (F (11, 19) = 32.389, MSE = 1205.191, p = .0001). Pair comparison ordering was: *p/-p>p/-q>q/-p>q/-q>p/q>-p/-q>-q/-p>q/p>-q/q>-p/q>-q/p>-p/p*. A linear contrast on this informativeness preference ordering showed a significant effect (F (1, 12) = 142.557, MSE = 5304.554, p = .0001).

The *-p* card was the preferred selection as regard informativeness when compared with *-q* (*-p* card mean informativeness preference selections were 14.000), but the *-p* card was less frequently selected as informative when compared with *p* (*-p* card mean informativeness preference selections were 2.650) and *q* (*-p* card mean informativeness preference selections were 4.200).

When the *-q* card was compared with *-p*, the *-q* card mean informativeness was 10.000, when compared with *p*, the *-q* card mean informativeness preferences selections were 3.400, and when compared with the *q* , the *-q* card mean informativeness preference selections were 4.550. The *-q* card was thus never the preferred selection as regard informativeness.

The card pair preferential selections data for each participant were transformed into z scores (the card pairing results for each participant having first been transformed into proportions (Guilford, 1954)). The purpose of transforming each participant's data into z scores was to produce a matrix of scaled informativeness values for each card pairing for each participant. For example, the z score matrix for participant 1 is set out in Table 4.5.2 below[4]:

Table 4.5.2: matrix of z scores for card pairings for participant 1 in Binary study

Comparison	*p*	*q*	*-p*	*-q*	Total
p	1.602	-.855	-.891	-.926	-1.07
q	-.214	1.602	-.819	-.891	-.320
-p	-.178	-.249	1.602	-.783	.392
-q	-.142	-.178	-.285	1.602	.997
Scaled Means	**1.068.**	**.320**	**-.393**	**-.998**	**0.003**

As the total equalled zero, rounded for error, this participant's scaled means may be taken as the scaled values for card pairing whose means are all at the zero point of the scale (Guilford, 1954 p. 161). Therefore, and as the above matrix presents scaled values in order of informativeness for one participant in this Binary study the *p* card was perceived as being the most informative card of a pair to select, the *q* card was perceived as being the next most informative card of a pair to select except when paired with the *p* card, the *-p* was not perceived as an informative card of a pair to select except when compared to the *-q* card, and the *-q* was perceived as the least informative card of a pair to select and was never the preferred selection.

4 Different participants may produce different scales of informativeness as far as each card/card pairing is concerned.

Scaled informativeness ratings for participant 1 in the Binary study also show that the selection of similarly valenced cards is associated as *p* and *q* have positive z scores and *-p* and *-q* have negative z scores. Similar results were produced when O&C modelled the independence of card selections by scaling the expected information of each card. (See Appendix 4.5.4 for each participant's scaled informativeness card/card pairing values. Except for participants numbered 10 and 20, *p* and *q* cards always had positive z scores and *-p* and *-q* always had negative z scores.)

Table 4.5.3 below is the **group** scaled informativeness values for each card pairing. This matrix was calculated by averaging 20 participants' card pairings scores. The card preference informativeness means have then been scaled for the group, i.e. the scaled means in the z score matrix are in the group's informativeness preference order.

Table 4.5.3: matrix of standard z scores in Binary study as a whole

Comparison	*p*	*q*	*-p*	*-q*	Total
p	1.602	-.784	-.865	-.838	-.855
q	-.285	1.602	-.809	-.796	-.289
-p	-.202	-.258	1.602	-.604	.538
-q	-.230	-.271	-.465	1.602	.636
Scaled Means	**.885**	**.289**	**-.538**	**-.636**	**0.000**

As the total equalled zero, rounded for error, these group scaled means may be taken as the scaled values for 16 card pairings whose mean is the zero point of the scale (Guilford, 1954 p. 161). For example and in similar terms to the scaled values produced by one participant in this Binary study, the *p* card was perceived, by the group, as being the most informative card of a pair to select, the *q* card was perceived as being the next most informative card of a pair to select, the *-p* was perceived as a slightly informative card of a pair to select, and the *-q* was perceived as the least informative card of a pair to select. Again, similarly valenced cards are associated as *p* and *q* have positive z scores and *-p* and *-q* have negative z scores.

The mean informativeness scores for p, *q*, *-p* and *-q* **cards** were also calculated and are summarised in the Means Table 4.5.4 below·

Means Table 4.5.4: *p*, *q*, *-p* and *-q* card selections (144 for each participant in total)

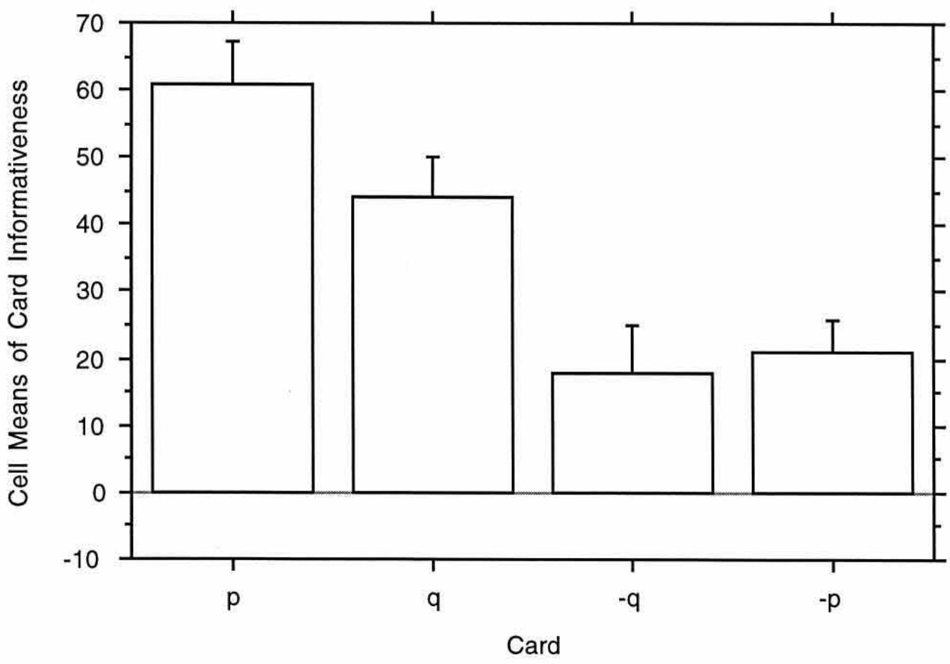
	Count	Mean	Std. Dev.	Std. Error
p	20	61.000	13.373	2.990
q	20	44.200	12.988	2.904
-q	20	17.950	15.483	3.462
-p	20	20.850	10.328	2.309

A one-way ANOVA on card informativeness data was performed and there was a significant effect in card informativeness depending on card type (F (3, 19) = 35.956, MSE = 8317.100 p = .0001).

Pairwise comparisons of means were carried out in order to investigate which cards were significantly different as far as informativeness was concerned in this Binary study. This analysis showed that at the .05 level there were significant effects between *q* and *p* (p = .0054); *-p* and *q*, *-p* and *p*, *-q* and *q*, and *-q* and *p* were all significantly different from each other (p = .0001 for these four means comparisons); but there was no significant effect between *-p* and *-q* card selections (p = .5489).

Figure 4.5.1 below illustrates the *p>q>-p>-q* card informativeness ordering produced in this Binary study, based on mean card informativeness.

Figure 4.5.1: *p>q>-p>-q* card informativeness ordering in Binary study



Before discussing the above results and as was the case in the Four Cards, Singles and Singles Ratings studies, analysis was carried out to see if there was a difference in card informativeness depending on whether the *p* **card in the selection task rule was a consonant, vowel, odd number or even number**. A 4 x 4 ANOVA was performed which showed no significant difference in card informativeness depending on *p* card type (i.e. consonant, vowel, odd or even number) ($F(9, 76) = .346$ $MSE = 5.953$, $p = .9583$). A one-way ANOVA was performed on each card and pairwise comparisons of means were carried in order to investigate which, if any, cards were significantly different from each other (depending on *p* card type) as far as informativeness was concerned. There were no significant effects at this level of analysis. As regards ordering, when *p* cards were consonants, card informativeness ordering was the consistently produced *p>q>-q>-p*, but when *p* cards were vowels and odd or even numbers the ordering was *p>q>-p>-q*.

Discussion of Binary Study

In this Binary study when card-pair preferences were analysed, the *p* and *q* cards were shown to be the most informative cards to select when compared to the *-q* and *-p* card. The *-q* card was the least informative card selection to make when paired with other cards. In addition, analysis of scaled card informativeness showed that similarly valenced cards were associated, as *p* and *q* cards on average produced positive z scores and *-p* and *-q* produced negative z scores.

When card informativeness (rather than card pair preferences) was analysed, the *p>q>-p>-q* card preference ordering in this Binary study was confirmed, and statistical analysis showed that there was no significant difference between the *-p* and *-q* selections. In general optimality terms, transitive ordering of the form *p>q>-p>-q* is assumed to reflect that preferences have been made about events in a given environment. When a consistent selection of events in a given environment is observed[5], it is assumed that a maximising principle is being applied, i.e. events which provide the most benefit or gain are consistently selected. In this Binary study therefore, the *p>q>-p>-q* ordering simply reflects that card preferences have been made where the *p* and *q* cards are optimal and therefore preferred selections and *-p* and *-q* are the least optimal and least preferred selections.

5 The *p>q>-q>-p* selection ordering is consistently observed in affirmative, abstract versions of the selection task.

In summary, the indirect scaling of card informativeness task has produced results which show that *p* and *q* are perceived as being more informative than *-p* and *-q* and that similar valenced cards are associated selections. Optimal data selection predicts that in order to elicit these specific responses, the probabilistic context must be one where *p* and *q* are rare events, there is thus uncertainty about whether MI or MD holds, and selection of *p* and *q* reduces this uncertainty the most.

In conclusion, the Binary study results provides further support for the O&C (1994) assumption that the decision rule to optimise expected information gain guides selection behaviour in the selection task, rather than a logic-based falsificationist decision rule which cannot account for relative increases and decreases in "*p* and *q* " and "*-p* and *-q*" card selections.

General Discussion

The studies in Section A of this chapter 4 have investigated: multiple and computerised task presentation, single card presentation, and how changing the task to be one of judging card informativeness affects behaviour. The different probabilities of *p* cards and the affect of probabilities on informativeness judgements and card selections were also analysed.

Controlling for multiple and computerised task presentation prior to their use in the probability studies (detailed in Section B of this chapter 4) was successful, as these factors did not change the $p>q>-q>-p$ card ordering in the Four Cards study.

The purpose of manipulating the number of cards presented at any one time was to control for single card presentation prior to running experiments which manipulate the probability and optimality of information. As the Singles study produced the consistently observed $p>q>-q>-p$ card ordering found in the Four Cards study and affirmative, abstract versions of the selection task, it was concluded that single card presentation makes no significant difference to card selection behaviour. Therefore single card presentation as used in the probability studies detailed in Section B has also been successfully controlled for.

The purpose of changing the task to one of judging card informativeness was to investigate whether there was a relationship between card selections and the informativeness of cards. In the Pilot and the Singles Ratings studies, direct rating of card informativeness was the task performed and in both of these studies the predicted $p>q>-q>-p$ informativeness ordering was produced. As this preferential informativeness ordering is also found in affirmative abstract versions of the selection task, it was concluded that, at a fundamental level, card selections reflect that cards are being selected on the basis of their expected informativeness.

Furthermore and in the Single Ratings study, analysis at the level of *p* card type added support to this conclusion. For example, there was a significant difference in *-q* selections when the *p* card was a vowel, in comparison to *-q* selections when the *p* card was a consonant (and when *p* cards were odd or even number). Such specific changes in card selections are predicted by optimal data selection, as the informativeness of cards and their selection is expected to change when the P(*p*) changes.

The Binary study produced results which again substantiated optimal data assumptions. For example, when forced to choose the most informative card from a pair of cards, the *p* and *q* cards were selected as providing the most gain in information and the *-p* and *-q* cards were perceived to be the least informative cards to select. In addition, when each participant's card pair preferences were scaled, *p* and *q* produced positive z scores and *-p* and *-q* produced negative z scores, which reflects the Pollard (1985) finding that similarly valenced cards are associated. I concluded from these results that card selection preferences are based on the selection of optimal information.

The card informativeness studies have therefore provided two forms of data (direct rating of informativeness and indirect scaling of informativeness preferences) both of which provide converging evidence for an optimal data explanation of the affirmative abstract version of the selection task.

In order to investigate further if the probabilistic context in affirmative abstract versions of the selection task is one where the P(*p*) and (P(*q*) are low (and therefore the selection of *p* and *q* cards is optimal as uncertainty about whether MI or MD holds is reduced the most by the selection of these two cards), studies in which the probabilistic context is controlled and then varied need to be carried out. Section B studies therefore explicitly manipulate the P(*p*) and P(*q*). It is hypothesised that different probabilistic contexts will produce different, but predictable, card preferences as the optimality (and therefore selection) of cards is probability dependent. Manipulation of probabilistic context is achieved by systematically increasing or decreasing the probability of cards in a learning phase, prior to the multiple selection task phase.

Most studies of the selection task change card selection behaviour by creating realistic contexts and changing the abstract selection task to a thematic version of the four card problem. The O&C model of optimal data selection explains the psychology underlying such "facilitative" selection task behaviour in terms of changes in the probability and optimality of information. A detailed rationale for the probability studies prefaces Section B.

Chapter 4

EXPERIMENTS - SECTION B

Experimental Rationale

The studies reported in Section A of this chapter did not manipulate the context of the abstract selection task. However, the way in which different probabilities influences the selection (and informativeness rating) of cards was made evident in the Single Ratings study. In this card informativeness ratings study, in the "P(*p*/vowel) condition" (i.e. when the P(*p*) is relatively high because vowels are more prevalent in English language usage than consonants), *q* and -*q* informativeness ratings were significantly different to *q* and -*q* informativeness ratings in the "P(*p*/consonant) condition" (i.e. when the P(*p*) is relatively low)[1]. Specifically, in the "P(*p*/vowel) condition", when compared to the other probabilistic contexts (i.e. when *p* cards were consonants, odd or even numbers), *q* was rated as less informative while the informativeness rating for -*q* increased. In the "P(*p*/consonant) condition" (and other *p* card conditions) the inverse occurred: the informativeness ratings for *q* increased and the informativeness ratings for -*q* decreased. These relative increases and decreases in card informativeness ratings are compatible with the O&C (1994) model of optimal data selection and it is predicted that when P(*p*) and P(*q*) are low, *p* and *q* will be optimal cards selections and/or rated the most informative cards as they reduce uncertainty the most about which model (MI or MD) holds[2]. This prediction is conditional is upon the *p* and *q* rarity assumption not being violated, i.e. that P(*p*) and P(*q*) remain low in comparison to the P(-*p*) and P(-*q*).

Parts VI and VII of Section B of chapter 4 describe two probability learning studies, the purpose of which is to investigate whether, at a fundamental level, the variables which affects selection behaviour and make card selections relevant are the probability of cards, and the consequent optimality of cards which O&C (1994) assume is probability-dependent.

---

1 See Figures 4.4.2 and 4.4.4 in Part IV of Section A of this chapter.

2 "MI" is a model of independence between *p* and *q* (i.e. there is no relationship between *p* and *q*) and "MD" is a model of dependency between *p* and q (i.e. if *p* holds then *q* holds).

In each of the two probability studies reported in Section B, there are two conditions. In one condition the P(p) and P(q) are low (which means that the optimality of these cards, or the amount of information to be gained by selecting these cards, is high). In this probabilistic context (which context optimal data selection assumes holds in affirmative abstract versions of the selection task), p and q are optimal card selections because these two cards reduce uncertainty the most about whether MI or MD holds. In the other condition, the P(p) and P(q) is high (which means that the optimality of p and q cards, or the amount of information to be gained by selecting these cards, is low). In this probabilistic context, p and q card are not optimal cards to select, but -q and -p are because their selection would reduce uncertainty the most, by providing the most gain in information, about whether MI or MD holds.

The First Probability study is reported next, in Part VI of Section B.

Part VI - First Probability Study

Introduction

The way in which the probability of information affects reasoning has only recently been studied in depth by reasoning psychologists. For example, how the probable likelihood of information relates to reasoning performance was investigated by Kirby (1994). As detailed in Part II of chapter 2, Kirby's view is that the way in which problem content is understood in the first place is important. He formalises his thesis by adopting signal detection principles, and proposes that card selections are influenced by the probabilities which people attach to making a "hit" (i.e. finding a certain card, say, the *p* card, in the first place and then finding a certain feature, say, a -*q*, on the reverse of that card).

An assumption underlying this hypothesis is that, in the conditional rule "if a card has a vowel on one side, then it has an even number of the other side", vowels are perceived as being a smaller letter set relative to the larger letter set of consonants. Given these different perceptions of vowels and consonants, Kirby proposes that there is a low probability of finding a *p* or vowel card in the first place (in signal detection terms, the probability of "making a hit" is low). But, if the probability of *p* cards were to increase (from small to medium to large probabilities), increased card availability would increase the likelihood of a "hit". In other words, increasing the likelihood of *p* cards will, in turn, increase the likelihood of finding a -*q* on their reverse sides.

To test his theory, Kirby designed experiments in which the probability of the *p* card in abstract versions of the selection task was manipulated by varying "*p* set". For example, three different experiments had three different selection task rules: the "small *p* set rule" was "if a card has a 1 on one side then it has a + on the other side". The "medium *p* set rule" was: "if a card has a number from 1 to 50 on one side then it has a + on the other side". The "large *p* set rule" was: "if a card has a number from 1 to 90 on one side then it has a + on the other side". Kirby's prediction that the proportion of -*q* selections would increase as the size of *p* set increased was substantiated and -*q* card selections did increase when *p* set increased, although he was not able to explain increased -*p* card selections[3].

---

3 Optimal data selection predicts that both -*q* and -*p* selections will increase when P(*p*) is high.

It was concluded from the above results that cards selections are influenced by the probability of making a hit. In more general terms, Kirby's view is that the interpretational component of cognition determines what inferences are made as the probability and utility of information will affect context and thus produce different outcomes. As regards inferential processes, Kirby assumption is that selection task behaviour is governed by truth-preserving logical rules such as falsification and he believes that the mental models theory provides a plausible account of the inferential component of the selection task.

The probability and utility of information are important issues for Manktelow and Over (1991, 1992, 1995), and Manktelow, Sutherland and Over (1995), who investigated the way in which subjective probabilities affect reasoning performance in a deontic version of the selection task, more specifically, Cheng and Holyoak's "Immigration Task". In this contextualised version of the selection task, participants are required to take the perspective of an immigration officer who has to check a certain form to ensure that passengers have been inoculated against cholera if they are entering into the country. The rule to be checked in Cheng and Holyoak's original task is: "if the form says ENTERING on one side [*p*], then the other side includes cholera among the list of diseases [*q*]". The four cards representing *p, q, -p* and *-q* are: ENTERING (*p*), TRANSIT (*-p*), CHOLERA, TYPHOID, HEPATITIS, (*q*) and TYPHOID and HEPATITIS (i.e. not inoculation against cholera) (*-q*).

Manktelow, Sutherland and Over's (1995) experiments are based on the assumption that *p* and *-q* (the logically correct card selections to make in order to test the Immigration Task rule for *violations*) would be more frequently selected if the immigration officer were given additional information about whether a passenger had travelled from either an area in which the probability of infection from cholera was higher or an area which had a comparatively lower incidence of the disease. These studies thus manipulate both the utility of keeping cholera out of a country, as well the probability, in the form of the incidence rate, of cholera in different parts of the world). Results from Experiment 4, in which both these factors were manipulated, produced increased *-q* selections.

These results are explained by Manktelow et al in terms of an extended mental models approach to reasoning, in which it is assumed that people have model preferences, and make judgements about the probability of models. In other words, mental models which incorporate subjective expected utility (where SEU is the combination of model preferences and subjective probability judgements about models) explain reasoning performance in deontic versions of the selection task. Within this theoretical framework, low probability items are assumed to remain implicit for most people, whereas high probability items form explicit representations which deliver required falsificationist (*p* and *-q*) selections.

The above studies have demonstrated that manipulation of subjective probabilities and utilities affects reasoning in abstract and deontic versions of the selection task. The two studies which I report in Section B manipulate probabilistic context in an affirmative abstract version of the selection task by systematically varying the amount of information which people have about *p, -p, q* and *-q* cards (vowels, consonants, even numbers and odd numbers, respectively) in a probability learning phase. A study on the role of prior learning experience was carried out by Pollard and Evans in 1983 when they sought to test whether "contrived experience" (i.e. learning in an experimental situation whether a conditional statement was usually true or whether a conditional statement was false) would produce different behaviour in the selection task.

For example, in Pollard and Evans (1983) first experiment, contrived experience about the truth or falsity of *two* abstract conditional selection task rules ("if the letter is A (B) [*p*] then the number is 1 (2) [*q*]") was achieved using a probability learning task during which participants were exposed to the contingencies of a deck of cards comprising A, B, C, 1, 2 and 3 cards. The contingencies learned were: A (*p*, or true antecedent "TA") was always paired with 1 (*q*, or true consequent "TC"); B (*p* , also a TA) was usually paired with 2 (*q* another TC) but was occasionally paired with 1 (*-q* , or false consequent "FC") or 3; C (*-p* , or false antecedent "FA") was usually paired with 3, but was occasionally paired with 1 or 2.

In order to learn these contingent probabilities, "one learning trial" of the deck of cards comprised 15 pre-programmed contingencies which were displayed on a computer a specific number of times. For example A→1 was programmed to be seen 3 times out of 15, B→2 and C→3 were programmed to be seen 4 times each out of 15, and B→1, B→3 and C→1 and C→2 were programmed to occur once each. Learning trials continued until each participant had reached a certain criterion (see Pollard and Evans (1993) at p. 292/3). Given the above contingency learning, the likelihood of occurrence (which Pollard and Evans called "the approximate probabilities") of the relevant contingencies was assumed to be 100% for A$\rightarrow$1, 53% for B$\rightarrow$2, 3% for B$\rightarrow$1 and 0% for A$\rightarrow$2 (the latter two rules having false consequents).

Given that participants had reached criterion and learned about the cards in the pack, Pollard and Evans assumed that certain beliefs about the conditional rules should hold. Specifically, because A was always paired with 1, it was assumed that the "if A then 1" conditional relationship should be perceived to be "always true", the "if B then 2" conditional relationship should be perceived as being "usually true", the "if B then 1" conditional relationship because it was seen occasionally should be perceived as being "usually false", and an "if A then 2" conditional relationship would be perceived as being "always false" as there was no probability learning about this contingency. Given this learning about the "truth values" (Evans, 1993 p. 121) of letter and number rules, predictions were that more *-q* or false consequents ("FC") would be selected when participants were tested on rules believed to be false and, in addition, more cards generally would be selected when rules were believed to be false.

The theoretical motivation underlying the above probability learning task and studies is quite different to the motivation underlying the manipulation of probabilistic context in order to vary the optimality of cards. Pollard and Evans (1983) derived their motivation from studies which they carried out in 1981 in order to test Van Duyne's (1976) availability hypothesis, that a rule is more likely to be falsified if it was believed in the first place to be a false rule rather than a true rule. Pollard and Evans (1981) found evidence that, in thematic versions of the selection task, more *-q* selections were made when the conditional statement or rule was believed to be false, rather than when the conditional statement was thought to be true. These results were interpreted as showing that, when a conditional is believed to be false it follows that the falsifying case (*p* and *-q*) is available from memory.

On the basis of the results of the 1981 study, Pollard and Evans (1983) designed the probability learning studies in order to make falsifying instances available or explicit, which way of theorising about selection task reasoning accords with a mental models explanation of the task. I have outlined above the first 1983 experiment and the way in which probability learning of letter/number rules was manipulated. This first experiment produced "ambiguous" results which Pollard and Evans interpreted as being due to participants assuming that there was a symmetrical relationship between *p* and *q*, i.e. that the $p \rightarrow q$ rule (material implication) was perceived as being a $p \leftrightarrow q$

rule (material equivalence). For example, selection orderings in the first 1983 study were: "always true condition: FC=FA>TC>TA; in the "usually true condition": FC>TA>TC>FA; in the "usually false condition: FC>TC>FA>TA; and in the "always false condition": FC>FA>TA>TC (see Pollard and Evans (1983 p. 293).

In order to correct for these ambiguous results, and in their second experiment there were four forms of conditional rules in which different symbols and colour combinations were used on the cards. I set out below, examples of the four forms of selection task rule, where the antecedent (front) side of the card was coloured red and the consequent (reverse) side of the card was coloured blue, and four different symbols sets represented TA, TC, FA and FC in the four different contingencies:

$p \rightarrow q$ If the pack of cards used had triangles on the front and stars on the back, $p \rightarrow q$ could represent a rule stating that: if there is a triangle on one side or the front (i.e. the true antecedent or "TA") then there is a star on the other side or the back (i.e. the true consequent "TC")[4].

$p \rightarrow -q$ If the pack of cards used rectangles on the front and a tick on the back of its cards, $p \rightarrow -q$ or TA and TC could represent a rule stating that: if there is a rectangle on the front (TA) there is not a tick (or no symbol) on the back (TC)[5].

$-p \rightarrow q$ If the pack of cards used diamonds on the front and crosses on the back of its cards, $-p \rightarrow q$ or TA and TC could represent a rule stating that: if there is not a diamond (or no symbol) on the front (TA) and there is a cross on the back (TC).

$-p \rightarrow -q$ The contingency $-p \rightarrow -q$ would represent a rule in which it was state that: if there is not a square on one side of the card (or no symbol), then there is not a circle (or again no symbol) on either side of the card.

The way in which the probabilities of rules were learned in this second experiment were assumed to make participants believe that a selection task rule was either

4 It is not clear to me whether the selection task used the terms "on one/other side of a card", or the words" on the front/back", or the words "on the red/blue" side.

5 It is also not clear if rules with negations stated "no symbol" or, for example, "not a tick". In addition, negated constituents (for example, "not a tick") could also be interpreted as representing "another symbol ". Pollard and Evans do not make contrast class explicit (see Oaksford & Stenning, 1992, and Oaksford and Chater, 1994 at p. 615-618).

"usually true" or "usually false". Pollard and Evans argued that in the first experiment, when it was known that a rule was "always" true or false, there was no need to select the card and the logic of the selection task was disrupted when "always" conditions are included.

In the second experiment therefore and when the selection task rule was about, say, $p \rightarrow q$ or material implication, in the learning phase in the ***usually true*** **condition,** q was seen on the reverse of p **seven** out of eight times (with $-q$ being on the reverse of p **once**), and q was also on the reverse of seven $-p$ cards and $-q$ was on the reverse of another seven $-p$ cards. In this usually true condition and as the contingency learned about was $p \rightarrow q$, in the selection task phase participants were presented with a "rule to be evaluated, which on the basis of their probability learning experiment was likely to hold" (Pollard and Evans, 1983 p. 295) , i.e. $p \rightarrow q$. This rule was assumed to hold true 77% of the time in the usually true condition (p. 296).

In the ***usually false*** **condition,** q was seen on the reverse of p **once**, out of eight times (with $-q$ being on the reverse of p **seven** times). In addition, and as in the usually true condition, q was also on the reverse of seven $-p$ cards and $-q$ was on the reverse of another seven $-p$ cards. In the usually false condition, the rule presented in the selected task phase was, given prior learning experience, unlikely to hold. In other words, as $p \rightarrow -q$ was more likely to hold, the rule to be evaluated in usually false selection task phase was $p \rightarrow q$ (i.e. the consequent of the rule most frequently learned about was negated). The usually false rule $p \rightarrow q$ was assigned a 6% probability of being true in the usually false condition.

This second experiment produced results where $-q$ and $-p$ selections increased when rules were believed to be false, but p and q selections decreased. Working within a mental models theoretical framework, the decreased selection of p and q cards could not be explained by Pollard and Evans, and they concluded that explicit instances of falsifying events make counterexamples more available in memory and therefore p and $-q$ selections increased. In terms of optimal data selection, however, decreased selection of p and q cards and increased $-q$ and $-p$ card selections simply reflects that the probabilistic context must be one where the P(p) and P(q) are increasing. In this probabilistic context the most optimal selections are predicted to be $-q$ and $-p$ as they reduce uncertainty the most about which model holds[6].

6 Evans and Over (1996) re-interpreted Pollard and Evans (1983) results in similar terms to Kirby (1994). They propose that the 1983 probability learning studies explicitly manipulate the probability of p. They argue that this re-interpretation provides evidence which falsifies

The Pollard and Evans (1983) studies are notable because the role of prior beliefs on selection task behaviour is investigated by way of a learning task prior to a selection task phase. However, the way in which probabilities and beliefs about conditional rules are assumed to be learned in their probability learning task is not systematic. This may be because the "tools" used by theories which assume that truth-preserving decision rules govern inferential behaviour, albeit in an interpretational stage, do not easily lend themselves, firstly to envisage and then to specify precisely and then formalise, different probabilistic contexts. What is evident, is that such theories constrain the way in which data are analysed and understood, for example, increased *-p* selections are difficult to explain, and it makes little sense to analyse and make inferences about changes in card preferences ordering (p>-q>q>-p) if it is assumed that truth-preserving decision rules are the basis of selection task inferential behaviour.

Gerd Gigerenzer (1991a) writes about the way in which tools (methods and instruments, both analytical and physical) shape theories, and how the familiarity with tools lays "the foundation for the general acceptance of the theoretical concepts and metaphors inspired by the tools" (p. 255). The O&C model of optimal data selection uses different tools to those traditionally used to investigate reasoning in the selection task. These tools are not neutral and they reflect the assumption that selection task behaviour is governed by an optimality-preserving decision rule (to optimize expected information gain) rather than logic or truth-preserving rules such MTT, which permit falsification.

As regards the manipulation of prior probabilities, the methodology which an optimality approach to cognition adopts in order to model behaviour requires the specification of all information in an environment, and that precise predictions be made about expected behaviour given that environment. If behaviour is as predicted, and if this pattern of behaviour is consistently observed (for example p>q>-q>-p preference ordering) in a given context, it can then be inferred that a specific probabilistic context evokes specific (maximising/optimising) behaviour.

---

the O&C (1994) hypothesis that optimal data selection governs selection task behaviour. The Pollard and Evans (1983) studies and the Evans and Over (1996) re-interpretation are fully discussed in chapter 5. The way in which optimal data selection may explain the results in Pollard and Evans (1983) second study is also detailed.

The O&C (1994) model of optimal data selection adopts the above methodological assumptions, and is thus the first, fully-specified optimality model of selection task behaviour. The studies in Section B were designed to test its assumptions and predictions, and the way in which the probability of cards is explicitly specified and manipulated in the first probability study, and the predictions made, are detailed in the Method section below.

Method

Participants

Forty participants aged between 18 and 50 from the subject panel of the Department of Psychology, University of Wales Bangor, comprising members of the general public in Gwynedd, North Wales, took part in this study. Twenty participants were randomly allocated to each of the high and low probability conditions. Participants were paid £2.50 for taking part and, if appropriate, a contribution towards travelling expenses of £1.50 was made.

Design

The dependent variable in the First Probability Study was the frequency of ***p, q, -q* and *-p*** card selections. The independent variables were **type of card** which had four levels for *p*, *q*, *-p* and *-q*, and the **probability of cards** in a learning phase.

The probability of cards was varied as set out in Tables 4.6.1.1 and 4.6.1.2 below. For example, the high probability Table 4.6.1.1 below shows that the *p* card (A) was never displayed with a *p* (A), *-p* (K) or *-q* (7) on its reverse, but there were 40 occasions where *q* (or 2) was on the reverse of the *p* card and 40 occasions where *p* was on the reverse of *q*.

TABLE 4.6.1.1: High (0.8) P(*p*) card presentation

Displayed Card	Reverse A	Reverse K	Reverse 2	Reverse 7	Total
Face A (*p*)	0	0	40	0	40
Face 2 (*q*)	40	8	0	0	48
Face 7 (*-q*)	0	2	0	0	2
Face K (*-p*)	0	0	8	2	10
Total	40	10	48	2	100

The low probability Table 4.6.1.2 below also shows that the *p* card (A) was never displayed with a *p* (A), *-p* (K) or *-q* (7) on its reverse, and in this probability condition there were 10 occasions where *q* (2) was on the reverse of the *p* card and 10 occasions where p was on the reverse of q. The probabilities of other cards also changed.

TABLE 4.6.1.2 Low (0.2) P(p) card presentation

Cards	Reverse p (A)	Reverse $-p$ (K)	Reverse q (2)	Reverse $-q$ (7)	Total
Face A (p)	0	0	10	0	10
Face 2 (q)	10	8	0	0	18
Face 7 ($-q$)	0	32	0	0	32
Face K ($-p$)	0	0	8	32	40
Total	10	40	18	32	100

Given the above high (.8) probabilities and low (.2) probabilities of the p and q cards, the predictions of the O&C (1994) model of optimal data selections are as follows:

(i) Generally, the probability of p card presentations in a learning phase will influence how many $-q$ cards are turned in the card selection phase.

(ii) Depending on the probabilities learned and in comparison to other studies of affirmative, abstract versions of the selection task, there will be an increase or decrease of other card selections besides increases and decreases in the selection of the $-q$ card. For example, when cards are informative, their selection is predicted to increase and when cards are less informative their selection is predicted to decrease. More specifically, infrequently presented cards are predicted to provide the most information gain and are optimal cards to select, and the most frequently presented cards are predicted to provide the least gain in information and are not optimal selections.

(iii) As regards this First Probability study and in the **high P(p)** condition: selection of p (A) cards is predicted to decrease in comparison to the low probability condition and other affirmative abstract selection task studies, and q (2) card selections are also predicted to decrease. As regards $-p$ (K) card selections, they are predicted to increase in comparison to low probability condition and other studies, and $-q$ (7) card selections are also predicted to increase. In the **low P(p)** condition: selection of p (A) cards are predicted to increase in comparison to the high probability condition, and q (2) card selections are also predicted to increase. As regards $-p$ (K) card selections, they are predicted to decrease in comparison to the high probability condition, and $-q$ (7) card selections are also predicted to decrease.

(iv) As regard card ordering, which is assumed to reflect card informativeness preference and that a maximization principle is being applied: in the **high P(*p*)** condition ordering is predicted to change to reflect card informativeness preferences *p*>-*q*>*q*>-*p*. In the **low P(*p*)** condition card ordering is predicted to remain the same as in affirmative, abstract versions of the selection task, the Four Card, Single Card, Pilot Ratings and Single Ratings studies and reflect the consistently observed card informativeness preference *p*> *q*>-*q*>-*p*.

Apparatus

PsyScope, a graphic interface experimental design application, was used to program experiments and record responses. A Macintosh LC475 computer was used and a keyboard on which relevant keys, colour-coded to match the colour of the selection task stimuli, recorded participant's responses. A jar of 100 plastic straws. Instruction sheets as detailed in the Procedure.

Procedure

Participants were tested individually and sat facing a Macintosh LC475 computer. As in all studies previously reported, the participant was handed a sheet of paper on which *Participant's Rights* as set out below (see Appendix 4.1 for precise format) were typed.

> Although you have agreed to participate in this study, you are at liberty to withdraw at any time. Any data which you produce will remain confidential and your anonymity is guaranteed. We are NOT interested in whether your responses are right or wrong. This is NOT a test of intelligence or ability, and you will not be timed. CLICK MOUSE ONCE FOR FURTHER DETAILS ABOUT THIS STUDY.

This statement was also on the computer screen.

When Participant's Rights had been read, it was explained that the study had two phases lasting in total between 20 to 30 minutes and all instructions about the task and the task itself would be displayed on the computer screen[7]. The apparatus to be used was also explained. For example, relevant response keys on the keyboard, "A", "K", "2" and "7" used in the learning phase (representing *p* , -*p* , *q* and -*q*, respectively)

---

7 As well as instructions appearing on the computer screen, all instructions in this study were typed on separate sheets of paper and could be read in this format if participant so wished.

were pointed out. These keys were highlighted with coloured stickers: the "A" key had a red sticker next to the letter A; the "K" key had a purple sticker; the "2" key a green sticker; and the "7" key a blue sticker). Other colour-highlighted keys on the keyboard were the "Y" key representing "Yes", and the "N" key representing "No" (both used during the selection task phase and colour coded with yellow stickers).

Having familiarised the participant with the apparatus, the experimenter advised that all further instructions would appear on the computer screen. The participant then clicked the mouse as instructed at the end of Participant's Rights and the study commenced.

General Instructions were then presented on the screen as set out below (see Appendix 4.6.1 for precise screen format):

> This study uses one pack of cards. Each card in the pack has a LETTER on one side and a NUMBER on the other side. In order to familiarise yourself with this rule, and other characteristics of the cards in this pack, you will be shown 100 cards. There will be a break every 20 cards. Your task during this familiarisation phase will be to press an appropriate key on the keyboard, in order to say what is on the reverse of the displayed card. A choice of responses, which will not change throughout the task, will be shown on the screen. Feedback as to whether your response is right or wrong will be given. We want you to monitor feedback by taking one straw at a time from the jar. If feedback is "YES" place a straw on the left of the desk. If feedback is "NO", place a straw on the right of the desk. Your final task involves 40 cards from the same pack. You will be given full details of what to do in this task later. When you are clear about these instructions, CLICK THE MOUSE TO PROCEED.

Regarding the above general instructions and on the same desk in between the monitor and keyboard, a jar containing 100 plastic straws was placed. These straws were used during the learning phase of the study to engage participant's attention by increasing interactivity during the learning phase of the experiment. To remind participants where to place straws during the learning phase of the study, a label on which was typed "NO straws" was taped to the desktop on the right of the keyboard.

Another label on which was typed "YES straws" was taped to the desktop on the left of the keyboard.

When the General Instructions had been read and when the participant clicked the mouse in order to proceed, the learning phase of the experiment comprising 100 trials commenced. The experimenter remained in the room with the participant for a short while, not more than three minutes, during this first part of the study to ensure that instructions had been understood and procedures were being correctly implemented. The experimenter sat immediately behind the participant and out of vision, but once satisfied that the participant understood what was required, as procedures were being properly carried out, she left the room.

Details of procedures during the learning phase are given below.

LEARNING PHASE IN FIRST PROBABILITY STUDY

The stimuli shown during *one* trial of the card learning phase of the experiment followed the screen sequences detailed below.

1. **First screen shown during learning phase**

   At the top of the screen the question "*What is on the reverse of this card* " appeared (see Appendix 4.6.2, screen diagram 1 for graphic format). Displayed below this question on the left-hand side of the screen was the face of a randomly selected card, being one of the four card stimuli[8]. On the right hand side of the face card and representing its reverse side was a card with a question mark thereon. On the far right and running from the top to the bottom of the screen, four, smaller in size, reverse card stimuli from which to choose a response to the question were displayed. The four choice stimuli remained the same throughout the learning phase.

   Participant's choice of reverse card was recorded by pressing either the "A" key (colour coded red on the keyboard), the "K" key (coloured coded purple

8 Four differently coloured stimuli representing *p*, *q* -*p* and -*q* were used. The stimuli on the cards were: two letters of the alphabet (an "A" stimulus coloured red representing *p*, and a "K" stimulus coloured purple representing -*p*); and two numbers (a "2" stimulus coloured green representing *q*, and a "7" stimulus coloured blue representing -*q*).

on the keyboard), the "2" key (colour coded green on the keyboard), or the "7" key (colour coded blue on the keyboard)[9].

2. **Second screen shown during learning phase**
   Immediately after a response key as described in 1 above was pressed, the question mark representing the unknown reverse of the face card disappeared and was replaced by the stimulus chosen by the participant as being on the reverse of the face card (see Appendix 4.6.2, diagram 2 for graphic format).

3. **Third screen shown during learning phase**
   Immediately after participant's response was displayed on the screen, feedback (either "YES" or "NO") was displayed on the screen stating whether participant's choice of reverse stimulus was correct or not (see Appendix 4.6.2, diagram 3 for graphic format).

4. **Fourth screen shown during learning phase**
   After "Yes" or "No" response feedback was given, the correct reverse stimulus was automatically displayed on the screen (or re-displayed if participant's response regarding the reverse card was correct) and the words "it is" appeared on the screen to the left of correct reverse card (see Appendix 4.6.2, diagram 4 for graphic format).

   If participant's response was incorrect, the participant took a straw from the jar and placed it on the right hand side of the desk near the "*"NO" straws"* label. If participant's response was correct, the participant took a straw from the jar and placed it on the left hand side of the desk near the "*"YES" straws"* label.

   The participant was prompted to click the mouse when ready to continue with the experiment (See Appendix 4.6.2 - screen diagram 4).

The instructions described above comprised one learning trial, and 100 learning trials followed the same sequence where the face cards were randomly selected and displayed.

---

9 Only relevant keys at a given time were active and all other keys (and mouse) if they were not the appropriate response keys at a given time were disabled.

During the card learning phase, breaks were programmed to occur every 20 trials, instructions for which are set out below (see Appendix 4.6.3 for precise screen format):

> You may now take a short break. Click the mouse when ready to proceed.

When 100 learning trials had been completed the screen instructions were as set out below (see Appendix 4.6.4 for precise format.):

> You have now completed the first task in this study and should be familiar with the characteristics of the cards in this pack. The same pack of cards will be used in the next task. When you are ready to begin the final task, CLICK MOUSE TO PROCEED.

SELECTION TASK PHASE IN FIRST PROBABILITY STUDY

When the participant clicked the mouse to proceed from the learning phase to selection task phase of the First Probability study, selection task instructions appeared on the screen as set out below (see Appendix 4.6.5 for precise format).

> On your understanding of the characteristics of cards in this pack, is the displayed card relevant to turn over in order to check: if a card has a VOWEL on one side, then it has an EVEN NUMBER on the other?
> [Press] "Y" = card is relevant to turn over in order to check the above.
> [Press] "N" = card is not relevant to turn over in order to check the above.
>
> [CARD]

Having read the selection task instructions, each selection response was recorded when either the yellow colour-coded "Y" key representing "YES, the card is relevant to turn over" or the yellow colour-coded "N" representing "NO, the card is not relevant to turn over" was pressed on the keyboard.

Immediately after the participant's response was recorded, there was an opportunity to change the selection response (see Appendix 4.6.6 for precise screen format).

> Do you want to change your mind?

Participant's responded to this by pressing either the "Y" key to record that they would like to change their previous response about the last card selection, or by pressing the "N" to record that they did not want to change their response.

The selection task phase lasted for 40 trials or selection tasks, where 10 trials of each of the *p* (A), -*p* (K), *q* (2) and -*q* (7) cards were randomly selected and displayed on the screen. A break was programmed to occur after the first 20 selection task trials and the selection task phase recommenced when the participant pressed any key to continue (see Appendix 4.6.3 for screen instructions and format).

When 40 trials of the selection task phase plus the opportunity to change each response had been completed, the final screen instructions were automatically displayed on the screen as set out below (see Appendix 4.6.7 for screen format):

> You have now completed both tasks! Thank you for participating. If you would like to know more about this study, we shall be happy to tell you.

This study took approximately 20 to 30 minutes for each participant to complete.

After completing the First Probability study, the participant notified the experimenter who was in an adjoining room. The experimenter then thanked each participant for their time and for participating, and offered debriefing about the rationale of the experiments. See Appendix 4.6.8 for example of one participant's raw data and how responses were recorded in this First Probability study.

Results and Discussion

In order to ascertain whether the probabilities of the cards were learned as expected in the learning phase, the mean frequency of correct and incorrect responses for each card was computed.

In the **high P(*p*) learning phase** mean frequency of correct responses as to what was on the reverse of the *p* card when it was displayed on the screen was 37.900 (out of a possible 40) and the mean frequency of incorrect responses was 2.200. When the *q* card was displayed, the mean frequency of correct responses regarding the reverse of this card was 35.250 (out of a possible 48) and the mean frequency of incorrect responses was 12.600. When the *-q* card was displayed, the mean frequency of correct responses regarding the reverse of this card was 1.500 (out of 2) and the mean frequency of incorrect responses was .650. When the *-p* card was displayed, the mean frequency of correct responses regarding the reverse of this card was 5.200 (out of a possible 10) and the mean frequency of incorrect responses was 4.700.

In the **low P(*p*) learning phase**, the mean frequency of correct responses regarding what was on the reverse of the *p* card when it was displayed was 8.600 (out of a possible 10) and the mean frequency of incorrect responses was 1.400. When the *q* card was displayed, the mean frequency of correct responses regarding the reverse of this card was 9.000 (out of a possible 18) and the mean frequency of incorrect response was 9.000. When the *-q* card was displayed, the mean frequency of correct responses regarding the reverse of this card was 28.100 (out of a possible 32) and the mean frequency of incorrect responses was 3.900 incorrect responses. When the *-p* card was displayed, the mean frequency of correct responses regarding the reverse of this card was 27.000 (out of a possible 40) and the mean frequency of incorrect responses was 13.000.

The above mean frequency data were transformed into proportions in order to ascertain if probabilities had been accurately learned in the learning phase of this study. The mean proportions of correct responses or predictions about what was on the reverse sides of each card are as set out in Table 4.6.1 below:

Table 4.6.1: mean proportion of correct responses about reverse side of *p, q, -q* and *-p* cards in high and low P(*p*) conditions[10]

Card	High P(*p*)	Low P(*p*)
p	.93 (.052, .012)	.84 (.153, .034)
q	.72 (.057, .013)	.50 (.089, .020)
-q	.74 (.363, .081)	.86 (.107, .024)
-p	.52 (.209, .047)	.67 (.104, .023)

Table 4.6.1 shows that in both the high and low P(*p*) conditions, the mean proportions of correct responses in the learning phase for what was on the reverse of the *p* cards is high .93 and .84, respectively). But in the low P(*p*) condition half the responses about what was on the reverse of the *q* were incorrect.

10 N = 20. Standard Deviations and Standard Errors are in brackets.

In order to see if the proportions of correct responses in the high P(*p*) condition were significantly different to the proportions of correct responses in the low P(*p*) condition, a 4 x 2 ANOVA was performed on the mean proportions data. This showed no significant difference between *overall* card selections depending on condition (F 1, 38) = .079 MSE =.003, p = .7798), however, there was a significant *interaction* between condition and card type, i.e. *p, q, -q* or *-p* cards (F 3, 38) = 12.570 MSE = .311, p = .0001)[11].

A one-way ANOVA was performed on the mean proportions of each card in order to investigate which, if any, card proportions were significantly different from each other depending on condition. Pairwise analysis of the proportions for each card in the two conditions revealed that at the .05 level there were significant effects between the mean proportions of *p* cards in the high P(*p*) condition and the mean proportion of *p* cards in the low P(*p*) condition (.0263) and this was the case for *q* cards (p = .0001), and *-p* cards (p = .0073), but there was no significant effect between conditions for the *-q* cards (p= .1567).

Accurate learning of the probability of *p* and *q* cards forms the basis of the predictions of the O&C model of optimal data selection. Predictions in the low P(*p*) condition rest on the specific assumption that *p* and *q* are rare events, i.e. that participants have learned that the probability of *p* and *q* cards is low in comparison to the probability of *-p* and *-q*. In the high probability condition, this rarity assumption no longer holds as *-p* and *-q* are the rare events, i.e. the probability of *-p* and *-q* cards is low in comparison to *p* and *q*. Therefore, if the probabilities of all cards in both conditions have not been accurately learned, the probability learning procedure will induce changes to the probabilistic context or state and produced selection behaviour different to that predicted by the O&C model of optimal data selection.

In order to ascertain the exact way in which the probabilities learned affected the frequency of card selections in the two probability conditions, one way ANOVAs were performed on each condition's card frequency selection data. The frequencies of card selections in the **high P(*p*) condition** in the First Probability Study are summarised in the Means Table 4.6.2 below

11 All ANOVA summaries are in Appendix 4.8.

Means Table 4.6.2: *p, q,-p* and *-q* card selections in high P(*p*) condition

	Count	Mean	Std. Dev.	Std. Error
p	20	5.750	4.447	.994
q	20	7.300	3.629	.811
-q	20	2.200	2.821	.631
-p	20	2.800	3.350	.749

A one-way ANOVA showed a significant (although not the predicted) effect between card selections depending on card type, i.e. whether the card was a *p, q, -q* or *-p* card (F (3, 19) = 8.217, MSE = 117.213, p = .0001). Pairwise comparisons were carried out in order to investigate which card selections were significantly different from each other. This analysis revealed that at the .05 level there were no significant effects between *p* and *q* (p = .1996) nor *-p* and *-q* card selections (p = .6174). But there were significant effects between *-p* and *q* (.0004), *-p* and *p* (p = .0165), *-q* and *q* (p = .0001), and *-q* and *p* (.0043).

The mean frequencies of card selections in the **low P(*p*) condition** are summarised in the Means Table 4.6.3 below

Means Table 4.6.3: *p, q,-p* and *-q* card selections in low P(*p*) condition

	Count	Mean	Std. Dev.	Std. Error
p	20	6.450	3.620	.809
q	20	6.550	3.502	.783
-q	20	3.900	3.110	.695
-p	20	3.650	3.265	.730

A one-way ANOVA was performed and revealed a significant (but again not a predicted) effect between card selections depending on card type (F (3, 19) = 4.444, MSE = 49.746, p = .0071). Pairwise comparisons were carried out in order to investigate which card selections were significantly different from each other. This analysis revealed that at the .05 level there were no significant effects between *p* and *q* (p = .9250) nor *-p* and *-q* card selections (p = .8140). But there were significant effects between *-p* and *q* (.0082), *-p* and *p* (p = .0105), *-q* and *q* (p = .0151), and *-q* and *p* (.0192).

Further analysis was carried out in order to ascertain whether *p, q, -q and -p* mean card selections were significantly different from each other depending on the P(*p*) condition. A 4 x 2 ANOVA showed that there was no difference in cards selections depending on probability condition (F (3, 38) = .812, MSE = 10.342, p = .4895). A one-way ANOVA was performed on each card in order to investigate simple effects, i.e. which, if any, card selections were significantly different from each other depending on probability condition. Selection of p cards in the high P(p) condition were no different from p card selections in the low P(p) condition((p = .5883), and this was the case for q card selections (p = 5100), -p card selections (p = .4215), although there were nearly significant differences between -q cards selections in the high P(p) condition and -q cards selections in the low P(p) condition (p = .0781). Figure 4.6.1 below illustrates cards selections, as well as the card preference orderings, in the high and low probability conditions more clearly.

Figure 4.6.1: p, q, -q and -p card selections and card preference ordering in both the high and low P(p) conditions

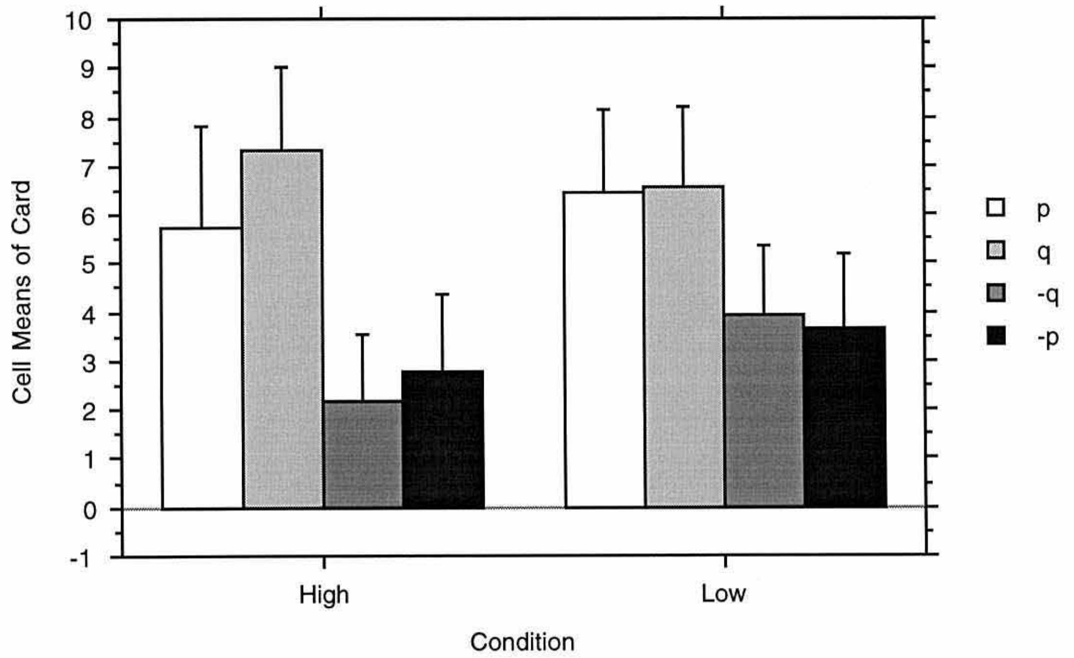


Figure 4.6.1 above illustrates that the precise predictions of the O&C model of optimal data selection were not substantiated in this First Probability Study as the predicted card informativeness orderings have not been produced. In the high P(p) condition a p>-q>q>-p ordering was predicted but a q>p>-p>-q ordering has been produced. In the low P(p) condition the consistently observed p>q>-q>-p was predicted but a q>p>-q>-p ordering has been produced. As optimal data selection assumes that card selections and card ordering are context-dependent, to produce unexpected results probabilities as specified by O&C have obviously not been accurately learned.

In order to assess if participants in the high and low P(p) conditions were predominately falsifiers, i.e. if on average they selected the -q card more than the q card, the consequent falsification index ("CFI") as constructed by Oaksford & Stenning (1992) was computed. Firstly, q and -q data for each participant (20 in each condition) were transformed into proportions (i.e. the number of times q and -q were selected by each participant was divided by the number of times each participant saw the cards: each card was seen 10 times). The arcsin transformation was then used so that q proportions and -q proportions data for each participant more closely met analysis of variance assumptions about normal distribution around the mean and homogeneity of variance (Howell (1989, p. 222)[12]. A composite variable, i.e. CFI, was then computed by subtracting q card proportions from -q proportions, where a positive CFI value means that there were more -q card selections than q card selections .

The above data transformations were also performed on data in the Single Card study as it acts as a control for the two P(p) conditions. Optimal data selections predicts that CFI values should increase as the P(p) increases, but as probabilities were not accurately learned, CFI comparative values are not expected to be compatible with predictions. Table 4.6.4 below illustrates the CFI values for the Single Card study, and the high and low P(p) condition in the First Probability study.

Table 4.6.4: CFI values in the Single Card study, and the high and low P(p) conditions in the First Probability study

	Count	Mean	Std. Dev.	Std. Error
Single Card	20	-1.389	1.609	.360
High P(p)	20	-1.037	1.278	.286
Low P(p)	20	-.579	1.112	.249

The CFI value in the low P(p) condition, contrary to predictions, indicates there were on average more -q selection than q selections in the "low probability condition" than either the control Single Card study or the "high probability condition". A one-way ANOVA was performed in order to see if there were any significant differences in CFI values across the three studies: there were none (F (2, 19) = 1.794, MSE = 3.298, p =.1802). Pairwise comparison of CFI values indicted that at the .05 level there was a nearly significant effect between CFI values in the Single Card study and Low P(p)

12 Formula used was: 2*arcsin(Sqrt(proportions))

condition of the First Probability study (p = .0666) but no significant effect between the Single Card study and High P(*p*) condition.

As regards predicted increases and decreases in card selections, and for information purposes only as the probabilistic contexts in both conditions in this study are not as specified by the O&C model of optimal data selection, Table 4.6.5 below sets out the mean proportions of cards selections in the O&C (1994) meta-analysis, Single Cards study and the two probability conditions in this First Probability study.

Table 4.6.5: *p, q, -p* and *-q* cards selections in O&C (1994) meta-analysis, Single Card study and First Probability study high and low P(*p*) condition in proportions (frequencies of card selections are in brackets)

Study[13]	*p* selections	*q* selections	*-q* selections	*-p* selections
Meta-analysis	.89 (745)	.62 (522)	.25 (215)	.16 (137)
Single Cards	.89 (214)	.73 (176)	.23 (56)	.21 (51)
High P(*p*)	.57 (115)	.73 (146)	.22 (44)	.28 (56)
Low P(*p*)	.64 (129)	.65 (131)	.39 (78)	.36 (73)

For example, in the **high P(*p*) condition** selection of ***p* cards** was predicted by the O&C model of optimal data selection to decrease in comparison to the low P(*p*) condition as well as in relation to other studies of affirmative abstract versions of the selection task. The selection of the *p* card did decrease in comparison to the other two studies and the low P(*p*) condition.

Selection of the ***q* card** was also predicted to decrease in comparison to *q* card selections in the low P(*p*) condition and the other studies, but *q* card selections increased in comparison to the meta-analysis and low P(*p*) condition and selection proportions were the same as the Single Card study (.73).

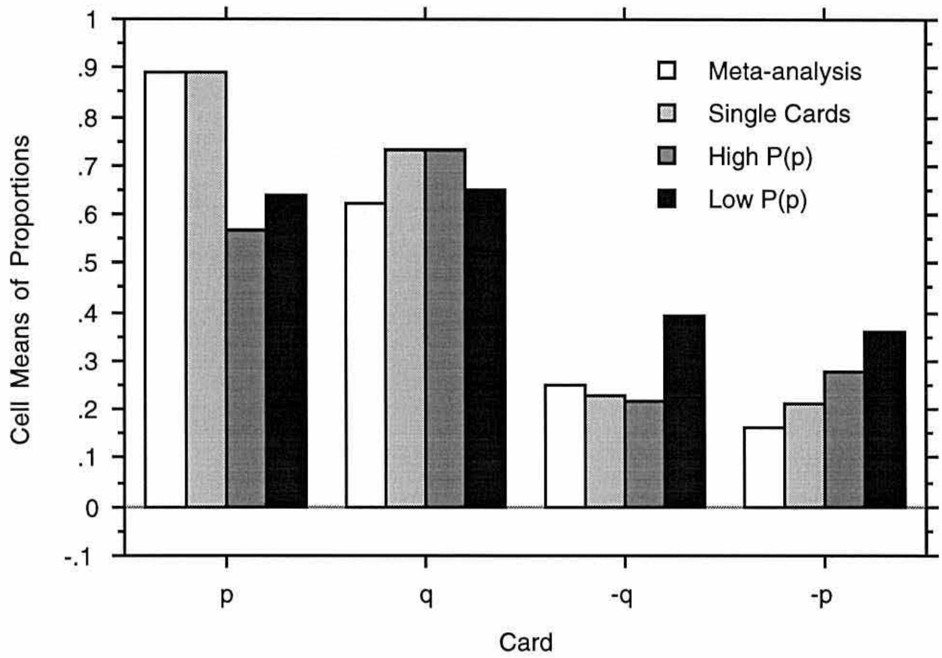
Selection of the ***-q* card** in the high P(*p*) condition was predicted to increase in comparison to the meta-analysis, Single Cards study and the low P(*p*) condition but selections were less than in the other three studies.

The selection of the ***-p* card** in the high probability condition was also predicted to increase in comparison to the Meta-analysis and Single Cards study which it did, but not in comparison to the low P(*p*) condition.

13 In the O&C (1994) meta-analysis, N = 845, in the Single Card study N = 240 (12 selection task trials x 20 participants) and the First Probability high and low P(*p*) condition, N = 200 (10 selection task trials x 20 participants).

Figure 4.6.2 below illustrates more clearly these comparative card selections which reflect relative increases and decreases in card informativeness or the amount of information expected to be gained by turning a card over.

Figure 4.6.2: across studies *p, q, -q* and *-p* card selections (in proportions)



As regards increases and decreases in card selections in the **low P(*p*) condition,** selection of *p, q -q* and *-p* cards was predicted to be similar to selection behaviour in the O&C (1994) meta-analysis and Single Cards studies, i.e. a *p>q>-q>-p* ordering would be produced and *p* and *q* were predicted to increase in comparison to the high P(*p*) condition and *-p* and *-q* were predicted to decrease. The above card ordering was not produced in the low probability condition (the ordering was *q>p>-q>-p*) therefore O&C predictions cannot be validly applied to selection behaviour in this (and the "high probability") condition, as probabilities have not been accurately learned.

In order to see if there is any relationship between probability learning accuracy and card selections, each participant's (20 in total) "correct responses" (in the learning phase) as to what was on the reverse of a card were transformed into *percentages* and correlated with *p, q, -q* and *-p* card selections. The correlation matrix for the high P(*p*) condition is summarised in Table 4.6.6 below

Table 4.6.6: correlation between learning accuracy and card selections in the high P(*p*) condition

	Learning Accuracy	p card	q card	-q card	-p card
Learning Accuracy	1.000	.222	.091	.139	.074
p card	.222	1.000	.344	-.122	-.558
q card	.091	.344	1.000	-.613	.010
-q card	.139	-.122	-.613	1.000	.322
-p card	.074	-.558	.010	.322	1.000

Table 4.6.6 above shows that in the high P(*p*) condition, there is a weak positive correlation between learning accuracy (as measured by percentage of correct responses made by a participant in the learning phase about what was on the reverse of a card) and *p* card selections (.222), -*q* card selections (.139)[14]. In other words, when learning accuracy is high (i.e. when the percentage of correct responses about the reverse sides of cards was high) *p* selections and, to a lesser extent, -*q* selections, were also high. This seems to go against overall card selection results in the high P(*p*) condition, where *p* card selections are less than *q* card selections and -*q* selections are less than -*p* selections (see Figure 4.6.1 above).

In addition and in similar terms to Pollard's (1985) finding regarding the independence of card selections, when selection of -*q* was high, selection of -*p* cards was also high (.322), but *q* selections were significantly lower (-.613) when -*q* selections were high.

Correlations between learning accuracy and card selections in the low P(*p*) condition, are summarised in Table 4.6.7 below:

Table 4.6.7: correlation matrix between learning accuracy and card selections in low P(*p*) condition

	Learning Accuracy	p card	q card	-q card	-p card
Learning Accuracy	1.000	-.007	.439	-.068	-.108
p card	-.007	1.000	.312	.070	-.320
q card	.439	.312	1.000	-.357	-.056
-q card	-.068	.070	-.357	1.000	.489
-p card	-.108	-.320	-.056	.489	1.000

In the low P(*p*) condition learning accuracy and *q* card selections were positively and nearly significantly correlated (.439), which means that when learning accuracy was high (as measured by percentage of "correct responses in the learning phase") there

14 When df = 18, i.e. N_{pairs} - 2, correlations are statistically significant at the .05 significance level when the observed value is greater than the critical value *r* = .444)

were high q card selections which would expected if the P(q) was low. The -q and -p card selections were negatively correlated with learning accuracy (-.068 and -.108, respectively), i.e. these card were not usually selected when learning accuracy was high. Again, these results seems to go against overall card selection results in the low P(p) condition, for example, Figure 4.6.1 shows that -q and -p card selections in this condition are on average quite high. As was the case in the high P(p) condition above, when selection of -q cards was high, selection of -p cards was also high (.489) but q selections were low (-.357).

In conclusion, the O&C predictions regarding increases and decreases in card selections and changes in ordering preferences have not been substantiated in this study. However, predicted behaviour can only be expected if the probabilistic context specified by the O&C model of optimal data selection is accurately learned. The card preference orderings produced in the high and low P(p) conditions of this study clearly reflect that this is not the case and different probabilistic models have evolved within which, of course, there are different optimal and non-optimal card selections.

The inaccurate learning of O&C specified probabilities may have been due to procedural problems. For example.

(i) The break given to participants every 20 cards in the learning and selection task phases was not a time-controlled break, and some participants may have taken breaks longer than the 30 to 40 seconds anticipated. This factor could have caused probabilities to be differentially assimilated in the learning phase of the study.

(ii) Or, if probabilities were accurately learned by some participants (although this is not a general finding given card ordering, and the mean proportion of correct responses regarding the reverse of each card - see Table 4.6.1), there may have been some dissociation between the learning and selection task phase. When debriefing participants, some advised that they had assumed that the learning and monitoring task was separate to the selection task phase. If this was the case, probabilities, if they were accurately learned by some participants, may have been re-assimilated prior to or during the selection task phase of the study.

(iii) Participants in this First Probability study were members of the general public which included participants who spoke Welsh, and other languages, as a first language. Most studies of the selection task use undergraduate students (usually psychology undergraduates) as participants and a change in population and language group may have had some affect on results.

Given the above procedural problems and the results which they may have produced, it is not possible, on the basis of data from this First Probability study, to assess whether an optimality approach to cognition and the O&C (1994) model of optimal data in particular is the appropriate way in which to explain behaviour in the selection task. In order to assess the value of optimal data selection, probabilities as specified by O&C need to be learned accurately and then applied to the selection task phase of a study.

To this end, the procedures in the learning and selection task phases of the Second Probability study detailed in Part VII of Section B have been changed. Using straws to increase participant's attentiveness when monitoring correct/incorrect feedback is not included in the learning phase. In order to increase probability learning accuracy, the number of trials in the learning phase has been increased to 200. The number of trials in the card selection phase has been increased to 48, to be in line with the 48 card selection trials in the Four Card, Single Card and Single Ratings studies. A 30 second break is programmed to occur after 50 learning trials and participants are prompted to proceed with the learning phase task after a programmed 30 seconds break.

Some additions have been made, for example, a form on which participants record their perceptions of what they have learned has been designed. In order to ensure that participants are clear about each card having a letter on one side and a number on the other, card stimuli are colour-discriminated in terms of letters and numbers only rather than there being four different colours for each of the *p, q, -p* and *-q* cards. Specific instructions to ensure that there is no dissociation between the learning and selection task phases have been included, and finally, all participants are first year psychology undergraduates at the University of Wales.

Part VII of this chapter reports the Second Probability study.

Part VII - Second Probability Study

Introduction

The way in which probabilities were learned in the First Probability study induced different probabilistic states to those expected by the O&C (1994) model of optimal data selection, as card selections, which are assumed to be related to the context-dependent informativeness of cards, were quite different to predictions. In order to assess whether an optimality approach to cognition is the appropriate way in which to explain behaviour in the selection task, a number of experimental procedures have been changed, and added, to the Second Probability study.

To ensure that probabilities are accurately learned, the number of trials in the learning phase has been increased to 200, and a form completed prior to the selection task phase on which participants' record their perceptions about what they learned has been designed. Specific instructions to ensure that there is no dissociation between the learning and selection task phases have also been included. For example, participants are instructed to advise a "hypothetical third person", who has no knowledge of the cards, which cards to select in order to be certain that the selection task rule is true or false.

Pollard and Evans (1983) in their first probability learning experiment instructed participants to advise a hypothetical person (who had no knowledge of the cards in the pack) which information was logically necessary to select in order to find out whether a selection task statement was true or false. More specifically, in the selection task phase, a hypothetical third person was offered the information: "the letter (or number) part of this pair is ... Would you like to know the number (letter)? and the participant's task, having acquired certain information about the cards, was to advise if this offer should be accepted or not.

Pollard and Evans introduced a hypothetical third person in their first experiment because the participant would have learned that, say, A cards were always paired with 1 (i.e. that this statement was "always true"). In these circumstances, the participant may have concluded that it was not necessary to acquire any further information about what was on the reverse side of the A card as this was already known A hypothetical subject who needs to acquire information about all the cards would therefore control for participants not selecting cards because they already knew that certain statements were "always true" or "always false"[1]. The use of a hypothetical subject was not used in Pollard and Evans second study and "always" rules were excluded. The theoretical motivation of a hypothetical person in this Second Probability study is simply to ensure that information acquired in the probability learning phase is seen to relate to and thus applied to the selection task phase, as this may not have been the case in the First Probability study where some participants may have dissociated the probability learning task from the selection task.

The above procedural changes should ensure that probabilities are accurately learned and then applied to the selection task phase of the Second Probability study. The Method of the Second Probability study is detailed below.

Method

Participants

In this study, there were forty participant aged between 18 and 55, all of whom were first year undergraduates from the Department of Psychology, University of Wales Bangor. Twenty participants were randomly allocated to the high probability condition (14 females and 6 males) and 20 to the low probability condition (12 females and 8 males)[2].

Design

The dependent variable was the frequency of ***p, q, -q* or *-p*** card selections. The independent variables were **card type**, i.e. *p, q, -q* or *-p* cards, and the **probability of cards** (high or low probability) in a learning phase. The probability of cards was varied as set out in Tables 4.7.1.1 and 4.7.1.2 below. For example, the high P(*p*) Table 4.7.1.1 below shows that the *p* card (A) was never displayed with a *p* (A), *-p* (K) or *-q* (7) on its reverse, but there were 80 occasions where *q* (or 2) was on the reverse of the *p* card and 80 occasions where *p* was on the reverse of *q*.

---

1 See the Introduction to the First Probability Study, where statements or rules used in Pollard and Evans (1983) first experiment, and whether they were "always true", "usually true", "usually false" or "always false", are detailed.

2 There were in total 44 participants but, after debriefing, the datasheets from four were excluded from analysis for the following reasons. In the low P(*p*)condition two female participants (one of whom was dyslexic) made no selections in the selection task phase, and the other female participant was not naive to the selection task. In the high P(*p*) condition one female participant was not naive to the selection task.

TABLE 4.7.1.1: high (0.8) P(*p*) card presentation

Card	Reverse A	Reverse K	Reverse 2	Reverse 7	Total
Face A (*p*)	0	0	80	0	80
Face 2 (*q*)	80	16	0	0	96
Face 7 (-*q*)	0	4	0	0	4
Face K (-*p*)	0	0	16	4	20
Total	80	20	96	4	200

The Low P(*p*) Table 4.7.1.2 below also shows that the *p* card (A) was never displayed with a *p* (A), -*p* (K) or -*q* (7) on its reverse, but in this probabilistic context there were 20 occasions where *q* (or 2) was on the reverse of the *p* card and 20 occasions where *p* was on the reverse of *q*. The probabilities of other cards also changed.

TABLE 4.7.1.2 low (0.2) P(*p*) *p* card presentation

Cards	Reverse *p* (A)	Reverse -*p* (K)	Reverse *q* (2)	Reverse -*q* (7)	Total
Face A (*p*)	0	0	20	0	20
Face 2 (*q*)	20	16	0	0	36
Face 7 (-*q*)	0	64	0	0	64
Face K (-*p*)	0	0	16	64	80
Total	20	80	36	64	200

Given the above high (.8) and low (.2) probabilities of the *p* card, the predictions of the O&C (1994) model of optimal data selections are the same as detailed in the First Probability study. For example:

(i) Generally, the probability of *p* card presentations in a learning phase will influence how many -*q* cards are turned in the card selection phase.

(ii) Depending on the probabilities learned and in comparison to other studies of affirmative, abstract versions of the selection task, there will be an increase or decrease of other card selections besides increases and decreases in the selection of the -*q* card. For example, when cards are informative, their selection is predicted to increase and when cards are less informative their selection is predicted to decrease. More specifically, infrequently presented cards (i.e. rare events) are predicted to provide the most information gain and are optimal cards to select, and the most frequently presented cards are predicted to provide the least gain in information and are not optimal selections.

(iii) Precise predictions are that in the **high P(*p*)** condition: selection of *p* (A) cards is predicted to decrease in comparison to the low probability condition and other studies, and *q* (2) card selections are also predicted to decrease. As regards -*p* (K) card selections, they are predicted to increase in comparison to low probability condition and other studies, and -*q* (7) card selections are also predicted to increase. In the **low P(*p*)** condition: selection of *p* (A) cards are predicted to increase in comparison to the high probability condition, and *q* (2) card selections are also predicted to increase. As regards -*p* (K) card selections, they are predicted to decrease in comparison to the high probability condition, and -*q* (7) card selections are also predicted to decrease.

(iv) As regard card ordering, which is assumed to reflect card informativeness preference and that a maximization principle is being applied: in the **high P(*p*)** condition ordering is predicted to change to reflect card informativeness preferences p>-q>q>-p. In the **low P(*p*)** condition card ordering is predicted to remain the same as in affirmative, abstract versions of the selection task, the Four Card, Single Card, Pilot Ratings and Single Ratings studies and reflect the consistently observed card informativeness preference p> q>-q>-p.

Apparatus

PsyScope, a graphic interface experimental design application, was used to program experiments and record responses. A Macintosh LC475 computer was used and a keyboard on which relevant keys, colour-coded to match the colour of the selection task stimuli, recorded participant's responses. Instruction sheets as detailed in the Procedure.

Procedure

Participants were tested individually and sat facing a Macintosh LC475 computer. As in all studies previously reported, the participant was handed a sheet of paper on which *Participant's Rights* as set out below (see Appendix 4.1 for precise format) were typed.

> Although you have agreed to participate in this study, you are at liberty to withdraw at any time. Any data which you produce will remain confidential and your anonymity is guaranteed. We are NOT interested in whether your responses are right or wrong. This is NOT a test of intelligence or ability, and you will not be timed. CLICK MOUSE ONCE FOR FURTHER DETAILS ABOUT THIS STUDY.

This statement was also on the computer screen[3].

When Participant's Rights had been read, it was explained that the study had two phases lasting in total between 30 to 40 minutes and all instructions about the study and the task itself would be displayed on the computer screen. The apparatus to be used was also explained. For example, relevant response keys on the keyboard, "A", "K", "2" and "7" (used in the learning phase and representing p , $-p$, q and $-q$, respectively) were pointed out. These keys were highlighted with coloured stickers: the "A" key had a red sticker next to the letter A; the "K" key had a red sticker next to the letter "K"; the "2" key a blue sticker, and the "7" key also had a blue sticker. Other task-relevant keys on the keyboard were the "Y" key representing a "Yes" response, and the "N" key representing a "No" response, both of which were used during the selection task phase and colour-coded with yellow stickers.

Having familiarised the participant with the apparatus, the experimenter advised that all further instructions would appear on the computer screen. When the participant clicked the mouse as instructed at the end of Participant's Rights the study commenced.[4]

General Instructions were then presented on the screen as set out below (see Appendix 4.7.1 for precise screen format):

> This study uses a large pack of cards. Each card in the pack has a letter on one side and a number on the other side. To become used to the cards in this pack, you will be shown 200 of them, with a 30 second break every 50 cards. During this learning stage, and when the face of each card is presented, you will be asked to predict what is on its reverse side by pressing a key. A choice of possible responses, which will not change throughout the task, will be shown on the screen. You will be told whether your prediction was right or wrong. Go through this learning stage AS QUICKLY AS POSSIBLE. During the second and final stage of this study you will be asked to advise someone whose task is to find out, and be absolutely certain about, whether a suggested rule "IF A CARD HAS A VOWEL ON

3 As well as instructions appearing on the computer screen, all instructions in this study were typed on separate sheets of paper and could be read in this format if participant so wished.

4 The experimenter remained in the room throughout the study and sat immediately behind the participant and out of vision.

ONE SIDE, THEN IT HAS AN EVEN NUMBER ON THE OTHER" does or does not apply to the pack. Unlike you, this person will not have seen the pack before nor will they have time to learn about any cards in the pack. Instead, they will be forced to pick out of the pack as few, highly informative face cards as possible. As you will have information on 200 cards in the pack, YOUR TASK will be to tell this person which face cards it is essential they look for and select from the pack in order to check their reverse sides, and so be absolutely certain that the ABOVE suggested rule is true or false. Further reminder instructions will be given later. In the meantime, and when you are clear about the above, HIT ANY KEY TO BEGIN LEARNING ABOUT THE PACK. NB: cards will be dealt RANDOMLY throughout.

When the general instructions had been read and when the participant hit any key in order to proceed, the learning phase of the experiment comprising 200 trials commenced. Details of procedures during the learning phase are given below.

LEARNING PHASE IN SECOND PROBABILITY STUDY

The stimuli shown during *one* trial of the card probability learning phase of the experiment followed the screen sequences detailed below.

1. **First screen shown during learning phase**

   At the top of the screen the question *What is on the reverse of this card?* appeared (see Appendix 4.7.2, screen diagram 1 for graphic format). Displayed below this question on the left-hand side of the screen was the face of a randomly selected card, being one of the four card stimuli[5]. On the right hand side of the face card and representing its reverse side was a card with a question mark thereon. On the far right and running from the top to the bottom of the screen, four, smaller in size, reverse card stimuli from which to choose a response to the question were displayed. In addition and above the letter choices "A" and "K", the heading "Letters" was written, and above the

---

5 Four stimuli representing *p*, *q* -*p* and -*q* were used. The stimuli on the cards were: two letters of the alphabet (an "A" stimulus coloured red representing *p*, and a "K" stimulus also coloured red representing -*p*); and two numbers (a "2" stimulus coloured blue representing *q*, and a "7" stimulus also coloured blue representing -*q*).

number choices "2" and "7", the heading "Numbers" was written. These four choice stimuli remained the same throughout the learning phase. The hypothesis "if a card has a vowel on one side, then it has an even number on the other" was displayed (in brackets) in the bottom left-hand corner of the screen and remained in this screen position throughout the learning trials[6].

Participant's choice of reverse card was recorded by pressing either the "A" key (colour coded red on the keyboard), the "K" key (coloured coded red on the keyboard), the "2" key (colour coded blue on the keyboard), or the "7" key (colour coded blue on the keyboard)[7].

2. **Second screen shown during learning phase**

Immediately after a response key as described in 1 above was pressed, the question at the top of the screen disappeared, as did the question mark representing the unknown reverse of the face card. The question mark was replaced by the stimulus chosen by the participant as being on the reverse of the face card (see Appendix 4.7.2, diagram 2 for graphic format).

3. **Third screen shown during learning phase**

Immediately after participant's response replaced the question mark on the screen, feedback (either "Yes" or "No") was also displayed on the screen (duration 1000 milliseconds) stating whether participant's choice of reverse stimulus was correct or not (see Appendix 4.7.2, diagram 3 for graphic format).

4. **Fourth screen shown during learning phase**

After "Yes" or "No" response feedback was given, the correct reverse stimulus was automatically displayed on the screen. Or, if participant's response regarding the reverse card was correct, the correct response was re-displayed. At the same time, the words "it is" were displayed to the left-hand side of correct reverse card (see Appendix 4.7.2, screen diagram 4 for graphic format).

---

6 This was included in order that participants were reminded why they were learning about cards. (In pencil and paper version of the selection task, the selection task rule remains visible to participants.)

7 Only relevant keys at a given time were active and all other keys (and mouse) if they were not the appropriate response keys at a given time were disabled.

> The next card learning trial was automatically programmed to commence 1000 milliseconds after the correct response was displayed on the screen (See Appendix 4.7.2 - screen diagram 4).

The instructions described above comprised one learning trial. A total of 200 learning trials followed the same sequence where the face cards (with pre-programmed probabilities and reverse stimuli - see Tables 4.7.1.1 and 4.7.1.2) were randomly selected and displayed.

During the card learning phase, breaks were programmed to occur every 50 trials when the instructions *You can take a break now* automatically appeared on the screen. After a 30 second break the learning phase was programmed to automatically recommence.

When 200 learning trials had been completed screen instructions advised that the first part of the study had been completed, as set out below:

> You have now completed the learning stage of this study. Before going on to the second and final stage, please complete the form which is face down on the desk you are sitting at.

The form required the participant to indicate what he or she observed was on the reverse of the *p, q, -p* and *-q* cards, and the four possible reverse sides of the cards were illustrated on the form. More than one reverse card could be ticked if this was appropriate. Next to the reverse stimuli which participants had ticked, they were asked to write how often they had observed the letter/number or number/letter combinations which they had ticked on the form: i.e. very frequently / quite frequently / not frequently / can't decide (see Appendix 4.7.3 for precise format).

When the participant had completed the form, instructions on the form and on the screen were to hit any key to proceed from the learning phase to selection task phase of the Second Probability study. Task reminder instructions then immediately appeared on the screen as set out below (see Appendix 4.7.4 for screen format):

> Imagine ALL (not just 200) cards are now randomly spread, face-up, across a table. Someone who has never seen nor learned about these cards before has to know with absolute certainty whether a suggested

> rule, "IF A CARD HAS A VOWEL ON ONE SIDE, THEN IT HAS AN EVEN NUMBER ON THE OTHER", does or does not apply to the pack. It will waste a lot of time and effort if this person turns over all the cards on the table to check their reverse sides. Your information about 200 of the pack's letter/number or number/letter combinations, and how frequently they occur, will save this person turning over unnecessary cards. Which face cards would you advise it IS necessary to look out for and select from the table in order to check their reverse sides, and so be absolutely certain that the ABOVE suggested rule is true or false. HIT ANY KEY TO BEGIN FINAL TASK.

When these task instructions had been read and immediately a key was pressed to begin the final task, selection task instructions appeared on the screen as set out below (see Appendix 4.7.5 for screen format):

> Would you advise it is necessary they look out for and select from the table the below face cards in order to check their reverse sides, and so be absolutely certain that a rule "IF A CARD HAS A VOWEL ON ONE SIDE, THEN IT HAS AN EVEN NUMBER ON THE OTHER", does or does not apply to the WHOLE pack?
>
> "Y" = Yes. They will be absolutely certain the ABOVE rule is true or false if the below face cards are selected and their reverse sides checked. "N" = "No. They will NOT be absolutely certain the ABOVE rule is true or false if the below face cards are selected and their reverse sides checked.
>
> [CARD]

Having read the selection task instructions, each selection response was recorded when either the "Y" key representing "Yes, [a third person] would be absolutely certain the ABOVE rule is true or false... " or the "N" representing "NO, [a third person] would NOT be absolutely certain the ABOVE rule is true or false... " was pressed on the keyboard.

Immediately after a response was recorded, there was an opportunity to change the card selection response when the question *Do you want to change your mind?* appeared on the screen. The participant responded to this by pressing either the "Y" key to record that they would like to change their previous response, or by pressing the "N" to record that they did not want to change their response.

The selection task phase lasted for 48 trials. During the 48 selection task trials, 12 trials of each of the *p* (A), -*p* (K), *q* (2) and -*q* (7) cards were programmed to be randomly selected and displayed on the screen. When 48 trials of the selection task phase plus the opportunity to change each response had been completed, the final screen instructions as set out below were automatically displayed:

> You have now completed this study! Thank you for participating. If you would like to know more about this work, we shall be happy to tell you.

This Second Probability study took approximately 30 to 40 minutes to complete, after which each participant was thanked for his or her time and for participating, and debriefed about the rationale of the experiments. See Appendix 4.7.6 for example of one participant's raw data and how responses were recorded in this Second Probability study.

Results and Discussion

In order to ascertain whether the probabilities of the cards were learned as specified, particularly the probability learning of the *p* and *q* cards on which predictions are based, the mean frequency of correct and incorrect responses regarding what was on the reverse of each card in both probability conditions was computed. Specifically, the total number of times each participant predicted correctly or incorrectly what was on the reverse of a displayed card was calculated. The total frequency (for 20 participants in total) of correct and incorrect responses was averaged to produce the group mean frequency of correct and incorrect responses about what was on the reverse of each displayed card.

In the **high P(*p*) learning phase** mean frequency of correct responses as to what was on the reverse of the *p* card when it was displayed on the screen was 78.800 (out of a possible 80) and the mean frequency of incorrect responses was 1.200. When the *q* card was displayed, the mean frequency of correct responses regarding the reverse of this card was 73.800 (out of a possible 96) and the mean frequency of incorrect responses was 22.200. When the *-q* card was displayed, the mean frequency of correct responses regarding the reverse of this card was 3.450 (out of 4) and the mean frequency of incorrect responses was .550. When the *-p* card was displayed, the mean frequency of correct responses regarding the reverse of this card was 9.850 (out of a possible 20) and the mean frequency of incorrect responses was 10.150.

In the **low P(*p*) learning phase**, the mean frequency of correct responses regarding what was on the reverse of the *p* card when it was displayed was 18.800 (out of a possible 20) and the mean frequency of incorrect responses was 1.110. When the *q* card was displayed, the mean frequency of correct responses regarding the reverse of this card was 19.100 (out of a possible 36) and the mean frequency of incorrect response was 16.850. When the *-q* card was displayed, the mean frequency of correct responses regarding the reverse of this card was 62.550 (out of a possible 64) and the mean frequency of incorrect responses was 1.450 incorrect responses. When the *-p* card was displayed, the mean frequency of correct responses regarding the reverse of this card was 60.100 (out of a possible 80) and the mean frequency of incorrect responses was 20.050.

The above means frequency data were transformed into mean proportions in order to ascertain if probabilities had been accurately learned in the learning phases of this Second Probability study. The mean proportions of correct responses or predictions about what was on the reverse sides of each card are as set out in Table 4.7.1 below:

Table 4.7.1: mean proportions of correct predictions about reverse side of *p, q, -q* and *-p* cards in both the high and low P(*p*) conditions[8]

Card	High P(*p*)	Low P(*p*)
p	.96 (.017, .004)	.92 (.050, .011)
q	.76 (.056, .013)	.53 (.069, .015)
-q	.85 (.181, ..040)	.95 (.017, .044)
-p	.49 (.142, .032)	.74 (.058, .013)

Table 4.7.1 above shows that in the high and low P(*p*) conditions, the mean proportion of correct responses in the learning phase for what was on the reverse of the *p* and *q* (and *-q*) cards is high (mean proportions .96 and .92, respectively). But in

8 N = 20. Standard Deviations and Standard Errors are in brackets.

the low P(p) condition nearly half the responses as to what was on the reverse of the q card were incorrect (mean proportions .53).

In order to see if the proportions of correct responses in the high P(p) condition was statistically different to the proportion of correct responses in the low P(p) condition, a 4 x 2 ANOVA was performed on the mean proportions data. This showed no significant difference between *overall* card selections depending on condition (F 1, 38) = 2.979 MSE =.019, p = .0925), however, there was a significant *interaction* between condition and card type, i.e. p, q, $-q$ or $-p$ cards (F 3, 38) = 45.500 MSE = .415, p = .0001)[9]. Pairwise analysis of the card proportions in the two conditions revealed that at the .05 level there were significant effects between the mean proportions of p cards in the high P(p) condition and the mean proportion of p cards in the low P(p) condition (.0009) and this was the case for q cards (p = .0001), $-q$ cards (p= .0104) and $-p$ cards (p = .0001).

Accurate learning of the probabilities of p and q cards forms the basis of the predictions of the O&C model of optimal data selection, and predictions in the low P(p) condition rest on the specific assumption that p and q are rare events, i.e. that participants have learned that the probability of p and q cards is low in comparison to the probability of $-p$ and $-q$. In the high probability condition, this rarity assumption no longer holds as $-p$ and $-q$ are the rare events, i.e. the probability of $-p$ and $-q$ cards is high in comparison to p and q.

In order to investigate if the above group mean proportions relate to what participants themselves perceived they had learned in the learning phase of this Second Probability study, data from the form on which participants wrote what they believed they observed and how frequently they made these observations were analysed.

In Table 4.7.2 below, the first and eighth columns headed "LOW" and "HIGH" detail the number of times each card dependency relationship or conditional rule was presented in either the "LOW" and "HIGH" P(p) conditions. In the next columns are participants' mean responses regarding which card combinations they believed they observed and how often ("Very often", "Quite often" "Not Often").

9 All ANOVA summaries are in Appendix 4.8.

Table 4.7.2: participants' mean responses about how frequently card combinations were observed in comparison to actual presentation of cards and their reverse sides

LOW	Very often	Quite often	Com-bined	Not often	Can't decide	No tick	HIGH	Very often	Quite often	Com-bined	Not often	Can't decide	No tick
p/q 20	6	8	**14**	5	0	1	**p/q 80**	20	0	**20**	0	0	0
q/p 20	4	10	**14**	4	1	1	**q/p 80**	12	8	**20**	0	0	0
q/-p 16	0	7	7	11	1	1	q/-p 16	0	7	7	12	0	1
-p/q 16	1	3	4	13	0	3	-p/q 16	1	9	10	9	0	0
-p/-q 64	15	4	**19**	1	0	0	**-p/-q 4**	0	6	**6**	11	0	3
-q/-p 64	17	3	**20**	0	0	0	**-q/-p 4**	4	3	**7**	12	0	1
-q/p 0	0	1	1	6	0	13	-q/p 0	0	1	1	2	1	16
p/-q 0	0	1	1	9	0	10	p/-q 0	0	0	0	1	0	19
	43	37	80	49	2	36		38	34	71	47	1	40

For example, in the **HIGH P(*p*)condition** in Table 4.7.2 above, the *p* card was presented with the *q* on its reverse 80 times. When completing the form about what they had learned, 20 out of 20 participants in this condition responded that they had seen the *p* card with a *q* on its reverse "Very Often". As regards what was on the reverse of the *q* card, which in total was observed 96 times (*q/p* 80 times and *q/-p* 16 times), 12 participants recorded that they "very often" observed a *p* on the reverse of *q*, and 8 participants recorded that they had observed this conditional relationship "quite often". In other words, all participants believed that the P(*p*)and (P(*q*) were high in comparison to the probabilities of other cards.

Referring again to Table 4.7.2 and as far the **LOW P(*p*) condition** is concerned, it can be seen (in the first column) that the *p* card in this condition was presented with a *q* on its reverse 20 times. But when completing the form, six participants out of 20 responded that they had seen the *p* card with a *q* on its reverse "Very Often", eight out of 20 participants said they had seen this combination "Quite Often", only five out of 20 participants said that this combination was "Not Often" observed, and one participant did not tick the form or make any comment about observing a *p/q* card dependency relationship. When both *p/q* and *q/p* "Very Often" and "Quite Often" responses are combined, 14 out of 20 participants perceived that *p* and *q* were observed frequently. O&C (1994) assumed that the probabilities attached to *p* and *q*, when learned, would be perceived as being be lower in the low P(*p*) condition, as illustrated below[10]:

low high

MI<------------------|-----|-----------|-->MD

10 "MI" refers to a model of independence between *p* and *q* and "MD" refers to a model of specific dependency where if *p* holds then *q* holds.

The participants in the low P(*p*) condition appear not to have learned the probabilities of *p* and *q* as specified by the O&C model of optimal data selection. As well as the analysis of participants' own perceptions about card probabilities showing this to be the case, Table 4.7.1 above in which the group mean proportions are detailed also shows that a different probabilistic state may have been produced in the low P(*p*) condition as the group mean proportion of correct responses about what was on the reverse of *q* cards was only .53 (the group mean proportions for *p, q* and *-p* were .92, .95 and .74, respectively). In other words, the way in which the conditional probabilities of *q/p* and *q/-p* [11] have been learned has necessarily changed the actual probabilities of *p* and *q*.

As the precise predictions of the O&C model rest on the accurate learning of the P(*p*) and the P(*q*), results predicted for the low P(*p*) condition cannot be expected. However, the probabilities as specified by optimal data selection have been accurately learned in the high P(*p*) condition, as measured by both the group mean proportions of correct responses for each card, as well as participants' mean subjective responses about *p* and *q* cards and what they perceived to be on the reverse of these cards. The precise predictions made by O&C can therefore be expected to be produced in the high P(*p*) condition.

In order to ascertain the exact way in which the probabilities learned affected the frequency of card selections in the two probability conditions, one way ANOVAs were performed on each probability condition's card frequency selection data. The mean frequencies of card selections in the **high P(*p*) condition** in the First Probability Study are summarised in the Means Table 4.7.3 below.

Means Table 4.7.3: *p, q,-p* and *-q* card selections in high P(*p*) condition

	Count	Mean	Std. Dev.	Std. Error
p	20	8.000	4.952	1.107
q	20	5.550	4.915	1.099
-q	20	7.050	4.707	1.053
-p	20	4.150	4.603	1.029

A one-way ANOVA showed no significant difference between card selections depending on card type, i.e. whether the card was a *p, q, -q* or *-p* card (F 3, 19) =

11 *q/-p* is parameter *b* in the O&C's model.

2.032, *MSE* = 57.246, *p* = .1195)[12]. A linear contrast was carried out on the *predicted p>-q>q>-p* ordering and this trend was found to be significant (*F* 1, 19) = 6.046, *MSE* = 170.303, *p* = .0170). Pairwise comparisons were carried out in order to investigate which, if any, card selections were significantly different from each other. This analysis revealed that at the .05 level there were significant effects between *-p* and *p* (p = .0255) but no significant effects were found between *p* and *q* (p = .1498), *-q* and *p* (.5736), *-q* and *q*, (p = .3752), *-p* and *q* (.4077), or *-p* and *-q* (p = .0894).

The frequencies of card selections in the **low P(*p*) condition** are summarised in the Means Table 4.7.4 below

Means Table 4.7.4: *p, q,-p* and *-q* card selections in the low P(*p*) condition

	Count	Mean	Std. Dev.	Std. Error
p	20	6.400	5.404	1.208
q	20	4.750	4.191	.937
-q	20	5.950	4.957	1.109
-p	20	6.650	4.760	1.064

A one-way ANOVA showed no significant difference between card selections depending on card type, i.e. whether the card was a *p, q, -q* or *-p* card (*F* (3, 19) = .561, *MSE* =14.212, *p* = .6427). A linear contrast was carried out on the unpredicted *-p>p>-q>q* ordering and this trend analysis was not found to be significant (*F* 1, 19) = 1.494, *MSE* = 37.822, *p* = .2266). Pairwise comparisons were carried out in order to investigate which, if any, card selections were significantly different from each other. This analysis revealed that at the .05 level there were no significant effects between any of the cards, i.e. *-p* and *p* (p = .8757), *p* and *q* (p = .3041), *-q* and *p* (.7401), *-q* and *q*, (p = .4539), *-p* and *q* (.2374), or *-p* and *-q* (p = .6617).

Further analysis was carried out in order to ascertain whether *p, q, -q and -p* mean card selections were significantly different from each other depending on the probability condition. A 4 x 2 ANOVA showed that there was no difference in card selections depending on probability condition (*F* (3, 38) = 1.298, *MSE* = 34.700, *p* = .2788). Pairwise comparisons were performed on each card in order to investigate simple effects, i.e. which, if any, card selections were significantly different from each

12 When the P(*p*) is high, fewer significant effects in card selections should be expected as the informativeness of *p* and *q* cards is decreasing and the informativeness of *-p* and *-q* cards is increasing.

other depending on probability condition. Selection of *p* cards in the high P(*p*) condition were no different from *p* card selections in the low P(*p*) condition (*p* = .3351), and this was the case for *q* card selections (p = .5829), -*q* card selections (p = .4762), and-*p* cards (p = .0995). Figure 4.7.1 below illustrates cards selections and the card preference orderings in the high and low probability conditions in this Second Probability study more clearly.

Figure 4.7.1: *p, q, -q* and *-p* card selections and card preference ordering in both the high and low P(*p*) conditions of the Second Probability study

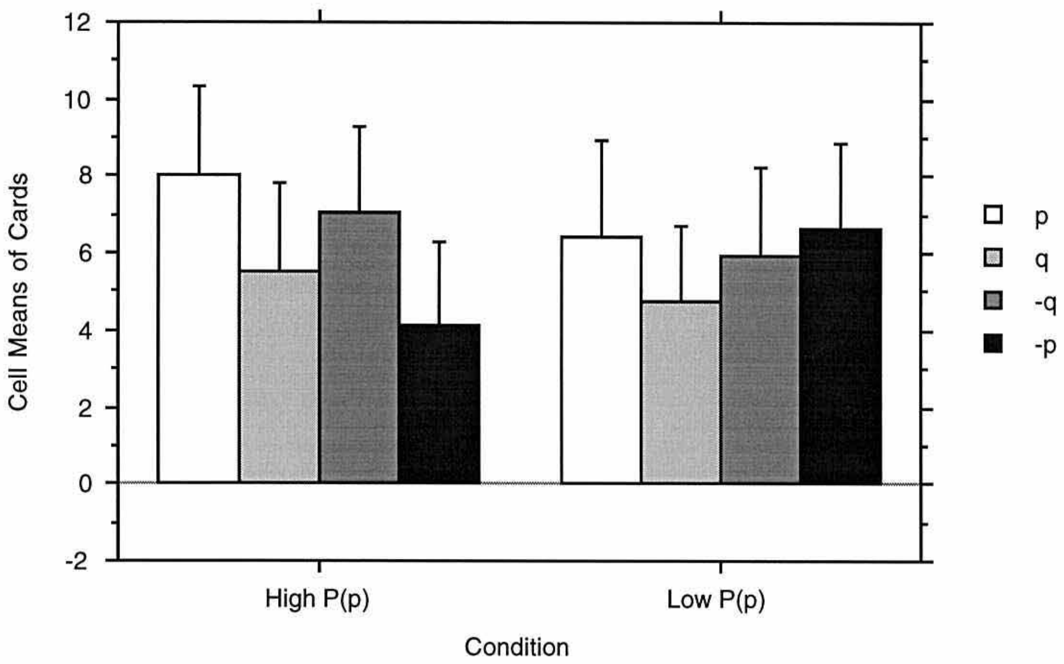


Figure 4.7.1 above illustrates that the precise predictions of the O&C model of optimal data selection were substantiated in the high P(*p*) condition as the predicted *p*>-*q*>*q*>-*p* ordering was produced. In the low P(*p*) condition a -*p*>*p*>-*q*>*q* ordering was produced rather than the consistently observed and expected *p*>*q*>-*q*>-*p* ordering. As the probabilistic context as specified by O&C was not accurately learned in the low P(*p*) condition, different card selection behaviour were not expected to be produced in this condition, but card selections are still assumed to reflect the informativeness of cards which is context-dependent.

In order to assess if participants in the high and low P(*p*) conditions were predominately falsifiers, i.e. if on average they selected the -*q* card more than the *q* card, the consequent falsification index ("CFI") as constructed by Oaksford &

Stenning (1992) was computed. Firstly, *q* and -*q* data for each participant (20 in each condition) were transformed into proportions (i.e. the number of times *q* and -*q* were selected by each participant was divided by the number of times each participant saw the cards: each card was seen 12 times). The arcsin transformation was then used so that *q* proportions and -*q* proportions data for each participant more closely met analysis of variance assumptions about normal distribution around the mean and homogeneity of variance (Howell (1989, p. 222)[13]. A composite variable, i.e. CFI, was then computed by subtracting *q* card proportions from -*q* proportions, where a positive CFI value means that there were more -*q* card selections than *q* card selections.

The above data transformations were also performed on data in the Single Card study as it acts as a control for the two P(*p*) conditions. Optimal data selections predicts that CFI values should increase as the P(*p*) increases, but as probabilities were not accurately learned in the low P(*p*) condition, CFI comparative values are not expected to produce results compatible with predictions. Table 4.7.5 below illustrates the CFI values for the Single Card study, and the high and low P(*p*) condition in the Second Probability study.

Table 4.7.5: CFI values in the Single Card study, and the high and low P(*p*) conditions in the Second Probability study

	Count	Mean	Std. Dev.	Std. Error
Single Card	20	-1.389	1.609	.360
High P(p)	20	.381	2.040	.456
Low P(p)	20	.294	1.651	.369

The CFI value in the high P(*p*) condition, as it is a positive value, indicates there were on average more -*q* selections than *q* selections in the this condition than in the Single Card study, in which there are more *q* than -*q* selections, which accords with optimal data selection predictions when the P(*p*) is perceived to be high. There is also positive CFI value in the low P(*p*) condition, contrary to predictions, although it is a lower value (indicating less -*q* selections) than the high P(*p*) condition.

A one-way ANOVA was performed in order to see if there were any significant differences in CFI values across the three studies, and there was (F (2, 19) = 5.924, MSE = 19.903, p =.0058). Pairwise comparison of CFI values indicated that at the

13 Formula used was: 2*arcsin(Sqrt(proportions))

.05 level there was a significant effect between CFI values in the Single Card and high P(*p*) condition of the Second Probability study (p = .0041) and between the Single Card and low P(*p*) condition (p = .0061) but, contrary to predictions, there was no significant effect between the high and low P(*p*) conditions (p = 8825)[14].

As regards predicted increases and decreases in card selections, Table 4.7.6 below sets out the mean proportions of cards selections in the O&C (1994) meta-analysis, Single Card study and the two probability conditions in this Second Probability study.

Table 4.7.6: *p, q, -p* and *-q* cards selections in O&C (1994) meta-analysis, Single Card and Second Probability study high and low P(*p*) conditions in proportions (frequencies of card selections are in brackets)

Study[15]	*p* selections	*q* selections	*-q* selections	*-p* selections
Meta-analysis	.89 (754)	.62 (522)	.25 (215)	.16 (137)
Single Cards	.89 (214)	.73 (176)	.23 (56)	.21 (51)
High P(*p*)	.66 (160)	.46 (111)	.58 (141)	.34 (83)
Low P(*p*)	.53 (128)	.39 (95)	.49 (119)	.55 (133)

For example, in the **high P(*p*) condition** selection of *p* cards was predicted by the O&C model of optimal data selection to decrease in comparison to the low P(*p*) condition as well as in relation to other studies of affirmative abstract versions of the selection task. The selection of the ***p* card** did decrease in comparison to *p* card selections in the Meta-analysis and Single Cards study, but not in comparison to the low P(*p*) condition.

Selection of the ***q* card** was also predicted to decrease in comparison to *q* card selections in the low P(*p*) condition and the other studies, and *q* card selections did decrease in comparison to the meta-Analysis and Single Card study, but not in comparison to the low P(*p*) condition.

Selection of the ***-q* card** in the high P(*p*) condition was predicted to increase which it did in comparison to the meta-analysis, Single Card study and the low P(*p*) condition.

The selection of the ***-p* card** in the high probability condition was also predicted to increase in comparison to the low P(*p*) and other studies, and selection of this *-p* card

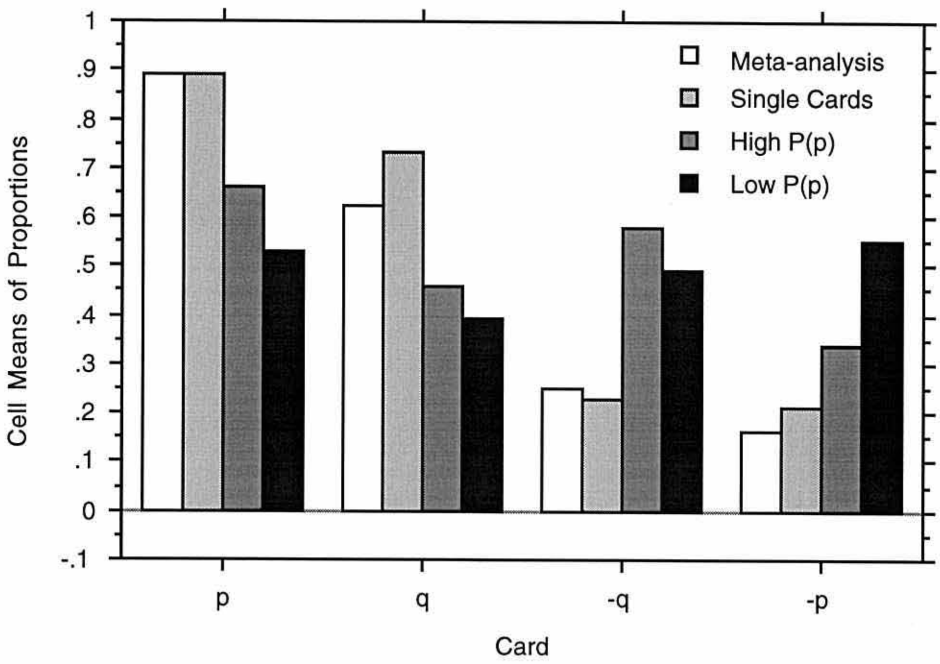
14 For information purposes only, the CFI value in the Four Cards study was -1.654, and in the Binary study the CFI value was -.832, i.e. there were more *q* than *-q* selections in these two studies.

15 In Meta-analysis, N = 845, in the Single Card and Second Probability studies N = 240.

did increase in comparison to the Meta-analysis and Single Card study, but not in comparison to the low P(*p*) condition.

Figure 4.7.2 below illustrates more clearly these comparative card selections which reflect relative increases and decreases in card informativeness or the amount of information expected to be gained by turning a card over.

Figure 4.7.2: across studies *p, q, -q* and *-p* card selections (in proportions)



In the high P(*p*) condition, therefore, and because the specific card selection behaviour was produced as predicted by the O&C model, it may be inferred that a maximization principle (i.e. the decision rule to optimize expected information gain) is being applied in this probability condition of the selection task, rather than the logical principle of *modus tollendo tollens* in order to falsify an hypothesis, which theoretical approach to selection behaviour cannot account for or predict changes in preferences.

As regards increases and decreases in card selections in the **low P(*p*) condition,** selection of *p, q -q* and *-p* cards was predicted to be similar to selection behaviour in the O&C (1994) meta-analysis and Single Card studies, i.e. a *p*>*q*>*-q*>*-p* ordering would be produced and *p* and *q* were predicted to increase in comparison to the high P(*p*) condition and *-p* and *-q* were predicted to decrease. These predictions were not supported but, in any event, they cannot be validly applied to selection behaviour in this condition as the probability of cards was not accurately learned.

Although results in the low P(p) condition are not as predicted, participants' mean responses regarding the perceived probabilities of cards in the low condition provide support for the optimality assumption that selection behaviour is related to card informativeness which is probability-dependent. For example and referring to Table 4.7.2 above, the probability of the p card is clearly not *perceived* by participants in the low P(p) condition as being .2 or "infrequent", therefore it is not possible to produce selection behaviour expected from a probabilistic context in which there is a .2 probability of p and q as specified by the O&C (1994) model. However, and if it is accepted that $-p$ has become a "first choice" selection in the "low probability" context, in optimality terms the $-p>p>-q>q$ ordering reflects that another probabilistic model or context has evolved which will necessarily produce card informativeness preferences or card selections different from those predicted. Model evolution and preference changes means the O&C rarity assumption (that p and q are rare events in comparison to other information in the environment, i.e. in comparison to $-p$ and $-q$ events) has been violated in the low P(p) context.

As regards model or rule change, when Pollard and Evans (1983) found in their Experiment 1 results where all cards were nearly equally selected, they argued that this was because symmetrical or equivalent relationships between p and q were assumed to exist. In other words, the dependency component of the rule in the selection task was assumed to have changed from $p \rightarrow q$ to $p \leftrightarrow q$. Given this symmetrical dependency relationship, it was argued that "all cards are potential falsifiers and the logically correct solution is to select all four cards" (Pollard and Evans, 1983 p. 294). While the probabilities learned in the low P(p) context may have changed the probabilistic context to one in which $p \leftrightarrow q$ rather than $p \rightarrow q$ holds, it need not be the case that equality of card selection means that all cards are potential falsifiers. In optimality terms, cards may be equally selected because the informativeness of cards is changing (see Figure 4.7.1 which reflects the increases and decreases in card selections). In addition, relative equality of card selections could reflect that cards may be equally preferred because they are equally informative (or equally uninformative).

The theoretical issue of cards being equally informative (i.e. because preferences cannot be discriminated) is discussed further in Chapter 5. For example, and in the low P(p) condition and as card selections are assumed to be related to the informativeness of cards, selection behaviour may be explained in terms of preferences regarding the informativeness of cards having been discriminated but these preferences are not related to the O&C model of dependency, i.e. *the decision rule to optimise expected information gain is being applied to another probabilistic context or model which has evolved* because of the way that probabilities have been learned. Another explanation may be that unpredicted card selections may reflect that transitional and/or ambiguous contexts or states have evolved. More specifically, the non-significant differences between card selections in the low P(p) condition may reflect that a different probabilistic model has evolved but that it is *not possible to distinguish card informativeness or other preferences in this model*[16]. A third explanation of selection behaviour in the low P(p) condition may be that a different probabilistic model may have evolved but *a different optimality-preserving, decision rule, for example a decision rule to optimize global fitness, is governing selection behaviour* rather than the O&C decision rule to optimize expected information gain.

Whilst the low probability condition may not have produced results as predicted, it has exposed conceptual rather than procedural problems relating to the learning of probabilities in this Second Probability (and First Probability) study. The theoretical implications of these results for the O&C model of optimal data selection are detailed in the next chapter. The procedural weakness of this Second Probability study are discussed further below.

As far as experimental procedures of future tasks in which probabilistic context is varied are concerned, it is important to ensure that probabilities are accurately learned. The probability manipulations in the First and Second Probability studies were calculated so that the probabilities of letter and number combinations as specified by the O&C model (see Tables 4.7.1.1 and 4.7.1.2 and Table 4.7.2 above) were learned. For example, and similar to the way in which Pollard and Evans (1983) designed their first probability learning task and rules, p (A) was "always" seen with a q (2), and $-p$ (K) was seen with q (2) and $-q$ (7) and so on. Unlike Pollard and Evans' first probability learning task, there was only one affirmative, abstract (material implication) selection task rule, and there were no exceptions to the rule, i.e. falsifying instances of the selection task rule were not learned or made explicit.

16 This explanation of non-preferential selection behaviour violates optimality assumptions about being able to distinguish choices or information and then have consistent preferences about those choices.

Learning about conditional probabilities is a weak link when studying the role of probability and optimality in reasoning. For example, and even though probabilities were accurately learned in the high P(p) condition Second Probability study (as reflected in participants' perceptions of what they learned and subsequent card selection ordering and CFI values, from which it is evident that making falsifying instances of a rule explicit is not necessary in order to change selection behaviour), the results of other probability studies reported in Section B reveal that participants do not only learn about the *actual* probabilities of cards. Participants also learn about *conditional* probabilities, i.e. the various dependency relationships between cards and this is where problems with probability learning tasks emanate[17].

The weakness of this method of (conditional) probability learning is most apparent in the low P(p) condition of the First and Second Probability studies where relatively similar frequencies of presentation of p and q dependency relationships were made. Probabilistic contexts in which conditional probabilities are not always easy to discriminate from each other, make it possible for different dependency relationships or conditional probabilities to evolve. For example, because the conditional probabilities $p \rightarrow q$ and $q \rightarrow p$ (and perhaps $q \rightarrow -p$) were perceived to be similar to each other, the specific (and distinguishing) dependency relationships between these cards was degraded, which degradation created a state conducive to the construction of new associations and dependency relationships, such as $p \leftrightarrow q$ or $p \& q \& -p$, and so on. The creation of *different conditional relationships* in turn, changes the *actual probabilities* of cards, especially P(p) and P(q), which then influences the way in which cards are selected because probabilistic context and the probabilities of p and q cards in particular have changed or are unclear.

Future experimental research into optimal data selection needs to ensure that *actual* probabilities are always clearly and precisely discriminated in order that probabilistic context does not evolve away from the equilibrium of the O&C models of high and low P(p). For example, and similar to the way in which Kirby (1994) relied on signal detection principles to manipulate the probability of the p card in his studies, experiments in which the actual probabilities of **events** (i.e. the probabilities of p, q $-q$ *and* $-p$ cards see Table 4.7.7 below) are learned, rather than probability learning about

17 By *actual* probabilities, I mean the *objective* probabilities of **events** p, q, $-p$ *and* $-q$ - see "Total" columns in Tables 4.7.1.1 and 4.7.1.2. However, probability learning of conditional rules such as "if p then q", rather than learning about objective probabilities of events, has produced different *subjective* probabilities (of both conditionals and events) in the low P(p) condition and an unpredicted probabilistic context has evolved as reflected in ordering.

dependency relationships between events (i.e. the *conditional* probabilities of cards) which dependency relationships can be degraded and reconstructed.

Table 4.7.7: appropriate **objective** probability learning of *p, q, -q* and *-p* events/cards

Cards	Low P(*p*)	High P(*p*)
p	20	80
q	36	96
-p	80	20
-q	64	04

Experimental design based on the learning of actual or objective p*robabilities of events* (not conditionals) will also be a refined test of the O&C prediction that the probability of *p* will affect the selection of *-q*. The role of the probabilities in information selection and cognition, and the way in which information is optimally represented in frequency formats in human cognition (Gigerenzer and Hoffrage, 1995), are discussed further in Chapter 5.

In conclusion, when experimental procedures produce both the predicted *p>q>-q>-p* preference ordering in the low P(*p*) condition as well as replicate the *p>-q>q>-p* card preferential ordering produced in the high P(*p*) condition, it can be confidently inferred that a maximizing principle in the form of the decision rule to optimise information gain is being applied in the selection task *as a whole*. At present, the most that can be validly inferred from the results of probability manipulation studies carried out and explained in Section B of Chapter 4 is that the high P(*p*) condition in the Second Probability study reflects that a maximizing principle is being applied, i.e. the decision rule "to optimize expected information gain" governs selections, rather than falsificationist principles. In the low P(*p*) condition, no valid inferences can be made about whether selection behaviour reflects that an optimising principles is being applied as the probabilities specified for the low P(*p*) and P(*q*) conditions were not accurately learned. However, detailed analysis of the results from the low P(*p*) condition have made it possible to understand more clearly how and why the probabilities as specified by the O&C model of optimal data selection[18] produce unpredicted probabilistic contexts, which, in turn, elicit selection behaviour incompatible with a probabilistic state in which the P(*p*) and P(*q*) are low.

The next Chapter 5 considers the theoretical implications of the above results for the O&C (1994) model of optimal data selection.

---

18 And the Pollard and Evans (1983) probability learning tasks, discussed in Chapter 5.

Chapter 5
DISCUSSION

In chapter 4, I reported seven studies carried out in order to test the O&C (1994) model of optimal data selection. The probability studies are of particular relevance to the model of optimal data selection and attention is focused on these in this discussion, especially the way in which probabilities were learned and how this may have affected selection task behaviour. For example, I concluded in the last chapter that learning about conditional probabilities may be inappropriate as actual probabilities of p and q are the parameters which influence cards selections. In Part I of this chapter 5, I consider the specific implications of experimental results and probability learning procedures for the O&C model, and how the model of optimal data selection may be refined. The Pollard and Evans (1983) probability learning studies are also re-analysed and, I show that results from their second experiment in particular substantiate an optimal data explanation of the selection task. In Part II, the general implications for the psychology of reasoning, of the O&C approach to reasoning and information selection and optimality modelling generally, are discussed.

Part I - Specific Discussion of O&C (1994) Model of Optimal Data Selection

In the discussion of the Second Probability study, I considered the way in which future probability learning studies should be designed, as in the low P(p) condition it was apparent that the O&C specified probabilities had not been learned so as to influence card selections as predicted in the selection task phase of the study. I concluded that because analysis showed (see Table 4.7.1 in Part VII of chapter 4) that participants were correct about what was on the reverse of the q card only 53% of the time, i.e. the *conditional* probabilities $q \rightarrow p$ and $q \rightarrow -p$ were not clearly discriminated, this changed the *actual* probability of p and the *actual* probability of q. Analysis of what participants perceived they had learned about the cards in the low P(p) condition supported this conclusion, and Table 4.7.2 in Part VII of chapter 4 shows that probabilities do not appear to have been learned as specified as the conditional probabilities $p \rightarrow q$ and $q \rightarrow p$ were perceived as being seen very or quite frequently, whereas they were actually presented on the computer screen only 20% of the time.

In chapter 4, I explained these results in terms of the inappropriateness of experimental procedures which require conditional probabilities rather than actual probabilities of cards to be learned. Assumptions of the O&C model may also contribute to this experimental procedural problem, as the P(p) is calculated by O&C from the *sum* of $p \rightarrow q$ and $q \rightarrow p$ (see Tables 4.7.1.1 and 4.7.1.2 in the Introduction of Part VII of chapter 4). Therefore when there is uncertainty about what is on the reverse of q (i.e. whether $q \rightarrow p$ or $q \rightarrow -p$ is the case), then the actual P(p) as well as the actual P(q) are necessarily affected. However, when the conditional probability of $p \rightarrow q$ and $q \rightarrow p$ and $q \rightarrow -p$ are discriminated, the actual probabilities of p and q are also discriminated, as is the case in the high P(p) condition (see Table 4.7.1 and 4.7.2 in the Results section of Part VII of chapter 4 which show that probabilities learned in this condition were accurately learned). The learning of *actual* probabilities at the outset would therefore avoid any loss of information about dependency or conditional relationships.

The learning of conditional probabilities was an experimental procedure in studies carried out by Pollard and Evans in 1983, details of which were given in the Introductions to Parts VI and VII of chapter 4. Results of the 1983 studies were interpreted by Pollard and Evans in 1983 in terms of logically correct card selections being facilitated when the statement "if p then q" was believed false, because counterexamples (p and $-q$ pairs) were available from memory in such cases. In 1996, Evans and Over reinterpreted the results of their probability studies using Kirby's theoretical assumption that when the probability of p increases the probability of an inconsistent outcome or hit, i.e. P($-q$), also increases. They propose that the learning probability procedures of the second Pollard and Evans (1983) experiment explicitly manipulate the probability of outcomes. For example, when the outcome $-q$ in the presence of a p was learned to be high and the rule being tested was in the material implication form $p \rightarrow q$, the selection of p and $-q$ increased, and when the outcome of $-q$ in the presence of p was learned to be low, the selection of p and $-q$ was low. Evans and Over thus argue that the 1983 studies provide evidence which falsifies the O&C optimal data explanation of selection task behaviour.

In their 1996 paper, Evans and Over reiterate the way in which probabilities were learned for the contingency $p \rightarrow q$. However, there were three other forms of rules or contingencies learned about, $p \rightarrow -q$, $-p \rightarrow q$ *and* $-p \rightarrow -q$[1] each of which in the

1 Using four forms of rule has is known as "the negations paradigm" selections tasks as antecedents and consequents of rules contain negations.

probability learning phase could have been learned to hold most of the time, i.e. be "usually true", or it could have been learned that a contingency did not usually hold, i.e. it was "usually false". In the Introduction to Part VI in chapter 4, the reason why other contingencies were used in the second experiment was explained[2]. I now explain below, the way in which all four contingencies were learned in both the "usually true" and "usually false" conditions.

Each pack of cards in the Pollard and Evans (1983) second experiment represented one of four possible contingencies: $p \rightarrow q$, $p \rightarrow -q$, $-p \rightarrow q$ or $-p \rightarrow -q$. Each participant was tested on all four types of contingencies in four separate selection tasks, each using a different pack of cards. Each pack used different symbol combinations to represent the constituents true antecedent (TA), true consequent (TC), false antecedent (FA) and false consequent (FA). In addition, the antecedent (front) side of the card was coloured red and the consequent (reverse) side of the card was coloured blue. For example:

$p \rightarrow q$ If the pack of cards used had triangles on the front and stars on the back, $p \rightarrow q$ would represent a rule stating that: if there is a triangle on one side or the (red) front (i.e. the true antecedent or "TA") then there is a star on the other side or the (blue) back (i.e. the true consequent "TC")[3].

$p \rightarrow -q$ If the pack of cards used rectangles on the front and a tick on the back of its cards, $p \rightarrow -q$ or TA and TC would represent a rule stating that: if there is a rectangle on the front (TA) there is not a tick (or no symbol) on the back (TC)[4].

$-p \rightarrow q$ If the pack of cards used diamonds on the front and crosses on the back of its cards, $-p \rightarrow q$ or TA and TC would represent a rule stating that: if there is not a diamond (or no symbol) on the front (TA) and there is a cross on the back (TC).

---

2 In the first experiment, it was concluded that results showed that the selection task rule had been interpreted as symmetrical, therefore the second study used four forms of contingency relationships in order to control for symmetry.

3 It is not clear whether the selection task used the terms "on one/other side of a card", or the words" on the front/back", or the words "on the red/blue" side.

4 It is also not clear if rules with negations stated "no symbol" or, for example, "not a tick". In addition, negated constituents (for example, "not a tick") could also be interpreted as representing "another symbol ". Pollard and Evans do not make contrast class explicit (see Oaksford & Stenning, 1992, and Oaksford and Chater, 1994 at p. 615-618)..

$-p \rightarrow -q$ The contingency $-p \rightarrow -q$ would represent a rule in which it was state that: if there is not a square on one side of the card (or no symbol), then there is not a circle (or again no symbol) on either side of the card.

In addition, in each probability learning phase (there were four probability learning phases for each participant, one for each pack of cards or contingency), the contingency being learned about could be either "usually true" or "usually false". Evans and Over (1996, p 360) table the conditional probabilities learned when the contingency being tested in the selection task phase was the material implication $p \rightarrow q$, as set out in Table 5.1 below:

Table 5.1: conditional probabilities learned in Pollard and Evans (1985) study for $p \rightarrow q$ contingency

Conditional probabilities learned	$\boldsymbol{p \rightarrow q}$ Usually True	$\boldsymbol{p \rightarrow q}$ Usually False
$p \rightarrow q$ or TA and TC	7	1
$p \rightarrow -q$ or TA and FC	1	7
$-p \rightarrow q$ or FA and TC	7	7
$-p \rightarrow -q$ or FA and FC	7	7

For example, in the "usually true" condition of the $p \rightarrow q$ pack of cards, q (TC) was learned to be on the reverse of p (TA) seven out of the eight times p (TC) was displayed on the computer screen, and $-q$ (FC) was on the reverse of p once out of these eight times. The inverse was the case in the "usually false" condition.

Having learned about the "$p \rightarrow q$ pack of cards" to criterion, in the selection phase participants were told to select two red cards (one with a symbol and one without, i.e. a p or TA and $-p$ or FA), and then to select two blue cards (one with a symbol and one without, i.e. a q or TC and $-q$ or a FC) without turning over the cards to see their reverse sides. The contingency $p \rightarrow q$ was then given to test, the task being to "decide which of the cards it would be necessary to turn over in order to find out whether the statement was true or false".

However, the probability of the $p \rightarrow q$ rule or contingency depended on prior learning experience. For example, in the "usually true" condition as participants had learned that the $p \rightarrow q$ contingency was usually true for the pack of cards, the $p \rightarrow q$ rule they evaluated was consistent with their learning experience. But in the "usually false" condition, the $p \rightarrow q$ rule evaluated in the selection task phase was unlikely to be true for the pack of cards learned about. In other words, in the "usually true" belief condition, it was learned that the probability of the rule $p \rightarrow q$ was high, but in the "usually false" belief condition, it was learned that the probability of **another** rule, $p \rightarrow \text{-}q$, was high (and the probability of $p \rightarrow q$ was low). In terms consistent with Bayesian methods, two different hypotheses about which symbols and colours went together were compared in the usually false belief condition. I illustrate this more clearly in Tables 5.2 to 5.5 below:

Table 5.2: P($p \rightarrow q$) *AA* selection task rule in "usually true" and "usually false" conditions

Contingency $\boldsymbol{p \rightarrow q}$ / Belief Condition	Rule learned to be usually true or highly probable	Rule evaluated, its probability, and subsequent card selection ordering
Usually true	P($p \rightarrow q$)	$p \rightarrow q$ - **high** *TA>TC>FC>FA*
Usually false	P($p \rightarrow \text{-}q$)	$p \rightarrow q$ - **low** *TA>TC>FC>FA*

For example, in the usually true condition, the probability of the $p \rightarrow q$ rule tested in the selection task phase being true, i.e. being the rule for the pack of cards learned about, was high as the rule was consistent with learning experience. In the usually false condition, however, the probability of the $p \rightarrow q$ rule tested in the selection task phase being true, i.e. being the rule for the pack of cards learned about, was low or inconsistent with learning experience. Card preference ordering in the usually true or high P(rule) condition was TA>TC>FC>FA and card selection for the low P(rule) condition is also TA>TC>FC>FA.

In the negations paradigm generally, it has been consistently found that when the consequent in the selection task rule is **affirmative**, card selection ordering is TA>TC>FC>FA. But when the consequent is a **negation**, ordering changes to TA>FC>TC>FA (i.e. the optimal data selection rarity assumption, that the probability of TA and TC are low, has been violated). The card ordering produced in the AA (affirmative antecedent and affirmative consequent) selection task of Pollard and Evans (1983) study thus replicates a consistent finding for AA rules. These selection orderings also reflect the optimal data assumption that rarity (or low P(p) and P(q), TA and TC) holds in AA rules. However, the selection orderings in Table 5.2 above mean that contrived learning experience did not vary probabilities of p and q and therefore change card selections to produce a TA>FC>TC>FA ordering. Although Pollard and Evans note that, in the "usually false" condition, TA and TC selections decreased and FC and FA selections increased (which optimal data selection would envisage if P(TA) or p and P(TC)) or q are increasing from low to high probabilities.

The three further tables 5.3 to 5.5 below set out the results for NA (negated antecedent and affirmative consequent), NN (negated antecedent and negated consequent) and AN (affirmative antecedent and negated consequent) rules, respectively.

The NA or $\boldsymbol{-p \rightarrow q}$ rule results show, in Table 5.3 below, that in both the "usually true" and "usually false" belief conditions, TA and TC are optimal selections, i.e. card selection ordering is consistent with rules in which the consequent in the selection task rule is affirmative, i.e. TA>TC>FC>FA.

Table 5.3: probability of $-p \rightarrow q$ selection task rule in "usually true" and "usually false" conditions

Contingency $\boldsymbol{-p \rightarrow q}$ Belief Condition	Rule learned to be usually true or highly probable	Rule, its probability and subsequent card ordering
Usually true	P($-p \rightarrow q$)	$-p \rightarrow q$ - **high** *TA>TC>FC>FA*
Usually false	P($-p \rightarrow -q$)	$-p \rightarrow q$ - low *TA>TC>FC>FA*

The next Table 5.4 below is concerned with the NN rule or $\boldsymbol{-p \rightarrow -q}$. Results show that in the "usually true" belief condition, TA and TC are optimal selections, and in the "usually false" condition, TA and FC are optimal. The card ordering in the "usually true" or high probability condition is therefore inconsistent with previous findings for rules with negated consequents, where TC>**FC**>TC>FA ordering is usually produced. The Pollard and Evans NN rule results are considered further in a later paragraph.

Table 5.4: (P-$p \rightarrow -q$) or NN selection task rule in "usually true" and "usually false" conditions

Contingency $\boldsymbol{-p \rightarrow -q}$ Belief Condition	Rule learned to be usually true or highly probable	Rule, its probability and subsequent card ordering
Usually true	P($-p \rightarrow -q$)	$-p \rightarrow -q$ - **high** *TA>TC>FC>FA*
Usually false	P($-p \rightarrow q$)	$-p \rightarrow -q$ - **low** *TA>FC>FA>TC*

Table 5.5 below is concerned with the AN rule or $\boldsymbol{p \rightarrow -q}$. Results show that in the "usually true" belief condition, TA and TC are optimal selections, and in the "usually false" condition, TA and FC are optimal selections.

Table 5.5: P($p \rightarrow -q$) AN selection task rule in "usually true" and "usually false" conditions

Contingency $p \rightarrow -q$ / Belief Condition	Rule learned to be usually true or highly probable	Rule, its probability and subsequent card ordering
Usually true	P($p \rightarrow -q$)	$p \rightarrow -q$ - **high** *TA>TC>FC>FA*
Usually false	P($p \rightarrow q$)	$p \rightarrow -q$ - **low** *TA>FC>TC>FA*

As in Table 5.4 above, the card ordering in the "usually true" or high probability condition is inconsistent with previous findings for rules with negated consequents, where TC>**FC**>TC>FA ordering is usually produced.

In order to explain the above results, Table 5.6 below sets out O&C's assumptions about the probability of the consequent card in each rule form, i.e. $p \rightarrow q$ or AA rule, $-p \rightarrow q$ or NA rule, $-p \rightarrow -q$ or NN rule, and $p \rightarrow -q$ or AN rule. The selections O&C predict for each contingency are also given, as well as actual selections depending on whether the probability of the selection task rule was low ("usually false" condition) or whether the probability of the selection task rule was high ("usually true" condition).

Table 5.6: O&C assumptions and (consequent) predictions in AA, NA, NN, and AN rules, and actual consequent selections depending on probability of selection task rule

Rule Form	O&C Assumptions about AA, NA, NN, AN	O&C Selection Predictions	Actual Card Selections **Low P(Rule)**	Actual Card Selections **High P(Rule)**
AA	P(TC) low	TC	TC	TC
NA	P(TC) low	TC	TC	TC
NN	P(TC) high	FC	FC	TC
AN	P(TC) high	FC	FC	TC

O&C **generally** assume that the probability of an **affirmative** constituent is low and the probability of a **negated** constituent is high. For example, if p represents a "specific symbol" and $-p$ represents "not a specific symbol" or "no symbol" there are more $-p$s in the world than p. Therefore, negated constituents have a higher probability than affirmative constituents.

As far as the **AA rule** specifically is concerned, and as emphasised throughout this thesis as experimental studies used modified version of the abstract AA selection task rule, O&C assume that rarity holds unless the P(TA) or p and P(TC) or q have been varied. In **AA and NA rules** therefore, as they both have affirmative consequents, the probability of the affirmative true consequent is low and TA and TC should be optimal selections as they reduces uncertainty the most. In **AN and NN rules**, which both have negated consequents, O&C assume that the probability of a negated consequent is high, for reasons already explained, and TA and FC should therefore be optimal selections as, when the P(TA) and P(TC) are high, FC reduces uncertainty more than TC.

Referring to Table 5.6 again, results of Pollard and Evans (1983) experiments show that when the probability of the selection task rule was low, i.e. **in the "usually false" belief condition or low P(rule) condition,** selection behaviour as predicted by O&C's model of optimal data selection was observed. For example, the AA and NA rules reflect the assumption that rarity holds and thus the above TA>**TC**>FC>FA ordering is produced. In contrast, NN and AN rules changed this ordering to TA>**FC**>TC>FA in the "usually false" condition and thus substantiate the assumption that the probability of negated antecedents is high (i.e. that rarity is violated).

In the "usually true" or high P(rule) condition, however, the card ordering produced by NN and AN rules changed to the consistently found ordering of TA>**TC**>FC>FA expected when rarity holds (i.e. when the probability of a consequent is low, specifically, affirmative constituents). Using the O&C model of the negations paradigm, I propose that card ordering change, where selection of FC has changed to selection of TC, has come about because the probability learning task has given participants in the usually true or high P(rule) condition sufficient time in which to *restrict* contrast classes in NN and AN rules, rather than probabilities having been manipulated as Pollard and Evans suggest.

For example, in the NN selection task, the rule could have been "if there is not a rectangle or no symbol on one side of the card (-*p*), then there is not a rectangle or no symbol on the other side of the card (-*q*)". As the antecedent and consequent sides of the cards in the Pollard and Evans second experiments were red and blue, respectively, the NN rule may have been *restricted* to an AA rule such as "if red on one side of the card (p = red), then blue is on the other side of the card (q = blue)". In this way, the probabilities of the antecedent and the consequent were reduced, i.e. rarity was restored, in which case TC rather than FC is an optimal selection and ordering will be that associated with rules in which the consequent is affirmative, i.e. TA>TC>FC>FA. The probabilities attached to the AN rule may have been similarly revised by restricting the rule to an AA rule such as "if a card has a rectangle on one side, then the other side is blue" rather than "if a card has a rectangle on one side then there is not a rectangle (or no symbol) on the other side".

Pollard and Evans in 1983 explained their experimental results in terms of logically correct card selections being facilitated because counterexamples were available from memory. Evans and Over (1996) re-interpreted the 1983 results within the theoretical framework of Kirby's hypothesis that when the probability of *p* increases then the probability of a hit or an inconsistent outcome, i.e. P(outcome -*q*), also increases. They propose that when the outcome -*q* in the presence of a *p* was learned to be high (i.e. "usually true") in the AA or $p \rightarrow q$ rule, selection of *p* and -*q* increases, and when the outcome of -*q* in the presence of *p* was learned to be low, the selection of *p* and -*q* decreased. Given this re-interpretation, Evans and Over (1996) propose that the Pollard and Evans experiments explicitly manipulated the probability of *p* in order to influence the probability of outcomes. Furthermore, they argue that the 1983 experimental results provide evidence which falsifies the O&C optimal data explanation of selection task behaviour. However, when *all* the results of the 1983 negations paradigm are considered in Bayesian optimal data terms, the Pollard and Evans studies provide evidence in support of the O&C optimality model of the negations paradigm as decisions do appear to be based on uncertainty reduction about contrast classes.

While Evans and Over's (1996) argument regarding the inappropriateness of an optimal data selection explanation of the selection task is not valid, their criticism of the O&C model because it does not take into account variations in the utility of evidence is a constructive point to make.

Manktelow and Over (1991), when discussing deontic reasoning, argue that the utility (or benefits and costs) of possible actions is an important factor to assess. Evans and Over (1996) when discussing the O&C (1994) optimality model specifically propose that the utility of evidence and/or selections may be differently weighted depending on a person's goal. They assume that "people's *subjective epistemic utility* is measured by the relevance of some data for them given their [epistemic, or knowledge-serving] goals... this utility is personal and therefore a rationality$_1$ concept" (p. 358). More specifically, Evans, Over and Manktelow (1993, p.167) write that maintaining a coherent and accurate belief system is necessary for our survival and achievement of goals in the world: this should be considered as a goal in itself.

Such goals are thus termed as epistemic or knowledge serving goals or inferences. It is these epistemic goals which Evans and Over propose should be incorporated into the O&C model of optimal data selection.

In my view, subjective epistemic utility is in some respects similar to the notion of "satiation". For example, in chapter 3, Part I, I gave an example of satiation used by Mazur (1994) to show the way in which the utility or value *to an individual* of a most favourite type book purchase decreases, and how the utility of the second favourite type book can become more than the value or utility of the most favourite book. In these circumstances, overall subjective utility is then maximised. In other words, an individual's belief system is continually being revised.

However, and as far as non-thematic versions of the selection task are concerned, I suggest that the O&C model needs rather to calculate decrements in the expected information gain of each card, as each card's information gain remains fixed. A clear distinction thus needs to been made: between variation in the utility of evidence to an individual, and decrements to information itself[5]. The latter distinction is concerned with calculating depletion of information in a set of properties in the (external) environment, for example, depletion of expected information gain over time. This form of decrement may provide an explanation of why models other than those specified by the O&C model of optimal data selection evolved in the First Probability study and in the low P(*p*) condition of the Second Probability study. In other words, dissipation of information may be the basic mechanism which underlies model evolution and attentional changes necessary to bring this about.

As a simple optimality approach to cognition is concerned with the specification of events in the environment, the notion of "satiation" (i.e. decrements in the subjective

5 When modelling *thematic* versions of the selection task, O&C (1994) calculate information gain for each card, and expected utilities are also calculated. This additional calculation for thematic selection tasks is made in order to take into account the perspective an individual adopts (Cosmides, 1989, Gigerenzer and Hug, 1992) which will influence optimal selection behaviour. In abstract versions of the selection task, "perspective" is not varied. But, as regards perspective, or in signal detection term, criterion: when watching a radar screen, and if the criterion is to look out for enemy planes, it is important that there is a clear and highly certain signal on the screen before taking action (firing missiles or bombs). But a doctor looking at an x-ray screen would need only the smallest of signals to prompt action if the criterion was to look out for cancer. When non-thematic versions of the selection task manipulate criterion or perspective in this sense then variations in the utility of information to an individual will also need to be calculated. The way in which mood and other affective states (such as *automatic negative thoughts* associated with clinical depression) influence reasoning may be another example of how different "perspectives" (or decrements in the utility of information to an individual) alter optimal data selection (Oaksford, Morris, Grainger and Williams, 1996).

utility of preferences and/or beliefs, to an individual) is less appropriate for the O&C model of optimal data selection as far as *non-thematic* selection task modelling is concerned For example, when modelling animal behaviour, Bell (1991, p. 95) writes that animals forage in the most profitable patches and leave when the profitability of resources in that patch declines. In selection task terms, reasoners will select the most informative card (profitable patch) until its information gain decreases to a threshold where it is more profitable to select another card with more expected information gain (i.e. move to another "patch"). This way of conceptualising decrements, i.e. as the decreasing informativeness of a card over time, in my view, is more suitable for an optimality model of behaviour in abstract versions of the selection task (unless mood, state or other criteria need to be modelled)[6].

In chapter 3, Part I, I mentioned that simple optimality modelling of animal behaviour used "prey" and "patch" models, where prey models are concerned with the decision rule of what prey to attack (what card to select), and a patch model is concerned with how long to stay in a patch (how long to select a card on the basis of its optimal information gain). The O&C (1994) model of optimal data selection is essentially a "prey" model as it is concerned with what information to select. Refinements to the O&C model to include the calculation of decrements in information gain would therefore seem to require that their model becomes a combination of a prey and patch model.

Regarding when to move patch, Bell (1991) writes that "... at some point in time even a high quality patch with a relatively high initial resource density [a highly informative card] turns into a lower quality patch as resources [expected information gain] are depleted. As to when to leave a patch, Stephens and Krebs (1986, p. 173) summarise four possible decision rules:

(i) A *number* decision rule, to leave after catching *n* prey.

(ii) A *time* decision rule, to leave after *t* seconds,

(iii) a *giving-up-time* decision rule, to leave after *g* seconds of unsuccessful search;

6 For example, Krebs and Kacelnik (1991, p. 119) cite an optimality study carried out by Milinski and Heller in 1978 who compared the behaviour of hungry and well-fed (i.e. "internal satiation") sticklebacks. Hungry fish were more likely to feed in a patch where the probable rate of food intake was higher but danger of predation was also higher.

(iv) a *capture rate* decision rule, to leave when instantaneous prey intake rate drops to a critical value *r*.

Bell (1991, p. 114) writes that decisions about when to leave a depleting patch (when a card is no longer an optimal selection) seem to be based on an animal's "perception" of an environment's *average prey density* (i.e. average EI_g*s* for all four cards). An animal will leave a particular patch in its environment or habitat when the prey density of a patch (probabilities and consequent EI_g of a card) reaches the average prey density of all patches in an animal's environment (i.e. average EI_gs of all four cards). When prey density increases, the probability of leaving a patch will decrease.

Talking about animals' perception of prey density, as measured indirectly by prey capture rates, is the same as saying that the frequency of information affects the decisions we make. This issue is considered by Gigerenzer and Hoffrage (1995) who propose that *frequency* formats are the ways in which information is acquired in natural sampling in animal foraging and in neural networks. Evolution has favoured this form of information representation in linguistic organisms, i.e. human beings, too (p. 142). The way in which the frequency of information, or prey density, was used to measure the decreasing utility of a property (expected energy or information gain) and when to leave a patch, is illustrated in a study carried out by Krebs, Erichson, Webber and Charnov (1977, in Bell p. 112):

Birds were given a chance to select large or small prey in the form of different sized pieces of mealworms which were presented sequentially on a moving conveyor belt. As mealworm pieces moved by, the bird could choose whether to pick one up or leave it. The prey encounter rate could be controlled by the rate of the conveyor belt such that prey encounter was low (0.025 prey per second) in one condition and higher (0.15 prey per second) in the other. When the encounter rate or probability of large and small prey types was low (resources nearly depleted), the birds were not selective and ate whatever prey was before them. When the encounter rates were higher, the birds selected the large mealworm pieces. This experiment is deemed to provide good evidence that animals are *aware of the number of prey consumed per unit time* (i.e. the probable frequency of information) which is an important proximate mechanism for estimating resource density and whether to leave a patch (refocus attention and selection behaviour).

In conclusion, Part I of this chapter 5 has been concerned with specific details of the O&C model of optimal data selection. I consider the weaknesses of learning about conditional rather than actual probabilities in the First and Second Probability studies. In this regard, animal learning studies provide support for the view that the frequency of information, i.e. the actual probability of information in the environment, is the way in which information should be presented in future experiments, and generally in decision making settings. The way in which optimal data selection accounts for results in the Pollard and Evans (1983) studies is then discussed.

I also consider the possible consequences of not calculating decrements in initial information gain, which may have exacerbated problems (such as model evolution) associated with learning about conditional probabilities which were not clearly distinguished. I propose that refinements of the O&C model of optimal data selection need to incorporate expected information gain decrement calculations in order to ensure that actual probabilities (rather than conditional probabilities) are learned as specified thus removing the likelihood of attentional shifts and/or new models evolving. To reach this conclusion, "distinctions" between decrements in the utility of information to an individual were compared with decrements to information itself in the (external) environment.

As outlined in Part VII of chapter 4, unpredicted results in the two conditions in the First Probability study and in the low P(*p*) condition in the Second Probability study may be due to a different decision rule being applied. For example, rather than a decision rule to optimise expected information governing behaviour, in certain contexts (i.e. when preferences about information gain cannot be distinguished), it may be more appropriate, i.e. adaptive, to use a decision rule which optimises global or other fitness. This is a broad optimality issue, and is not specific to the O&C model of optimal data selection. For this reason, the role of decisions rules and other optimality issues are discussed in Part II of this chapter 5.

In Part I, I considered the specific implications of experimental results and probability learning procedures for the O&C (1994) model of optimal data selection and how this optimality model may be refined. In Part II, the implications of optimality modelling for the psychology of reasoning generally are discussed.

Part II - General Discussion An Optimality Approach to Information Selection and Reasoning

In Part VII of chapter 4, I proposed that unpredicted results in the two conditions in the First Probability study and in the low P(*p*) condition in the Second Probability study may be explained in three different ways:

(a) The decision rule to optimise expected information gain is governing selection behaviour but in a new probabilistic context or model not specified or expected by O&C.

(b) A new probabilistic context may have evolved in which it is not possible to distinguish card informativeness or other preferences The issue of whether preferences about information gain can be distinguished is of particular importance to an optimality approach to cognition, because without any preference being distinguished it cannot be inferred that a maximisation principle (in whatever form) is being applied (see Part I of chapter 3).

(c) A different optimality-preserving decision rule, for example, to optimise global fitness, may be governing selection behaviour in either the O&C specified probabilistic context or in the newly evolved probabilistic context.

In Part I, I explained the evolution of new models, the issue raised in (a) above, in terms of conditional probabilities not being clearly discriminated and learned as specified. This problem of model evolution was probably compounded by decrements in the information gain of each card over time, which decrements the O&C model does not calculate. The remaining two issues, (b) and (c) above, raise other important questions about distinguishing preferences, and which decisions rules govern behaviour in different context.

For example, the O&C model of optimal data selection rejects the view that the goal or (truth-preserving) decision rule in the selection task is to falsify and look for counterexamples. An optimality approach, in turn, questions whether an optimality-preserving decision rule where one set of properties in the environment (expected information gain) is the appropriate decision rule in all contexts. These issues are of importance to the psychology of information selection and reasoning and are considered further below.

In Part II, I use further examples from animal behaviour studies to illustrate the complexity of context-dependent selection behaviour more clearly. For example, the work of Provenza and Cincotta (1993) is outlined as it considers transitional states where perhaps preferences cannot be clearly distinguished and where optimisation need not always be the principle governing behaviour. A further study by Custard (1977) is then described as it shows the conditions in which the most nutritious resources are not always selected. These optimality studies are relevant to the psychology of reasoning because they provide simple explanations of the conditions or probabilistic contexts which produce behaviour observed in the selection task.

Besides model evolution, another explanation of equal card selection may be that the context is one of transition in which no discrimination between the informativeness of cards can be made. If this is the case, it is not be possible to infer that a maximization principle is being applied, as in transitional states optimizing behaviour is only a possible outcome[1]. Theoretical argument in support of this explanation of selection task behaviour is provided by Provenza and Cincotta (1993), who argue that assuming that a state of transition exists is a reasonable option when explaining unpredicted behaviour. They propose a hill climbing model of behaviour and the specific analogy used is that of a hill climber on a foggy day who has to rely on memory of past experience to determine which direction to climb as well as relying on additional feedback that occurs while actually climbing in the fog. The important point is that, even though a number of individuals are climbing the same foggy hill at the same time as each other, because the conditions are unclear and there are differences in past learning experience, different paths will be taken. Selection behaviour in these

---

1 Manktelow and Over (1992) and Evans, Over and Manktelow (1993) do not assume that preferences are always maximised as normative decision theory assumes is the case. Over and Manktelow (1993, p. 236) assume that people are expressing subjective utility judgement when they make preferences. But subjective utility need not always be maximised, perhaps because preferences are unclear, confused, or even inconsistent (Manktelow and Over, 1993, p248). Ideal preferences may change because of overriding prudential, social or moral rules.

ambiguous circumstances may or may not lead to the top of the hill (or optimizing expected information gain).

Provenza and Cincotta argue that the *process* of adaptation, which involves transitional states in which learning from past experience is necessary, is ignored by optimality models of cognition. They emphasise that behaviour in the long term and in transitional or "foggy" states is not always stable, optimal, or predictable, as behaviour is a dynamic or stochastic process not a static state composed of fixed events and solutions which can always be determined. In other words, transitional states (in which preferences may or may not be discriminated) are probably the circumstances in which, in selection task terms, new, unexpected models evolve which may or may not be governed by the decision rule to optimise expected information gain.

While it may not always be possible to discriminate preferences about the informativeness of cards and thereafter optimize expected information gain, it may be that a decision rule based on optimising longer term or more global preferences (rather than expected information gain) can be more adaptively applied. In Part I of chapter 3, section 3.1.7, I detailed John-Goss Custard's (1977) first study of the foraging behaviour of redshank ,whose main prey were large worms, as an example of classical optimal diet selection. I now outline a follow-up study in which Goss-Custard (1977) found that when *amphipos crustacean corophium* was available in addition to *polychaete* worms, redshank fish tended to select *corophium.* He discounted the possibility that the habitat typical of *corophium* was one in which *polychaete* worms were hard to find as some birds concentrated on worms while the majority fed on *corophium.* Prior to the second study, Goss-Custard assumed that redshank would achieve a higher rate of net energy intake by feeding on *corophium* than by taking worms, but subsequent analysis of the energy content of the prey and the energy cost of obtaining prey showed that two and three times more energy per minute would have been obtained by the redshanks taking worms exclusively than by feeding on *corophium.* Energy was clearly not the only factor relevant to foraging redshank when *corophium* was available. Presumably, *corophium* contains something other than energy that is important to the redshank (see McFarland, 1993, p. 443) . As this latter study shows, optimal diet selection can be *traded-off* with other-than-energy benefits.

In Part I of chapter 3, I detailed the way in which Krebs and Kacelnik 1991) model trade-offs (energy gain traded off with risk of predation) in order to compute expected terminal reward which decision rule incorporates more than one set of properties in an environment rather than the classical optimality decision rule which maximises only one set of properties, e.g. energy or information gain. Krebs and Kacelnik's model of cognition predicts that when food resources are high a patch with a low risk of predation is an optimal selection. But if reserves are low, the model predicts that it pays to select a risky patch with a higher probability of food, and if reserves are neither high nor low, selection of the safe patch followed by the risky patch is optimal behaviour.

Selection behaviour is also affected by the overall scarcity of information. For example, Werner and Hall (1974)[2] observed how bluegill sunfish perceived their habitat or context in terms of "capture rates". They tested to see if selectivity of prey increased as prey density increased (see also the study of Krebs, Erichson, Webber and Charnov, 1977, described in Part I of this chapter). In the Werner and Hall study, the nutrients ingested per unit time were nearly equal for all prey, and the following context-dependent results were found:

(i) Bluegill sunfish were not selective when hunting in a mixture of small, medium and large size prey at a *low* density (20 of each class), but instead consumed *all three size classes* according to how often each was encountered.

(ii) The bluegill sunfish became more selective at *high* prey densities (350 of each class), consuming mainly the *largest size prey.*

(iii) At an *intermediate* density bluegill sunfish chose mainly the *two largest size classes* of prey.

Relating these conclusions to the selection task and in particular to the predictions of the O&C model of optimal data selection, performance in the high P(p) condition in the Second Probability study is comparative to selectivity of the bluegill sunfish in high prey density contexts: when resources are in abundance, the decision rule is to select the largest and therefore most nutritious prey. In selection task terms, when the

2 In Bell, 1991 p.111.

P(p) and P(q) are high, the O&C optimality model similarly predicts that the decision rules to select the most informative cards (-p and -q) will apply[3].

As regards performance in the low P(p) condition in the Second Probability study, equality of card selections (i.e. non-significant differences in card selections) may be similar to prey selection behaviour of the bluegill sunfish in a low prey density context. In other words, when *all* resources are perceived to be relatively scarce, the decision rule governing selection behaviour may change to optimize global fitness or long-term survival, and all sizes of prey and not only the largest prey, are consumed. No significant differences in card selections in the low P(p) experimental condition in the Second Probability study (and in the Pollard and Evans (1983) first experiment) may therefore reflect that when *all* cards are perceived to give little or equal information, selection of apparently sub-optimal, less "nutritious" but available information is an adaptive, long-term fitness response rather than waiting to encounter scarce but highly-optimal information (short-term fitness), and this decision rule applies until a model evolves in which there is optimal information to select.

Other more global decision rules are also part of human behaviour. For example, selection behaviour can be governed by a decision rule to optimise the utility of group rather than individual preferences, as is the case in altruistic behaviour (Cosmides, 1989; Manktelow and Over, 1993, p. 255). McFarland (1989) writes that at the level of an individual, the costs of altruism are usually in terms of time and energy, and in complex societies altruism may be in the form of lending tools, baby-sitting or other favours which may or may not be quantified in terms of money. Parental care is a form of altruism because the parent diminishes his or her own fitness by investing time and energy in the care of offspring, but this is regarded as a selfish altruism (Dawkins, 1976 in McFarland, 1989 p. 59). Reciprocal altruism, on the other hand, is the decision rule which governs families and hunter-gatherer societies which do not use coercive exchange. These co-operative situations are based on sharing, and cheaters (or those who do not always contribute to, say, food production) are subject to subtle forms of disapproval but demands are not made upon them to produce in *particular* cases. In contexts in which an egalitarian contribution by all is important, cheaters are less tolerated and formal books of accounts would be kept of contributions made in order that a fair balance is achieved. And, finally, in societies

3 As already discussed, Kirby (1994) when explaining performance on the selection task does not argue that cards are selected because they are most informative or optimal cards to select. Instead Kirby assumes that the probability of p being high is the factor which causes the selection of the -q card to increase, as the probability of there being a hit (a -q) increases when the probability of p increases. O&C argue that the probability of information *and* the optimality of information are both factors in selection.

in which only certain people contribute to production, violation of rules and regulations by these specific producers would be policed[4].

The above optimality studies of animal behaviour reveal an advantage of using optimality modelling: it is possible to distinguish different decision rules which may be more or less appropriate in different contexts. However, one of the major criticisms of an optimality approach to cognition is that, in theory, all behaviour can be explained if everything in that environment is specified. In Part I of chapter 3, I point out that McFarland is a rationalist in this strict sense as he assumes that behaviour can be determined as rational if an environment is fully determined and specified. However, the issue of whether the whole array or all information in an environment can very be specified is debatable because unpredictable and unknown events, stochasticity, occur. For example, in order to refine the O&C model of optimal data selection by specifying and accounting for loss of information in the selection task environment, I suggested that it needs to become a "prey combined with patch" model of selection behaviour. However, when these events in the revised selection task environment are taken into account, additional refinements may again be necessary in order to specify and account for other events or information in (either a broader or narrower) environment or internal to an individual.

The view that it is not possible to specify the whole array of information in an environment (or personal to an individual) because the environment and behaviour are not deterministic, is related to another issue for which an optimality approach is criticised: neglecting the role of mechanisms underlying behaviour. For example, Provenza and Cincotta (1993) point out that in order to move from a state of incomplete information to a different state with more information, three basic steps are assumed. Firstly, an optimality approach assumes that an organism will remain fairly stable yet make enough errors to explore alternatives without becoming extinct; secondly, memories of these errors needs to be kept thus creating a new transitional state in which different alternatives are explored; and thirdly, the adaptive value of the new state must be recognised so that the first step can be repeated (i.e. seeing the necessity of revising beliefs), and so on. The *process of adaptation,* which includes

4 Cosmides (1989) explains the selection task in terms the costs and benefits of social contracts, as well as in terms of innate Darwinian algorithms which have evolved to enable the detection of cheaters. Most relevantly, she highlights the notion that different perspectives, i.e. environments or economies, can have different views on cheaters and therefore different subjective utilities are assigned to the violation of regulations by cheaters.

transition and memory, is thus the implicit basis for optimization, although this process is ignored in most optimality modelling.

Different individual past learning experience can thus produce diversity in behaviour when events in an environment are perceived to be ambiguous or indistinct from each other, and in this way there is "creativity". A strict optimality approach to cognition has problems in explaining individual diversity, although Anderson (1991) proposes two solutions and assumes there are more. The first solution to the question of diversity he gives is that different people may select different decision rules or strategies, but Anderson does not say "why" (Provenza and Cincotta's view that the process of adaptation, which includes transitions and memories, is one way in which diversity may occur). Anderson's second solution is that the same decision rules apply to all people, but "parameters of the model" (i.e. metabolic expenditure and cost of memory search given different capabilities) may be different for individuals. In both instances, people are assumed to behave optimally depending on their capabilities.

Notwithstanding the above weaknesses, optimality models do have explanatory and inferential value as they provide a way of comparing observed with predicted behaviour. For example, Stephens and Krebs (1986, p. 213) write that models of "ideal swimming" and "ideal digging" behaviours can be compared in order to ascertain whether a flattened appendage is designed for swimming or digging. As the different models make different predictions about how to use the appendage, it can be seen whether behaviour conforms to one model more than the other model, or whether the appendage is a compromise between swimming and digging functions. In similar terms, it is my view that selection task behaviour has been shown to conform to the predictions made by optimal data selection, rather than predictions made by models of reasoning which assume that truth-preserving inferential decision rules (in the form of mental logics or mental models) govern behaviour in an analytic or other stage (or component) of reasoning.

More specifically, most theories of reasoning considered in this thesis (mental logics, mental models, and the heuristic-analytic approach) assume that the goal of a reasoner in the selection task (i.e. the decision rule being applied) is to look for counterexamples, or to falsify (or look for violations). This theoretical assumption is based on the traditional view that falsification, or the application of truth-preserving rules or principles such as *modus tollendo tollens*, are the means (processes) by which correct inferences can be made at an inferential or analytic stage of reasoning. However, none of these theories is able to give a full account of abstract selection task results whereas explanation is possible within the framework of Bayesian optimal data selection.

As regards basic optimality models, of which the O&C model of optimal data selection is one, Krebs and Kacelnik (1991 p. 126) conclude that they give a simple intuitive feel of what is going on and often generate successful predictions. But their simplicity is also their weakness as relationships between information gain and other fitness criteria are ignored. The O&C model was formulated in order to demonstrate the role of information gain in reducing uncertainty about which of two models holds in the selection task. But unpredicted results have also made it possible to investigate the role of information loss and increasing uncertainty which, in turn, may permit the evolution of states of transition where creativity based on past learning is an appropriate adaptive response.

As far as scientific hypothesis testing and the psychology of reasoning are concerned, an optimality approach provides evidence that scientific methods need to acknowledge the psychology of information selection in hypothesis construction, and as far as hypothesis testing is concerned, to understand that looking for falsifying instances is not what people naturally do, as behaviour is optimally adapted to the environment. An optimality approach to reasoning shows that it need not be the case that irrational or too many unnecessary inferences are made, rather there are often too many contexts in each of which what is optimal selection changes and, in some circumstances, decision rules themselves may change.

Generally, analysing behaviour in optimality terms can provide methods of researching, say, the difference between novice and expert reasoning, and what selection mechanisms are implicated in stereotyping and prejudice. For example, does a novice clinician select optimal information but from a context where information or experience is scarce or ambiguous, whereas the expert-clinician makes decisions in an information or experience rich, unambiguous context? Or are different decision rules being applied by novices and experts? Is stereotyping a function of selecting optimal information, and does prejudice reflect non-adaptation to different contexts and gains in information and, if necessary, to apply different decision rules?

Another issue, which has developed out of researching an optimality approach to cognition, is concerned with the format, probabilistic or frequency (i.e. natural numbers), in which information should be represented or conveyed (Gigerenzer, 1994). This point is of particular relevance in the growing area of genetic testing and genetic counselling, where advice about risk of disease may be better conveyed using frequency formats in order than individuals may optimally assess whether or not to have a genetic test in order to ascertain their risk or vulnerability to a certain disease or illness.

In a related field, the basic optimality assumption regarding specification of events and their probabilities (and subsequent optimality) in an environment is most applicable to clinical interventions designed to reduce anxiety and the hopelessness which cancer patients feel when diagnosed as terminally ill. For example, Parle, Jones and Maguire (1996) identify the kind and amount of information which facilitates prevention of serious affective disorders in cancer patients. Their interventions explicitly seek to specify all the events of concern to a patient (for example, emotional stress and short term and long term worries), not only the events which a clinician may think are relevant and appropriate to discuss (for example, objective risk and physical pain). Maguire, Faulkner, Booth, Elliott and Hillier (1996) further investigate the way in which health professionals communicate information to and receive information from patients. The rationale of this research is to improve intervention methods and communication techniques used by health professionals so that full disclosure (i.e. full specification of an environment) by patients is made possible, thereby decreasing the likelihood of severe clinical depression.

In conclusion, the way in which we theorise about rationality and how people reason, make decisions, communicate and assimilate information, has implications for the way in which we live and understand our everyday lives in the world: rationality should be about the ability to adapt to a changing environment by choosing the most relevant and appropriate decision rules and information in given circumstances.

LIST OF APPENDICES

APPENDIX 3.1
O&C MODEL OF OPTIMAL DATA SELECTION EQUATIONS

O&C (1994) calculate the prior probabilities or information gain of data (D) using Shannon-Wiener (Shannon and Weaver, 1949, Weiner 1948 - see O&C, 1994 p. 610) information measure as follows:

(1) information before receiving D: $I(H_i) = -\sum_{i=1}^{n} P(H_i)\log_2 P(H_i)$

(2) information after receiving D: $I(H_i|D) = -\sum_{i=1}^{n} P(H_i|D)\log_2 P(H_i|D)$

(3) Bayes Theorom $P(H_i|D) = \dfrac{P(D|H_i)P(H_i)}{-\sum_{j=1}^{n} P(D|H_j)P(H_j)}$

(4) information gain: $I_g = I(H_i) - I(H_i|D)$

(5) expected information gain[1]: $E(I_g) = E[I(H_i) - I(H_i|D)]$

(6) E(Ig) conditional upon: $P(p\backslash q, MD) = \dfrac{P(p, q|MD)}{P(p, q|MD) + P(-p, q|MD)}$

(7) $b = \dfrac{P(q) - P(p)P(MD)}{1 - P(p)P(MD)}$

$[P(q) \geq P(p(P(MD))]$[2]

---

1 O&C rewrite this $E(I_g)$ equation as allowed by probaiblity theory (see O&C , 1996, their equation (6) on page 610) for full details)

2 Inequality between p and q must be respected.

APPENDIX 3.2
O&C MODEL
MD and MI CONTINGENCY TABLES

The below contingency tables for the model of dependency and model of independence[1] are reproduced from Oaksford and Chater (1994) Table 1 (Probabilities for the Dependence model, MD and the Independence Model, MI) at page 610.

Using these contingency tables O&C model the conditional rule "if p then q" by calculating $E(I_g)$s in terms of paramaters a, b (see below) and $P(MD)$. In the MD contingency table there are no exceptions to the p and q dependency relationship.

	MD		MI	
	q	***-q***	***q***	***-q***
p	a	0	ab	a(1-b)
-p	(1-a)b	(1-a)(1-b)	(1-a)b	(1-a)(1-b)

where:

a corresponds to the probability of p occurring regardless of q, i.e. P(p)
a is the same in both models, i.e. row marginals are the same

b corresponds to the probability of q in the absence of p i.e. P(q|-p)
b is the same in both models.

O&C set the prior probability of MD and MD to be equiprobable, i.e. P(MI) = .5 and (MD) = 1 - P(MI)[2] But in MD, when p occurs, q must occur, whereas in MI the probability of p is not relevant to the probability of q.

NB The probability of q i.e. P(q) is not the same in both models because probability of q depends on whether rule is true of false.

---

1 MD is specified as being a model or context in which if p holds then q must hold
MI is specified as being a context or model in which p and q are not related.

2 In other terms, there is maximum entropy or uncertainty for each model.

APPENDIX 3.3
EXPERIMENTAL CONDITIONS
MI probabilities and
MD probabilities and frequencies

MI - probabilities

	q	**-q**	**Total**
p	0.04	0.16	0.2
-p	0.16	0.64	0.8
	0.2	0.8	1

<u>MD LOW</u> - probabilities

	q	**-q**	**Total**
p	0.2	0	0.2
-p	0.16	0.64	0.8
	0.36	0.64	1

MD LOW - frequencies

	Total	**p**	**-p**	**q**	**-q**
q	18	10	8	0	0
-q	32	0	32	0	0
p	10	0	0	10	0
-p	40	0	0	8	32
	100	10	40	18	32

<u>MD HIGH</u> - probabilities

	q	**-q**	**Total**
p	0.64	0.16	0.8
-p	0.16	0.04	0.2
	0.8	0.2	1

MD HIGH - frequencies

	Total	**p**	**-p**	**q**	**-q**
q	48	40	8	0	0
-q	2	0	2	0	0
p	40	0	0	40	0
-p	10	0	0	8	2
	100	40	10	48	2

APPENDIX 4.1
PARTICIPANT'S RIGHTS

YOUR RIGHTS AS A PARTICIPANT

Although you have agreed to participate in this study, you are at liberty to withdraw at any time. Any data which you produce will remain confidential and your anonymity is guaranteed.

We are NOT interested in whether your responses are right or wrong. This is NOT a test of intelligence or ability, and you will not be timed.

CLICK THE MOUSE ONCE FOR FURTHER DETAILS ABOUT THIS STUDY.

APPENDIX 4.2
GENERAL TASK INSTRUCTIONS

This study uses several packs of cards.

All the cards in these packs have a LETTER on one side and a NUMBER on the other side.

There are rules about what letters and numbers can go together.

For example:

"If a card has a 2 on one side then it has a T on the other side".

CLICK MOUSE FOR MORE INSTRUCTIONS ABOUT 'YOUR TASK'...

APPENDIX 4.3
NEW CARDS AND NEW RULE INSTRUCTIONS

New pack! New rule....

APPENDIX 4.4
END OF TASK INSTRUCTIONS

You have now completed this task.

If you would like to know more about this study, we shall be happy to answer any questions.

Thank you for your time.

CLICK THE MOUSE ONCE TO END THIS SESSION.

APPENDIX 4.1.1
SPECIFIC INSTRUCTIONS
FOUR CARDS STUDY

FOUR cards at a time will be dealt from one of several packs used in this study. Only one side of each card will be displayed on the screen. A rule will also be shown.

Your task will be to name those cards, and only those cards, which need to be turned over in order to determine whether a rule is true or false of the pack then being used.

You will be prompted to press one, or more keys on the keyboard in order to record your card selection.

Your task will continue until you have made card selections from several packs. You will be prompted when you have reached the end.

If you would like to review these instructions, PRESS THE "A" KEY. If you are clear about what to do CLICK THE MOUSE ONCE TO BEGIN.

APPENDIX 4.1.2
SELECTION TASK INSTRUCTIONS
FOUR CARDS STUDY

Rule: If a card has a J on one side then it has a 6 on the other side

Your task is to name those cards and only those cards which need to be turned over in order to determine whether the above rule is true or false of the pack now in use.

To record your selection, PRESS one, or more, or all of the equivalent keys on the keyboard.

CLICK MOUSE ONCE when you have finished your card selection.

6	J	9	W

APPENDIX 4.1.3
EXAMPLE PARTICIPANT'S DATASHEET
FOUR CARDS STUDY

Trial	Stimulus					Responses	
1	*General Task Instructions*						
2	*Specifc Selection Instructions - Control*						
3	**".GW79 - 9G"**	i GW79 - 9G			46583	**g**	*q*
3	**7**	**1st** GW79		46150			
3	**W**	**2nd** GW79		45650			
3	**G**	**3rd** GW79		45150			
3	**9**	**4th** GW79		44650			
3	".GW79 - 9G"	i GW79 - 9G			79350	**9**	*p*
4	*New Pack! New Rule...*						
5	**".AE25 - E5"**	i AE25 - E5			16800	**e**	*p*
5	**A**	**1st** AE25		16384			
5	**2**	**2nd** AE25		15867			
5	**E**	**3rd** AE25		15384			
5	**5**	**4th** AE25		14884			
5	".AE25 - E5"	i AE25 - E5			23366	**5**	*q*
7	**".UX29 - X9"**	i UX29 - X9			15833	**9**	q
7	**9**	**1st** UX29		15400			
7	**X**	**2nd** UX29		14900			
7	**2**	**3rd** UX29		14400			
7	**U**	**4th** UX29		13900			
7	".UX29 - X9"	i UX29 - X9			18783	**x**	*p*
9	**".DF31 - D3"**	i DF31	5366			**d**	*p*
9	**1**	**1st** DF31		4933			
9	**F**	**2nd** DF31		4433			
9	**D**	**3rd** DF31		3933			
9	**3**	**4th** DF31		3433			
9	".DF31 - D3"	i DF31	6233			**3**	*q*
11	**".6JW9 - J6"**	i 6JW9	26250			**j**	*p*
11	**6**	**1st** 6JW9		25816			
11	**W**	**2nd** 6JW9		25316			
11	**J**	**3rd** 6JW9		24816			
11	**9**	**4th** 6JW9		24316			
11	".6JW9 - J6"	i 6JW9	29284			**9**	*-q*
13	**".AU51 - 1A"**	i AU51 - 1A			4800	**1**	*p*
13	**U**	**1st** AU51		4366			
13	**1**	**2nd** AU51		3866			
13	**5**	**3rd** AU51		3350			
13	**A**	**4th** AU51		2866			
13	".AU51 - 1A"	i AU51 - 1A			7384	**a**	*q*
15	**".QL64 - 4Q"**	Four i	5417			**4**	*p*
15	**4**	**1st** QL64			5000		
15	**L**	**2nd** QL64			4500		
15	**6**	**3rd**QL64			4000		
15	**Q**	**4th** QL64			3500		
15	".QL64 - 4Q"	Four i	7450			**q**	*q*

17	**".AK27 - A2"**	i AK27	9834		**a**	*p*
17	**K**	**1st** AK27	9400			
17	**7**	**2nd** AK27	8900			
17	**A**	**3rd** AK27	8400			
17	**2**	**4th** AK27	7900			
17	".AK27 - A2"	i AK27	11234		**2**	*q*
19	**".AB31 - 3B"**	i AB31	5116		**b**	*q*
19	**A**	**1st** AB31	4700			
19	**B**	**2nd** AB31	4183			
19	**3**	**3rd** AB31	3683			
19	**1**	**4th** AB31	3183			
19	".AB31 - 3B"	i AB31	6316		**3**	*p*
21	**".EC48 - 8E"**	i EC48 - 8E				
21	**E**	**1st** EC48	8117		**8**	*p*
21	**C**	**2nd** EC48	7617			
21	**8**	**3rd** EC48	7117			
21	**4**	**4th** EC48	6617			
21	".EC48 - 8E"	i EC48 - 8E		10117	**e**	*q*
23	**".UA86 - U8"**	i UA86	6034		**8**	*q*
23	**8**	**1st** UA86	5600			
23	**6**	**2nd** UA86	5100			
23	**U**	**3rd** UA86	4600			
23	**A**	**4th** UA86	4100			
23	".UA86 - U8"	i UA86	7434		**u**	*p*
25	**".PZ18 - 8Z"**	i PZ81	7150		**8**	*p*
25	**8**	**1st** PZ81	6734			
25	**P**	**2nd** PZ81	6234			
25	**Z**	**3rd** PZ81	5734			
25	**1**	**4th** PZ81	5234			
25	".PZ18 - 8Z"	i PZ81	9600		**z**	*q*

26 Completion of Study Instructions

APPENDIX 4.2.1
SPECIFIC TASK INSTRUCTIONS
SINGLE CARD STUDY

ONE card will be dealt from a pack. Only one side of this card will be displayed on the screen. A rule will also be shown.

Your task will be to decide if the card needs to be turned over in order to determine if a rule is true or false of the pack then in use.

You will be prompted to press an appropriate key on the keyboard in order to record your decision.

The task will continue until you have made decisions about cards in several packs. You will be prompted when you have reached the end.

If you would like to review these instructions, PRESS THE "A" KEY. If you are clear about what to do CLICK THE MOUSE ONCE TO BEGIN.

APPENDIX 4.2.2
SELECTION TASK INSTRUCTIONS
SINGLE CARD STUDY

Rule: If a card has a J on one side then it has a 6 on the other side.

Must the card below be turned over in order to determine whether the above rule is true or false of the pack now in use?

If it is necessary to turn the card below over, press the "Y" key.

If it is not necessary to turn this card over, press the "N" key.

$\boxed{6}$

APPENDIX 4.2.3
EXAMPLE DATASHEET
SINGLE CARD STUDY

Trial	Condition	Card		Response
3	A-2 **3**	**2***q*	60717	**n**
4	A-2 **4**	**7**-*q*	27767	**n**
5	A-2 **1**	**K**-*p*	13950	**n**
6	A-2 **2**	**A***p*	7517	**y**
8	3-B **1**	**3***p*	11433	**y**
9	3-B **4**	**A**-*q*	7484	**n**
10	3-B **2**	**1**-*p*	6867	**n**
11	3-B **3**	**B***q*	18083	**n**
13	X9 **1**	**X***p*	9566	**y**
14	X9 **4**	**2**-*q*	8316	**n**
15	X9 **2**	**U**-*p*	6534	**n**
16	X9 **3**	**9***q*	13484	**n**
18	4-Q **4**	**4***p*	7434	**y**
19	4-Q **1**	**Q***q*	50384	**n**
20	4-Q **3**	**6**-*p*	7117	**n**
21	4-Q **2**	**L**-*q*	2333	**n**
23	8-Z **3**	**8***p*	5767	**y**
24	8-Z **4**	**Z***q*	8916	**n**
25	8-Z **1**	**1**-*p*	2233	**n**
26	8-Z **2**	**P**-*q*	2633	**n**
28	E-5 **2**	**A**-*p*	4750	**n**
29	E-5 **3**	**2**-*q*	4000	**n**
30	E-5 **4**	**5***q*	5900	**n**
31	E-5 **1**	**E***p*	2733	**y**
33	9-G **1**	**G***q*	5400	**n**
34	9-G **2**	**W**-*q*	1850	**n**
35	9-G **3**	**7**-*p*	1783	**n**
36	9-G **4**	**9***p*	2300	**y**
38	D-3 **1**	**D***p*	6483	**y**
39	D-3 **2**	**3***q*	4284	**n**
40	D-3 **3**	**F**-	2950	**n**
41	D-3 **4**	**1**-*q*	2083	**n**
43	U-8 **1**	**U***p*	5633	**y**
44	U-8 **3**	**8***q*	3400	**n**
45	U-8 **2**	**A**-*p*	2500	**n**
46	U-8 **4**	**6**-*q*	4367	**n**
48	J-6 **4**	**9**-*q*	4683	**n**
49	J-6 **2**	**J***p*	2700	**y**
50	J-6 **1**	**6***q*	2133	**n**
51	J-6 **3**	**W**-*p*	1984	**n**
53	1A **3**	**5**-*p*	3733	**n**
54	1A **4**	**1***p*	3084	**y**
55	1A **2**	**U**-*q*	2816	**n**
56	1A **1**	**A***q*	1534	**n**
58	8E **1**	**4**-*p*	5250	**n**
59	8E **4**	**C**-*q*	4666	**n**
60	8E **3**	**E***q*	1833	**n**
61	8E **2**	**8***p*	1950	**y**

Good afternoon!

This study involves four cards, depicted below.

9 U X 8

Each card has a letter on one side and a number on the other side.

Your task is to rate how useful or informative each card would be if it were turned over to test a rule.

The rule is:

if there is a vowel on one side of the card then there is an even number on the other side.

Please <u>circle</u> the informativeness rating you choose for each card.

For example:

an **8** rating means the card is **extremely useful**
a **6** rating means the card is **very useful**
a **4** rating means the card is **quite useful**
a **2** rating means the card is **not so useful**
a **0** rating means the card is **no use at all**

9	U	X	8
8	8	8	8
7	7	7	7
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1
0	0	0	0

Thank you for participating.

APPENDIX 4.4.1
SPECIFIC INSTRUCTIONS
SINGLE RATINGS STUDY

ONE card at a time will be been dealt from one of several packs used in this study. Only one side of a card will be displayed on the screen. A rule will also be shown.

Your task will be to rate (on a scale from 1 to 5) how much information a card, if turned over, provides about whether a rule is true or false of the pack then being used.

You will be prompted to press an appropriate key on the keyboard in order to record your rating of each card.

When you have assessed the informativeness of a small sample of cards in several packs, the task will end.

If you would like to review these instructions, PRESS THE "A" KEY.If you are clear about what to do CLICK THE MOUSE ONCE TO BEGIN.

APPENDIX 4.4.2
SELECTION TASK INSTRUCTIONS
SINGLE RATINGS STUDY

Rule: If a card has a J on one side then it has a 6 on the other side.

Please rate how much information the card below, if turned over, provides about whether the above rule is true or false of the pack now in use.

Select either "1", "2", "3", "4" or "5" where:

"1" = card is USELESS	(provides NO INFORMATION about truth or falsity of rule)
"2" = card is INADEQUATE	(provides LITTLE INFORMATION...)
"3" = card is HELPFUL	(provides SOME INFORMATION ...)
"4" = card is IMPORTANT	(provides LOTS OF INFORMATION...)
"5" = card is VITAL	(provides MOST INFORMATION...)

6

APPENDIX 4.4.3
EXAMPLE DATASHEET
SINGLE RATINGS STUDY

Trial	Condition				RATINGS Response key
3	9-G 1	G	34334	0	5
4	9-G 3	7	27317	0	1
5	9-G 4	9	5550	0	5
6	9-G 2	W	7083	0	1
8	E-5 2	A	10933	0	1
9	E-5 4	5	5433	0	1
10	E-5 3	2	3884	0	5
11	E-5 1	E	3750	0	5
13	U-8 2	A	5083	0	1
14	U-8 1	U	2517	0	5
15	U-8 3	8	4984	0	5
16	U-8 4	6	3566	0	1
18	D-3 4	1	4400	0	1
19	D-3 3	F	4617	0	1
20	D-3 2	3	3017	0	5
21	D-3 1	D	3850	0	5
23	1A 3	5	7967	0	1
24	1A 1	A	3333	0	5
25	1A 4	1	4417	0	5
26	1A 2	U	4184	0	1
28	8E 1	4	19183	0	3
29	8E 1	8	4950	0	5
30	8E 4	C	5617	0	1
31	8E 3	E	3816	0	5
33	4-Q 3	6	5300	0	1
34	4-Q 1	Q	3200	0	5
35	4-Q 4	4	2966	0	5
36	4-Q 2	L	3083	0	1
38	A-2 2	A	10900	0	5
39	A-2 3	2	6700	0	5
40	A-2 1	K	3900	0	1
41	A-2 4	7	35583	0	2
43	X9 2	U	7167	0	2
44	X9 4	2	11717	0	2
45	X9 3	9	8183	0	5
46	X9 1	X	4983	0	1
48	J-6 2	J	4267	0	5
49	J-6 4	9	7533	0	2
50	J-6 3	W	14900	0	2
51	J-6 1	6	2483	0	5
53	8-Z 1	1	4133	0	2
54	8-Z 3	8	2916	0	5
55	8-Z 2	P	2367	0	2
56	8-Z 4	Z	3500	0	5
58	3-B 4	A	3383	0	2
59	3-B 1	3	2683	0	5
60	3-B 2	1	2367	0	2
61	3-B 3	B	2684	0	5

APPENDIX 4.5.1
SPECIFIC INSTRUCTIONS
BINARY STUDY

You will see TWO cards on the screen, both dealt from the same pack. Only one side of each card will be displayed on the screen. A rule will also be shown.

Your task is to decide which of the two displayed cards, if turned over, provides more information about whether a rule is true or false of the pack then being used.

You will be prompted to press an appropriate key on the keyboard in order to record your decision.

The task will continue until you have compared a number of pairs of cards from several packs. You will be prompted when you have reached the end.

If you would like to review these instructions, PRESS THE "A" KEY. If you are clear about what to do CLICK THE MOUSE ONCE TO BEGIN.

APPENDIX 4.5.2
SELECTION TASK INTRUCTIONS
BINARY STUDY

Rule: If a card has a 4 on one side then it has a Q on the other side.

Which of the two cards below, if turned over, is more helpful in establishing whether the above rule is true or false of the pack now in use?

Press "1" if the card on the LEFT of the screen provides more information about whether the above rule is true or false of the current pack.

Press "*" if the card on the RIGHT of the screen provides more information about whether the above rule is true or false of the current pack.

Q	OR	L

APPENDIX 4.5.3
EXAMPLE DATASHEET
BINARY STUDY

1 *General Task Instructions*
2 *Specifc Selection Instructions - Binary*

Trial	Rule/Pair	Displayed Stimuli	Stimuli on right or left of screen	Select card on right? or left (l = left * = right)
3	4Q 2	6	r	1
3	4Q 2	Q	l	
4	4Q 10	Q	r	1
4	4Q 10	4	l	
5	4Q 7	Q	r	*
5	4Q 7	6	l	
6	4Q 11	L	r	1
6	4Q 11	4	l	
7	4Q 9	4	r	*
7	4Q 9	6	l	
8	4Q 5	6	r	1
8	4Q 5	L	l	
9	4Q 4	Q	r	*
9	4Q 4	L	l	
10	4Q 8	L	r	*
10	4Q 8	6	l	
11	4Q 12	6	r	1
11	4Q 12	4	l	
12	4Q 3	4	r	*
12	4Q 3	Q	l	
13	4Q 1	L	r	1
13	4Q 1	Q	l	
14	4Q 6	4	r	*
14	4Q 6	L	l	

New Pack! New Rule...

17	J6 10	6	6JW9 r	4684	0	*
17	J6 10	9	J 6	4667	0	*
18	J6 7	9	6JW9 r	5500	0	*
18	J6 7	W	J 6	5500	0	*
19	J6 11	J	6JW9 r	2317	0	*
19	J6 11	9	J 6	2300	0	*
20	J6 8	6	6JW9 r	4050	0	*
20	J6 8	W	J 6	4034	0	*
21	J6 4	W	6JW9 r	3934	0	1
21	J6 4	J	J 6	3917	0	1

APPENDIX 4.5.3
EXAMPLE DATASHEET
BINARY STUDY

22	J6 6	6	6JW9 r	6000	0	1
22	J6 6	J	J 6 5983	0	1	
23	J6 5	9	6JW9 r	3600	0	1
23	J6 5	J	J 6 3583	0	1	
24	J6 1	J	6JW9 r	2233	0	*
24	J6 1	6	J 6 2217	0	*	
25	J6 9	J	6JW9 r	2584	0	*
25	J6 9	W	J 6 2567	0	*	
26	J6 2	W	6JW9 r	2000	0	1
26	J6 2	6	J 6 1983	0	1	
27	J6 12	W	6JW9 r	4450	0	1
27	J6 12	9	J 6 4434	0	1	
28	J6 3	9	6JW9 r	3467	0	1
28	J6 3	6	J 6 3450	0	1	

New Pack! New Rule...

31	8E 2	4	48EC right	2650	0	1
31	8E 2	E	8 E 2634	0	1	
32	8E 12	C	48EC right	3733	0	1
32	8E 12	8	8 E 3717	0	1	
33	8E 7	8	48EC right	2584	0	*
33	8E 7	4	8 E 2567	0	*	
34	8E 10	4	48EC right	2434	0	1
34	8E 10	8	8 E 2417	0	1	
35	8E 9	E	48EC right	2050	0	*
35	8E 9	4	8 E 2034	0	*	
36	8E 3	8	48EC right	5800	0	*
36	8E 3	E	8 E 5783	0	*	
37	8E 11	E	48EC right	3050	0	1
37	8E 11	8	8 E 3034	0	1	
38	8E 5	8	48EC right	2350	0	*
38	8E 5	C	8 E 2333	0	*	
39	8E 6	E	48EC right	2167	0	*
39	8E 6	C	8 E 2150	0	*	
40	8E 4	4	48EC right	5450	0	*
40	8E 4	C	8 E 5434	0	*	
41	8E 8	C	48EC right	5233	0	*
41	8E 8	4	8 E 5216	0	*	
42	8E 1	C	48EC right	2233	0	1
42	8E 1	E	8 E 2216	0	1	

New Pack! New Rule...

45	UA 3	6	UA86 r	3317	0	1
45	UA 3	U	U 8 3300	0	1	
46	UA 5	8	UA86 r	1217	0	*
46	UA 5	A	U 8 1217	0	*	
47	UA 1	A	UA86 r	2017	0	1
47	UA 1	U	U 8 2000	0	1	
48	UA 2	8	UA86 r	4700	0	1
48	UA 2	U	U 8 4684	0	1	
49	UA 8	A	UA86 r	2250	0	1
49	UA 8	8	U 8 2233	0	1	
50	UA 11	A	UA86 r	3950	0	1
50	UA 11	6	U 8 3934	0	1	
51	UA 12	8	UA86 r	2450	0	*

APPENDIX 4.5.3
EXAMPLE DATASHEET
BINARY STUDY

51	UA 12	6	U 8 2450	0	*		
52	UA 6	6	UA86 r	2150	0		*
52	UA 6	A	U 8 2134	0	*		
53	UA 7	U	UA86 r	2100	0		1
53	UA 7	8	U 8 2084	0	1		
54	UA 9	6	UA86 r	1900	0		1
54	UA 9	8	U 8 1883	0	1		
55	UA 4	U	UA86 r	3083	0		*
55	UA 4	A	U 8 3066	0	*		
56	UA 10	U	UA86 r	3400	0		*
56	UA 10	6	U 8 3383	0	*		

New Pack! New Rule...

59	3B 3	1	BA31 r	2983	0		1
59	3B 3	B	3 B 2967	0	1		
60	3B 5	3	BA31 r	2200	0		*
60	3B 5	A	3 B 2200	0	*		
61	3B 12	3	BA31 r	2417	0		*
61	3B 12	1	3 B 2400	0	*		
62	3B 4	B	BA31 r	1967	0		*
62	3B 4	A	3 B 1967	0	*		
63	3B 9	1	BA31 r	2617	0		*
63	3B 9	3	3 B 2600	0	*		
64	3B 8	B	BA31 r	3134	0		1
64	3B 8	3	3 B 3117	0	1		
65	3B 10	A	BA31 r	4500	0		*
65	3B 10	1	3 B 4483	0	*		
66	3B 2	3	BA31 r	2684	0		*
66	3B 2	B	3 B 2667	0	*		
67	3B 6	1	BA31 r	1734	0		1
67	3B 6	A	3 B 1734	0	1		
68	3B 7	A	BA31 r	2200	0		1
68	3B 7	3	3 B 2184	0	1		
69	3B 11	B	BA31 r	1633	0		*
69	3B 11	1	3 B 1617	0	*		
70	3B 1	A	BA31 r	2133	0		1
70	3B 1	B	3 B 2117	0	1		

New Pack! New Rule...

73	A2 5	2	AK27 r	1616	0		*
73	A2 5	K	A==2 1600	0	*		
74	A2 11	K	AK27 r	6033	0		1
74	A2 11	7	A==2 6016	0	1		
75	A2 6	7	AK27 r	2500	0		*
75	A2 6	K	A==2 2483	0	*		
76	A2 3	7	AK27 r	4217	0		1
76	A2 3	A	A==2 4200	0	1		
77	A2 8	K	AK27 r	1683	0		*
77	A2 8	2	A==2 1667	0	*		
78	A2 9	7	AK27 r	1333	0		1
78	A2 9	2	A==2 1317	0	1		
79	A2 4	A	AK27 r	2650	0		*
79	A2 4	K	A==2 2634	0	*		
80	A2 2	K	AK27 r	1350	0		1
80	A2 2	A	A==2 1334	0	1		

APPENDIX 4.5.3
EXAMPLE DATASHEET
BINARY STUDY

81	A2 1	2	AK27 r	2167	0	1
81	A2 1	A	A==2	2150	0	1
82	A2 7	A	AK27 r	1350	0	*
82	A2 7	2	A==2	1333	0	*
83	A2 12	2	AK27 r	1416	0	*
83	A2 12	7	A==2	1400	0	*
84	A2 10	A	AK27 r	1367	0	*
84	A2 10	7	A==2	1350	0	*

New Pack! New Rule...

87	E5 12	2	EA25 r	1917	0	1
87	E5 12	5	E 5	1900	0	1
88	E5 5	2	EA25 r	4867	0	*
88	E5 5	A	E 5	4850	0	*
89	E5 7	E	EA25 r	3017	0	*
89	E5 7	2	E 5	3017	0	*
90	E5 4	E	EA25 r	2600	0	*
90	E5 4	A	E 5	2583	0	*
91	E5 11	A	EA25 r	1883	0	1
91	E5 11	5	E 5	1867	0	1
92	E5 1	A	EA25 r	1500	0	1
92	E5 1	E	E 5	1484	0	1
93	E5 10	E	EA25 r	1350	0	*
93	E5 10	5	E 5	1333	0	*
94	E5 9	5	EA25 r	1400	0	*
94	E5 9	2	E 5	1383	0	*
95	E5 8	A	EA25 r	2817	0	1
95	E5 8	2	E 5	2800	0	1
96	E5 3	5	EA25 r	1650	0	1
96	E5 3	E	E 5	1633	0	1
97	E5 6	5	EA25 r	2633	0	*
97	E5 6	A	E 5	2616	0	*
98	E5 2	2	EA25 r	2517	0	1
98	E5 2	E	E 5	2500	0	1

New Pack! New Rule...

101	8Z 6	8	PZ18 r	3233	0	*
101	8Z 6	Z	1 Z	3216	0	*
102	8Z 3	Z	PZ18 r	1267	0	*
102	8Z 3	P	1 Z	1250	0	*
103	8Z 7	8	PZ18 r	3534	0	*
103	8Z 7	1	1 Z	3517	0	*
104	8Z 2	8	PZ18 r	2617	0	*
104	8Z 2	P	1 Z	2600	0	*
105	8Z 12	Z	PZ18 r	2833	0	1
105	8Z 12	8	1 Z	2816	0	1
106	8Z 9	Z	PZ18 r	1183	0	*
106	8Z 9	1	1 Z	1167	0	*
107	8Z 8	P	PZ18 r	5433	0	1
107	8Z 8	1	1 Z	5417	0	1
108	8Z 4	P	PZ18 r	2217	0	1
108	8Z 4	Z	1 Z	2200	0	1
109	8Z 1	1	PZ18 r	2033	0	*
109	8Z 1	P	1 Z	2017	0	*
110	8Z 11	P	PZ18 r	1867	0	1

110	8Z 11	8	1 Z	1867	0	1
111	8Z 5	1	PZ18 r	1917	0	1
111	8Z 5	Z	1 Z	1900	0	1
112	8Z 10	1	PZ18 r	2216	0	1
112	8Z 10	8	1 Z	2200	0	1

New Pack! New Rule...

115	1A 2	5	AU51 r	2817	0	1
115	1A 2	A	1-A	2800	0	1
116	1A 3	1	AU51 r	4017	0	*
116	1A 3	A	1-A	4017	0	*
117	1A 5	5	AU51 r	4567	0	*
117	1A 5	U	1-A	4550	0	*
118	1A 7	1	AU51 r	2316	0	*
118	1A 7	5	1-A	2300	0	*
119	1A 10	5	AU51 r	1034	0	1
119	1A 10	1	1-A	1017	0	1
120	1A 9	U	AU51 r	2350	0	1
120	1A 9	5	1-A	2333	0	1
121	1A 12	U	AU51 r	1583	0	1
121	1A 12	1	1-A	1566	0	1
122	1A 6	1	AU51 r	1466	0	*
122	1A 6	U	1-A	1450	0	*
123	1A 4	A	AU51 r	1550	0	*
123	1A 4	U	1-A	1550	0	*
124	1A 11	A	AU51 r	1284	0	1
124	1A 11	1	1-A	1267	0	1
125	1A 8	A	AU51 r	1683	0	*
125	1A 8	5	1-A	1667	0	*
126	1A 1	U	AU51 r	2583	0	1
126	1A 1	A	1-A	2567	0	1

New Pack! New Rule...

129	D3 11	D	DF31 r	2234	0	*
129	D3 11	1	D 3	2217	0	*
130	D3 2	3	DF31 r	1067	0	1
130	D3 2	D	D 3	1050	0	1
131	D3 1	F	DF31 r	733	0	1
131	D3 1	D	D 3	717	0	1
132	D3 3	1	DF31 r	1984	0	1
132	D3 3	D	D 3	1967	0	1
133	D3 9	1	DF31 r	1700	0	1
133	D3 9	3	D 3	1683	0	1
134	D3 4	D	DF31 r	1233	0	*
134	D3 4	F	D 3	1216	0	*
135	D3 7	D	DF31 r	2483	0	*
135	D3 7	3	D 3	2467	0	*
136	D3 8	F	DF31 r	1533	0	1
136	D3 8	3	D 3	1517	0	1
137	D3 5	3	DF31 r	1267	0	*
137	D3 5	F	D 3	1250	0	*
138	D3 12	F	DF31 r	4100	0	*
138	D3 12	1	D 3	4083	0	*
139	D3 6	1	DF31 r	1417	0	1
139	D3 6	F	D 3	1417	0	1

APPENDIX 4.5.3
EXAMPLE DATASHEET
BINARY STUDY

140	D3 10	3	DF31 r	1434	0	*
140	D3 10	1	D 3	1417	0	*

New Pack! New Rule...

143	X9 6	2	2XU9 right	2850	0	1
143	X9 6	X	X 9	2833	0	1
144	X9 1	X	2XU9 right	1216	0	*
144	X9 1	2	X 9	1200	0	*
145	X9 3	9	2XU9 right	1234	0	*
145	X9 3	2	X 9	1217	0	*
146	X9 11	U	2XU9 right	1133	0	1
146	X9 11	9	X 9	1133	0	1
147	X9 2	U	2XU9 right	5650	0	1
147	X9 2	2	X 9	5633	0	1
148	X9 5	9	2XU9 right	1566	0	1
148	X9 5	X	X 9	1550	0	1
149	X9 8	2	2XU9 right	3133	0	*
149	X9 8	U	X 9	3116	0	*
150	X9 9	9	2XU9 right	1883	0	*
150	X9 9	U	X 9	1867	0	*
151	X9 4	U	2XU9 right	1733	0	1
151	X9 4	X	X 9	1716	0	1
152	X9 7	X	2XU9 right	1300	0	*
152	X9 7	U	X 9	1300	0	*
153	X9 12	2	2XU9 right	1933	0	1
153	X9 12	9	X 9	1917	0	1
154	X9 10	X	2XU9 right	2234	0	*
154	X9 10	9	X 9	2217	0	*

New Pack! New Rule...

157	9G 12	7	WG79 r	1400	0	1
157	9G 12	9	9 G	1383	0	1
158	9G 7	W	WG79 r	3250	0	1
158	9G 7	7	9 G	3233	0	1
159	9G 1	G	WG79 r	1300	0	*
159	9G 1	W	9 G	1283	0	*
160	9G 6	9	WG79 r	1233	0	1
160	9G 6	G	9 G	1216	0	1
161	9G 5	7	WG79 r	1500	0	1
161	9G 5	G	9 G	1483	0	1
162	9G 3	9	WG79 r	2017	0	*
162	9G 3	W	9 G	2000	0	*
163	9G 2	7	WG79 r	2800	0	*
163	9G 2	W	9 G	2800	0	*
164	9G 9	9	WG79 r	1733	0	*
164	9G 9	7	9 G	1717	0	*
165	9G 8	G	WG79 r	1683	0	*
165	9G 8	7	9 G	1683	0	*
166	9G 11	G	WG79 r	1417	0	1
166	9G 11	9	9 G	1400	0	1
167	9G 10	W	WG79 r	1116	0	1
167	9G 10	9	9 G	1100	0	1
168	9G 4	W	WG79 r	6416	0	1
168	9G 4	G	9 G	6400	0	1

APPENDIX 4.5.4
Z SCORE MATRICES
BINARY STUDY

Participant 1: scaled values for card pairings

Comparison	*p*	*q*	*-p*	*-q*	Total
p	1.602	-.855	-.891	-.926	-1.07
q	-.214	1.602	-.819	-.891	-.322
-p	-.178	-.249	1.602	-.783	.392
-q	-.142	-.178	-.285	1.602	.997
Scaled Means	**1.068.**	**.320**	**-.393**	**-.998**	**0.003**

Participant 2: scaled values for card pairings

Comparison	*q*	*p*	*-q*	*-p*	Total
p	-.426	1.596	-.957	-.957	-.744
q	1.596	-.641	-.922	-.957	-.924
-p	-.105	-.105	-.534	1.596	.852
-q	-.140	-.105	1.596	-.534	.817
Scaled Means	**.925**	**.745**	**-.817**	**-.852**	**0.001**

Participant 3: scaled values for card pairings

Comparison	*q*	*p*	*-q*	*-p*	Total
p	-.429	1.607	-.928	-.892	-.642
q	1.607	-.645	-.892	-.964	-.894
-p	-.105	-.177	-.429	1.607	.896
-q	-.177	-.141	1.607	-.645	.644
Scaled Means	**.896**	**.644**	**-.642**	**-.894**	**0.004**

Participant 4: scaled values for card pairings

Comparison	*p*	*q*	*-q*	*-p*	Total
p	1.586	-.881	-.952	-.916	-1.163
q	-.175	1.586	-.952	-.916	-.457
-p	-.140	-.140	-.424	1.586	.882
-q	-.104	-.104	1.586	-.637	.741
Scaled Means	**1.167**	**.461**	**-.742**	**-.883**	**0.003**

Participant 5: scaled values for card pairings

Comparison	*p*	*q*	*-q*	*-p*	Total
p	1.573	-.945	-.945	-.945	-1.262
q	-.104	1.573	-.945	-.945	-.421
-p	-.104	-.104	-.386	1.573	.979
-q	-.104	-.104	1.573	-.663	.702
Scaled Means	**1.261**	**.420**	**-.703**	**-.980**	**-0.002**

Participant 6: scaled values for card pairings

Comparison	*p*	*q*	*-p*	*-q*	Total
p	1.572	-.909	-.944	-.944	-1.225
q	-.139	1.572	-.944	-.944	-.455
-p	-.104	-.104	1.572	-.768	.596
-q	-.104	-.104	-.280	1.572	1.084
Scaled Means	**1.225**	**.455**	**-.596**	**-1.084**	**0.000**

Participant 7: scaled values for card pairings

Comparison	*p*	*q*	*-p*	*-q*	Total
p	1.603	-.644	-.962	-.854	-.857
q	-.429	1.603	-.926	-.926	-.678
-p	-.105	-.141	1.603	-.747	.610
-q	-.213	-.141	-.321	1.603	.928
Scaled Means	**.856**	**.677**	**-.606**	**-.924**	**0.003**

Participant 8: scaled values for card pairings

Comparison	*p*	*q*	*-q*	*-p*	Total
p	1.603	-.963	-.963	-.963	1.286
q	-.106	1.603	-.783	-.675	.039
-p	-.106	-.393	-.429	1.603	.675
-q	-.106	-.285	1.603	-.644	.568
Scaled Means	**1.285**	**-.038**	**-.572**	**-.679**	**-0.004**

Participant 9: scaled values for card pairings

Comparison	*q*	*p*	*-q*	*-p*	Total
p	-.662	1.650	-.625	-.768	-.405
q	1.650	-.440	-.842	-.842	-.474
-p	-.256	-.330	-.588	1.650	.476
-q	-.256	-.477	1.650	-.514	.403
Scaled Means	**.476**	**.403**	**-.405**	**-.474**	**0.000**

Participant 10: scaled values for card pairings

Comparison	*p*	*-q*	*q*	*-p*	Total
p	1.623	-.829	-.793	-.975	-.974
q	-.289	-.289	1.623	-.720	.325
-p	-.107	-.325	-.362	1.623	.829
-q	-.253	1.623	-.793	-.757	-.180
Scaled Means	**.974**	**.180**	**-.325**	**-.829**	**0.000**

Participant 11: scaled values for card pairings

Comparison	*p*	*q*	*-p*	*-q*	Total
p	1.584	-.845	-.951	-.951	-1.163
q	-.211	1.584	-.809	-.916	-.352
-p	-.105	-.247	1.584	-.880	.352
-q	-.105	-.140	-.176	1.584	1.163
Scaled Means	**1.163**	**.352**	**-.352**	**-.1.163**	**0.000**

Participant 12: scaled values for card pairings

Comparison	*p*	*q*	*-p*	*-q*	Total
p	1.567	-.941	-.941	-.941	-1.256
q	-.103	1.568	-.941	-.941	-.417
-p	-.103	-.103	1.567	-.801	.560
-q	-.103	-.103	-.244	1.567	1.117
Scaled Means	**1.258**	**.421**	**-.559**	**-1.116**	**0.004**

Participant 13: scaled values for card pairings

Comparison	*p*	*q*	*-p*	*-q*	Total
p	1.585	-.881	-.952	-.916	-1.164
q	-.176	1.585	-.881	-.881	-.353
-p	-.105	-.176	1.585	-.845	.459
-q	-.140	-.176	-.211	1.585	1.058
Scaled Means	**1.164**	**.352**	**-.459**	**-1.057**	**0.000**

Participant 14: scaled values for card pairings

Comparison	*q*	*p*	*-q*	*-p*	Total
p	-.442	1.653	-.769	-.769	-.327
q	1.653	-.664	-.769	-.769	-.549
-p	-.331	-.331	-.442	1.653	.549
-q	-.331	-.331	1.653	-.664	.327
Scaled Means	**.549**	**.327**	**-.327**	**-.549**	**0.000**

Participant 15: scaled values for card pairings

Comparison	*p*	*-q*	*-p*	*q*	Total
p	1.632	-.546	-.834	-.907	-.655
q	-.181	-.254	-.363	1.632	.834
-p	-.254	-.290	1.632	-.724	.364
-q	-.546	1.632	-.797	-.834	-.545
Scaled Means	**.651**	**.542**	**-.362**	**-.833**	**-0.002**

Participant 16: scaled values for card pairings

Comparison	*p*	*q*	*-p*	*-q*	Total
p	1.562	-.868	-.938	-.938	-1.182
q	-.173	1.562	-.938	-.938	-.487
-p	-.103	-.103	1.562	-.938	.418
-q	-.103	-.103	-.103	1.562	1.253
Scaled Means	**1.183**	**.488**	**-.417**	**-1.252**	**0.002**

Participant 17: scaled values for card pairings

Comparison	*p*	*q*	*-p*	*-q*	Total
p	1.578	-.912	-.947	-.947	-1.228
q	-.139	1.578	-.947	-.947	-.455
-p	-.104	-.104	1.578	-.563	.807
-q	-.104	-.104	-.493	1.578	.877
Scaled Means	**1.231**	**.458**	**-.809**	**-.879**	**0.001**

Participant 18: scaled values for card pairings

Comparison	*p*	*q*	*-p*	*-q*	Total
p	1.656	-.846	-.628	-.697	-.515
q	-.257	1.656	-.697	-.734	-.032
-p	-.480	-.406	1.656	-.697	.073
-q	-.406	-.369	-.406	1.656	.475
Scaled Means	**.513**	**.035**	**-.075**	**-.472**	**0.001**

Participant 19: scaled values for card pairings

Comparison	*p*	*q*	*-p*	*-q*	Total
p	1.578	-.912	-.948	-.912	-1.194
q	-.140	1.578	-.948	-.948	-.458
-p	-.104	-.104	1.578	-.700	.670
-q	-.140	-.104	-.352	1.578	.982
Scaled Means	**1.194**	**.458**	**-.670**	**-.982**	**0.000**

Participant 20: scaled values for card pairings

Comparison	*-p*	*-q*	*p*	*q*	Total
p	-.179	-.216	1.623	-.615	.613
q	-.179	-.216	-.470	1.623	.758
-p	1.623	-.506	-.901	-.901	-.685
-q	-.579	1.623	-.865	-.865	-.686
Scaled Means	**.686**	**.685**	**-.613**	**-.758**	**0.000**

APPENDIX 4.6.1
GENERAL INSTRUCTIONS
FIRST PROBABILITY STUDY

This study uses one pack of cards. Each card in the pack has a LETTER on one side and a NUMBER on the other side.

In order to familiarise yourself with this rule, and other characteristics of the cards in this pack, you will be shown 100 cards. There will be a break every 20 cards.

Your task during this familiarisation phase will be to press an appropriate key on the keyboard, in order to say what is on the reverse of the displayed card. A choice of responses, which will not change throughout the task, will be shown on the screen.

Feedback as to whether your response is right or wrong will be given. We want you to monitor feedback by taking one straw ata time from the jar.

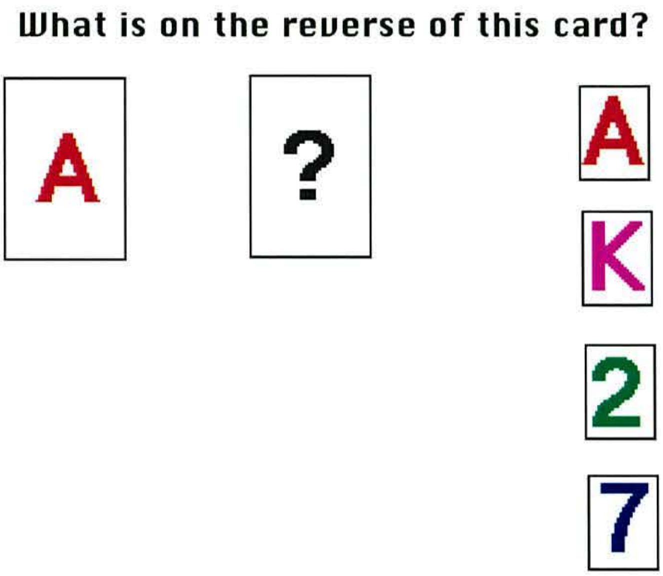
If feedback is "YES" place a straw on the left of the desk.
If feedback is "NO", place a straw on the right of the desk.

Your final task involves 40 cards from the same pack. You will be given full details of what to do in this task later.

When you are clear about these instructions, CLICK THE MOUSE TO PROCEED.

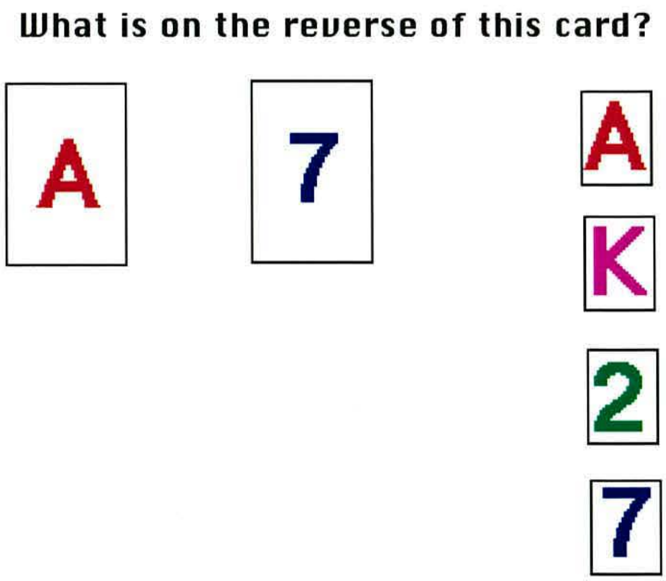
APPENDIX 4.6.2 - DIAGRAM 1
LEARNING PHASE INSTRUCTIONS
FIRST PROBABILITY STUDY

For example, when "A" is the face card



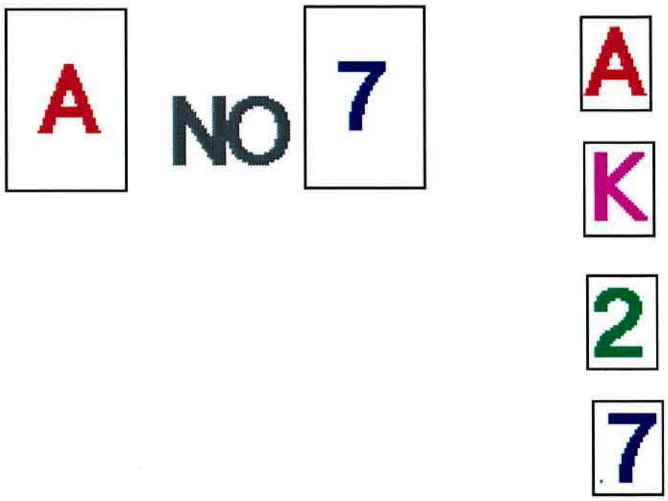
APPENDIX 4.6.2 - DIAGRAM 2
LEARNING PHASE INSTRUCTIONS
FIRST PROBABILITY STUDY

For example, if "7" was selected by the participant as being on the reverse side of "A" and this choice was recorded by pressing the 7" key on the keyboard.



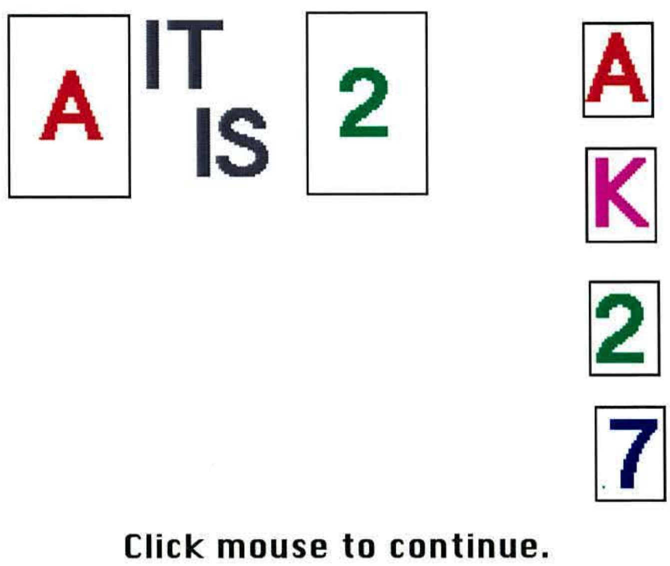
APPENDIX 4.6.2 - DIAGRAM 3
LEARNING PHASE INSTRUCTIONS
FIRST PROBABILITY STUDY

Example of the way in which feedback given to participant about choice of reverse side of face card, if A was face card and 7 was selected as its reverse.



APPENDIX 4.6.2 - DIAGRAM 4
LEARNING PHASE INSTRUCTIONS
FIRST PROBABILITY STUDY

Example of screen showing correct reverse stimulus of face card.



APPENDIX 4.6.3
BREAK INSTRUCTIONS
FIRST PROBABILITY STUDY

You may now have a short break.

CLICK THE MOUSE when ready to proceed.

APPENDIX 4.6.4
END OF LEARNING PHASE INSTRUCTIONS
FIRST PROBABILITY STUDY

You have now completed the first task in this study.

The final task uses a pack of cards with exactly the same letter and number combinations as you have just observed. Your familiarity with this and other card characteristics should help you with the task.

CLICK MOUSE TO PROCEED.

APPENDIX 4.6.5
SELECTION TASK INSTRUCTIONS
FIRST PROBABILITY STUDY

On your understanding of the characteristics of cards in this pack, is the displayed card relevant to turn over in order to check:

> if a card has a VOWEL on one side, then it has an EVEN NUMBER on the other?

"Y" = card is relevant to turn over in order to check the above.

"N" = card is not relevant to turn over in order to check the above.

APPENDIX 4.6.6
CARD CHOICE QUERY
FIRST PROBABILITY STUDY

Do you want to change your mind?

APPENDIX 4.6.7
FINAL INSTRUCTIONS
FIRST PROBABILITY STUDY

You have now completed both tasks!

Thank you for participating.

If you would like to know more about this study,
we shall be happy to tell you.

APPENDIX 4.6.8
EXAMPLE DATASHEET
FIRST PROBABILITY STUDY

P(*p*): 0.8 (HIGH)
LEARNING PHASE (100 trials)

Trial	Face and Reverse[1]			Participant's Prediction about Reverse
1	K7	11066	0	7
2	2K	5584	0	a
3	A2	6066	0	2
4	2A	4567	0	a
5	A2	4967	0	2
6	2K	2583	0	a
7	A2	6300	0	7
8	7K	3967	0	k
9	2A	2433	0	a
10	2A	2034	0	a
11	2A	1650	0	a
12	A2	1383	0	2
13	A2	1883	0	2
14	2A	2250	0	a
15	K2	1450	0	7
16	2A	1483	0	k
17	A2	2650	0	2
18	2A	6750	0	k
19	2A	1166	0	a
20	A2	1083	0	2
21	2A	1067	0	a
22	A2	1166	0	2
23	A2	2000	0	2
24	A2	1483	0	2
25	2K	1217	0	a
26	A2	1783	0	2
27	K7	4917	0	7
28	A2	1017	0	2
29	2A	2234	0	a
30	2A	1366	0	a
31	2K	1133	0	a
32	2A	1817	0	k
33	2A	1900	0	a
34	2A	834	0	a
35	A2	1333	0	2
36	A2	1134	0	2
37	2A	2017	0	a
38	2A	1066	0	a
39	A2	1083	0	2
40	2A	1250	0	a
41	2A	750	0	a
42	A2	867	0	2
43	2A	1216	0	a
44	2K	1050	0	a
45	2A	1350	0	a

1 In other words, the face card actually displayed, and the unseen reverse, i.e. the conditional probability learned.

46	2K	917	0	a
47	K2	3150	0	7
48	A2	1384	0	2
49	2A	1400	0	a
50	2A	967	0	a
51	K2	4467	0	2
52	A2	916	0	2
53	2A	1300	0	a
54	2A	1216	0	a
55	2A	1250	0	a
56	A2	1334	0	2
57	A2	1067	0	2
58	2A	666	0	a
59	A2	800	0	2
60	A2	917	0	2
61	A2	1750	0	2
62	2A	1133	0	a
63	A2	884	0	2
64	K2	3216	0	7
65	A2	650	0	2
66	A2	534	0	2
67	2A	1316	0	a
68	A2	800	0	2
69	2K	1134	0	a
70	A2	2334	0	2
71	7K	1750	0	k
72	A2	900	0	2
73	A2	1034	0	2
74	K2	2867	0	2
75	A2	1400	0	2
76	A2	1750	0	2
77	2A	734	0	a
78	2A	3917	0	a
79	2A	1183	0	a
80	2A	867	0	a
81	A2	2117	0	2
82	A2	1367	0	2
83	2A	950	0	a
84	A2	1050	0	2
85	2A	550	0	a
86	2A	916	0	a
87	A2	517	0	2
88	K2	4566	0	2
89	2A	767	0	a
90	A2	950	0	2
91	2A	883	0	a
92	2A	1100	0	a
93	K2	2083	0	2
94	A2	1167	0	2
95	K2	817	0	2
96	A2	1150	0	2
97	A2	1967	0	2
98	2K	917	0	a
99	2A	1250	0	a
100	2A	2000	0	a

SELECTION TASK PHASE (40 trials)

	Face Card			Turn over? Change Mind?
102	K	47950	0	n
102	K	3283	0	n
103	K	16183	0	n
103	K	10450	0	n
104	A	12684	0	y
104	A	1933	0	n
105	K	12216	0	n
105	K	1233	0	n
106	2	12683	0	y
106	2	1566	0	n
107	2	11967	0	y
107	2	1666	0	n
108	K	8800	0	n
108	K	1383	0	n
109	7	19700	0	n
109	7	967	0	n
110	K	8217	0	n
110	K	916	0	n
111	A	6216	0	y
111	A	1550	0	n
112	A	11100	0	y
112	A	1500	0	n
113	A	5517	0	y
113	A	1600	0	n
114	A	6900	0	y
114	A	1317	0	n
115	7	3167	0	n
115	7	1184	0	n
116	2	3133	0	y
116	2	1283	0	n
117	7	4200	0	n
117	7	1200	0	n
118	A	7133	0	y
118	A	1233	0	n
119	A	7517	0	y
119	A	1317	0	n

120	K	4416	0	n
120	K	1033	0	n
121	A	3500	0	y
121	A	1533	0	n
122	7	7350	0	n
122	7	1050	0	n
123	7	3017	0	n
123	7	917	0	n
124	2	2750	0	y
124	2	1167	0	n
125	2	2467	0	y
125	2	1100	0	n
126	2	3050	0	y
126	2	1300	0	n
127	7	1917	0	n
127	7	1084	0	n
128	K	2117	0	n
128	K	817	0	n
129	2	3416	0	y
129	2	1250	0	n
130	2	1883	0	y
130	2	1183	0	n
131	7	1500	0	n
131	7	950	0	n
132	A	1750	0	y
132	A	1450	0	n
133	2	1450	0	y
133	2	1400	0	n
134	K	1417	0	n
134	K	1034	0	n
135	K	2850	0	n
135	K	750	0	n
136	A	1683	0	y
136	A	1233	0	y
137	2	4850	0	y
137	2	2034	0	n
138	7	1283	0	n
138	7	750	0	n

139	7	817	0	n
139	7	750	0	n
140	7	1300	0	n
140	7	717	0	n
141	K	1250	0	n
141	K	733	0	n

APPENDIX 4.7.1
GENERAL INSTRUCTIONS
SECOND PROBABILITY STUDY

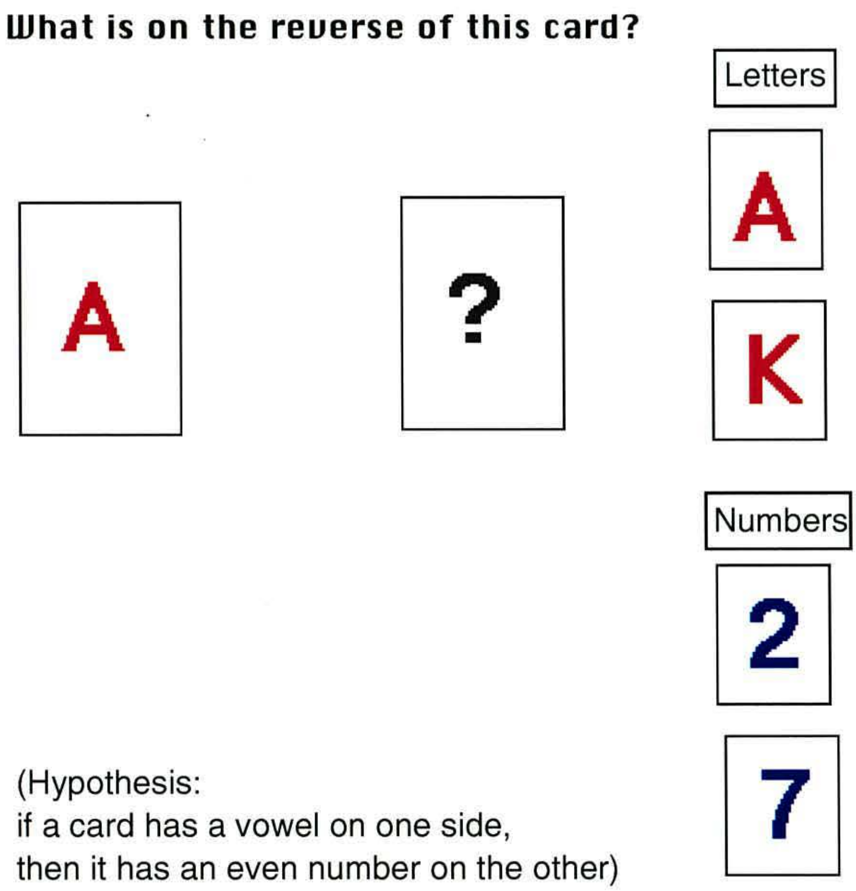
This study uses a large pack of cards. Each card in the pack has a letter on one side and a number on the other side. To become used to the cards in this pack, you will be shown 200 of them, with a 30 second break every 50 cards. During this learning stage, and when the face of each card is presented, you will be asked to predict what is on its reverse side by pressing a key. A choice of possible responses, which will not change throughout the task, will be shown on the screen. You will be told whether your prediction was right or wrong. Go through this learning stage AS QUICKLY AS POSSIBLE.

During the second and final stage of this study you will be asked to advise someone whose task is to find out, and be absolutely certain about, whether a suggested rule "IF A CARD HAS A VOWEL ON ONE SIDE, THEN IT HAS AN EVEN NUMBER ON THE OTHER" does or does not apply to the pack.

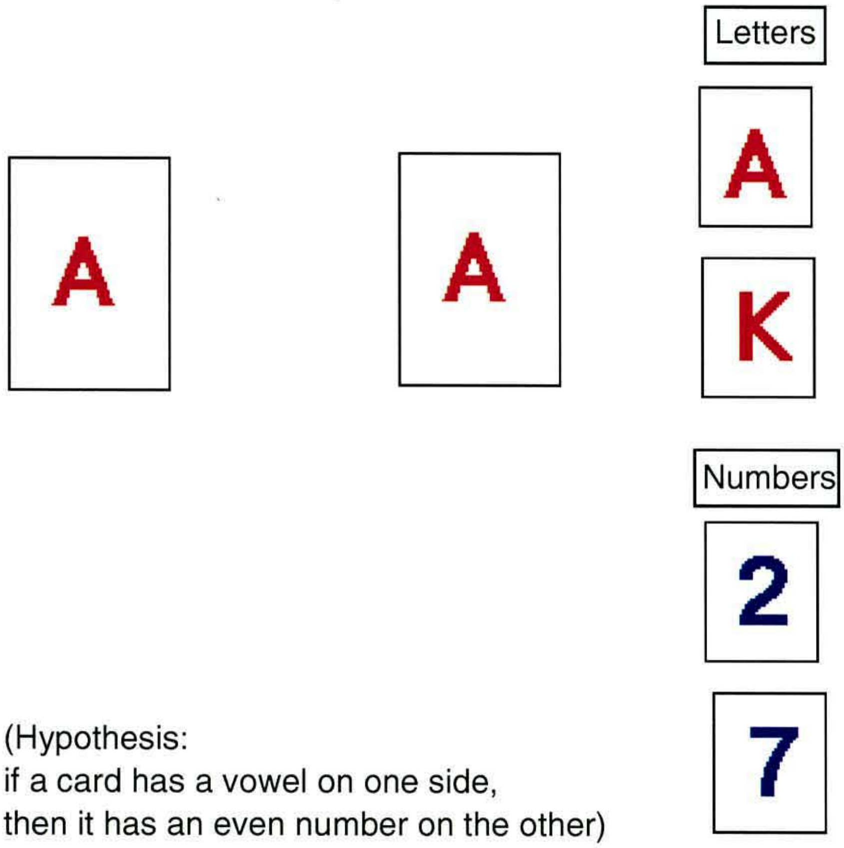
Unlike you, this person will not have seen the pack before nor will they have time to learn about ANY cards in the pack. Instead, they will be forced to pick out of the pack as few, highly informative face cards as possible. As you will have information on 200 cards in the pack, YOUR TASK will be to tell this person which face cards it is essential they look for and select from the pack in order to check their reverse sides, and so be absolutely certain that the ABOVE suggested rule is true or false. Further reminder instructions will be given later. In the meantime, and when you are clear about the above, HIT ANY KEY TO BEGIN LEARNING ABOUT THE PACK.

NB: cards will be dealt RANDOMLY throughout.

APPENDIX 4.7.2
SCREEN DIAGRAM 1
SECOND PROBABILITY STUDY

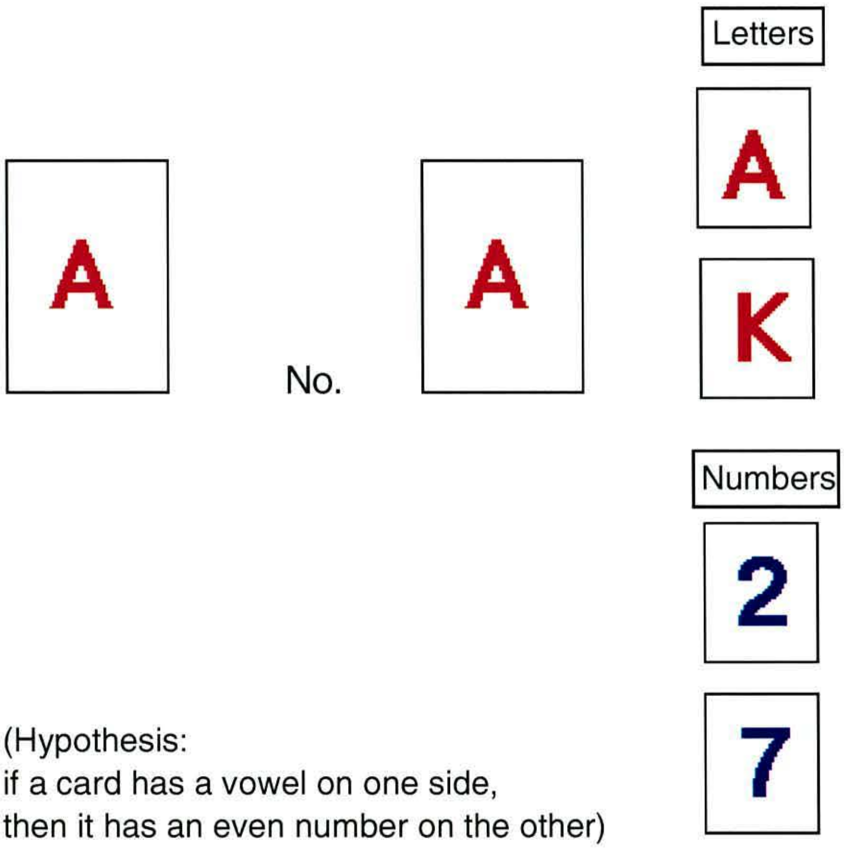


APPENDIX 4.7.2
SCREEN DIAGRAM 2
SECOND PROBABILITY STUDY



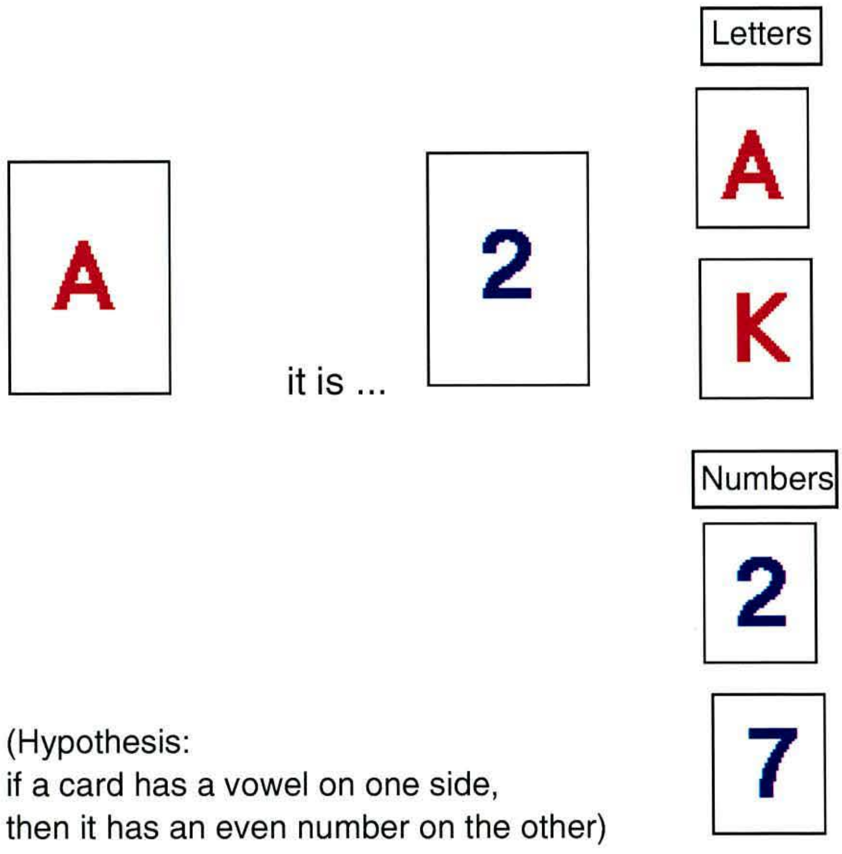
(Hypothesis:
if a card has a vowel on one side,
then it has an even number on the other)

APPENDIX 4.7.2
SCREEN DIAGRAM 3
SECOND PROBABILITY STUDY



(Hypothesis:
if a card has a vowel on one side,
then it has an even number on the other)

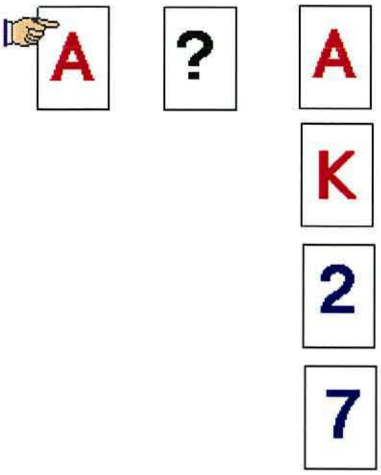
APPENDIX 4.7.2
SCREEN DIAGRAM 4
SECOND PROBABILITY STUDY

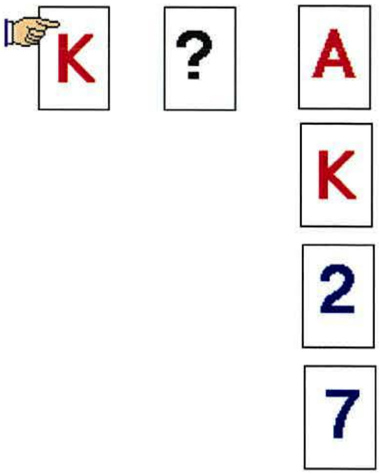


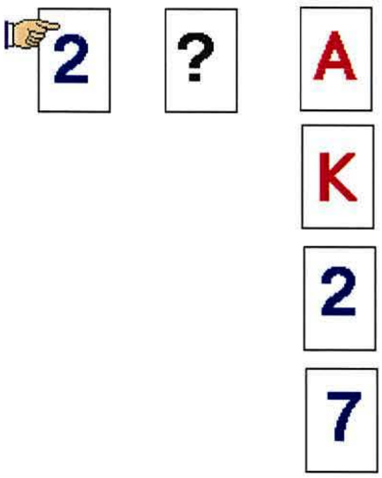
APPENDIX 4.7.3
FORM FOR COMPLETION
SECOND PROBABILITY STUDY

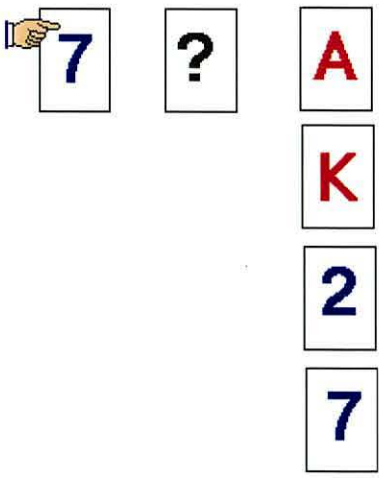
Complete this as **quickly and automatically** as possible.

1) Tick what you believe is on the reverse of the four cards pointed out below.
2) Next to your tick/s write how often you think you observed the letter/number or number/letter combinations:
very frequently / quite frequently / not frequently / can't decide.









When you have finished,
return to the screen and proceed with the last part of the study,
which makes use of your familiarity with the characteristics of the cards.
Thank you.

APPENDIX 4.7.4
TASK REMINDER INSTRUCTIONS
SECOND PROBABILITY STUDY

Imagine ALL (not just 200) cards are now randomly spread, face-up, across a table. Someone who has never seen nor learned about these cards before has to know with absolute certainty whether a suggested rule, 'IF A CARD HAS A VOWEL ON ONE SIDE, THEN IT HAS AN EVEN NUNBER ON THE OTHER", does or does not apply to the pack.

It will waste a lot of time and effort if this person turns over all the cards on the table to check their reverse sides. Your information about 200 of the pack's letter/number or number/letter combinations, and how frequently they occur, will save this person turning over unnecessary cards. Which face cards would you advise it IS necessary to look out for and select from the table in order to check their reverse sides, and so be absolutely certain that the ABOVE suggested rule is true or false.

HIT ANY KEY TO BEGIN FINAL TASK.

APPENDIX 4.7.5
SELECTION TASK INSTRUCTION
SECOND PROBABILITY STUDY

Would you advise it is necessary they look out for and select from the table the below face cards in order to check their reverse sides, and so be absolutely certain that a rule "IF A CARD HAS A VOWEL ON ONE SIDE, THEN IT HAS AN EVEN NUMBER ON THE OTHER", does or does not apply to the WHOLE pack?

"Y" = Yes. They will be absolutely certain the ABOVE rule is true or false if the below face cards are selected and their reverse sides checked.

"N" = "No. They will NOT be absolutely certain the above rule is true or false if the below face cards are selected and their reverse sides checked."

APPENDIX 4.7.6
EXAMPLE DATA SHEET
SECOND PROBABILITY STUDY

P(p): 0.8 (HIGH)
LEARNING PHASE (200 trials)

Trial	Face and Reverse[1]			Participant's Prediction about Reverse
1	2A	19017	0	a
2	A2	5833	0	2
3	7K	8900	0	k
4	A2	2483	0	2
5	A2	2583	0	2
6	K7	4200	0	7
7	2A	4583	0	a
8	A2	2200	0	2
9	2A	3267	0	a
10	2A	3084	0	a
11	K2	1733	0	7
12	2K	4767	0	a
13	2A	19767	0	a
14	A2	3700	0	2
15	2A	5650	0	a
16	2A	5017	0	a
17	2A	3384	0	a
18	K7	5350	0	7
19	A2	3250	0	2
20	2K	2783	0	a
21	K2	9317	0	7
22	2K	3600	0	a
23	K2	3833	0	7
24	A2	9150	0	2
25	A2	8134	0	2
26	2A	3216	0	a
27	2A	4000	0	a
28	2A	4666	0	a
29	A2	2917	0	2
30	2A	2300	0	a
31	2K	2066	0	a
32	A2	1833	0	7
33	K2	2200	0	2
34	A2	2800	0	7
35	2A	1416	0	a
36	K7	3217	0	7
37	A2	3933	0	2
38	A2	2317	0	2
39	A2	1150	0	2
40	2A	1817	0	a
41	2A	933	0	a
42	2A	583	0	a
43	A2	1750	0	2
44	K7	1433	0	7
45	A2	1133	0	2
46	2A	1750	0	a
47	2A	1933	0	a

1 In other words, the face card actually displayed, and the unseen reverse, i.e. the conditional probability learned.

48	A2	1750	0	2
49	A2	2733	0	2
50	A2	1467	0	2
51	2A	2200	0	a
52	2A	1550	0	a
53	K2	1616	0	7
54	A2	3900	0	2
55	A2	1283	0	2
56	A2	784	0	2
57	A2	667	0	2
58	2A	1217	0	a
59	2A	1183	0	a
60	A2	1550	0	2
61	2A	1950	0	a
62	A2	1267	0	2
63	K2	550	0	2
64	2K	1100	0	a
65	2K	3850	0	a
66	A2	2600	0	2
67	2A	2767	0	a
68	2A	3084	0	a
69	A2	1650	0	2
70	K2	1817	0	2
71	2A	4084	0	k
72	A2	1483	0	2
73	2A	3217	0	a
74	K2	2567	0	2
75	2A	1333	0	a
76	A2	1550	0	2
77	2K	1067	0	a
78	2A	1134	0	a
79	A2	1900	0	2
80	K2	2366	0	2
81	A2	5283	0	2
82	A2	817	0	2
83	K2	466	0	2
84	2K	2350	0	k
85	A2	2517	0	7
86	2A	1066	0	a
87	A2	1550	0	2
88	A2	967	0	2
89	A2	1000	0	2
90	2A	1266	0	a
91	A2	1617	0	2
92	2A	2050	0	a
93	A2	1400	0	2
94	A2	1066	0	2
95	A2	1150	0	2
96	2A	1584	0	a
97	A2	850	0	2
98	A2	900	0	2
99	2A	884	0	a
100	A2	950	0	2
101	2A	2800	0	a
102	A2	866	0	2
103	A2	817	0	2
104	2A	1183	0	a
105	A2	883	0	2

106	A2	1100	0	2
107	2A	1384	0	a
108	A2	866	0	2
109	A2	1217	0	2
110	2A	1816	0	a
111	2K	1733	0	a
112	A2	2500	0	7
113	2A	1100	0	a
114	A2	1334	0	2
115	2A	1033	0	a
116	2K	3084	0	a
117	2K	3250	0	k
118	2A	1817	0	k
119	7K	2616	0	k
120	2A	1117	0	a
121	A2	1300	0	2
122	2A	1667	0	a
123	A2	1100	0	2
124	K2	2917	0	7
125	A2	1283	0	2
126	2A	1283	0	a
127	2K	6383	0	a
128	A2	3383	0	2
129	2A	2567	0	a
130	A2	1733	0	2
131	2A	1133	0	a
132	2A	1116	0	a
133	2A	1633	0	a
134	2A	784	0	a
135	A2	583	0	a
136	A2	883	0	2
137	2A	1450	0	a
138	2A	1583	0	a
139	2A	1450	0	a
140	A2	1350	0	2
141	2K	1067	0	a
142	2A	1217	0	k
143	A2	1284	0	2
144	2A	2867	0	a
145	2A	1284	0	a
146	A2	1050	0	2
147	A2	983	0	2
148	A2	950	0	2
149	K2	2650	0	2
150	2A	1517	0	a
151	2A	1683	0	a
152	A2	1783	0	2
153	A2	1500	0	2
154	2A	1066	0	a
155	2A	917	0	a
156	K2	2783	0	2
157	2A	2500	0	a
158	2A	1600	0	a
159	A2	1433	0	2
160	A2	1217	0	2
161	2A	950	0	a
162	K2	867	0	2
163	2A	1266	0	a

164	2A	1234	0	a
165	2A	866	0	a
166	A2	1417	0	2
167	K2	966	0	2
168	2A	966	0	a
169	2A	1033	0	a
170	A2	1134	0	2
171	2A	967	0	a
172	2A	1633	0	a
173	7K	3017	0	a
174	2A	967	0	a
175	2K	4133	0	a
176	2K	2166	0	a
177	2A	1150	0	a
178	A2	3383	0	2
179	A2	3017	0	2
180	A2	3167	0	2
181	A2	3084	0	2
182	A2	950	0	2
183	2A	1734	0	a
184	2A	1550	0	a
185	A2	1033	0	2
186	2A	2583	0	a
187	A2	1317	0	2
188	7K	1750	0	k
189	2K	1934	0	a
190	2A	2050	0	k
191	A2	1050	0	2
192	2A	950	0	a
193	A2	1000	0	2
194	2A	950	0	a
195	2A	1267	0	a
196	A2	1267	0	2
197	K2	1683	0	2
198	2A	1766	0	a
199	2A	2667	0	a
200	A2	1150	0	2

SELECTION TASK PHASE (48 trials)

Trial	Face Card			Turn over? Change Mind?
202	A	60217	0	n
202	A	18634	0	y
203	7	17784	0	y
203	7	7950	0	n
204	2	12400	0	n
204	2	1567	0	n
205	A	8567	0	n
205	A	1050	0	n
206	K	6166	0	n
206	K	683	0	n

207	K	16716	0	y
207	K	1617	0	n
208	2	42500	0	y
208	2	1450	0	n
209	7	9050	0	y
209	7	950	0	n
210	A	8900	0	y
210	A	450	0	n
211	2	1133	0	y
211	2	483	0	n
212	2	33734	0	y
212	2	1117	0	n
213	7	7017	0	y
213	7	1033	0	n
214	K	16467	0	y
214	K	550	0	n
215	A	5983	0	y
215	A	533	0	n
216	7	53116	0	y
216	7	567	0	n
217	7	3883	0	n
217	7	1267	0	n
218	7	4100	0	n
218	7	1583	0	y
219	A	46550	0	y
219	A	1000	0	n
220	K	9850	0	y
220	K	5900	0	n
221	2	39467	0	y
221	2	384	0	n
222	K	10234	0	y
222	K	467	0	n
223	7	5433	0	y
223	7	483	0	n
224	K	3383	0	y
224	K	533	0	n
225	A	8417	0	y
225	A	567	0	n

226	K	4067	0	y
226	K	417	0	n
227	2	4833	0	y
227	2	433	0	n
228	A	8533	0	y
228	A	550	0	n
229	K	2234	0	n
229	K	467	0	y
230	2	1650	0	y
230	2	334	0	n
231	2	1150	0	y
231	2	300	0	n
232	7	3450	0	n
232	7	766	0	y
233	A	1300	0	y
233	A	333	0	n
234	K	1683	0	n
234	K	400	0	y
235	2	3483	0	y
235	2	633	0	n
236	K	9400	0	n
236	K	1183	0	y
237	K	1634	0	n
237	K	667	0	y
238	A	983	0	y
238	A	450	0	n
239	7	8617	0	n
239	7	416	0	y
240	2	1267	0	y
240	2	434	0	n
241	2	24934	0	y
241	2	1034	0	n
242	K	1100	0	n
242	K	383	0	y
243	7	22383	0	y
243	7	416	0	n
244	2	517	0	y
244	2	550	0	n

245	A	1017	0	y
245	A	300	0	n
246	7	8033	0	y
246	7	350	0	n
247	7	533	0	y
247	7	350	0	n
248	A	466	0	y
248	A	300	0	n
249	A	600	0	y
249	A	400	0	n

APPENDIX 4.8
ANOVA SUMMARIES
ALL EXPERIMENTS

I FOUR CARDS STUDY

ANOVA relating to Means Table 4.1.1: *p, q, -q* and *-p* card selections

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	19	227.950	11.997		
Cards	3	1406.450	468.817	40.516	.0001
Cards * Subject	57	659.550	11.571		

Dependent: Cards

ANOVA relating to Means Table 4.1.2: *p, q, -q* and *-p* card selections depending on *p* card type (consonants, vowels, odd or even numbers)

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
p Card Type	3	.163	.054	.067	.9772
Subject(Group)	76	61.325	.807		
Card	3	351.613	117.204	139.343	.0001
Card * p Card T...	9	.613	.068	.081	.9998
Card * Subject(...	228	191.775	.841		

Dependent: Card Selections

II SINGLE CARD STUDY

ANOVA relating to Means Table 4.2.1: *p, q, -q* and *-p* card selections

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	19	398.138	20.955		
Card	3	1037.838	345.946	28.113	.0001
Card * Subject	57	701.413	12.305		

Dependent: Card

ANOVA relating to *p, q, -q* and *-p* card selections depending on card presentation mode (i.e. single or four card presentation)

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Study	1	35.156	35.156	2.134	.1523
Subject(Group)	38	626.088	16.476		
Card	3	2427.119	809.040	67.769	.0001
Card * Study	3	17.169	5.723	.479	.6973
Card * Subject(...	114	1360.963	11.938		

Dependent: Card

ANOVA relating to Means Table 4.2.3: *p, q, -q* and *-p* card selections depending on *p* card type (consonants, vowels, odd or even numbers)

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
p Card	3	.134	.045	.030	.9928
Subject(Group)	76	112.212	1.476		
Card	3	259.459	86.486	88.411	.0001
Card * p Card	9	2.253	.250	.256	.9852
Card * Subject(...	228	223.038	.978		

Dependent: Cards

II SINGLE CARD STUDY (continued)

ANOVA relating to *p, q, -q* and *-p* card selections depending on card presentation mode (i.e. single or four card presentation) and *p* card type (consonants, vowels, odd or even numbers)

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
p Card	3	.205	.068	.060	.9808
Presentation	1	8.789	8.789	7.698	.0062
p Card * Presentation	3	.092	.031	.027	.9940
Subject(Group)	152	173.537	1.142		
Cards	3	606.780	202.260	222.343	.0001
Cards * p Card	9	.964	.107	.118	.9993
Cards * Presentation	3	4.292	1.431	1.573	.1952
Cards * p Card * Presentati...	9	1.902	.211	.232	.9898
Cards * Subject(Group)	456	414.813	.910		

Dependent: Card

III PILOT RATINGS STUDY

ANOVA relating to Means Table 4.3.1: *p*, *q*, *-q* and *-p* card informativeness ratings

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	29	437.867	15.099		
Card	3	235.133	78.378	12.245	.0001
Card * Subject	87	556.867	6.401		

Dependent: Informativeness Rating

IV SINGLE RATINGS STUDY

ANOVA relating to Means Table 4.4.1: *p, q, -q* and *-p* card informativeness ratings

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	19	11.973	.630		
Card	3	135.894	45.298	1.71E2	.0001
Card * Subject	57	15.131	.265		

Dependent: Card Informativeness

ANOVA relating to Means Table 4.4.2: *p, q, -q* and *-p* card selections depending on *p* card type (consonants, vowels, odd or even numbers)

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
p card type	3	.426	.142	.189	.9038
Subject(Group)	76	57.224	.753		
Card	3	542.784	180.928	534.551	.0001
Card * p card ty...	9	17.320	1.924	5.686	.0001
Card * Subject(...	228	77.171	.338		

Dependent: Card Informativeness

RATINGS REPLICATION:

ANOVA relating to Means Table 4.4.3: *p, q, -q* and *-p* card informativeness ratings

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	34	44.168	1.299		
Card	3	167.992	55.997	1.06E2	.0001
Card * Subject	102	54.115	.531		

Dependent: Card Informativeness

ANOVA relating to card informativeness ratings depending on card type (*p, q, -q* or *-p)* and study (Single Ratings or Ratings Repliction)

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Study	1	.109	.109	.103	.7501
Subject(Group)	53	56.141	1.059		
Card	3	292.740	97.580	224.057	.0001
Card * Study	3	2.392	.797	1.831	.1437
Card * Subject(...	159	69.247	.436		

Dependent: Card Informativeness

IV **SINGLE RATINGS STUDY** and **RATINGS REPLICTION** (continued)

ANOVA relating to Means Table 4.4.4: *p, q, -q* and *-p* card selections depending on *p* card type (consonants, vowels, odd or even numbers)

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
p card type	3	2.134	.711	.491	.6892
Subject(Group)	136	197.061	1.449		
Card	3	674.576	224.859	316.167	.0001
Card * p card ty...	9	22.734	2.526	3.552	.0003
Card * Subject(...	408	290.170	.711		

Dependent: Card Informativeness

ANOVA relating to ard informativeness ratings depending on study (i.e. single or four card presentation) and *p* card type (consonants, vowels, odd or even numbers) and card type (*p, q, -q* or *-p* card)

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
p card type	3	1.865	.622	.518	.6701
Study	1	.516	.516	.430	.5128
p card type * Study	3	.229	.076	.064	.9789
Subject(Group)	212	254.285	1.199		
Card	3	1171.995	390.665	676.383	.0001
Card * p card type	9	37.673	4.186	7.247	.0001
Card * Study	3	9.422	3.141	5.438	.0011
Card * p card type * Study	9	.905	.101	.174	.9966
Card * Subject(Group)	636	367.341	.578		

Dependent: Card Informativeness

V BINARY STUDY

ANOVA relating to Means Table 4.5.1: frequencies of card pair comparisons

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	19	0.000	0.000		
Card Comparis...	11	13257.100	1205.191	32.389	.0001
Card Comparis...	209	7776.900	37.210		

Dependent: Card Pairings

ANOVA relating to Means Table 4.5.4: *p, q, -q* and *-p* card selections

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	19	-1.110E-16	-5.843E-18		
Card	3	24951.300	8317.100	35.956	.0001
Card * Subject	57	13184.700	231.311		

Dependent: Card Informativeness

ANOVA relating to *p, q, -q* and *-p* card informativeness ratings depending on *p* card type (consonants, vowels, odd or even numbers)

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
p card type	3	0.000	0.000		
Subject(Group)	76	-1.388E-17	-1.826E-19		
Card	3	6237.825	2079.275	120.981	.0001
Card * p card ty...	9	53.575	5.953	.346	.9583
Card * Subject(...	228	3918.600	17.187		

Dependent: Card Informativess

VI FIRST PROBABILITY STUDY

ANOVA relating to comparison of mean proportions of correct responses about reverse side of cards in learning phases of high and low P(*p*) conditions

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Condition	1	.003	.003	.079	.7798
Subject(Group)	38	1.638	.043		
Cards	3	2.457	.819	33.062	.0001
Cards * Condition	3	.934	.311	12.570	.0001
Cards * Subject...	114	2.824	.025		

Dependent: Proportions

ANOVA relating to Means Table 4.6.2: *p, q, -q* and *-p* card selections in high P(*p*) condition

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	19	177.237	9.328		
Card	3	351.638	117.213	8.217	.0001
Card * Subject	57	813.113	14.265		

Dependent: Cards

ANOVA relating to Means Table 4.6.3: *p, q, -q* and *-p* card selections in low P(*p*) condition

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	19	230.238	12.118		
Card	3	149.238	49.746	4.444	.0071
Card * Subject	57	638.013	11.193		

Dependent: Cards

ANOVA relating to *p, q, -q* and *-p* card selections depending on probability condition

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Condition	1	15.625	15.625	1.457	.2348
Subject(Group)	38	407.475	10.723		
Card	3	469.850	156.617	12.304	.0001
Card * Condition	3	31.025	10.342	.812	.4895
Card * Subject(...	114	1451.125	12.729		

Dependent: Card

VI FIRST PROBABILITY STUDY (continued)

ANOVA relating to CFI values depending on study (Single Card study, and high and low P(*p*) conditions of First Probability study)

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	19	33.836	1.781		
CFI	2	6.596	3.298	1.794	.1802
CFI * Subject	38	69.869	1.839		

Dependent: CFIs

VII SECOND PROBABILITY STUDY

ANOVA relating to comparison of mean proportions of correct responses about reverse side of cards in learning phases of high and low P(*p*) conditions

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Condition	1	.019	.019	2.979	.0925
Subject(Group)	38	.237	.006		
Card	3	3.444	1.148	125.511	.0001
Card * Condition	3	1.246	.415	45.400	.0001
Card * Subject(...	114	1.043	.009		

Dependent: Proportions

ANOVA relating to Means Table 4.7.3: *p, q, -q* and *-p* card selections in high P(*p*) condition

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	19	142.937	7.523		
Cards	3	171.737	57.246	2.032	.1195
Cards * Subject	57	1605.513	28.167		

Dependent: Cards

ANOVA relating to Means Table 4.7.4: *p, q, -q* and *-p* card selections in low P(*p*) condition

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	19	342.938	18.049		
Cards	3	42.637	14.212	.561	.6427
Cards * Subject	57	1443.113	25.318		

Dependent: Cards

ANOVA relating to *p, q, -q* and *-p* card selections depending on probability condition

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Condition	1	2.500	2.500	.196	.6609
Subject(Group)	38	485.875	12.786		
Card	3	110.275	36.758	1.375	.2542
Card * Condition	3	104.100	34.700	1.298	.2788
Card * Subject(...	114	3048.625	26.742		

Dependent: Cards

VII SECOND PROBABILITY STUDY (continued)

ANOVA relating to CFI values depending on study (Single Card study, high and low P(*p*) conditions of Second Probability study)

Source	df	Sum of Squares	Mean Square	F-Value	P-Value
Subject	19	52.362	2.756		
CFI	2	39.807	19.903	5.924	.0058
CFI * Subject	38	127.669	3.360		

Dependent: CFIs

References

Almor A. & Sloman, S. A. (1996). Is Deontic Reasoning Special? *Psychological Review 103(2),* 374-380.

Anderson, J. R. (1990a). *The Adaptive Character of Thought.* Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Anderson, J. R. (1990b). *Cognitive psychology and its implications.* New York: W. H. Freeman and Company.

Anderson, J. R. (1991). Is human cognition adaptive? *Behavioural and Brain Sciences, 14,* 471-517.

Anderson, J. R. (1993). *Rules of the Mind.* Hillsdale, New Jersey: Lawrence Erlbaum Associates Inc.

Attneave, F. (1959). *Applications of information theory to psychology: A summary of basic concepts, methods, and results.* New York: Holt, Rinehart and Winston.

Bell, W. J. (1991). *Searching Behaviour. The behavioural ecology of finding resources.* London: Chapman and Hall.

Chapman, M. (1992). Equilibration and the Dialectics of Organization. In: H. Beilin, & P. B. Pufall (Eds.), *Piaget's Theory. Prospects and Possibilities.* Hillsdale, New Jersey: Lawrence Erlbaum Associates .

Cheng, P. W., & Holyoak, K. J. (1985). Pragmatic reasoning schema. *Cognitive Psychology, 17,* 391-416.

Cheng, P. W., & Holyoak, K. J. (1989). On the natural selection of reasoning theories. *Cognition, 33,* 285-313.

Christiansen, M. H., & Chater, N. (1992). Connectionism, Learning and Meaning. *Connection Science,* (4), 227-252.

Churchland, P. M. (1990). *Matter and Consciousness.* London: The MIT Press.

Churchland, P. C. (1989). *Neurophilosophy: Toward a Unified Science of the Mind/Brain.* London: The MIT Press.

Cohen, L. J. (1981). Can human irrationality be experimentally demonstrated? *The Behavioral and Brain Sciences, 4,* 317-370.

Cohen, L. J. (1989). *An Introduction to The Philosophy of Induction and Probability.* Oxford: Clarendon Press.

Cooke, H. P., & Tredennick H. (1938). *Aristotle. The Organon - I.* London: William Heinemann Ltd.

Cooke, H. P., & Tredennick, H. (1960). *Aristotle - Posterior Analytics and Topica.* London: William Heinemann Ltd.

Coombs, C. H., Dawes, R. M., & Tversky A. (1970). *Mathematical Psychology. An Elementary Introduction.* Englewood Cliffs, New Jersey: Prentice-Hall, Inc.

Cosmides, L. (1989). The logic of social exchange: Has natural selection shaped how humans reason? Studies with the Wason selection task. *Cognition, 31,* 187-276.

Cosmides, L., & Tooby, J. (1987). From Evolution to Behaviour: Evolutionary Psychology as the Missing Link. In: J. Dupre (Ed.). *The Latest on the Best. Essays on Evolution and Optimality.* Cambridge, Massachusetts: The MIT Press.

Cosmides, L., & Tooby, J. (1992). Cognitive Adaptations for Social Exchange. In: J. H. Barkow, L. Cosmides, & J. Tooby (Eds.), *The Adapted Mind. Evolutionary Psychology and the Generation of Culture.* Oxford: Oxford University Press.

Cosmides, L., & Tooby, J. (1996). Are humans good intuitive statisticians after all? Rethinking some conclusions from the literature on judgment under uncertainty. *Cognition, 58,* 1-75.

Earman, J. (1983). *Testing Scientific Theories.* Minneapolis: University of Minnesota Press.

Evans J. St.B.T. (1972). Interpretion and matching bias in a reasoning task. *Quarterly Journal of Experimental Psychology, 24,* 193-199.

Evans J. St.B.T. (1977). Linguistic Factors in Reasoning. *Quarterly Journal of Experimental Psychology, 29,* 297-306.

Evans J. St.B.T. (1980). Current issues in the psychology of reasoning. *British Journal of Psychology, 71,* 227-239.

Evans J. St.B.T. (1983). Linguistic determinants of bias in conditional reasoning. *Qaurterly Journal of Experimental Psychology, 35A,* 635-644.

Evans J. St.B.T. (1984). Heuristics and analytic processes in reasoning. *British Journal of Psychology, 75,* 451-468.

Evans J. St.B.T. (1989). *Bias in human reasoning: Causes and consequences.* Hove: Lawrence Erlbaum Associates.

Evans J. St.B.T. (1991). Theories of Human Reasoning: The Fragmented State of the Art. *Theory of Psychology, 1 (1),* 83-105.

Evans J. St.B.T. (1993a). Bias and rationality. In: K. I. Manktelow, & Over. D.E. (Eds.), *Rationality. Psychological and philosophical perspectives.* London: Routledge.

Evans J. St.B.T. (1993b). The Cognitive Psychology of Reasoning: An Introduction. *The Quarterly Journal of Experimental Psychology,* (46A (4)), 561-567.

Evans J. St.B.T. (1995). Relevance and Reasoning. In Newstead S. E., & Evans Jonathan St. B.T. (Eds.), *Perspectives on Thinking and Reasoning : Essays in Honour of Peter Wason.* Hove, East Sussex: Lawrence Erlbaum and Associates, Ltd.

Evans J. St.B.T., & Lynch, J. S. (1973). Matching bias in the selection task. *British Journal of Psychology,, 64(3),* 391-397.

Evans J. St.B.T., Newstead Stephen E., & Byrne, R. M. J. (1993). *Human Reasoning. The Psychology of Deduction.* Hove, East Sussex: Lawrence Erlbaum Associated Ltd.

Evans J. St.B.T., & Over, D. E. (1996). Rationality in the selection task: epistemic utility versus uncertainty reduction. *Psychological Review, 103 (2)*, 356-363.

Evans J. St.B.T., Over. D.E., & Manktelow, K. I. (1993). Reasoning, decision making and rationality. In: P. N. Johnson-Laird & E. Shafir (Eds.), *Reasoning and Decision Making.* Oxford: Blackwell.

Fedorov, V. V. (1972). *Theory of Optimal Experiments.* New York: Academic Press.

Flanagan, O. (1991). *The Science of the Mind.* London: MIT Press.

Fodor, J. A. (1975). *The Language of Thought.* New York: Thomas Y. Crowell.

Fodor, J. A. (1983). *The Modularity of Mind.* Cambridge, MA: The MIT Press.

Ford, M. (1994). Two models of mental representation and problem solution in syllogistic reasoning. *Cognition, 54,* 1-71.

Galotti, K. M. (1989). Approaches to studying formal and everyday reasoning. *Psychological Bulletin, 105 (3),* 331-351.

Garcia, R. (1992). The Structure of Knowledge and the Knowledge of Structure. In H. Beilin, & P. B. Pufall (Eds.), *Piaget's Theory. Prospects and Possibilites.* Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Gardner, H. (1985). *The Mind's New Science.* New York: Basic Books, Inc.

Garner, W. R. (1962). *Uncertainty and structure as psychological concepts.* New York: John Wiley and Sons Inc.

Gigerenzer, G. (1991a). From tools to theories: A heuristic of discovery in cognitive psychology. *Psychological Review, Vol. 98, No. 2,* 254-267.

Gigerenzer, G. (1991b). Probabilistic mental models: A Brunswikian theory of confidence. *Psychological Review, Vol. 98, No. 4,* 506-528.

Gigerenzer, G. (1994). Why the distinction between single-event probabilities and frequencies is important for psychology (and vice versa). In: G. Wright, & P. Ayton (Eds.), *Subjective Probability.* Chichester: John Wiley & Sons.

Gigerenzer, G., & Hoffrage, U. (1995). How to improve Bayesian reasoning without instruction: frequency formats. *Psychological Review, 102 (4),* 684-704.

Gigerenzer, G., & Hug, K. (1992). Domain-specific reasoning: Social contracts, cheating, and perspective change. *Cognition, Vol. 43,* 127-171.

Gigerenzer G., & Murray D. J. (1987). *Cognition as Intuitive Statistics.* Hillsdale, New Jersey: Lawrence Erlbaum Associates, Inc.

Gigerenzer, G., Swijtink, Z., Porter, T., Daston, L., Beatty, J., & Kruger, L. (1989). *The Empire of Chance: how probability changed science and everyday life.* Cambridge: Cambridge University Press.

Gilhooly, K. J. (1988). *Thinking.* London: Academic Press.

Good, I. J. (1950). *Probability and the weighing of evidence.* London: Charles Griffin & Company Limited.

Goodwin, R. Q., & Wason, P. C. (1972). Degrees of insight. *British Journal of Psychology, 63,* 205-212.

Green, D. W. (1995). Externalization, Counter-examples, and the Abstract Selection Task. *The Quarterly Journal of Experimental Psychology, 48A (2),* 424-446.

Green, D. W., & Larking, R. (1995). The Locus of Facilitation in the Abstract Selection Task. *Thinking and Reasoning, 1(2),* 183-199.

Griggs R. A., & Cox J. R. (1982). The elusive thematic materials effects in the Wason selection task. *British Journal of Psychology, 73,* 407-420.

Guilford, J. P. (1954). *Psychometric Methods.* New York: McGraw-Hill Book Company.

Hacking, I. (1975). *The Emergence of Probability.* London: Cambridge University Press.

Haken, H. (1983). *Synergetics.* Berlin: Springer-Verlag.

Hey, J. D. (1992). Experiments in economics - and psychology. In S. E. G. Lea, P. Webley, & B. M. Young (Eds.), *New Directions in Economic Psychology. Theory, Experiment and Application.* Aldershot, England: Edward Elgar Publishing Limited.

Holyoak K. J., & Cheng, P. W. (1995). Pragmatic Reasoning about Human Voluntary Action: Evidence from Wason's Selection Task. In: S. E. Newstead, & J. S. B. Evans (Eds.), *Perspectives on Thinking and Reasoning.* Hove, East Sussex: Lawrence Erlbaum Associates Ltd.

Hospers, J. (1989). *An Introduction to Philosophical Analysis.* London: Routledge.

Houston, A. I. (1993). The Importance of State. In: R. N. Hughes (Ed.), *Diet Selection. An Interdisciplinary Approach to Foraging Behaviour.* Oxford: Blackwell Scientific Publications.

Houston.A.I., & Staddon, J. E. R. (1981). Optimality principles and behaviour: It's all for the best. *The Behavioural and Brain Sciences, 4*, 395-396.

Hughes, G. E., & Cresswell, M. J. (1968). *An Introduction to Modal Logic.* London: Methuen and Co. Ltd.

Johnson-Laird, P. N. (1983). *Mental Models.* Cambridge: Cambridge University Press.

Johnson-Laird, P. N. (1995). Inference and Mental Models. In: S. E. Newstead, & J. S. B. Evans (Eds.), *Perspectives on Thinking and Reasoning. Essays in Honor of Peter Wason.* Hove, East Sussex: Lawrence Erlbaum Associates Ltd.

Johnson-Laird, P. N., & Byrne, R. M. J. (1991). *Deduction.* Hove: Lawrence Erlbaum Associates.

Johnson-Laird, P. N., & Byrne, R. M. J. (1993a). Precis of Deduction. *Behavioural and Brain Sciences, 16,* 323-380.

Johnson-Laird, P. N., & Byrne, R. M. J. (1993b). Models and deductive rationality. In: K. I. Manktelow, & Over D.E. (Eds.), *Rationality. Psychological and Philosophical Perspective.* London: Routledge.

Johnson-Laird, P. N., Legrenzi, P., & Legrenzi, M. S. (1972). Reasoning and a sense of reality. *British Journal of Psychology, 63,* 395-400.

Johnson-Laird, P. N., & Wason, P. C. (1970a). Insight into a logical relation. *Quarterly Journal of Experimental Psychology, Vol. 22,* 49-61.

Johnson-Laird, P. N., & P. C. Wason. (1970b). A theoretical analysis of insight into a reasoning task. *Cognitive Psychology, Vol. 1,* 134-148.

Kirby, K. N. (1994). Probabilities and utilities of fictional outcomes in Wason's four-card selection task. *Cognition, 51,* 1-28.

Kirby, K. N. (1994b). False alarm: a reply to Over and Evans. *Cognition, 52,* 245-250.

Kneale, W., & Kneale, M. (1962). *The Development of Logic.* Oxford: Clarendon Press.

Krebs, J. R., & Kacelnik, A. (1991). Decision-making. In: J. R. Krebs, & N. B. Davies, *Behavioural Ecology. An Evolutionary Approach.* Oxford: Blackwell Scientific Publications.

Laming, D. (1996). On the analysis of irrational data selection: a critique of Oaksford and Chater (1994). *Psychological Review, 103(2),* 364-373

Lea, S. E. G., Webley, P., & Young, B. M. (1992). Economic psychology: A new sense of direction. In S. E. G. Lea, P. Webley, & B. M. Young (Eds.), *New Directions in economic Psychology. Theory Experiment and Appliction.* Aldershot, England: Edward Elgar Publishing Limited.

Lemmon E. J. (1977). *The "Lemmon Notes". An Introduction to Modal Logic.* Oxford: Basil Blackwell.

Lemmon E. J. (1965). *Beginning Logic.* Wokingham: Van Reinhold (UK) Co. Ltd.

Levine, R. L., & Fitzgerald H. E. (1992). *Analysis of Dynamic Psychological Systems.* . New York: Plenum Press.

Lewes, G. H. (1857). *The Biographical History of Philosophy.* London: John W. Parker and Son, West Strand.

Lintern, G., & Kugler, P. N. (1991). Self-organization in connectionist models: Associative memory, dissipative structures and thermodynamic law. *Human Movement Science, 10,* 447-483.

Mackie, J. L. (1973). *Truth, Probability and Paradox.* Oxford: The Clarendon Press.

Maguire, P., Faulkner, A., Booth, K., Elliott, C. & Hillier, V.(1996).Helping cancer patients disclose their concerns. *European Journal of Cancer 32A (1),* 78-81.

Manktelow, K. I. (1993). Models for deontic deduction. *Behavioral and Brain Sciences, 16 (2),* 357.

Manktelow, K. I., & Evans, J. S. B. (1979). Facilitation of reasoning by realism: Effect or non-effect? *British Journal of Psychology, 70,* 477-488.

Manktelow, K. I., & Over. D.E. (1990). *Inference and Understanding.* London: Routledge.

Manktelow, K. I., & Over. D. E. (1991). Social roles and utilities in reasoning with deontic conditions. *Cognition, 39,* 85-105.

Manktelow, K. I., & Over, D. E. (1992). Utility and deontic reasoning. Some comments on Johnson-Laird and Byrne. *Cognition, 43,* 183-188.

Manktelow, K. I., & Over, D. E. (1993). *Rationality. Psychological and Philosophical Perspectives.* London: Routledge.

Manktelow K.I., Sutherland E.J., & Over D.E. (1995). Probabilistic Factors in Deontic Reasoning. *Thinking and Reasoning, 1(3),* 201-219.

Marr, D. (1982). *Vision.* San Fransisco: W. H. Freeman and Company.

Maturana, H. R. (1991). Science and Daily Life: The Ontology of Scientific Explanation. In F. Steir (Ed.), *Research and Reflexivity.* London: Sage Publications.

Mazur E. J. (1994). *Learning and Behaviour.* Englewood Cliffs, New Jersey: Prentice-Hall, Inc.

McFarland, D. (1989). *Problems of Animal Behaviour.* Harlow, Essex: Longman Scientific & Technical.

McFarland, D. (1993). *Animal Behaviour.* Harlow, Essex: Longman Scientific & Technical.

McKenzie, C. R. M. (1994). The accuracy of intuitive judgment strategies: Covariation assessment and Bayesian inference. *Cognitive Psychology, 26, 209-239,*

Oakhill, J. V., & Johnson-Laird P. N. (1985). Rationality, memory and the search for counterexamples. *Cognition, 20,* 79-94.

Oaksford, M. (1993). Mental models and the tractability of everyday reasoning. *Behavioral and Brain Sciences, 16 (2),* 360-361.

Oaksford, M., & Chater, N. (1992). Bounded Rationality in Taking Risks and Drawing Inferences. *Theory and Psychology, 2(2),* 225-230.

Oaksford M., & Chater, N. (1993). Reasoning theories and bounded rationality. In: Manktelow K.I., & Over. D. E. (Eds.), *Rationality. Psychological and Philosophical Perspectives,*London: Routledge.

Oaksford, M., & Chater, N. (1994). A rational analysis of the selection task as optimal data selection. *Psychological Revew, 101 (4),* 608-631.

Oaksford, M., & Chater, N. (1995a). Theories of Reasoning and the Computational Explanation of Everyday Inference. *Thinking and Reasoning, 1 (2),* 121-152.

Oaksford M., & Chater N. (1995b). Information gain explains relevance which explains the selection task. *Cognition, 57,* 97-108.

Oaksford M., & Chater N. (1996). Rational explanation of the selection task. *Psychological Review, 103, No. 2,* 381-391.

Oaksford M., Chater, N., Grainger, R., & Larkin J. (in press). Optimal data selection in the reduced array selection task (RAST).

Oaksford, M., Morris, F., Grainger, B., & Williams J.M.G. (1996). Mood, Reasoning and Central Executive Processes. *Journal of Experimental Psychology. Learning, Memory and Cognition, 22,* 477-493.

Oaksford M., & Stenning, K. (1992). Reasoning with conditionals containing negated constituents. *Journal of Experimental Psychology: Learning, Memory and Cognition, 18,* 835-854.

O'Brien, D. P. (1993). Mental logic and human irrationality. We can put a man on the moon, so why can't we solve those logical-resoning problems? In: K. I. Manktelow, & D. E. Over (Eds.), *Rationality. Psychological and Philosophical Perspectives.* London: Routledge.

O'Brien, D. P. (1995). Finding logic in human reasoning requires looking in the right places. In: *Perspectives on Thinking and Reasoning. Essays in Honour of Peter Wason.* Hove, East Sussex: Lawrence Erlbaum Associates Ltd.

Ormerod, T. C., Manktelow, K. I., & Jones, G. V. (1993). Reasoning with Three Types of Conditional: Biases and Mental Models. *The Quarterly Journal of Experimental Psychology, 46A (4),* 653-677.

Over, D. (1993). Deduction and degrees of belief. *Behavioral and Brain Sciences, 16 (2),* 361-362.

Over, D. E., & Evans, J. S. B. (1994). Discussion. Hits and misses: Kirby on the selection task. *Cognition, 52,* 235-243.

Over D. E., & Manktelow K. I. (1993). Rationality, utility and deontic reasoning. In: K. I. Manktelow, & D. E. Over (Eds.), *Rationality. Psychological and Philosophical Perspectives.* London: Routledge.

Parle, M., Jones, B. & Maguire, P. (1996). Maladaptive coping and affective disorders among cancer patients. *Psychological Medicine, 26,* 735-744

Phillips L. D. (1973). *Bayesian statistics for social scientists.* England: The Whitefriars Press Ltd.

Platt, R. D., & Griggs, R. A. (1995). Facilitation and matching bias in the abstract selection task. *Thinking and Reasoning, 1(1),* 55-70.

Politzer, G., & Nguyen-Xuan, A. (1990). Reasoning about Conditional Promises and Warnings: Darwinian Algorithms, Mental Models, Relevance Judgements or Pragmatic Schemas? *The Quarterly Journal of Experimental Psychology, 44A (3),* 401-421.

Pollard P. (1982). Human reasoning: Some possible effects of availbility. *Cognition, 12,* 65-69.

Pollard, P. (1985). Nonindependence of selections of the Wason selection task. *Bulletin of the Psychonomic Society, 23,* 317-321.

Pollard, P. (1990). Natural selection for the selection task: Limits to social exchange theory. *Cognition, 36,* 196-204.

Pollard P., & Evans J. St. B. T. (1981). The effects of prior beliefs in reasoning: An associational interpretation. *British Journal of Psychology, 72,* 73-81.

Pollard, P., & Evans J. St. B. T. (1983). The Effect of Experimentally Contrived Experience. *Psychological Research, 45,* 287-301.

Popper, K. R. (1959). *The Logic of Scientific Discovery.* London: Hutchinson & Co.

Prigogine, I., & Stengers, I. (1990). Consciousess and the Physical World. In: Pickering J., & M. Skinner (Eds.), *From Sentience to Symbols.* Hemel Hempstead, England: Harvester Wheatsheaf.

Provenza, F. D., & Cincotta, R. P. (1993). Self-Organizational Learning Process: Accepting Adaptability at the Expense of Predictability. In: R. N. Hughes (Ed.), *Diet Selection. An Interdisciplinary Approach to Foraging Behaviour.* Oxford: Blackwell Scientific Publications.

Rachlin, H., Battalio, R., Kagel, J., & Green, L. (1981). Maximization theory in psychology. *The Behavioural and Brain Sciences, 4,* 371-417.

Rips, L. J. (1983). Cognitive processes in propositional reasoning. *Psychological Review, 90,* 38-71.

Rips, L. J. (1990). Reasoning. *Annual Review of Psychology, Vol 41,,* 321-353.

Russell, J. (1984). *Explaining Mental Life.* London: Macmillan Press.

Shafir, E., Simonson, I., & Tversky, A. (1994). Reason-Based Choice. In P. N. Johnson-Laird, & E. Shafir (Eds.), *Reasoning and Decision Making.* Oxford: Blackwell Publishers.

Shanks, D. R. (1995). Is human learning rational? *Quarterly Journal of Experimental Psychology, 48A 92),* 257-279.

Shettleworth, S. J., Reid, P. J., & Plowright, C. M. C. (1993). The Psychology of Diet Selection. In: Hughes R. N. (Eds.), *Diet Selection. An Interdisciplinary Approach to Foraging Behaviour.* Oxford: Blackwell Scientific Publictions.

Sibly, R., & McFarland, D. (1974). A State-Space Approach to Motivation. In McFarland D. J. (Ed.), *Motivational Control Systems Analysis,*London: Academic Press Inc.

Simon, H. A. (1959). Theories of decision-making in economics and behavioural science. *The American Economic Review, XLIX No. 3,* 253-283.

Simon, H. A. (1983). *Reason in Human Affairs.* Oxford: Basil Blackwell.

Simon, H. A. (1986). Rationality in Psychology and Economics. In: R. M. Hogarth, & M. W. Reder (Eds.), *Rational Choice. The Contrast between Economics and Psychology.* Chicago: The University of Chicago Press.

Smalley, N. S. (1974). Evaluating a rule against possible instances. *British Journal of Psychology, 65,* 293-304.

Sperber, D., Cara, F., & Girotto, G. (1995). Relevance theory explains the selection task. *Cognition, 56,* 31-95.

Sperber, D., & Wilson, D. (1986). *Relevance: Communication and cognition.* Oxford: Basil Blackwell.

Stenning, K., & Oberlander, J. (1995). A cognitive theory of graphical and linguistic reasoning: Logic and implementation. *Cognitive Science, 19 (1),* 97-140.

Stephens, D. W., & Krebs, J. R. (1986). *Foraging Theory.* Princeton, New Jersey: Princeton University press.

Sternberg, R. J. (1990). *The psychologist's companion: A guide to scientific writing for students and researchers.* Cambridge: Cambridge University Press.

Stevenson, R. J., & Over, D. E. (1995). Deduction from Uncertain Premises. *The Quarterly Journal of Experimental Psychology, 48A (3),* 613-643.

Stillings, N. A., Feinstein, M. H., Garfield, E. L., Rosenbaum, D. A., Weisler, S. E., & Baker-Ward, L. (1987). *Cognitive Science. An Introduction.* Cambridge, Massachusetts: The MIT Press.

Townsend, J. T. (1992). Chaos Theory: A Brief Tutorial and Discussion. In: A. F. Healy, S. M. Kosslyn, & R. M. Shiffrin (Eds.), *From Learning Theory to Connectionist Theory. Essays in Honor of William K. Estes.* (Vol. 1.). Hillsdale, New Jersey: Lawrence Elbaum.

Varela, F. J. (1979). *Principles of Biological Automony.* New York: North Holland.

Wason, P. C. (1966). Reasoning. In B. Foss (Ed.), *New Horizons in Psychology.* Harmondsworth: Penguin Books.

Wason, P. C. (1968). Reasoning about a rule. *Quarterly Journal of Experimental Psychology, 20,* 273-281.

Wason, P. C. (1969). Regression in reasoning? *British Journal of Psychology, 60 (4),* 471-480.

Wason, P. C. (1979). Novelty and tradition in cognitive research: A case study. *Italian Journal of Psychology, 6(1),* 1-7.

Wason, P. C., & Evans J. St.B. T. (1975). Dual processes in reasoning? *Cognition, 3(2),* 141-154.

Wason, P. C., & Golding, E. (1974). The language of inconsistency. *British Journal of Psychology, 65(4),* 537-547.

Wason, P. C., & Johnson-Laird, P. N. (1970). A conflict between selecting and evaluating information in an inferential task. *British Journal of Psychology, 61,* 509-515.

Wason, P. C., & Johnson-Laird, P. N. (1972). *Psychology of Reasoning.* London: B. T. Batsford Ltd.

Wason, P. C., & Shapiro, D. (1971). Natural and contrived experience in a reasoning problem. *Quarterly Journal of Experimental Psychology, 23,* 63-71.

Weber, B. H., Depew, D. J., & Smith, J. D. (1988). *Entropy, Information, and Evolution. New Perspectives on Physical and Biological Evolution.* Cambridge Massachusetts: The MIT Press.