DOCTOR OF PHILOSOPHY

Construction of growth models for Pinus nigra var. maritima (Ait.) Melville (Corsican pine) in Great Britain

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Award date:
1998

Awarding institution:
University of Wales, Bangor

Construction of growth models for *Pinus nigra* var. *maritima* (Ait.) Melville (Corsican pine) in Great Britain

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A thesis submitted to the university of Wales Bangor

for

the degree

of

Doctor of Philosophy

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Bangor, United Kingdom

November, 1998

I would like to dedicate this thesis to two special people for their continued love and encouragement despite having suffered loneliness and partial neglect, in order that I could finish this work successfully;

to my loving wife Sujatha and little son Madhawa.

ACKNOWLEDGEMENTS

I wish to express my sincere gratitude and thanks to my two supervisors, Mr. T.A.R. Jenkins (School of Agricultural and Forest Sciences, University of Wales Bangor) for his precious guidance, help, advice and critical comments throughout this study and Dr. G.J. Mayhead (School of Agricultural and Forest Sciences, University of Wales Bangor) for his valuable guidance, supervision and comments especially on the manuscript for the completion of this thesis.

My sincere gratitude is also extended to Mr. C.J. Whitaker (Department of Mathematics, University of Wales Bangor) for his advice and help whenever I required detailed statistical advice.

I also wish to thank staff at the Forestry Commission Research Agency in Great Britain, Mrs. Janet Methley (Head of Mensuration Branch), Mr. Robert Matthews (Growth Modeller) and Mr. Ian Craig (in charge of sample plot data) for their kind assistance in providing sample plot data for this study.

I wish to thank Mr. Sam Foster (Manager, Computer Laboratory, School of Agricultural and Forest Sciences) and Mr. David O'Brian (Former Assistant Manager, Computer Laboratory, School of Agricultural and Forest Sciences) for their help with computer software problems.

My thanks are also extended to the British Council and Overseas Development Administration(ODA) for the financial support to study in Great Britain.

Finally I would like to thank my loving parents who always encouraged me for further studies and forced to come to Great Britain leaving them in Sri Lanka.

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Ph.D. Thesis, University of Wales Bangor, UK.

ABSTRACT

The British Forestry Commission (FC) provided data for 49 permanent sample plots of Corsican pine (*Pinus nigra* var. *maritima* (Ait.) Melville) in Great Britain. They covered various thinning types (low, intermediate, neutral, crown, exploitation), general yield classes (10-22) and initial planting densities (1736-6944 trees per ha). They had been measured at one to six year intervals and thinning was carried out at four to eight year intervals.

The FC follows a detailed procedure for recording sample plot data for various measurements. Computer programmes were written to read these data to do the calculations for the construction of models. Models were initially constructed for separate thinning types by partitioning the data by thinning types (27 sample plots). Later, the possibility of using one set of parameters for each model for all thinning types was tested. However, there were only enough data to construct models for intermediate and neutral thinning types. Each data set was divided into two sets: 75% for constructing the models and 25% for validation.

All models were constructed using regression analysis after determining the basic model structure by examining the scatter distributions and the correlation of selected explanatory variables with the corresponding response variable. All possible combinations of the explanatory variables were tested in order to obtain the best models. It was assumed that there was no natural mortality when thinning was carried out. The performance of the models was tested using statistical tests and standard residual distributions. Two models were constructed initially for each response variable, and after many tests, the best of the two models was selected.

The growth models were constructed to predict the future diameter at breast height (dbh), future total height, current timber height, current total volume and current merchantable volume of individual trees of the main crop trees of Corsican pine growing in Great Britain. The dbh and total height prediction models used the present value of the same variables, a factor to represent the site and the duration of the simulation period. The timber height prediction model used an exponential function developed by multiplying dbh and total height. The total volume prediction model was constructed using basal area and total height of individual trees. The merchantable volume prediction model was a derivation of the selected total volume prediction model. A set of models was also constructed to predict the mean tree basal area, mean dbh and mean total height of the trees removed in thinning. The only explanatory variable of these models was the same value as the response variable but just before thinning. A general procedure was described to estimate the number of trees removed in each thinning.

Three selected models developed outside Great Britain for other species were re-calibrated to Corsican pine in local British conditions without adding new factors or variables to compare the predictability of the new set of models. Bias was highlighted for many re-calibrated models indicating the necessity of new growth functions or variables. Finally the predictions of all the newly constructed and re-calibrated models were tested with the observed values against plantation age.

All the newly constructed models indicated a very low bias and a high modelling efficiency of over 0.9. The signs of the estimated parameters of selected models were corrected to be compatible with the possible biological reality. When compared with the actual data, predictions of the newly constructed models were much closer to the actual values than the predictions from the re-calibrated models.

CHAPTER 1: GENERAL INTRODUCTION

Corsican pine, *Pinus nigra* var. *maritima* (Ait.) Melville was introduced to Great Britain in 1759. It is a light demanding, wind-firm, frost hardy species which has persistent branches. Yield class normally varies from 6 to 20. The best growth rates can be obtained in areas where annual rainfall is low and temperatures are high in summer. The best soil types for Corsican pines are light sandy or heavy clays such as those in the Midlands, south and east of England. Corsican pine also grows successfully on the north-east coast of Scotland and notably in Culbin. This species tolerates more air pollution than other commonly grown conifers but is susceptible to die back caused by *Gremmeniella abietina* (Largerb.) Morelet. Rotation age is typically 45-80 years (Hart, 1994).

In forestry, growth and yield are usually predicted by tables, graphs or mathematical models. Most of the time, the graphs and tables are constructed using growth and yield models which may comprise many separate, but interrelated components, each of which may influence and be influenced by other components and by assumptions in the model (Soares *et al.*, 1995). Growth and yield models in forestry are divided into many categories by different authors (e.g. Clutter *et al.*, 1992; Korzukhin *et al.*, 1996; Philip, 1994; Vanclay, 1994; Voit and Sands, 1996). There is not a clear definition for the correct method of classification but it is acceptable to use any method for the categorisation which is dependent on the requirement of the user or the modeller.

According to Kimmins (1997) a model can take many different forms, but basically it is either an abstract or a physical entity that represents in some way the form and/or the function of real world entities and processes. Models may therefore be constructed as predictive tools and relatively little new knowledge may be acquired in their development. For the users, the value of models lies in what they can do, not in how the models were made.

The models constructed for the present study will ultimately be re-calibrated for other *Pinus* species grown in Sri Lanka. Because of the lack of suitable data (e.g. physiological and climatic) in Sri Lanka for process based and stochastic modelling, it was decided to construct empirical models at this stage. Because the models will be used under a range of conditions in Sri Lanka, and to allow for detailed output, individual tree level models were constructed.

The most important requirement for sound modelling is serial data for many growth seasons. Without such data from permanent sample plots, the modeller cannot easily surmise the history of the plantation, removals from thinning etc. It is important to remember that the extrapolation or projection of model predictions is frequently difficult to justify.

For the present study, the Forestry Commission in Great Britain has given access to Corsican pine data for 49 sample plots re-measured over the period 1920-1992. All these plots were established in even-aged, monocultures. The plots were subjected to different thinning types: low, intermediate, neutral and exploitation. Some plots were maintained without thinning. General yield class varied from 10 to 22.

Biological knowledge of the various relationships between tree characteristics is very important in modelling, especially for selecting the explanatory variables, determining the correct sign of the parameters etc. For the growth of trees, the quality of the particular site plays a major role, irrespective of other stand characteristics such as stand density. In plantations, one efficient way to measure the quality of the site is top height[1] which is largely independent of stand density (Jenkins, pers. comm.; Philip, 1994) and therefore, gives a clear idea about the quality of the site for the particular tree species. Top height is also easily measured in the field. Many modellers have used the total height[2] of plantations at a particular age as a measure of site quality known as "site index" (Alder, 1980; Burkhart and Tennent, 1977; Trousdell *et al.*, 1974).

For the present study, top height was estimated by developing total height and breast height diameter (dbh) relationships. For each five year age class, a family of parallel lines was developed relating breast height diameter and total height for the prediction of top height.

[1] Definition of top height is given in page 10.
[2] Height of the tree from the ground level to the highest growing point (Philip, 1994).

In yield or growth prediction in forestry, volume is the most crucial variable. Various approaches have been used to address the problem of predicting present and future stem volume yield for management purposes. Prediction of future yield also requires the prediction of the relevant future stand characteristics such as the number of trees per hectare, the basal area per hectare and the average dominant height[1] (Pienaar, 1989).

For the set of growth models newly constructed in this study, data were first partitioned by the thinning types to estimate separate sets of parameters for the same models. However, for the construction of growth models, only two thinning types namely, intermediate and neutral were sufficient in the data obtained from the Forestry Commission. There were 19 and 18 sample plots in the data which were maintained under intermediate and neutral thinning types respectively. For each thinning type, 75% of plots were used for model construction and the remaining 25% for the validation of the constructed models. In this study, some assumptions had to be made during the construction of some of the model structures such as: there is no natural mortality in plantations if thinning is carried out regularly; the shape of the crown of Corsican pine trees is conical; the rate of photosynthesis is dependent on the crown structure and volume.

For each response variable, two models were developed so that the best model could be selected finally. The models for the main crop trees were developed using multiple linear regression except for the models constructed for the prediction of timber height of individual trees. For models predicting future growth, i.e. dbh and total height of individual trees, the explanatory variables selected were the present size of the response variable, passage of time between assessment (plantation age difference between the present time and the future time), and a factor to represent the site quality. For one total volume prediction model of individual trees, crown characteristics were tried as explanatory variables. Merchantable volume[2] prediction models were derived from the results of the total volume prediction models. Models for timber height prediction were constructed using non-linear regression between total height and dbh. For many models, number of existing trees per hectare or total basal area per hectare were tested as explanatory variables

[1] Definition of dominant height is given in page 10.
[2] Merchantable volume is the tree stem volume from the ground level to 7 cm over bark top diameter.

to represent the stand density. However, these variables could be not included either because they were not significant or changed the sign of more important parameters without improving the statistical properties of the model.

For the prediction of the mean values of basal area, diameter at breast height and total height of the trees removed in thinnings, two models, i.e. one linear and one non-linear, were constructed for each variable.

The predictability of the newly constructed models was compared with three of the well recognised empirical growth and yield models developed in the past. The selected models were developed by other authors outside of Great Britain and therefore, re-calibration was done to adapt these models to local conditions.

For each model, graphs of residuals (standard residuals, whenever possible) versus fitted values were carefully examined for possible outliers or a particular pattern of distribution. When the number of residuals were very high, standard deviations of the residuals at selected fitted values were checked for an expected even distribution. Precision and bias were quantitatively tested using average model bias, mean absolute difference and the modelling efficiency in addition to the coefficient of variation (R^2). These tests were also helpful for comparing the different models constructed to predict the same response variable. For the finally selected models, the test described by Weisburg (1985) was done to observe if there was any lack of fit. These models were then validated with the reserved data by overlaying the predictions on the raw data.

After selecting the final models, the possibility of using one set of parameters for each model for intermediate and neutral thinning types, instead of separate parameter sets was tested. If the attempt was unsuccessful, separate parameter sets were selected for use in otherwise similar models.

Finally, all the selected newly constructed and re-calibrated models were compared directly with the particular observed values for two sample plots selected from the plots reserved for validation for two thinning types. The models constructed in this study appear to provide more reliable outputs than the re-calibrated existing models.

CHAPTER 2: REVIEW OF LITERATURE

2.1 Yield prediction of forests

In timber management the predictions of volume production for forest managers have traditionally taken the form of yield tables which are tabular records showing expected volume of wood (board feet, cords, cubic feet, cubic metres etc.) per unit of land area (acre, hectare) by combinations of measurable characteristics of the forest stand (site quality, stand density) (Clutter *et al.*, 1992).

2.1.1 History of yield predictions

In the past, yield prediction was normally based on the projection forward of a simple historical bioassay, the pattern of biomass accumulation in merchantable biomass components over past rotations of many similar crops. As long as the future growth and economic conditions remain very similar to those of the past, it is difficult to imagine a better yield prediction method (Kimmins *et al.*, 1990). Indeed a yield table is one of the oldest approaches to yield estimation. The concept was apparently first applied in the Chinese "Lung Chuan codes" some 350 years ago (Vuokila, 1965).

The technique as we know it today in commercial forestry was devised in Europe in the 18th century. The first yield tables were published in Germany in 1787 and within a hundred years, over a thousand yield tables had been published (Vanclay, 1994). The first conventional yield table for south Australian plantation stands was produced in 1931 by Grey from temporary plots (spot plots) (Lewis *et al.*, 1976). Modern yield tables often include not only yields, but also stand height, mean diameter, number of stems, stand basal area and current and mean annual volume increments (Vanclay, 1994).

More sophisticated calculations and analytical techniques enabled additional variables to be included in yield calculation. Stand density was an obvious choice for a third variable after volume and site indices as it enables data from partially stocked plots to be used and means that the yield table can be applied to any stand. In 1981, Edwards and Christie published yield tables, which are used today for management purpose in plantation forestry in Great Britain. This set of tables provides height, stems per hectare, volume per hectare after thinning, mean annual and cumulative volume production at five year intervals for many species and site management regime combinations.

The approach has also been applied to mixed stands, especially selection forests in central Europe. There are several ways to build compact tables for natural forests. The basal area of the dominant species may be expressed as a percentage of total stand basal area in mixed forests (e.g. MacKinney *et al.*, 1937). The same technique could be used for uneven forests after identifying the main stand (e.g. Duerr and Gevorkiantz, 1938).

One of the important milestones in growth modelling in the 1960s was the understanding that growth and yield models must be compatible (Buckman, 1962; Clutter, 1963). Forest managers had a need for both growth and yield models (or tables) and it was important that these guides provided compatible results (Vanclay, 1991).

2.2 Growth and yield of trees and forests

Growth refers to the increase in size of a population or an individual over a given period of time (e.g. growth in volume of a stand in m^3 ha^{-1} y^{-1}). Yield refers to the final size of a population or individual at the end of a certain period (e.g. total volume produced by a stand in m^3 ha^{-1}), and usually includes any removals (e.g. thinnings) (Vanclay, 1994). Growth can be expressed and measured in several ways. One can look at number of stems; at biomass; at dry biomass; at volume; at size e.g. length and diameter. The variable chosen as the most appropriate for modelling the growth or yield depends on one's interest, and also on the process itself (Doucet and Sloep, 1992).

Individual trees are the basic units of the forest (Liu and Ashton, 1995), and tree growth depends both on a tree's own dimensions and the effect of other trees (Sievanen, 1993). The trees are usually different from each other in location, size and behaviour such as response to environmental stress, growth and reproduction patterns (DeAngelis and Gross, 1992). Growth rates are greatly influenced by site conditions and interaction among individual trees. The major type of interaction is competition for root and shoot space, a process which occurs when resources such as light and nutrients are in short supply (Liu and Ashton, 1995).

2.2.1 Basal area

Generally the change in basal area of individual trees with age is an exponential increase early on while in later years it is more or less linear so that a curve drawn of basal area against age for the early years can be extrapolated as a straight line continuing at the same slope as that found in the period just before culmination of the current annual increment (Fraser, 1980).

2.2.2 Crown and canopy

The crown structure is now often considered as a component of growth and yield models. Crown development and recession are determined by the tree interactions and its size is used as the predictor of future stem growth (Houllier *et al.*, 1995).

There have been many relationships developed between the crown dimensions and other tree characteristics. A trees crown reflects the cumulative level of competition over time (Mitchell, 1975). Increasing number of trees per unit area reduces crown length and reducing stand density through thinning slows the recession of the crown base (Assman, 1970; Makela, 1997; Short and Burkhart, 1992). Consequently, the present crown length is strongly influenced by growing conditions in the past and this suggests its use as an integrator for competition previously experienced by the tree (Hasenaur and Monserud, 1996). The use of the average crown diameter of the mean tree[1] allows a reasonable estimate to be made of the degree of crown competition (Christie, 1994).

Horizontal crown development can be measured by crown diameter or crown projection area. These are indirect and crude methods of assessing photosynthetic area. Age and immediate stocking levels surrounding a tree affect the size and the growth of crown diameter (CD) and crown projection area (CPA); however, within a stand density and an age class, CD or CPA is highly related to stem diameter or basal area, respectively (Sprinz and Burkhart, 1987).

The efficiency of tree crown production, defined as net assimilation, can be expressed as stem wood production per unit of leaf weight (Larson and Isebrands, 1972; Shelburne and Hedden, 1996).

2.2.3 Stand density

Stand density is a measure of the degree of crowding of live trees (Ayhan, 1978) which changes with thinning (Wenk, 1994), and within stocked areas is commonly expressed by various growing space ratios. Stand density is also a quantitative measure of live tree stocking expressed either relatively (as of unity), or absolutely (per unit area) in terms of number of trees, basal area, or volume (Ayhan, 1978).

[1] Mean tree value is the average value corresponding to the total number of trees per unit area.

2.2.4 Diameter

The size class distribution of bh diameters is primarily dependent on the age structure of the stand. Multi-aged stands such as in irregular and selective forestry tend to have reversed *J* shaped diameter distributions, while even-aged stands exhibit mount shaped distributions with varying degrees of left or right skewness (Clutter *et al.*, 1992). Diameter may be measured over or under bark - in the latter case either by measuring bark thickness or by removing the bark at the point of measurement (Philip, 1994).

2.2.5 Competition

As the individual trees in the stand grow in size, trees begin to compete for resources such as water, light and mineral nutrients (Tang *et al.*, 1994). A tree's ability to survive in the stand can be related to its supply of available photosynthates (West, 1987), growth rate and size or some other measures of vigour, e.g. the rate of change of foliage dry weight (Makela and Hari, 1986).

The roots of neighbouring trees begin to intermingle and eventually the overlap becomes sufficiently great to reduce the stem diameter growth of the tree and competition begins (Ayhan, 1978). More rapidly apparent is above ground competition for light. A tree's lower branches are shaded by its own upper crown and by the crowns of neighbouring trees. The ability of foliage to withstand shade varies greatly between species (Evans, 1996).

Competition between trees in a forest is indicated by competition indices in many models. The philosophy behind the competition indices is that they can reasonably reflect the impacts of the amount of resources that a subject tree cannot obtain because of the competitive effect of the neighbouring individuals; and that tree growth is directly influenced by the degree of the competition (Daniels *et al.*, 1986).

2.2.6 Height

Height is an important variable as at a given age it reflects the quality of the site. Sites with tall trees of a given age and species are more fertile and productive than the sites with shorter trees of the same age (Philip, 1994). Chhetri and Fowler (1996b) wrote that total heights of trees are normally required for the estimation of growth and yield such as wood volume or number of trees. Low light intensity stimulated tree height growth at the expense of diameter growth (Ayhan, 1978).

Top height and Dominant height

Top height is the average total height of the hundred trees of the largest girth per hectare (Eriksson *et al.*, 1997; Philip, 1994; Rollinson, 1985). However, some authors (Edwards, 1976) define it as the total height of the tree of the average basal area or diameter at breast height of the hundred largest girth trees per hectare. Dominant height is the total average height of hundred tallest trees per hectare (Philip, 1994). These definitions are not universal and recently top height and dominant height have been accepted as synonyms (Philip, 1994).

2.2.7 Mortality

Some researchers divide mortality into two major categories: regular and irregular. Regular mortality results from suppression or competition for limited resources such as light, water and nutrients. Irregular mortality occurs because of density independent forces including insect and pathogen attack and catastrophic factors such as hurricanes, windstorms, floods and fires (Liu and Ashton, 1995).

In his experiments, Alder (1978) observed that in permanent sample plots after three years of planting, simple mortality due to suppression was not found to be a significant occurrence over the range of management practices if the thinning is carried out.

2.2.8 Stand volume

Tree volume is the most crucial variable in most forest management systems. After planting, the annual volume increment of even-aged plantations increases with age, reaches a peak after some years and then falls off. Since the more productive crops produce both a higher volume, and a higher proportion of it earlier, substantial increases in early yields can be obtained by concentrating a thinning programme in the most productive crops (Fraser, 1980).

In their experiments, McClain *et al.* (1994) found that total and merchantable volume per tree increased for all species (i.e. black spruce, white spruce, and red pine) as initial spacing increased from 1.8m to 3.6m. However, volume production per unit area decreased significantly for all species as spacing increased.

Current annual increment and mean annual increment

The volume increment of a tree or a forest stand in the present year is called current annual volume increment (CAI) while its average increment over a period of years is called mean annual volume increment (MAI) (Hart, 1994).

It is generally found that the peak level of CAI is a more or less constant proportion of the maximum value of MAI. This peak of CAI generally occurs at about 60% of the age of the maximum MAI, so that if the age when CAI reached its peak is known, it is reasonable to assume that the maximum MAI will occur at about 1.7 times that age (Fraser, 1980).

2.3 Thinning

Thinning is the removal of a proportion of the trees in a crop (Hart, 1994) in order to provide more growing space for the remaining trees and thereby enhance their diameter increment, but also to provide an intermediate yield of timber (Hamilton, 1980). Thinning normally improves the final crop quality.

Estimated number of trees per hectare is dependent on the initial planting density and the type and the intensity of thinning which is carried out. In a low thinning, the average volume per tree of the thinnings will be about 75-80% of the crop mean volume, but in a crown thinning the mean volume per tree of thinnings may be about equal to or greater than the mean volume per tree for the main crop (Fraser, 1980).

Normally, the number of trees in the main crop after each thinning can be calculated as the residual when the number of trees removed in thinning is known (Fraser, 1980).

Self-thinning

In forests where thinning is not carried out a higher number of trees are removed by natural mortality due to the higher level of competition which is called self-thinning. The self-thinning of stands follows a typical pattern, where the slope of the curve decreases with age, indicating declining mortality with age (Kuuluvainen, 1991). According to Kimmins (1997), the self-thinning rule says that if the logarithm of mean total mass of individual plants is plotted against the logarithm of the number of plants per unit area (stand density) for fully stocked stands, a straight line with a slope of -3/2 results. The conceptual basis for this relationship is that any site has a maximum plant biomass-carrying capacity; as the present population approaches this limit, individual tree growth can continue only if the number of individuals is reduced (Kimmins, 1997).

2.3.1 Thinning cycle

The thinning cycle is the interval in years between successive thinnings. The choice of thinning cycle will usually depend on local management considerations and on the yield class of the crop. The usual length in temperate climates (Mayhead, pers. comm.) is from 4 to 6 years in young and fast growing crops and about 10 years for older and slower growing species (Rollinson , 1985).

2.3.2 Thinning intensity

Thinning intensity is the rate at which volume is removed, e.g. 10 $m^3ha^{-1}yr^{-1}$. It should not be confused with the thinning yield which is the actual volume per hectare removed in one thinning (Rollinson, 1985).

The maximum thinning intensity which can be maintained without causing a loss of cumulative volume production is known as the marginal thinning intensity. The marginal thinning intensity is reasonably close to an intensity which in terms of annual rate of volume removal is 70% of the maximum mean annual volume increment (Hart, 1994).

2.3.3 Thinning yield

Thinning yield is the actual volume per unit area removed in any thinning (Rollinson, 1985). It has been found experimentally, in Great Britain and elsewhere, that the marginal thinning intensity is 70% of maximum mean annual increment. If the thinning cycle is five years, then each thinning will remove 350% of the maximum MAI.

There is little to be gained in planting the tree species closely and the thinning of them unless there is a market for small timber (Mayhead, pers. comm.). This reflects the fact that the intensity of thinning assumed is such that cumulative volume production is reduced to levels more appropriate to wider spacing such as 2.4 – 3.0m (Christie, 1994).

2.4 Forest site

One very important factor in model construction is the quality of the site. Site quality is defined as the sum of all the environmental factors affecting the biotic community of an ecosystem (Daniels *et al.*, 1979b; Spurr and Barnes, 1980; Ford-Robertson, 1983). Productivity is defined as the maximum amount of timber that a site can produce over a given time (Davis and Johnson, 1987; Wang and Klinka, 1996).

It is well recognised that many factors which are physical or physiological in origin, contribute to stand growth. While some of these factors have been included in models of site index or productivity, they have been generally excluded from empirical systems of growth and yield. While there are some exceptions, it was, and in some cases still is, almost impossible to measure many of these factors in a forest with an intensity sufficient for them to be included with permanent sample plot data used to construct growth and yield models (Woollans *et al.*, 1997).

Some sites support luxuriant forests whilst others are capable only of supporting 'poor' growth. These differences may be due to soil (fertility, drainage, etc.), climate (temperature and rainfall patterns), topography (elevation, aspect, etc.) and other factors and may be reflected in the species composition and the growth patterns (Vanclay, 1994).

Whether a forester views a site in the ecological sense as a unit of a stable combination of site factors or in the management sense, as the primary production unit of forest produce (Shonau, 1988), the main aims of site evaluation are similar. In the first case the emphasis is on the identification of environmental factors related to tree growth and the prediction of forest yield, while in the second case, the importance of species choice and the development of growth models are stressed. In commercial forestry, dealing with exotic tree species, site evaluation is usually carried out by studying a considerable number of sample plots of a certain species under varying conditions, measuring tree growth on these plots and quantifying the various relevant site factors. When undertaking such a site factor analysis, it is assumed that the species in question has been planted on a wide scale, covering many different site conditions distributed in a normal pattern. This is seldom the case, for a poor representation of the various site conditions occurs frequently and that can lead to erroneous and misleading conclusions (Shonau and Purnell, 1988).

2.4.1 Classification of site quality

Killian (1984) described the goal of site classification as clarifying the possibilities and risks to forest management and allowing the prediction of yield. Rennie (1963) and Carmean (1975) divided determination of site quality into direct and indirect methods: the production capacity is either measured directly from forest growth or estimated indirectly from site attributes expressing this capacity (Shonau, 1988).

Direct methods of site quality evaluation have been used since the early 19th century. They measure site quality in terms of various expressions of tree growth such as height, basal area, timber volume, timber mass or production of resins, bark, cork and so forth (Tajchman and Waint, 1983). But, in commercial forestry MAI at culmination is the most meaningful. The main drawback of using MAI is its dependency on stand density as well as genotype, competing vegetation, disease, insects, site preparation and fertilisation (Shonau, 1988).

Direct methods of evaluation require the existence, either now or in the past, of the species of interest at the particular location where site quality is to be evaluated. When on-site measurements of the species of interest are not available, indirect methods must be employed. Direct methods almost invariably provide better evaluations of the site quality than indirect methods (Clutter *et al.*, 1992).

2.4.1.1 Direct methods for evaluating site quality

(i) Estimation site quality from historical yield records

In agricultural enterprises the site quality of a given field for a particular crop is most commonly measured by simply averaging prior annual yields of the crop in question from that field using cases where the genetic constitution of the crop remains relatively constant. There are, however, few areas of the world where such procedures can be successfully employed in forestry today (Clutter *et al.*, 1992).

(ii) Estimation of site quality from stand volume data

Since volume production is usually the growth parameter of greatest interest to the forest manager, an evaluation of site productivity in terms of volume is desirable, but the method of measuring volume must be standardised. Utilisable volume is inadequate because utilisation standards vary in time and place (Vanclay, 1994).

The volume attained by a stand at any given age can be greatly affected by factors other than site quality and unless the factors are controlled or adjustments are made to reflect their effects, volumetric production differences among forest stand will have little relationship to true site quality differences (Clutter *et al.*, 1992).

Yield class

The Forestry Commission in Great Britain uses the yield class system to classify the quality of forest sites. Yield class is an estimate of the maximum mean annual increment (MMAI) of stem volume per hectare per year. It is a specific growth rate category to which a crop can be assigned relatively easily (Hart, 1994). Yields of forest tree variables will vary depending on such factors as soil type, exposure, elevation and management treatment (Hart, 1994). Determination of the yield class of sample plots or forests is done by inspecting a graph of current annual increment and mean annual increment versus the plantation age (fig. 2.1).

The MAI curve reaches a maximum where it crosses the CAI curve (Hart, 1994). This point (X in the figure 2.1) defines the maximum average rate of annual volume increment, which a particular stand can achieve and this indicates the yield class (Edwards and Christie, 1981).

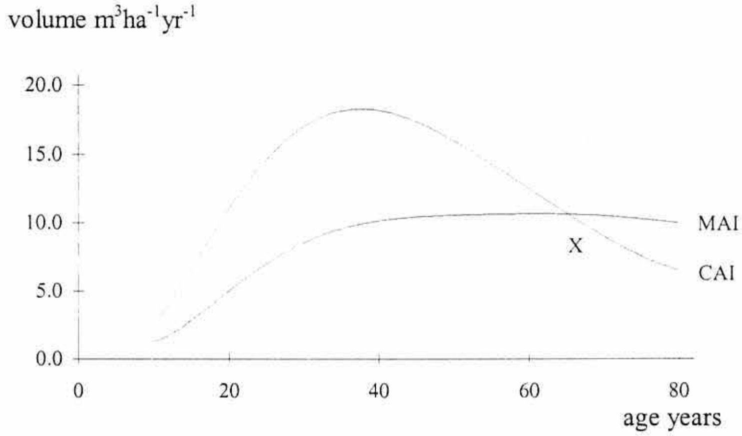


Figure 2.1: Patterns of volume increment in even-aged stands. CAI - current annual increment; MAI - mean annual increment. Source - Edwards and Christie, 1981.

General yield class

Producing graphs like figure 2.1 is difficult for most forest stands and impossible for field managers. Fortunately a good relationship exists between top height and the cumulative volume production of stands and this can be used to avoid actually measuring or recording cumulative volume production. The logical sequence for managers wishing to assess yield class would thus be to measure top height, convert this to cumulative volume production, and divide this by the age of the stand to derive MAI. This procedure has been simplified by constructing top height/age curves from which yield class can be read directly. Yield class obtained through top height and age of the stand alone is termed general yield class (GYC) (Edwards and Christie, 1981).

(iii) Estimation of site quality from stand height data

For many species, areas of good site quality are areas where height growth rates are high. In other words, for these species, volume production potential and height growth are positively correlated (Edwards and Christie, 1981; Philip, 1994). The practical utility of the volume-potential height growth correlation stems from the fact that the height development pattern of the larger trees in an even-aged stand is little affected by stand density and intermediate thinning (Clutter *et al.*, 1992).

Site index

The potential for wood production in even-aged monoculture forest stands is frequently assessed by an index of site quality (Magnussen and Penner, 1996) usually as the base of the height-age relationship which is referred to simply as the site index (SI) (Alder, 1980).

Most height based methods of site quality evaluation involve the use of site index curves. Construction of site index curves is a fundamental task in much forestry yield research (Elfving and Kiviste, 1997). Any set of SI curves is simply a family of height development patterns with qualitative symbols or numbers associated with the curves for referencing purposes (e.g. Fraser, 1980). The most common method of referencing uses the heights achieved at some specified reference age. This reference age, referred to as the "index age" or "base age", is commonly selected to lie close to the average rotation age. However, for many families of height development curves it makes little difference in practice what age is selected as the index age (Clutter *et al.*, 1992).

There are two fundamental uses for base-age specific site index equations: (a) to estimate height at any given age from the site index, and (b) to estimate site index at any given age (Wang and Payandeh, 1995). Site index is highly correlated with volume and is relatively insensitive to moderate variations in stand density (Nigh and Sit, 1996).

2.4.1.2 Indirect methods for evaluating site quality

(i) Estimation of site quality from over-story inter-species relationships

This method for evaluating site quality must be applied when the species (or forest type) of interest is not present on the land area under evaluation. In this situation where other trees or vegetation are present, measurement made on present vegetation can be used to evaluate site quality for the species of interest (Clutter *et al.*, 1992).

(ii) Estimation of site quality from lesser vegetation characteristics

Since many environmental factors affect both over- and under-story vegetation, it is not unreasonable to expect that under-story vegetation characteristics could provide information on site quality for tree growth. The species composition of under-story vegetation present on a given site is often an excellent indicator of surface soil moisture availability, and the degree of luxuriousness of lower vegetation commonly reflects the fertility of the top-most horizon or horizons present in the soil surface. However, the characteristics of deeper soil horizons may have little impact on under-story vegetation, but still have great influence on the quality of the site for tree growth (Clutter *et al.*, 1992).

Killian (1984) agreed that vegetation is a very sensitive site indicator but wrote also that purely floristic systems such as ground vegetation types, plant communities and forest cover types gave satisfactory results only in natural or slightly altered forests. The use of plant indicators or communities is most suitable in the more temperate regions and it is difficult to apply in areas with a destroyed or disturbed vegetation that has been harvested or burnt rapidly, or in areas that have been used for agriculture or pastures with intensive cultivation or fertilisation (Shonau, 1988).

(iii) Estimation of site quality from topographic and climatic data

These methods divide the land surface into units with uniform characteristics and distinguish primary and secondary site factors which relate to tree growth. Primary factors such as microclimate, elevation, topography, parent material, surface water and ground water are independent from the ecosystem or forest community. Secondary site factors such as forest microclimate, forest soil, litter layer and moisture regime are developed and influenced by components of the ecosystem. Both primary and secondary site factors can be used to predict site quality (Shonau, 1988).

Climatic factors have generally been difficult to factor into silvicultural and ecological analysis. This is primarily because climate is recorded at a sparse irregularly network of meteorological stations. The problem is how to extrapolate from these few points for reliable estimates of climate at any location within a forest, region, province or continent (Mackey *et al.*, 1996). This is even a problem in Great Britain with its long history of meteorological data (Mayhead, pers. comm.).

The potential for wood production in even-aged monoculture forest stands is frequently assessed by an index of site quality (Magnussen and Penner, 1996). Any single estimate of site productivity must be approximate, because it summarises several multi-dimensional factors of the environment as a single index. The vegetation itself reflects most of the important site factors and the height growth of pure even-aged stands provides a good measure of site productivity for forest management purposes. The volume production is difficult to measure, and it is convenient to use an alternative which is easier to measure. In even-aged stand of a single species the most common alternative is site index, the expected height at nominated index age (Vanclay, 1994).

2.5 Growth and yield models

Growth estimation of living trees and stands is needed by managers for many purposes including:

a. yield prediction,
b. health monitoring,
c. long term productivity monitoring,
d. socio-economic analysis of forest influences[1] (Adlard, 1995),
e. marketing,
f. planning harvesting and
g. planning long term machinery requirements.

[1] For example, the time of introduction of grazing animals may be constrained by the height growth of trees present. The time of "safe" introduction of animals could therefore be predicted from a height growth model.

Most forest growth models are constructed by several equations independently fitted to data (Soares *et al.*, 1995) and these may comprise many separate but interrelated components, each of which may influence, and be influenced by other components and assumptions of the model (Vanclay, 1994; Jenkins, pers. comm.). These models usually describe growth rate as a regression function of variables such as site index, basal area and stem density. In most growth and yield models, site index is used to determine the growth potential or maximum growth rate (Liu and Ashton, 1995).

In the 1970s researchers started to develop mathematical and computer models to simulate the development of stands and individual trees within the stands (Stage, 1973; Clutter and Allison, 1974; Johnstone, 1976).

2.5.1 Role of growth and yield models

Growth models provide a reliable way to quantify silvicultural, roading and harvesting options to determine the sustainable timber yield, and examine the impact of forest management and harvesting on other values of the forest (Vanclay, 1994).

2.5.2 Classification of growth and yield models

Different authors categorise the yield models in different ways, e.g.:

a. whole stand; size class and single tree level models (Clutter *et al.*, 1992; Davis and Johnson, 1987; Mitchell, 1988; Philip, 1994; Vanclay, 1994),
b. empirical, process-based and hybrid (Kimmins *et al.*, 1988; Kimmins *et al.*, 1990; Korzukhin *et al.*, 1996; Voit and Sands, 1996),
c. deterministic and stochastic (Vanclay, 1994),
d. distance dependent and distance independent (Clutter *et al.*, 1992; Philip, 1994; Vanclay, 1994).

2.5.2.1 Whole stand models

Whole stand models are often simple and robust, but may involve problems not possible in other approaches (Vanclay, 1994). Population parameters such as stocking (number of trees per unit area), plantation age, site index, stand basal area per hectare, number of trees per hectare (Clutter *et al.*, 1992), standing volume are used to predict the growth or yield of the whole forest. No detail of individual trees in the stand is determined (Vanclay, 1994).

It should be noted that some stand level models (e.g. diameter distribution models) produce tree level outputs (frequencies and average heights by dbh classes). However, they are still classified as stand level models because the inputs are stand level statistics (Clutter *et al.*, 1992).

2.5.2.2 Size class models

Size class models provide some information regarding the structure of the stand. Several techniques are available to model stand structure, but one of the most widely used is the method of stand table projection, which essentially produces a histogram of stem diameter (Vanclay, 1994).

2.5.2.3 Single tree level models

The most detailed approach is that of single tree models which use the individual tree as the basic unit of modelling. The minimum data input required is a list specifying the characteristics of each tree in the stand. Some models also require the relative spatial position of the tree or tree height and crown class. Single tree models may be very complex, modelling branches and internal stem characteristics and may be linked to harvesting and conversion simulators (e.g. Mitchell, 1988; Vanclay, 1988).

Single tree growth has been found to be a better measure of stand growth than alternatives based on averages and predicting growth on a stand basis (Ayhan, 1978).

2.5.2.4 Distance independent and distance dependent models

A distance independent model relates stocking and competition through average and summed terms such as number of stems per hectare, basal area per hectare, angle counts etc. A distance dependent model however, uses the distances from the subject tree to its surrounding competitors as one of the independent variables to predict the growth.

It has been argued that distance dependent tree level growth models provide more details on tree development and incorporate relationships expressing biological and ecological interactions at a more fundamental level than is possible with other model types. Some modellers seem to have drawn one or both of the following conclusions from this argument (Clutter *et al.*, 1992):

a. Stand level yield estimates obtained by accumulating predicted individual tree yields will have greater statistical precision than comparable estimates generated by stand level models.

b. Distance dependent tree level models can be used to predict reliably growth on stand types for which no empirical data are available.

2.5.2.5 Empirical and process-based models

While a rigorous categorisation of models is difficult to define, it seems that there are two major classes of models in forestry. One class is categorised by empirical yield prediction models and the other by process-based physiological models (PBMs). A typical empirical yield prediction model is based on data from a few management regimes and attempts to use the current information about a forest to extrapolate overall and specific growth patterns. Under controlled conditions such empirical yield models are robust and amenable to rigorous statistical analysis, they often lead to solid, empirical relationships and tables of stand properties that have proved to be reliable tools for the forest manager (Voit and Sands, 1996).

Process-based models simulate the biological processes that convert carbon dioxide, nutrients and moisture into biomass through photosynthesis (Sievanen and Burk, 1993; Sievanen and Burk, 1994). These estimates have not yet been developed to the stage where biomass and biomass growth can be identified as individual cells and cell wall thickening and aggregated into trees with detailed dimensions for forest managers (Sievanen, 1993).

One of the more empirical aspects of many process-based models has been the partitioning of photosynthates between leaves, roots and shoots (Vanclay, 1994). West (1987) assumed that 20% of net photosynthates would be used for new leaves, 20% for stem and branch development, and 60% for root growth. West (1993) developed the model further to examine more realistic ways to model photosynthate partitioning in response to functional relationships between tree parts. He assumed that the general growth strategy of trees is to maximise leaf production subject to a few constraints.

Recent advances in forest growth modelling have indicated the high potential of process-oriented models for examining a variety of questions ranging from standard management problems to more complex issues of environmental change (Ek and Dudek, 1982; Shugart, 1984; Valentine, 1985; Voit and Sands, 1996). However, due to a number of difficulties their use has been rather limited (Makela, 1988). For example, rigorous testing of a PBM will require special measurements, such as determination of the components of stand biomass. The cost and labour intensity of obtaining such data are high. Lack of suitable data has evidently been an obstacle to testing PBMs (Sievanen, 1993; Sievanen and Burk, 1993).

Although empirical growth models differ widely, common basic elements appear in most of them. Estimates are made of the changes with time of tree diameter, height, form, volume or all of these variables and also change in the number of trees per unit area. There are also other functions or variables, such as volume estimates based on tree species, age, land quality, climate, area history and vegetation present. If the estimate is for a single species in a limited geographic area, other species are excluded and the effect of the current climate and prevalent soils of the area are included by default (Bruce and Wensel, 1988).

2.5.2.6 Hybrid models

The hybrid simulation approach involves combining the above two approaches (empirical and process-based) using the major strength of each approach to compensate for the major shortcoming of the other (Kimmins *et al.*, 1988). This is done mainly by improving the empirical growth models by including additional explanatory variables such as growth indices derived from process-based models (Woollans *et al.*, 1997).

2.5.2.7 Deterministic and stochastic models

A deterministic model predicts the expected values under a given set of conditions, but a stochastic model incorporates uncertainty in the outcome by generating a random variable or variables from a prescribed probability function and adjusts the prediction by including the effect of this stochastic element (Philip, 1994). In other words, in deterministic modelling, processes are identified and understood in terms of basic mathematical and physical laws and axioms. In stochastic modelling, a random element is permitted and modelling is done by empirical probability distribution (Henderson-Sellers, 1996). This confers on the prediction a degree of variation to match reality. For example, a very sophisticated growth model might incorporate a variable representing the occurrence of abnormally dry periods. Then the prediction of growth and survival would be adjusted by using a value for the degree of drought in a particular period drawn from a probability distribution (Philip, 1994).

Deterministic models will not be replaced by stochastic models; the efficiency and usefulness of deterministic models in providing information for forest management have been demonstrated and cannot be currently matched by stochastic models. Deterministic models are more efficient at predicting the main response, and can be used to determine the optimum management strategies for forest management in a way not possible with stochastic models. Deterministic and stochastic models are complementary and used in concert may both prove useful in forest management (Vanclay, 1991).

2.5.2.8 Explicit and implicit prediction models

Explicit prediction systems are those which include equations to predict volume per unit area directly (i.e. some whole stand models). Implicit systems predict basic information on stand structure, and stand volume is obtained indirectly (e.g. from tree or class mean diameters in single tree and size class models respectively) (Vanclay, 1994).

Daniels *et al.* (1979a) compared the predictive ability of two empirical whole stand models and a empirical single tree model. The most accurate yield estimates (in terms of minimum mean square error) were provided by the whole stand distribution model. However, all three models provided estimates of sufficient accuracy for most plantation management uses. The relative costs of the predictions were 1:25:1400 for the whole stand yield model, the whole stand distribution model and the single tree model respectively.

Mowrer (1989) demonstrated that computational efficiency is one cost of complex models, and that complex models may propagate greater variances than more simple whole stand models. This means that any error in the inventory of initial stand condition may be magnified by methods such as single tree models, whereas they may remain comparatively unaltered by less complex ones, but should be designed to provide specific information needs (Vanclay, 1994).

At present, single tree models are preferred over stand models by many forest scientists who are dealing with stand growth. In addition to this, the trend to complex ecosystem models is evident. Even so, stand level models will not be ruled out because of their simplicity, general applicability and reliability (Wenk, 1994).

The modeller of the biological phenomenon has a choice of several investigative approaches, and how this choice is exercised depends on at least three items: the state of knowledge about the system being modelled, the nature of the responses exhibited by the system, and the objectives of the modeller (Thornley, 1991).

2.5.3 Predicting current growth and future yield

Current growth predictions do not involve a projection of stand density, while predictions of future yield do involve such a projection, either explicitly or implicitly (Clutter *et al.*, 1992).

Harrison and Daniels (1988) wrote that the forest growth should be predicted on the basis of an understanding of the determinants of the forest growth, and estimates of how these determinants will change in the future rather than on the record of past tree growth. However, process oriented models have yet to be accepted by forest managers as a practical means of predicting yield. These models of forest growth have tended to be either too simple to account for all the significant determinants of growth (and therefore inflexible), or they have become extremely complex where attempts have been made to include all (or a large number of) significant determinants.

2.6 Construction of growth and yield models

A good model does not simply happen; it is planned that way. Modellers cannot combine several haphazardly formulated relationships and expect to get reliable predictions. Instead careful thought should be given to the design of the model at the outset of model construction. The following parts should be considered (Vanclay, 1994):

a. what the model will be used for,
b. what inputs will be provided,
c. what outputs are required,
d. the data available for fitting the model,
e. the resources available to construct, test and use the model.

Dixon *et al.* (1990) wrote that there are three major components essential to the development of models:

a. an understanding of the process or relationship being modelled,
b. mathematical, statistical, computational techniques and equipment capable of handling the problem,
c. experimental or survey data.

Gilchrist (1984) divided the procedure of statistical modelling into 5 steps:

(i) Identification: this is the process of finding or choosing an appropriate model for a given situation.

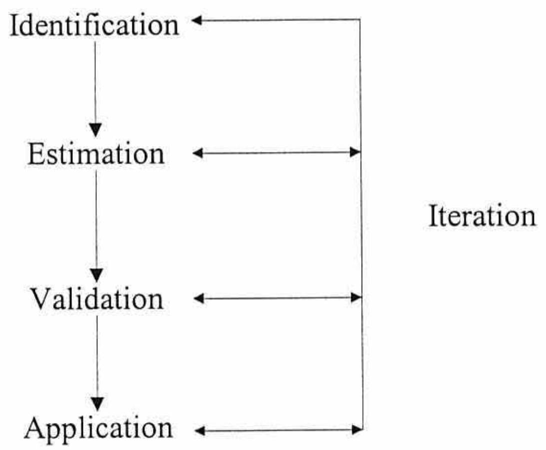
(ii) Estimation and fitting: though the general form of a model will be of interest to us, in practice, it must be put into a detailed numerical form (parameters must be identified and quantified). This is the stage of moving from a general model to the specific numerical form which is called model fitting. The process of assigning numerical values to parameters is called estimation before using in the field.

(iii) Validation: although the model meets assumptions satisfactorily, relative contributions of each model element indicated by the signs of the parameters, the procedure of the construction and the accuracy of the predictions must be tested.

(iv) Application: after completing the above tasks, the model can be applied to real populations for the predictions of particular variables.

(v) Iteration: this is a process of continuous development, of going back a stage or two to make use of additional information.

The following flow chart illustrates the connection between the five steps listed above.



(Source: Gilchrist, 1984)

If the entire model construction procedure was designed at the outset, the following would have to be assumed as known: (i) which variables were the most important, (ii) over what ranges the variables should be studied, (iii) on what scale the variables and responses should be considered (e.g. linear, logarithmic, or reciprocal scales), and (iv) what multi-variable transformations should be made (Box *et al.*, 1978).

The structure of the forest growth model reflects the model objectives and different types of models are required to satisfy different purposes (Kimmins *et al.*, 1990). The first step in model construction is to prepare an outline of the model, formulate the functional relationships required, and fit the functions to data (Vanclay, 1994).

Chen *et al.* (1988) wrote that when multiple factors are involved, it becomes difficult to analyse the cause and effect relationship by conventional statistical procedures. In nature one factor may have a positive impact on growth while another factor neutralises its effect.

One extreme approach to modelling is to derive the form of the model on the basis of an understanding of the situation (Gilchrist, 1984). Garcia (1984) and Weisburg (1985) wrote that it is wise to examine the relationships using plots before developing models. The most common diagnostic is the scatter plot of the variables (Weisburg, 1985), and this will also help to prove the assumptions (Dewar, 1993). It is also useful to plot residuals against explanatory variables in order to look for any transformations that may be required, and to check for the requirement of additional variables. In models of intensively managed plantations, mortality and recruitment may frequently be ignored (Vanclay, 1994).

An efficient way to see the major dependencies is to use stepwise regression. This is an alternative to examining a large number of residual plots, but is not a substitute for graphical inspection, and should serve as a way to highlight explanatory variables against which residuals should be plotted (Soares *et al.*, 1995).

The improvement of high-speed computing equipment has made it possible for growth modellers to use the individual tree rather than the stand as the basic prediction unit. However, the fact that this is possible does not necessarily mean it is desirable, and considerable thought has been given to the relative merits of stand level versus tree level growth models (Clutter *et al.*, 1992).

The measurements of the predictive variables are compared with the corresponding model predictions. In its most elementary form this comparison is performed mainly qualitatively by inspecting visually the agreement between the observed data and model predictions. More sophisticated approaches express the agreement between data and model quantitatively in terms of misfit measures which typically are functions of the error between measurements and the model predictions (Janssen and Heuberger, 1995).

Success in developing models depends on carefully identifying the needs, selecting the important variables, formulating a suitable model, collecting good data (both quantity and quality), using reliable coefficient estimation procedures, and carefully evaluating the model. Good modellers rely as much on their knowledge of silviculture and on the biological principles of growth, as they do on statistical tests when selecting models and developing algorithms. Any relationship that violates accepted biological principles should be rejected, even if it results in efficient predictions for a particular data set. The model should be kept simple. Unnecessary complexity does not improve a model, and may create many problems. Every model is an abstraction of reality and will be wrong in some sense. Users should remember that all models may be wrong, and some may be more useful than others (Vanclay, 1994).

2.6.1 Requirement of data

Data requirements for the different approaches to model construction vary widely. Direct predicting models for yields can be developed from inventory data collected from temporary plots. Equations or systems of equations that explicitly or implicitly predict the growth require at least some re-measured plot data. Elaborate single tree growth models are the most demanding of data (Clutter *et al.*, 1992). In forestry, trees are measured at 3 to 10 year intervals, but users of the growth models developed with the data may desire projections for intervals as short as one year (Amateis and McDill, 1993).

The ideal basis of constructing stand development and yield functions is long term re-measurement data of permanent sample plots. When single examination data are used, information of a past development of a given stand is usually unreliable, and therefore the stand cannot be placed in a sequence of stand development in relation to the other stands (Johnstone, 1976).

According to Oderwald and Hans (1993), predictions outside the range of observed data are not generally considered in model building or validation. Predictions are rightly made only for the range of data on which the model is based, since statistical fitting procedures cannot take account of non-existent data.

2.6.1.1 Transformation of data

Suitable transformations can be determined graphically or analytically. The best way to obtain an idea about the transformation is the observation of the residual plots (e.g. Aitkin *et al.*, 1989; Kassab, 1987).

If a transformation has been used, predictions will contain transformation bias, the magnitude of which depends upon the variability of the data. Often the bias may be small enough to be ignored. However, where a poor fit is obtained, an adjustment should be made for this transformation bias when performing the back transformation. Weighted regression avoids the need for these transformations and corrections (Vanclay, 1994).

2.6.1.2 Partition of data

In addition to overall appraisals, it is desirable to partition data (e.g. by age, site index or stand density), and examine model performance in each of several strata (e.g. Mayer and Butler, 1993). The most revealing insights may be obtained by devising strata based on a knowledge of the biological system, the model and the characteristics of the data. However, the absence of any visible inadequacies in any particular stratification does not imply that weaknesses cannot be found in an alternative stratification (Vanclay, 1994).

According to Vanclay (1994), the temptation to use all the available data for the development of the model must be avoided, as it is equally important to have an independent set of data available for benchmark testing. The need for such testing is not diminished through the use of "self-calibrating" models.

2.6.2 Equations

The level of complexity of the approaches of the different kinds of equations has varied from the simplicity of single regression equation expressing the yield per unit area as a function of age, site class, and basal area, to the detailed intricacy of equation systems that simulate the growth of each individual tree in a stand as a function of its own characteristics, the characteristics of neighbouring trees and the distances to neighbouring trees (Clutter *et al.*, 1992).

2.6.2.1 Empirical and theoretical equations

Empirical equations are expressions which describe the behaviour of the response variable without attempting to identify the causes or to explain the phenomenon. This does not mean that empirical functions provide biologically unrealistic predictions, nor does it mean that they are inferior to supposedly biologically-based equations. They can and should be formulated to behave in a biologically realistic way across a wide range of possible conditions (Vanclay, 1994).

In contrast to empirical equations, theoretical equations have an underlying hypothesis associated with the cause or function of the phenomenon described by the response variable. There are few theoretical equations formulated specially for forestry applications. Most theoretical growth and yield equations have been borrowed from other disciplines, and as a result may be rather empirical for some forestry applications. However, some general principles govern the behaviour of many systems, and provide the basis for these theoretical equations (Vanclay, 1994).

An empirical study by Martin and Ek (1984), found that carefully formulated equations could be more accurate than theoretical equations for a wide range of data. However, according to Vanclay (1994), theoretically based equations may be more reliable for predictions which involve extrapolations beyond the range of the data.

2.6.2.2 Stand Table Projection

Stand table projection predicts the future stand table from the present stand table by adjusting each entry in the table with the estimated diameter and mortality increments.

If no appropriate model exists for predicting the future yield, the usual recourse is use of one of the forest inventory procedures that estimate future stand growth from increment core measurements of past growth. These estimation procedures are generally referred to as 'stand table projection' (Clutter *et al.*, 1992).

Although stand table projection methods and equation prediction systems share the objective of predicting future yield, their physiological approaches are quite different. With equation prediction systems, the prediction of growth and future yield is obtained by comparing the subject stand with other similar stands whose growth rates have been measured over time. Stand table projection methods, on the other hand, attempt to predict the future growth rates of trees in the same time (Clutter *et al.*, 1992). Pienaar (1989) attempted to produce a stand table following this method and discussed the possibilities.

Stand table projection is a valuable tool for growth estimation for stands where no alternative procedure is available for the prediction of future yield. However, Clutter *et al.* (1992) believed that stand table projections are generally used insufficiently and are often applied incorrectly to give grossly inaccurate estimates of future growth and yield. Also serious errors can often be introduced into stand table projection estimates through incorrect selection of sample trees, application of invalid growth assumptions and mis-estimation of mortality (Clutter *et al.*, 1992).

2.6.3 Regression analysis

There are two main types of regressions i.e. linear and non-linear. The linear regression equation has the form:

$$Y = \alpha_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon \qquad 2.1$$

where the Greek letters α, β_1, β_2 represent unknown parameters believed to be constant for a given model/data set combination, whereas X_1, X_2 are variables (often called regressor, predictor, or independent variable) which may represent experimental settings, predetermined conditions, or uncontrolled observed values assumed to be measured without error. The response variable *Y*, is called the dependent variable, deviating from the expected (i.e. mean) value given by the regression line by an amount ε, which is an unobservable random error term whose values are unknown but which is assumed to have a mean value of zero (Ratkowsky, 1983).

Linear regression implies that explanatory variables enter the objective function in a linear and additive way. It in no way implies that the resulting relationships must be straight lines. This form of regression is widely used for fitting equations to data (Vanclay, 1994).

A non-linear regression model is one in which the parameters appear non-linearly, e.g.:

$$Y = \theta_1 X^{\theta_2} + \varepsilon \qquad 2.2$$

where: θ_1 and θ_2 are the parameters to be estimated (Ratkowsky, 1983).

The linearity or non-linearity of the model is determined by the way the parameters enter into the model and not by the way of the explanatory variables.

Linear models are widely used in growth and yield studies, and offer several advantages. Most computer systems and many pocket calculators incorporate reliable algorithms to fit such equations to data. The solution to the equation is unique, easily obtained and rather robust, even when assumptions implicit in the method are violated (Vanclay, 1994).

Many theoretical and asymptotic models are of non-linear form. Whilst non-linear regression allows great flexibility in formulating models to ensure extrapolation, it does have some limitations. One problem is that, unlike linear regression, non-linear regression does not necessarily provide a unique best unbiased solution for a given set of variables. Non-linear solutions are determined iteratively, and may be influenced by the estimating method and the starting conditions specified by the user (Soares *et al.*, 1995).

The coefficient of determination

The coefficient of determination (R^2), measures the proportion of total variation about the mean explained by the regression. R is the correlation between the observed and predicted response variable and is usually called the correlation coefficient (Draper and Smith, 1981).

R^2 is expressed as a proportion in the range 0-1 or a percentage in the range 0-100. The closer R^2 is to one (or 100%) the better the fit. In such situations, the residual sum of squares, $RSS = \Sigma(y_i - \hat{y}_i)^2 = 0$ which implies $y_i = \hat{y}_i$ for all *i*'s, i.e. the observed values are equal to the corresponding fitted values (Kassab, 1987). In 1981 Draper and Smith wrote that R^2 can take values as high as one (or 100%) when all the *X* values are different. When repeat runs exist in the data, the value of R^2 cannot attain one no matter how well the model fits. R^2 gives a higher when number of explanatory variables are high. This is because no however good, can explain the variation in the data due to the pure error.

ıls

ıls (e) are the differences between the observed (y) and predicted ($\hat{y}$) of the response variable. Residuals provide information regarding tions about the error term and the appropriateness of the model. Any te data analysis requires the examination of residuals (Weisburg, 1985; ; *et al.*, 1988). The most common method, especially useful in simple ıon, is a plot of errors (e_i) versus the fitted values ($\hat{y}_i$). Isolated points in lots far from the expected values will be indicative of possible outliers urg, 1985).

s

s (points away from the others), should be investigated to see if they are ılt of human, instrumental or gross experimental errors, in which case they be discarded. If they are genuine, they may provide useful information. It ıe that an important independent variable has been omitted from the model the error variances are not equal (Kassab, 1987).

Fitting the equations to data

are many techniques available for fitting equations to data and the riate one to use depends on the relationship chosen to represent the , the nature of the data, and on the resources available to fit the model and Wattes, 1988; Draper and Smith, 1981; Gilchrist, 1984; Ratkowsky, ;eber and Wild, 1989). Models are often fitted using the growth increment pendent variable in a regression model (Rennolls, 1995; Sievanen *et al.*, ;oares *et al.*, 1995). The regression models can be linear or non-linear, but, removed by redefining the predictors into new linear combinations that are easier to interpret (Weisburg, 1985).

2.9 Model errors

Errors in independent variables may be random or non-random and they can occur for a number of reasons (Gertner, 1991):

a. measurement errors,
b. sampling errors,
c. grouping errors,
d. process or prediction errors.

Sample error occurs when an independent variable of a model is estimated with a sample procedure. Grouping errors are due to classification of the data, example: to size classes. Process errors occur when an independent variable of one model is predicted with another model without testing the bias. If the model used is linear, random errors with 0 means in the independent variables do not cause bias in the predictions. Random errors, however, increase the variance of the predictions (Kangas, 1996).

2.10 Model evaluation

Model evaluation is an important part of model building, and some examination of the model should be made at all stages of model design, fitting and implementation (Vanclay and Skovsgaard, 1997). Evaluation should not merely be an afterthought or an acceptance trial. A thorough evaluation of a model involves several steps, including two which are often called qualitative and quantitative tests in forest growth modelling (Vanclay, 1994). Model evaluation should extend to all model components and assumptions, and this requires a thorough understanding of the structure of the model and the interrelationships between components (Soares *et al.*, 1995).

Model evaluation should reveal any errors and deficiencies of the model, and should establish (Vanclay, 1994):

a. whether the equations used adequately represent the processes involved,
b. if the equations have been combined in the model correctly,
c. that the numerical constants obtained in fitting the model are the best estimates (unbiased minimum variance estimators),
d. the range of site and stand conditions over which the model applies,
e. if the model satisfied specified accuracy requirements,
f. whether the model provides realistic predictions throughout this range,
g. how sensitive model predictions are to errors in estimated coefficients and input variables.

One of the most effective ways to examine model performance is to plot residuals for all possible combinations of tree and stand variables, and to look for patterns which may indicate serial correlation, dependencies on initial conditions or on projection length, or other systematic patterns (e.g. Soares *et al.*, 1995). It is common to plot observed values (y) against predicted values ($\hat{y}$), but in many cases it is more revealing to plot residuals ($e = y - \hat{y}$) versus observed values (Vanclay, 1994).

Although model verification, calibration and validation are usually done by the modeller, model evaluation should be done by the user, who is responsible for the accuracy of the predictions (Buchman and Shifley, 1983).

2.10.1 Model validation

Even if yield tables or models are available, and early assessments have shown them to be reasonably applicable, it is still advisable to check the actual standing volume periodically, both to ensure that unexpected change in the growth pattern is not affecting the performance of the crops, and also that the form of control over thinning to ensure that the intensity of thinning is about right (Fraser, 1980).

"Model validation is the process of substantiating that the behaviour of the model represents that of the problem entity to satisfactory levels of confidence and accuracy consistent with the independent application of the model within its application domain" (Brown and Kulasiri, 1996). This involves the comparison of data obtained from the real system of interest with corresponding data generated from simulating of any model (Kassab, 1987; Reynolds and Chung, 1986).

If sufficient independent data are available, the model should be validated by comparing model predictions with data. In the absence of such validation data, errors in the uncertainty in the model structure cannot be detected. However, it is possible to quantify the uncertainty in the model prediction associations with the uncertainties in the model inputs and often to identify the inputs that are primarily responsible (Voet and Mohren, 1994).

The validation process ends with one of four outcomes for a particular decision (Newberry and Stage, 1988):

a. the model is adequate,
b. the model needs revision using the available data identified in the process,
c. the data appear inadequate to evaluate model, and new data are required,
d. the model is inadequate.

Benchmark tests

For benchmarking in its purest form some data are set aside, or new data are obtained for benchmark tests. The most convincing test would use a set of data drawn from an independent population measured over a long period, but such data are rarely available. Growth modellers are frequently faced with the decision of having to partition a data set from a single population into two subsets, one for development and the other for the testing the model. Where ample data are available, this partitioning causes few problems. However, when data are scarce, there is a temptation to use all the available data for development in an attempt to improve the model (Vanclay, 1994).

2.10.2 Re-calibration of models

Re-calibration refers to the search for adjustment to improve model predictions for a specific locality. Specific features in the growth model may require modification. This might involve development of local growth functions within the existing framework to improve accuracy; or it might involve the incorporation of subroutines for, for example, proportional losses from fire or wind fall possibly on a stochastic basis (Alder, 1978).

Re-calibration also implies adjusting a growth model so that it provides good predictions for a new population. This may entail estimating new parameters for some or all of the equations of the model, or may use a scaling factor to adjust predictions (Vanclay, 1994).

The creation of a variant of a growth model for a new locality may involve several steps. Firstly the model should be benchmarked using data from the new locality to determine if any re-calibration is needed. Given that some adjustment is necessary, the residuals about predictions should be examined to see if a single scaling factor would be adequate, or if a more sophisticated adjustment is necessary. If inspection of residuals indicates that a simple adjustment to increment rates would provide satisfactory predictions (e.g. analogous to a better site productivity), then such re-calibration may be attempted. However, if a more complex adjustment to growth patterns is indicated, it may be preferable to abandon re-calibration attempts and to estimate new parameters for all coefficients in the model (Vanclay, 1994).

2.11 Model predictions

Kimmins *et al.* (1988) indicated that in many applications of a model, it will not be necessary to include a large number of determinants of growth. Other applications may require a much more complex set of simulations. It is not possible to include a simulation of all determinants of growth, and even if it was, it would probably result in a model of such size and complexity that the model would have little value for forest managers as a yield predictor. Also because of the complexity of biological populations it is not possible to describe mathematically all the important interactions that affect the growth of single trees (Ayhan, 1978).

The important thing is whether or nor the model will provide useful predictions, assessed by an appropriate suite of diagnostic tests. Prominent among these criteria is the requirement that the model provides biologically reasonable predictions for the whole range of possible conditions (Vanclay, 1994).

Error dependencies on projection length or initial forest condition can be shown graphically, or by indicating the precision of simulations from different starting conditions or projection lengths. Temporal trends may be revealed by plotting residuals against the year of measurement (Soares *et al.*, 1995). Direct graphical comparisons with the data appear to be much more useful for assessing the reliability of predictions (Garcia, 1984) than the quantitative tests because the former tests make it easier to identify the trends of predictions from the actual data.

2.12 Conclusions for the review of literature

The major points arising from the above review of the literature are summarised below:

(i) If the management regimes of plantations have not changed with time, projection of past trends of growth and yield are adequate as complex models for successful predictions of future yield.

(ii) Growth and yield models in forestry can be classified into many categories. However, there is no clear and obvious prescription of the correct method. It will be seen that process-based models are still in the development stage although they describe the relationships between the tree variables better than empirical models. The single tree level empirical models are more difficult to construct than whole stand or size class models due to the need for detailed data. However, some modellers confirmed that stand level empirical models are adequate for most management conditions.

(iii) Most yield prediction systems are expressed as mathematical equations or systems of inter-relating equations rather than as tables so that computers can be used to generate predictions for any desired combinations of inputs. Regression analyses are the commonest techniques for fitting the equations to data. Linear regression equations are suitable for most occasions. For process-based models, non-linear regression techniques are used.

(iv) When constructing models, there are several factors which may guide the selection of possible explanatory variables. Obviously if a certain variable is not present in the data available for model development, then that variable cannot be included in regression analyses leading to the development of the growth model. The variables used in growth models should not be an arbitrary collection of those correlated with growth or yield in a forest stand, but should be carefully chosen to ensure biologically realistic predictions across the whole range of possible conditions.

(v) Reliable data are an important factor in model construction. The only way to obtain long term re-measured data is by maintaining permanent sample plots. If the modeller has enough data, it is common to partition them by thinning type, age range or age class. in order to improve accuracy. A sub-set is also reserved for validation of constructed models.

(vi) Plantations can either be maintained unthinned or under a specific well defined thinning regime. Plantations can be maintained properly under a well defined thinning regime than changing the thinning regime time to time. Thinning affects growth relationships by providing more space per tree. Therefore, caution must be exercised when models are developed for plantations with different thinning regimes because the growth and mortality rates may be different.

(vii) Site quality plays a major role in tree growth and yield modelling. However, assessment of the site quality is difficult. Most of the time, modellers use a factor to represent the site quality. Top height related factors are the obvious choice because it is easy to measure and also independent from the stand density.

(viii) The parameter signs must be explained biologically after estimations have been made. If any parameter violates acceptable theory, then it must be removed along with the variable from the model.

(ix) Growth models must be able to be rejected through the normal process of experimental testing. Model evaluation is an important part of model construction and will indicate the nature of the forests for which the model may be expected to yield reliable results, as well as areas in which further research and data collection are required.

(x) Model evaluation involves several techniques. The coefficient of determination is not a good measure for assessing the predictive ability of models. There are formal tests available for such purpose to analyse the model performance quantitatively. Examination of the residual distributions with fitted values is a good assessment for model behaviour and possible outliers.

(xi) If an existing model is adapted for a different geographical locality, all or some of the parameters should be re-estimated to improve accuracy. Sometimes, different functions may need to be estimated to improve the predictions.

2.13 Objectives of the present study

Bearing in mind the literature reviewed above, and the available Corsican pine data, the following objectives were defined for the thesis:

(i) To construct a set of empirical growth models to predict the following variables of individual trees;

a. future growth of diameter at breast height,

b. future total height growth,

c. current timber height growth,

d. current total tree volume,

e. current merchantable volume, and

to predict the following mean variables of trees removed in thinning;

f. mean tree basal area,

g. mean tree diameter (dbh),

h. mean tree total height.

(ii) To re-calibrate three empirical growth and yield models for Corsican pine constructed out side of Great Britain for different species by Pienaar and Harrison (1989), Soares *et al.* (1995) and West and Mattay (1993).

(iii) To compare the predictive ability of newly constructed growth models constructed for the present work with the re-calibrated models of paragraph (ii) above and observed data.

CHAPTER 3: MANIPULATION OF THE RAW DATA

3.1 Introduction

For a good model the very first priority is a sound data base. Even if the assumptions and the structures of the models are developed precisely, the efficiency and the reliability of these models can be very low if the data are measured, collected or grouped incorrectly. Collection and preparation of the data before fitting to the equations can take a long time and therefore often acts as the limiting factor in the modelling process.

When a model is built to suit various site conditions, whatever the predictor and explanatory variables, it is necessary to obtain data which cover these conditions. Most models require long term re-measured data and therefore it is desirable to have access to permanent sample plots. Data resulting from re-measured sample plots or trees are referred to as a real growth series (Turnbull, 1963). There are many advantages in having permanent sample plots, rather than temporary ones, because continuous inventory data sample the forest on successive occasions, thus quantifying growth and change (Soares *et al.*, 1995).

3.2 Source of the data used in this study

The British Forestry Commission kindly provided access to the data for 49 permanent sample plots of Corsican pine distributed in many parts of Great Britain (Figure 3.1). The data cover various thinning types, general yield classes etc. had been measured at one to six year intervals with thinning carried out at four to eight year intervals. Some sample plots were maintained unthinned. These data were used to construct and validate new models and also to re-calibrate and validate the selected models built in the past in other countries.

3.3 Description of the data

3.3.1 The sample plots

The Forestry Commission in Britain has a very specific way of recording sample plot details outlined in the code of sample plot procedure (Edwards, 1976). Various numbers are used to indicate various types of measurements and the qualities of sample plots and trees; these are shown in table 3.1.

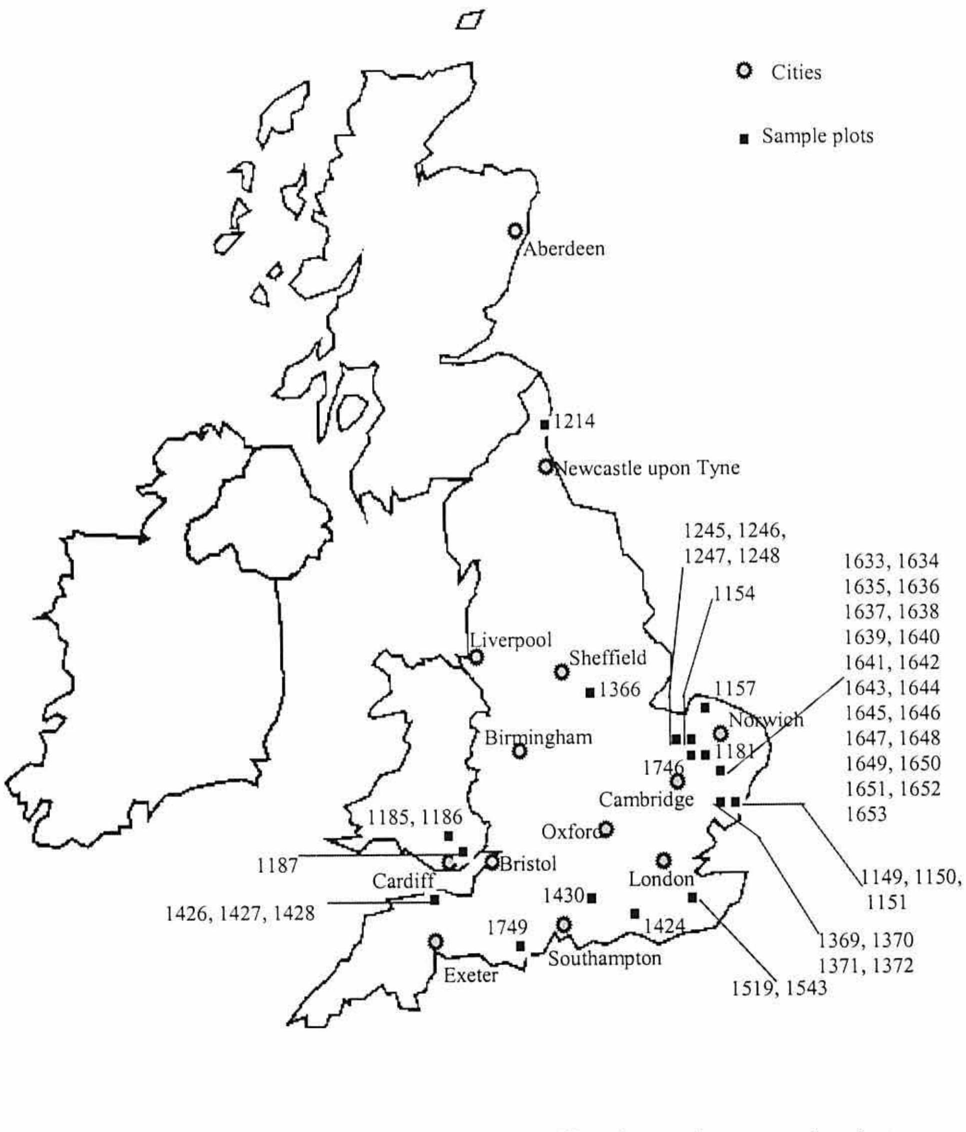


Figure 3.1: Map showing the locations of the 49 Corsican pine sample plots obtained from the Forestry Commission in Great Britain.

Plot number	Planting year	Plot size, ha	General yield class, $m^3 ha^{-1} yr^{-1}$	Local yield class, $m^3 ha^{-1} yr^{-1}$	Thinning type & intensity	Space between rows, m	Space between trees, m
1149	1920	0.3642	14	13	9125	1.2	1.2
1150	1920	0.1012	14	13	9100	1.2	1.2
1151	1920	0.0809	16	15	1075	1.2	1.2
1154	1926	0.2865	14	11	9000	1.4	1.4
1157	1924	0.4047	16	14	9125	1.4	1.4
1181	1927	0.2023	16	13	3100	1.4	1.4
1185	1924	0.1056	18	18	3100	1.5	1.5
1186	1926	0.0947	14	12	3100	1.5	1.5
1187	1924	0.0967	18	19	3100	1.5	1.5
1214	1924	0.1825	12	14	3100	1.5	1.5
1245	1928	0.1352	14	14	6100	1.4	1.4
1246	1928	0.1202	14	15	3100	1.4	1.4
1247	1928	0.1202	16	14	0	1.4	1.4
1248	1928	0.1210	16	15	3100	1.4	1.4
1366	1928	0.1651	10	10	3100	1.4	1.4
1369	1935	0.1295	12	15	0	0.9	0.9
1370	1935	0.1012	14	17	3075	1.4	1.4
1371	1935	0.0967	14	16	3100	1.8	1.8
1372	1935	0.1360	14	13	3125	2.4	2.4
1424	1922	0.1558	20	00	3100	1.8	1.8
1426	1935	0.1352	18	13	0	0.9	0.9
1427	1935	0.1437	14	12	3100	1.4	1.4
1428	1935	0.1271	12	12	3100	1.8	1.8
1430	1937	0.2003	18	19	3100	1.4	1.4
1519	1934	0.0890	20	17	3100	1.2	1.2
1543	1951	0.0558	22	18	3100	1.4	1.4
1633	1951	0.0626	16	18	0	1.4	1.4
1634	1951	0.0626	16	17	5350	1.4	1.4
1635	1951	0.0658	16	17	5130	1.4	1.4
1636	1951	0.0658	16	16	5230	1.4	1.4
1637	1951	0.0658	16	16	5230	1.4	1.4
1638	1951	0.0626	16	16	5120	1.4	1.4
1639	1951	0.0626	16	18	0	1.4	1.4
1640	1951	0.0626	14	14	5340	1.4	1.4
1641	1951	0.0658	16	17	5130	1.4	1.4
1642	1951	0.0626	16	17	5240	1.4	1.4
1643	1951	0.0626	16	16	5350	1.4	1.4
1644	1951	0.0626	16	16	5240	1.4	1.4
1645	1951	0.0626	14	16	5120	1.4	1.4
1646	1951	0.0626	14	15	5340	1.4	1.4
1647	1951	0.0658	16	16	5130	1.4	1.4
1648	1951	0.0626	16	18	5120	1.4	1.4
1649	1951	0.0626	14	11	5340	1.4	1.4
1650	1951	0.0626	16	17	5240	1.4	1.4
1651	1951	0.0658	16	16	5230	1.4	1.4
1652	1951	0.0626	16	16	5350	1.4	1.4
1653	1951	0.0626	16	19	0	1.4	1.4
1746	1964	0.0999	18	22	3100	1.6	1.6
1749	1970	0.1020	22	24	3100	2.0	2.0

Table 3.1: Description of the 49 Corsican pine sample plots obtained from the Forestry Commission.

3.3.1.1 Recording of thinnings

"0" is used to indicate plots where thinning is not carried out (control experiments). Thinning, when done, is recorded by four digit numbers. The first digit indicates the type of thinning as listed below:

1 - low thinning,
3 - intermediate thinning,
5 - neutral thinning (systematic thinning),
6 - crown thinning,
7 - heavy crown thinning,
9 - exploitation (Edwards, 1976).

The next three digits are used to show the thinning intensity as a percentage of marginal thinning intensity (Jenkins, pers. comm.). The number 9125 would therefore indicate 125% of marginal thinning intensity of an exploitation thinning.

3.3.2 The tree measurement data in sample plots

Four major measurement types can be found for individual trees in each sample plot measured by the Forestry Commission. These four types (listed below) have different kinds of measurements for calculations of different forest tree variables. At the beginning of each measurement type, plot number, measurement year and the month and the type of the measurement are recorded.

3.3.2.1 Measurement type 1 - general register

All the living trees in the plot, both main crop and thinning, are recorded. Each tree has a tree number, classification number and the diameter in millimetres rounding to the nearest millimetre (Edwards, 1976). In the data file, measurements of five trees are recorded in one row (15 columns). Tree number is specific for a particular tree in the plot. The tree classification number contains three digits as follows (Hummel *et al.*, 1959):

1st digit:	1,2,3, or 4 to denote the position in the canopy (dominant trees, co-dominant trees, sub-dominant trees and suppressed trees respectively),
2nd digit:	1,2, or 3 to denote stem quality (good stem, slightly defective stem, and very defective stem respectively),
3rd digit:	1,2, or 3 to denote crown shape and size (good crown, slightly defective crown and very defective crown respectively).

Trees which are marked for subsequent thinning are indicated by minus sign (-) in front of the diameter value (Hummel *et al.*, 1959).

3.3.2.2 Measurement type 5 - standing height measurements

Twenty trees (or less) where the diameter at breast height is 7.0 cm or above are systematically selected from the main crop trees of measurement type 1 using the sampling fraction for the height measurements (Edwards, 1976). Each tree has three descriptions: tree number, diameter at breast height in millimetres and total height in ten centimetre steps. On the data file, details of five trees are recorded in one row.

However, the total number of trees for height measurements can be different; for example, according to Hummel *et al.* (1959) up to 40 trees could be selected.

3.3.2.3 Measurement type 2 - main crop volume measurements

Normally ten trees are selected from the measurement type 5 for standing tree volume measurements (Edwards, 1976; Hummel *et al.,* 1959). At least two rows are required to record the details for each tree. In the first row, tree number, diameter at breast height in millimetres, total height in ten centimetre steps, timber height in ten centimetre steps, height to the lower crown (lowest whorl of branches with dead ones), height to the upper (live) crown (lowest whorl of branches all alive) and crown diameter in ten centimetre steps are recorded respectively. Finally the number of sections measured for volume calculation is entered (Jenkins, pers. comm.).

The following numbers of rows are dependent on the number of stem sections measured; i.e. up to 5 sections - one row, up to 10 sections - 2 rows, up to 15 sections - 3 rows etc. Each section has three measurements: the length of each section in ten centimetre steps; then the mid section diameter in millimetres; and finally the bark thickness at the mid point of each section in millimetres (Edwards, 1976). The final section always extends up to the point at which the stem diameter reduces to 7.0 cm over bark diameter (Jenkins, pers. comm.).

3.3.2.4 Measurement type 3 - thinning tree volume measurements

All the trees felled for thinning of 7.0 cm diameter at breast height and over are measured for volume provided that they number less than 40. If there are more, a sample of approximately 30 may be measured, if such sampling would save time (Edwards, 1976).

Measurements are similar in every way to measurement type 2. But in the first row only five recordings i.e.; tree number, diameter at breast height in millimetres, total height in ten centimetre steps, timber height in ten centimetre steps and the number of sections measured respectively.

A specimen of the sample plot measurement file is shown in the Appendix 1.1.

3.4 Calculations used for the computer programs and model building

3.4.1 Age at time of measurement

The Forestry Commission uses the first of July as the operative date for an increase in age (Hummel *et al.*, 1959). Following this procedure, between July in one year and June in the next year is considered as one growing year. The plantation age is determined by subtracting the planting year from the current year.

3.4.2 Mean diameter at breast height

Mean diameter at breast height is defined for this study is:

$$dbh_{\bar{g}} = \sqrt{\frac{\Sigma dbh_i^2}{n}} \qquad 3.1$$

where:
$dbh_{\bar{g}}$ = mean tree diameter at the breast height, cm
dbh_i = diameter at breast height of the ith tree, cm
$\bar{g}$ = mean basal area tree, m^2
n = number of trees (Philip, 1994)

Many authors (Philip, 1994; Vanclay, 1994) indicated that the arithmetic mean values are not suitable for tree volume calculations because they do not represent the real mean according to the tree size. Size of the individuals is important when calculating the values such as total volume because bigger trees contribute more to the total than the smaller trees. Therefore, the means of diameter at breast height and total height were determined by using equations 3.1 and 3.4 respectively. The mean diameter calculated using equation 3.1, is also known as the quadratic mean diameter.

3.4.3 Basal area

3.4.3.1 Individual tree basal area

Calculation of individual tree basal area was done by using the equation 3.2.

$$g_i = \frac{\pi \ dbh_i^2}{40000} \qquad 3.2$$

where:
dbh_i = diameter at breast height of the ith tree, cm
g_i = basal area of the ith tree, m^2 (Philip, 1994)
$\pi = 3.142$

3.4.3.2 Mean tree basal area

The mean tree basal area is defined for this study as:

$$\bar{g} = \frac{\Sigma g_i}{n} \qquad 3.3$$

where: $\bar{g}$ = mean tree basal area of the ith tree, m^2 (Philip, 1994)

3.4.4 Mean total tree height

For the present study, mean total tree height is defined as:

$$\bar{h} = \frac{\sum(h\, g_i)}{\sum g_i} \qquad 3.4$$

where: h = total height of the ith tree, m

$\bar{h}$ = mean total tree height, m (Philip, 1994)

3.4.5 Tree bole volume

There are many equations to calculate the tree bole volume. However, the most compatible equation with the data obtained from the Forestry Commission was Huber's formula (3.5) even though it sometimes causes bias. Smalian's formula (Appendix 1.2) tends to introduce more bias than Huber's formula (Jenkins, pers. comm.). Newton's formula (Appendix 1.2) is more accurate than either Huber's or Smalian's formulae. However, both Newton's and Smalian's formulae are impossible to apply to the Forestry Commission data.

Total volume of a log of wood can be defined by Huber's formula as:

$$v_{s_i} = \left(\frac{\pi\ l_i \left(d_{m_i}^2\right)}{40000} \right) \qquad 3.5$$

where: d_{m_i} = mid diameter of the ith log, cm

l_i = length of the ith log, m

v_{s_i} = total volume of the ith log of the tree, m^3

(Philip, 1994)

3.4.5.1 Merchantable volume

In Great Britain, the merchantable volume is taken to be from the base of the tree to the point of 7.0 cm over bark diameter. To calculate the merchantable volume of the whole tree the volume of the each section was calculated separately using Huber's formula (3.5) and then added together (equation 3.6) using one of the computer programs written for the current work (Appendix 1.6). Merchantable volume of a tree is thus:

$$v_m = \Sigma v_{s_i} \tag{3.6}$$

where: v_m = merchantable volume of the tree, m^3

3.4.5.2 Total stem volume

Considering the final section of the tree above the 7.0 cm over bark diameter as a cone, total stem volume was calculated using formula 3.7.

$$v = v_m + \left(\frac{\pi \ d^2 (h - h_m)}{120000} \right) \tag{3.7}$$

where:
d = 7.0 cm
h_m = timber height of the tree, m
h = total height of the tree, m
v = total volume of the tree, m^3

3.4.6 Crown volume

Volume of the live crown was calculated treating the crown of Corsican pine trees as a cone, using the following formula:

$$v_c = \left(\frac{\pi \ d_c^2 \ h_c}{12} \right) \tag{3.8}$$

where: d_c = diameter at the base of the live crown of the tree, m

h_c = height to the top of the tree from the live crown base, m

v_c = volume of the live crown of the tree, m^3

3.4.7 Number of trees

The number of trees in one sample plot was determined using the general register. Total number per hectare was calculated using equation 3.9.

$$N = \frac{n}{a} \qquad 3.9$$

where: a = area of the plot, ha

n = total number of trees

N = total number of trees per hectare

3.5 Computer programs written for the current work

FORTRAN (FORmula TRANsformation) is a computer programming language commonly chosen as the standard for forest modelling (Adman, 1984; Ashcroft *et al.*, 1986; Balfour and Marwick, 1986; Hammond *et al.*, 1988; Hughes *et al.*, 1978; Monro, 1983). For the current work several computer programs were written using FORTRAN 77 in order to read the Forestry Commission sample plot data and to do the necessary calculations prior to model construction.

3.5.1 Program 1

This program reads different data types which were recorded in the different format in sample plot data files (flow chart - Figure 3.2; detailed program - Appendix 1.3). The following sub-routines were written for the data calculation.

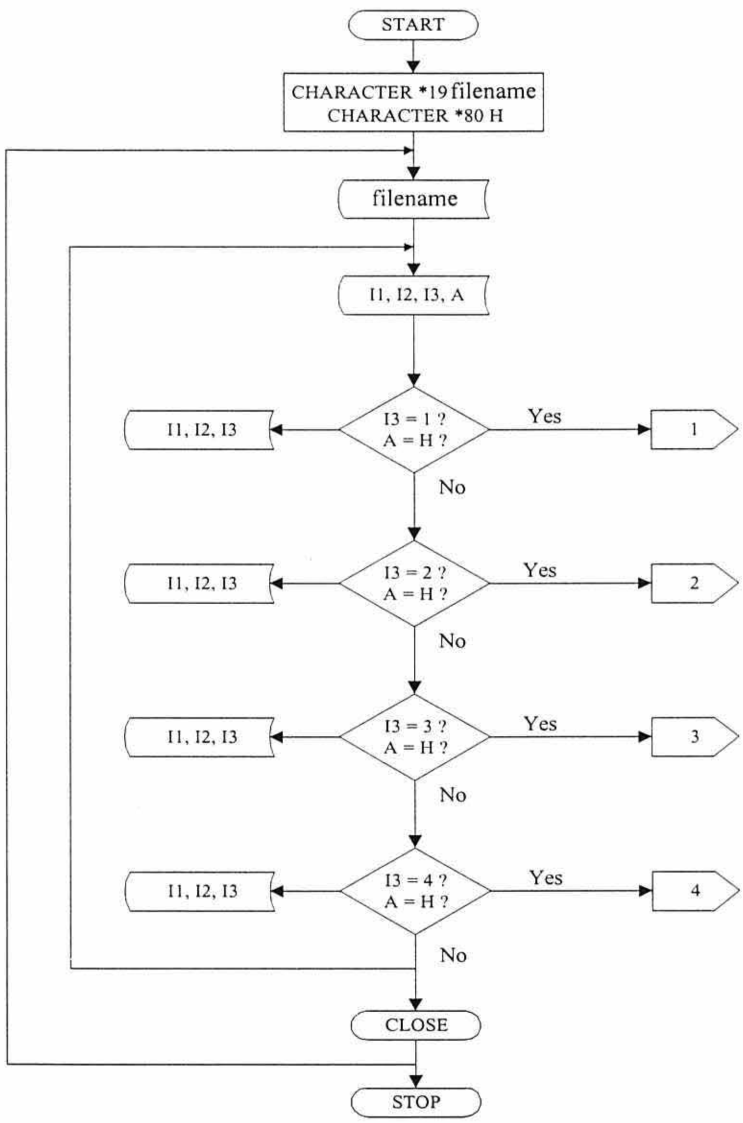


Figure 3.2: Flow chart for program 1.

3.5.1.1 Sub-routine 1

This sub-routine was written to separate the main crop and thinning trees using measurement type 1 data (general register) (flow chart - Figure 3.3; detailed sub-routine - Appendix 1.4).

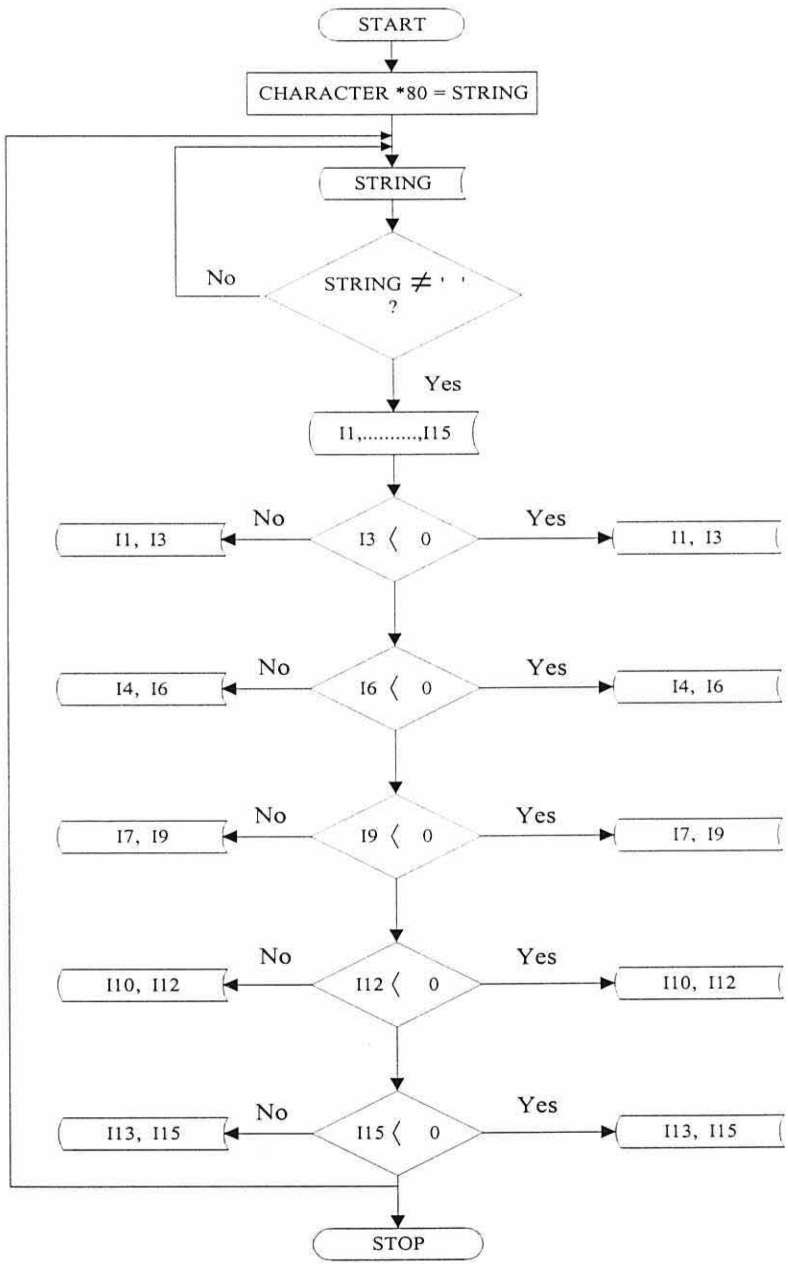


Figure 3.3: Flow chart for sub-routine 1.

3.5.1.2 Sub-routine 2

This sub-routine calculates the total volume, basal area and the total height of individual trees in main crop and thinning trees without the forked trees using the data from measurement types 2 and 3. It also calculates the total volume per plot (flow chart - Figure 3.4; detailed sub-routine - Appendix 1.5).

Stems that fork below breast height are considered as two separate trees and, if the tree is forked immediately above the dbh point, it is considered as one tree. (Avery and Burkhart, 1994; Cailliez, 1980; Hamilton, 1988). However, for the volume estimations, both limbs in forked trees are measured up to the 7.0 cm over bark diameter. If these trees were added to the volume calculations, it would over estimate the volume per tree. Therefore, such trees were avoided when the programs were written. The following procedure was used to detect the trees which were forked above the breast height:

$$La_j = \left[h_j - \sum (L_i)\right] \qquad 3.10$$

where:

h_j = total height of the *j*th tree

La_j = additional length of the *j*th tree

L_i = length of the *i*th log

If *La* is a negative number, the tree has two or more stems which all contribute to the total tree volume, and if it is not a negative number, the tree was considered to have a single bole.

3.5.1.3 Sub-routine 3

Sub-routine 3 was written to calculate the merchantable volume, basal area, total height and the timber height of individual trees in main crop and thinning trees, without forked trees, and the total merchantable volume per plot using the data from measurement types 2 and 3 (flow chart - Figure 3.5; detailed sub-routine - Appendix 1.6).

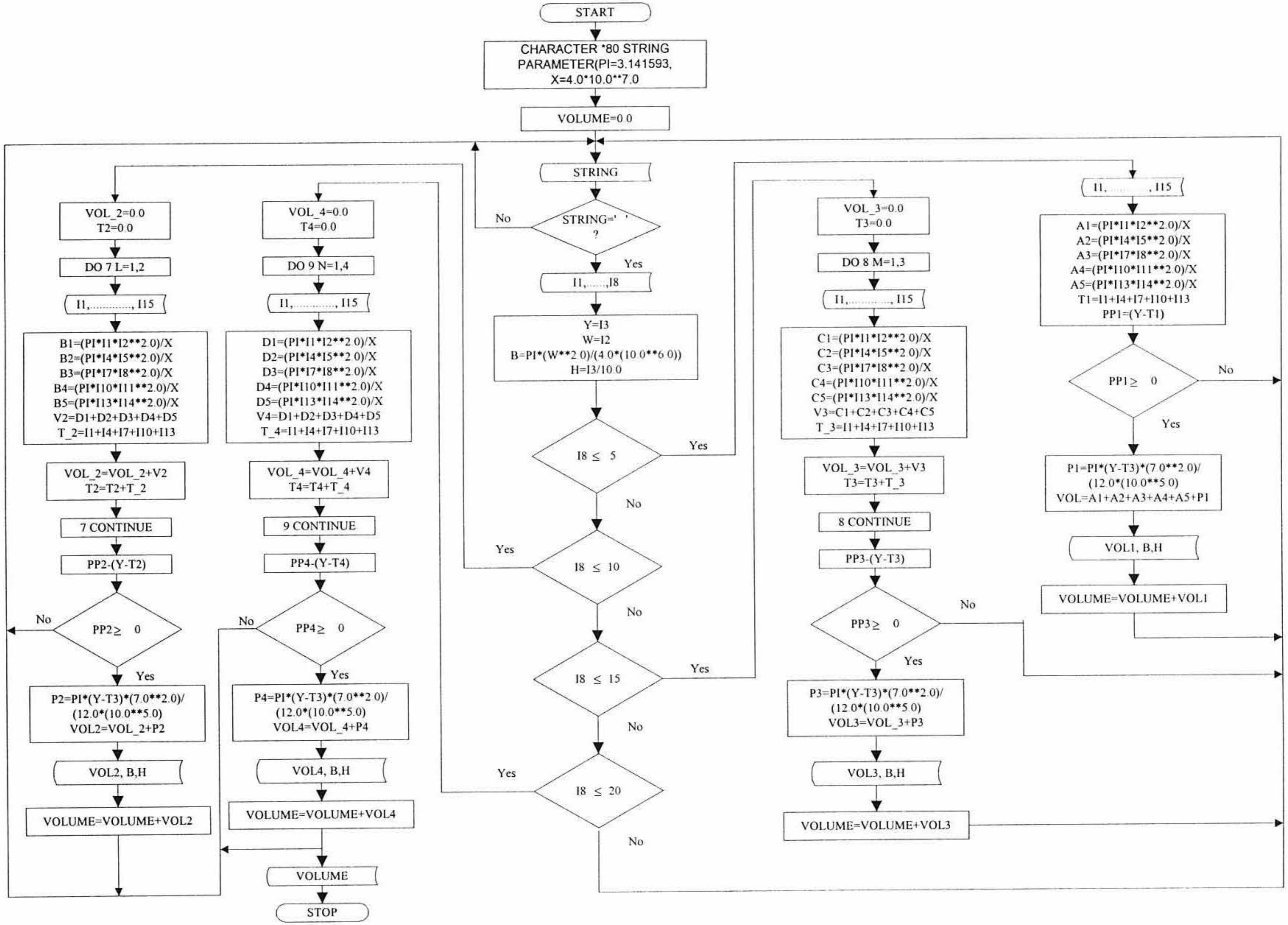


Figure 3.4: Flow chart for sub-routine 2.

3.5.1.4 Sub-routine 4

This was written to calculate the total height, basal area, and total height*basal area of individual trees from measurement type 5 data (flow chart - Figure 3.6; detailed sub-routine - Appendix 1.7).

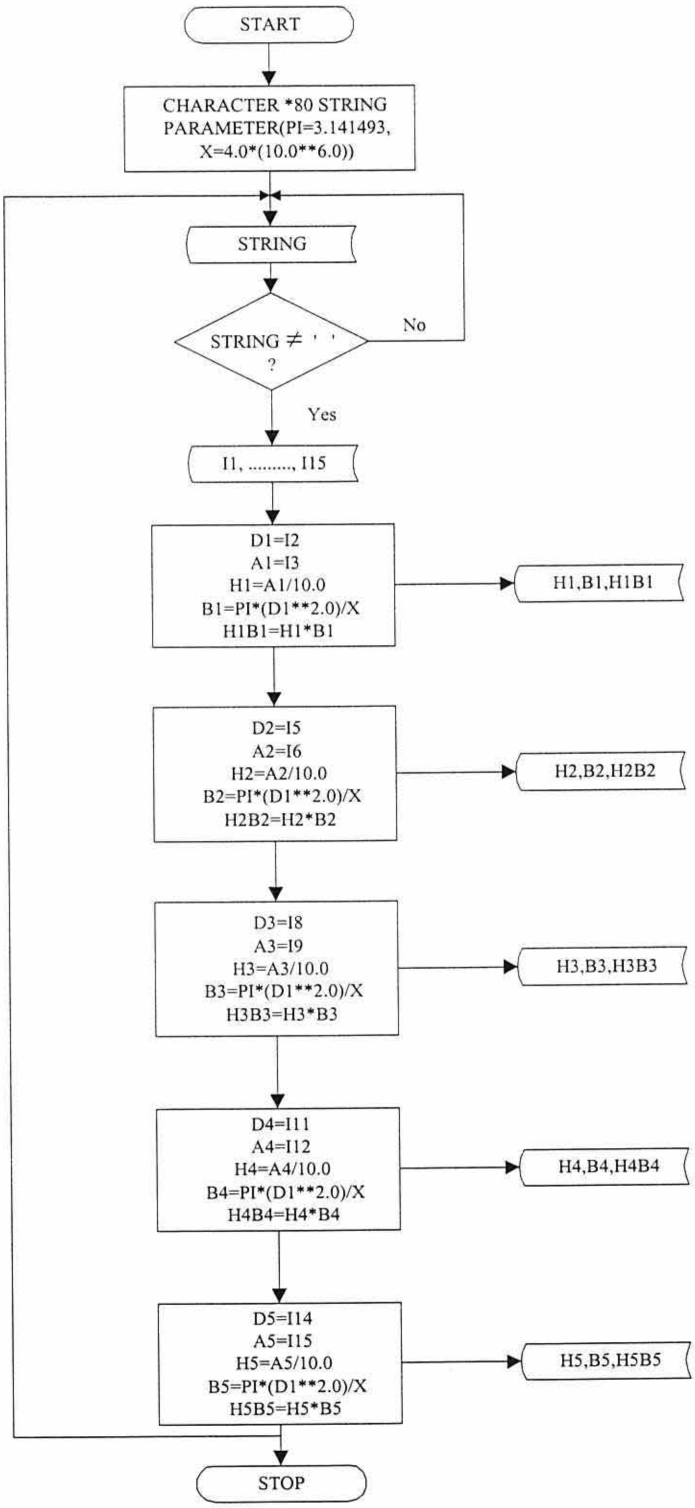


Figure 3.6: Flow chart for sub-routine 4.

3.5.1.5 Sub-routine 5

Length of the lower crown and upper crown, upper crown diameter, volume of the live crown, height, diameter and basal area of individual trees were calculated by sub-routine 5 using the data from measurement type 2 (flow chart - Figure 3.7; detailed sub-routine - Appendix 1.8).

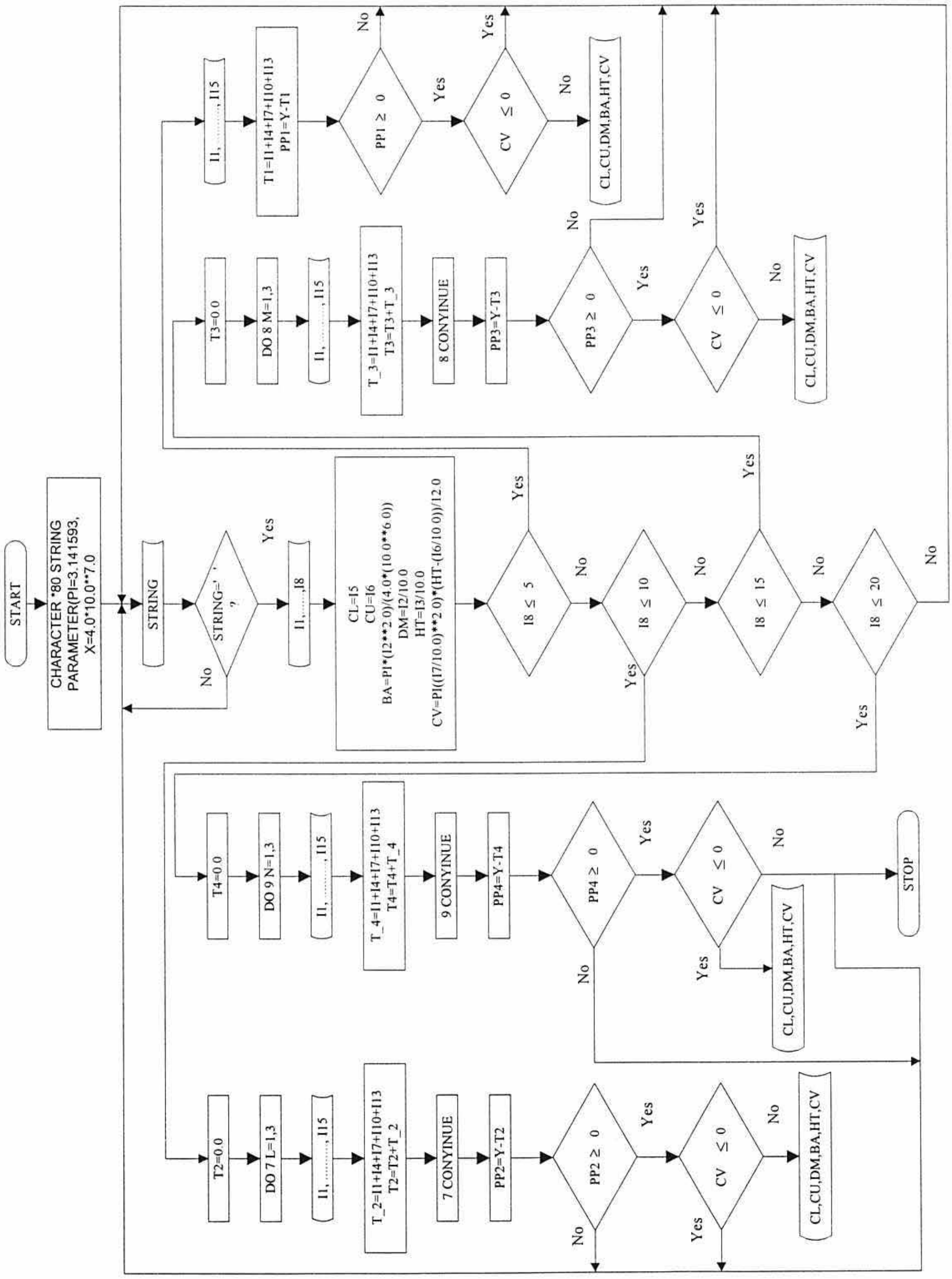


Figure 3.7: Flow chart for sub-routine 5.

3.5.2 Program 2

This was written to calculate the total basal area, total diameter squared at breast height and total number of trees per plot and to write the diameter at breast height and basal area of individual trees using the result files from sub-routine 1 in program 1 (flow chart - Figure 3.8; detailed program - Appendix 1.9)

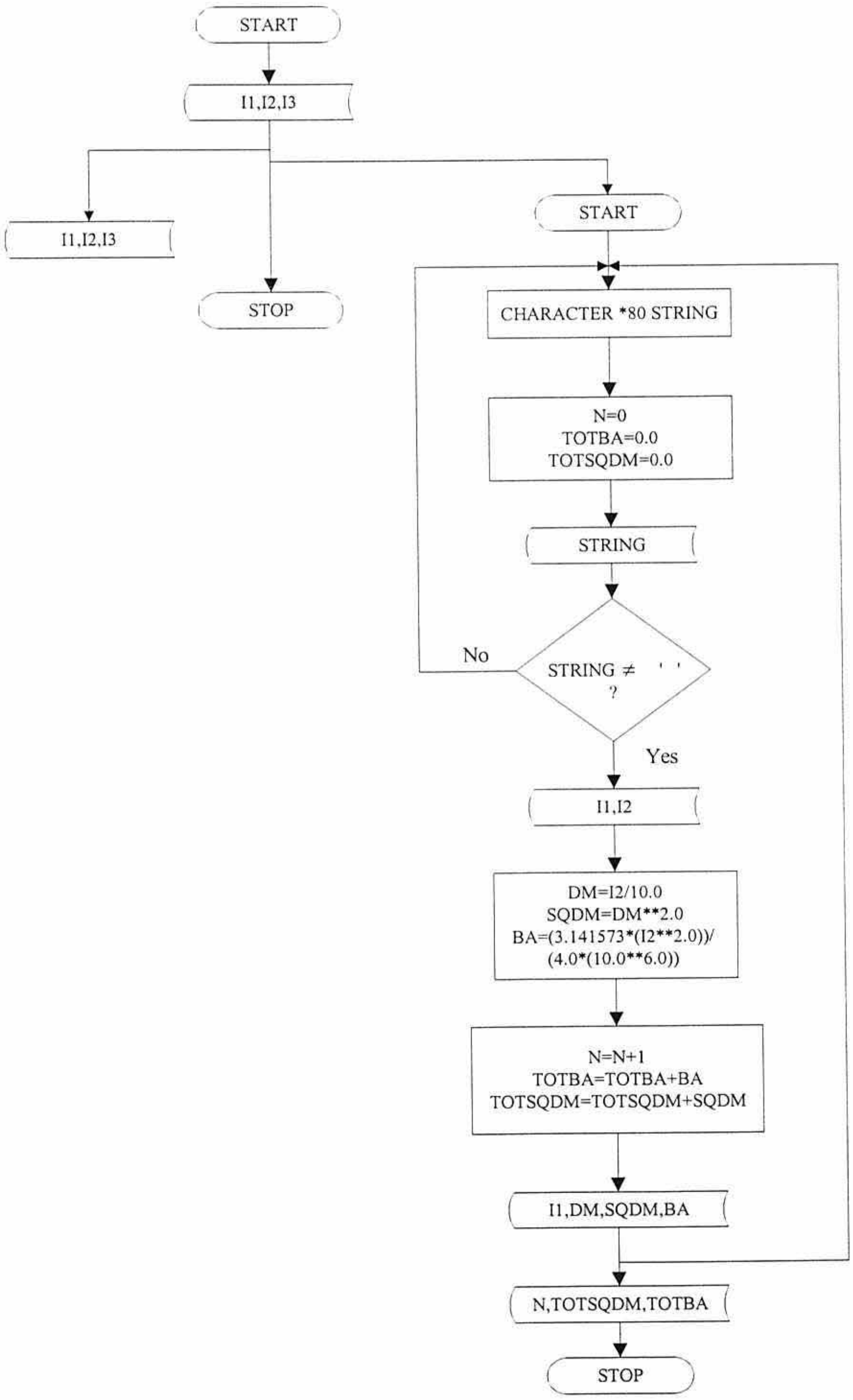


Figure 3.8: Flow chart for program 2.

3.6 Summaries of the sample plot data

Summaries of the sample plot data, expressed as the mean values, were required for two reasons. Firstly for the thinning prediction models, and secondly to obtain a general idea about the growing pattern of tree variables along with the thinnings. In order to facilitate subsequent analysis, the sample plot summary data were entered into standardised data files as follows (A diagram of the summary file for plot 1149 is given Appendix 1.10):

(i) The year and the month of the measurements were taken from sub-routine 1 and entered in the first column. The number of rows were dependent on the number of measurement occasions for each plot.

(ii) In the second column, number of trees was entered.

(iii) Mean diameter (cm) and mean height (m) were recorded to one decimal place in the next two columns respectively using data generated by sub-routine 1, program 2 and sub-routine 4. For the calculation of mean values, equations 3.1 and 3.4 were used.

(iv) Mean basal area (m^2) and the total basal area (m^2) were calculated to three decimal places in program 2 and were entered in columns 5 and 6 respectively.

(v) Mean total volume (m^3) and total volume (m^3) were calculated to three decimal points using sub-routine 2. These values were recorded in the 7th and 8th columns.

(vi) In the 9th and 10th columns, mean and total merchantable volume (m^3) at each measurement time were entered using sub-routine 3.

For thinning trees in the same plot, steps (ii) to (vi) were repeated in the next 9 columns. Finally, planting year, general yield class, thinning type and plot size were recorded.

3.7 Discussion

If data handling errors are minimised then the precision of the model will be maximised. Therefore, individual numbering of the sample trees is very important because it is the only way of detecting measurement errors (Vanclay, 1994). Before the fitting process was started, all the data were examined to find unusual characters, errors, or omissions.

Considering the importance of re-measured data from permanent sample plots, Vanclay (1994) wrote that dynamic inventories should satisfy the data requirements for growth models for decades ahead. In order to provide for this next generation of growth models, it is appropriate to appraise critically the utility of the present dynamic inventory and to establish new plots specifically directed at collecting data for such future growth models. Such a series of elite plots should sample the range of forest conditions (and should include thinning studies), but should be established in limited numbers so that appropriate care and attention can be given to detail and accuracy.

When calculating volume, Huber's, Newton's and Smalian's formulae give correct results for a frustum of a quadratic paraboloid and a cylinder. If the log is not a frustum of a quadratic paraboloid and not a cylinder, then the use of either Huber's or Smalian's formula will introduce errors (Philip, 1994). Studying the tests done by Young *et al.* (1967), Jenkins (pers. comm.) found that for 3 m tree logs, Smalian's formula over-estimated the tree bole volume by 1.4 % while Huber's formula under-estimated by 0.7 % for Sitka spruce. However, these values could not be included to determine the bias of the volume in the present data because the length of the logs varied in Forestry Commission tree measurements. The errors given by both Huber's and Smalian's formulae are proportional to the length of the log and the square of the difference between the diameters of the two ends. However, the errors in the estimation of tree and log volumes are expected to be reduced by using Huber's formula and summing the volumes of sections which should be as short as practicable (Philip, 1994), typically 3 m in this study.

There are a number of forked trees in any pine plantation. However, determination of the number of such trees in a particular plantation is very difficult unless a visual observation. Removal of the forked trees can under-estimate the total volume but the prediction of individual tree volume using other variables cannot be affected. Therefore, it was decided to remove the forked trees from the process of modelling in this study. The distorted trees of plantations are removed in the fist thinning. After the first thinning, there would be a negligible number of such trees in managed plantations. Therefore, the distorted trees were not considered in this study.

It was difficult to write a program for separating the main crop and thinning trees from the general register in order to calculate the total basal area values in one step. To overcome this problem, firstly the separation was done by using sub-routine 1 and then a second program (program 2) was written for the essential calculations. In the data type 3 (measurements for the thinning volume calculations) there were only 5 columns in the 1st row, while this number was 8 in the main crop measurements (refer section 3.5). Therefore when the programs and sub-routines were used for the thinning trees, I8 was replaced by I5.

CHAPTER 4: CONSTRUCTION OF THE NEW SET OF GROWTH MODELS

4.1 Introduction

In general, the two crucial requirements for a good model are that the relationships should be significant and the assumptions should be satisfied, in which case inferences and predictions from the fitted model are likely to be reliable (Kassab, 1987).

Prior knowledge about the relationships between forest tree variables is very important in model building. Many variables have been used for modelling over time but some variables, such as crown dimensions have only recently been used. Crown structure has also been widely studied in recent years and is recognised to influence tree growth greatly and also stand dynamics (Deleuze, 1996; Hasenaur and Monserud, 1996; Maguire and Hann, 1989; Peterson, 1997; Valentine *et al.*, 1994).

4.1.1 Constructing or developing growth and yield models

Modelling the real world involves problem analysis, model building, and/or model validation, model selection and then application of the selected models (Henderson-Sellers, 1996).

A first major step in regression analysis is to decide on the mathematical form of the model to be fitted to the data at hand (Kassab, 1987). The benefits of the mathematically presented model are that it is clearly defined and thus easily communicated, so that its strengths and weaknesses may be analysed (Gilchrist, 1984).

Developing more than one model to predict a particular variable is important because it allows the modeller a good comparison of the performance of each model and the importance of each parameter. By identifying the good and poor part of each model, it is easier to develop one model by using only the good parts.

4.1.2 Advantage of using a combination of tests

A regression analysis on its own is not a very good indicator of the accuracy of a model. Standardised residual plots of fitted values do not give a quantitative result although they are a useful indicator of bias. The coefficient of determination (R^2) is not a very reliable indicator of model performance (Weisburg, 1985). When the number of data or the number of explanatory variables are high, R^2 tends to indicate a number very close to 1 (or 100%) even if the model does not fit with the data well. Therefore the necessity of some other test of model performance such as lack of fit is clearly highlighted (Price, pers. comm.).

4.1.3 Role of thinning in yield prediction

Thinning is well known to increase the stem diameter growth of residual trees, but the mechanism behind this response is not well understood, and long term physiological responses to thinning are largely unknown (Peterson *et al.*, 1997; Smith, 1986). In their experiments, Peterson *et al.* (1997) found that the growth responses of residual trees are generally attributed to increased crown volumes (i.e. increased photosynthetic area). Taking this into account, it is wise to first develop separate models for forests under different thinning regimes and then to combine these into one model if practicable.

4.2 Methods used for the construction of models

As stated in Chapter1, empirical models were decided to construct because of the lack of data for process-based modelling. The constructed models were individual tree level in order to obtain detailed predictions.

4.2.1 Building the relationships for main crop trees

4.2.1.1 Basal area

Individual tree basal area (at breast height) at any age can be calculated if the diameter at breast height for the same tree at the particular age is known using the formula described below.

$$g_i = \frac{\pi \ dbh_i^2}{40000} \qquad (3.2)\ 4.1$$

where: dbh_i = diameter at breast height of the ith tree, cm

g_i = basal area of the ith tree m^2 (Philip, 1994)

4.2.1.2 Diameter at breast height

In this work it is assumed that the future growth of the individual tree diameter at breast height (dbh) can be predicted as a function of the present dbh, current age, age at the time the prediction is required, current stand density and the quality of the site (equation 4.2).

$$dbh_{t+\Delta t} = f(dbh_t, a_t, a_{t+\Delta t}, s, d_t) \qquad 4.2$$

where: a = age of the plantation, years

d = density of the plantation (number of surviving trees, ha^{-1})

dbh = diameter at breast height, cm

s = quality of the site

t = time at the beginning of the simulating period, years

Δt = duration of the simulating period, years

4.2.1.3 Total tree height

The height of individual trees at any time in future can be predicted as a function of the current height of those trees, current age and age at the end of the simulating period, site quality, and the number of surviving trees per hectare (equation 4.3).

$$h_{t+\Delta t} = f(h_t, a_t, a_{t+\Delta t}, s, d) \qquad 4.3$$

where:

a = age of the plantation (years)

d = density of the plantation (number of surviving trees), ha^{-1}

h = total height, m

s = quality of the site

t = time at the beginning of the simulating period

Δt = Duration of the simulating period, years

Competition is a very important factor affecting increases of dbh and height. Adding a competition index to the models will increase the complexity. Therefore it is avoided here by assuming the competition is represented by the present time measurement of the particular variable and the number of trees per ha.

4.2.1.4 Timber height

In Great Britain, timber height is usually taken as the height of the tree from the ground (uphill side) to the point at which the over bark diameter is 7.0 cm. Timber height can be predicted as a function of the total height and diameter (equation 4.4).

$$h_{tim} = f(h, dbh) \qquad 4.4$$

where: h_{tim} = timber height of the tree, m

Timber height is affected by the form of the tree. It is assumed in this thesis that the total height and the stem diameter at breast height represent the rate of taper.

4.2.1.5 Total tree volume

(i) Total tree volume prediction model *a*

For production forestry, individual tree volume is the most crucial variable. The basic equation of the current work is the common equation used to calculate the total volume of individual trees.

$$v_i = g_i * h_i * ff_i \qquad 4.5$$

where:

g_i = basal area of the ith tree, m^2

ff_i = form factor of the ith tree

h_i = total height of the ith tree, m

v_i = total tree volume of the ith tree, m^3

(Philip, 1994)

The best and the shortest definition of the form of a tree or log is its shape. The shape may be regular, as for a solid of revolution, or - more commonly - irregular (Philip, 1994).

The comparison of tree bole forms with various solids of revolution (cylinders, paraboloids etc.) may be expressed in numerical terms as form factors. Such ratios are derived by dividing stem volume by the volume of a chosen solid (Avery and Burkhart, 1994). For example, the form may be expressed by the cylindrical form factor, that is the ratio of the volume of the tree or log to that of a cylinder of equal basal cross-sectional area and height (Philip, 1994).

During the past century, the stem form of many tree species was studied by researchers in an attempt to explain the shape of the tree stems. Additional work is still needed in this area and no single theory has been developed that adequately explains the variety of shapes that trees can assume (Figueiredo-Filho *et al.*, 1996).

Form factor is highly correlated with site variation, stand density, growth of crown and competition from the neighbouring trees. In this study form factor was replaced using the variables mentioned above which are easily measurable. Thus expanding equation 4.5:

$$v = f(g,\ h,\ a,\ s,\ N,\ crown\ growth,\ competition) \qquad 4.6$$

Dbh and total height at the beginning of the simulating period were used as explanatory variables for dbh and total height prediction models respectively. Therefore, a separate variable was not used for the competition assuming the current growth of dbh and total height for the particular models could replace it. However, the total volume prediction model is a current growth prediction model and therefore a separate variable for competition was tested.

Growth responses of individual trees are generally attributed to increased crown volumes increasing photosynthetic surface area (Ginn *et al.*, 1991). The shoot growth, cambial growth and root growth are initiated, controlled and maintained primarily by photosynthates and growth substances produced in the crown (Kozlowski, 1971) and it is well known that the quantity of carbohydrates produced by a tree depends primarily on the size of the main crown structure, crown leaf surface area, and the spacing of the roots to absorb water and mineral nutrients etc (Biging and Gill, 1997).

In this study the shape of the crown of *Pinus nigra* is assumed to be conical and the calculation of the volume of the live crown (equation 3.8 - page 55) was made easier by this assumption. Sievanen *et al.* (1988) and Sievanen and Burk (1993; 1994) assumed that the leaves in the live crown of the *Pinus* species are evenly distributed. Combining these two assumptions it can be concluded that the rate of photosynthesis is dependent on the size of the crown and the amount of solar radiation received by the crown. The rate of photosynthesis of the tree determines the rates of increase of the other variables. Thus the stem volume can be predicted as a function of the following variables as 4.6 but ignoring the factor competition.

$$v = f(g,\ h,\ a,\ s,\ N,\ crown\ volume) \qquad 4.7$$

Many authors (Deleuze, 1996; Hasenaur and Monserud, 1996; Maguire and Hann, 1989; Peterson, 1997; Sprinz and Burkhart, 1987) have written about the influence of the crown dimensions other than crown volume on tree growth. Therefore, the following measurements were used in addition to the crown volume and crown diameter for a better prediction

Crown depth

This is defined as:

$$h_c = h - h_{cb} \qquad 4.8$$

where: h = total height of the tree, m
h_c = length of crown, m
h_{cb} = height to the live crown base from the ground, m
(Hasenaur and Monserud, 1996; Philip, 1994)

Live crown base is the position of tree stem where the first whorl of live branches arises.

Crown ratio

This is defined as:

$$c_r = h_c / h \qquad 4.9$$

where: c_r = crown ratio (Hasenaur and Monserud, 1996)

(ii) Total volume prediction model *b*

A second total volume prediction model (4.10) originally developed by Schumacher and Hall (Avrey and Burkhart, 1994) was selected for comparison of the predictability of the model described above. The non-linear form of this model is:

$$v = a * (dbh^{b1} h^{b2}) \qquad 4.10$$

where: *a, b1* and *b2* are unknown parameters

(Avrey and Burkhart, 1994)

The above model is frequently linearised as:

$$\log v = b_0 + b_1^{/} \log(dbh) + b_2^{/} \log(h) \qquad 4.11$$

where: b_0, $b_1^{/}$ and $b_2^{/}$ are unknown parameters

(Avery and Burkhart, 1994; Philip, 1994)

4.2.1.6 Prediction of the future volume

It is assumed that in a stand where thinning is being carried out, the self-thinning rate or natural mortality is zero or very close to zero (equation 4.12) i.e.:

No. trees just after thinning at time t_1 = *No. trees just before thinning at time* t_2

4.12

The variables such as total height and basal area used to construct the total volume prediction models can be predicted at any time in the future using dbh and total height prediction models. Substituting these values in the volume prediction models, the future volume can be predicted.

4.2.1.7 Merchantable volume

(i) Merchantable volume prediction model *a*

As the current study progressed, the total volume prediction model *a* indicated that the form factor of *Pinus nigra* var. *maritima* trees is 0.5. It indicates that the average form factor is the same as would be expected from a quadratic paraboloid (considering the shape of Corsican pine trees, approximation of a paraboloid). If a frustum of a paraboloid is considered which has a 7 cm top diameter:

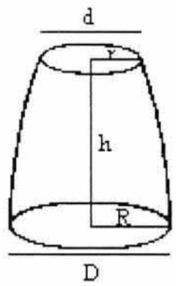


Figure 4.1: Diagram of a frustum of paraboloid. (Source: Hamilton, 1988).

The volume of a frustum of a paraboloid as in Figure 4.1 can be expressed by the following two formulae:

$$\frac{\pi\ h}{2}(R^2 + r^2) \qquad 4.13$$

$$\frac{\pi\ h}{8}(D^2 + d^2) \qquad 4.14$$

(Hamilton, 1988)

Equation 4.14 was rearranged by removing $\pi\ /8$ (to reduce the complexity) and substituting h with timber height to obtain the equation 4.15.

$$v_{mer} = b * \left\{ h_{tim}\left(\frac{dbh^2}{10000} + \frac{49.0}{10000}\right)\right\} \qquad 4.15$$

where: h_{tim} = timber height (height to the 7.0 cm over bark diameter from the ground level), m

v_{mer} = merchantable volume of the tree, m^3

b = unknown parameter

$49.0 = 7.0^2$ (the top over bark diameter), cm

(ii) Merchantable volume model *b*

A derivation of the total volume prediction model *a* (equation 4.77 - page 124) was used as the second model. The upper part from the 7.0 cm over bark diameter of the tree stem was assumed to be a cone and the volume of this cone was subtracted from the total volume of the tree using the equation 4.16.

$$vol_{mer} = b * \left\{ (g\,h) - \left(\frac{\pi \;\; 49.0}{40000} * \left(\frac{h - h_{tim}}{3} \right) \right) \right\} \qquad 4.16$$

where: h = total height of the tree, m

b = unknown parameter

4.2.2 Prediction models of thinning tree variables

The unit size of individual trees removed in thinning is closely related to the type of thinning. Therefore the size of the variables of the removed trees in thinning (y_{bt}) can be predicted by the same tree variables in the stand at just before thinning (y_{th}) (equation 4.17).

$$y_{th} = f\,(y_{bt}) \qquad 4.17$$

This equation was substituted for each variable as expressed below to predict the thinned values of the particular variable.

However, the models were not constructed to predict the distribution of total height or dbh at this stage because the interest was simply to build the relationships between thinned and main crop tree variables.

4.2.2.1 Basal area

Models were not developed for the prediction of the basal area of individual trees for the main crop in this research work because it can easily be calculated using the equation 4.1. However, basal area prediction models were constructed for the trees removed in thinning for the user to select the appropriate model when determining the basal area (use of the basal area model or calculation of basal area using the dbh model).

$$\overline{g}_{th} = a + b * \overline{g}_{bt} \qquad 4.18$$

where: $\overline{g}_{bt}$ = average basal area per tree just before thinning, m^2

$\overline{g}_{th}$ = average basal area per tree to be thinned, m^2

a, b = unknown parameters

4.2.2.2 Diameter at breast height

Mean diameter at breast height of trees removed in thinning is expressed as:

$$\overline{dbh}_{th} = a + b * \overline{dbh}_{bt} \qquad 4.19$$

where: $\overline{dbh}_{bt}$ = average diameter at breast height per tree just before thinning, cm

$\overline{dbh}_{th}$ = average diameter at breast height per tree to be thinned, cm

a, b = unknown parameters

4.2.2.3 Total height

Mean total height of trees removed in thinning can be expressed as:

$$\bar{h}_{th} = a + b * \bar{h}_{bt} \qquad 4.20$$

where: $\bar{h}_{bt}$ = average total height per tree just before thinning, m

$\bar{h}_{th}$ = average total height per thinned tree, m

a, b = unknown parameters

4.2.2.4 Total tree volume

Preliminary tests indicated that the main crop and thinned trees contain the same parameter for total volume prediction models highlighting the same form factor. Therefore, equations 4.18, 4.19 and 4.20 can be used to predict the volume of thinned trees. By substituting these values in the volume prediction models developed for the total volume prediction in main crop trees (equations 4.77 and 4.78 - page 124), the average volume per tree removed in thinning can be estimated.

4.2.2.5 Merchantable volume

Timber height can be predicted using models 4.75 and 4.76 (page 119), if both the total height and the diameter at breast height are known. The models built for the prediction of merchantable volume of main crop trees can be substituted by the thinning variables so that the merchantable volume of the trees to be thinned can be predicted.

4.2.2.6 Number of trees

The volume of the trees removed in thinning can be calculated using the volume prediction models (equations 4.77 and 4.78 - page 124). The number of the trees removed in thinning is calculated using the procedure described below with equations 4.21-4.23.

$$\frac{volume\ removed\ in\ thinning}{volume\ just\ before\ thinning} = k \qquad 4.21$$

Knowing the thinning intensity, the volume removed in thinning can be calculated easily. The volume before thinning is given by the volume model because, within one thinning cycle, the number of trees remains constant (equation 4.12). Substituting these values into equation 4.21 the value of k can be calculated. If the left hand side of the equation 4.21 is expanded and re-arranged, then:

$$n_{th} = \frac{k * \bar{v}_{bt} * n_{bt}}{\bar{v}_{th}} \qquad 4.22$$

where:

n_{bt} = number of surviving trees just before thinning

n_{th} = number of trees removing in thinning

$\bar{v}_{bt}$ = average total volume per tree just before thinning, m^3

$\bar{v}_{th}$ = average total volume per tree removing in thinning, m^3

However, simplifying the equation 4.21, the number of trees removed in thinning can be calculated using the equation 4.23:

$$n_{th} = \frac{total\ volume\ removed\ in\ thinning}{\bar{v}_{th}} \qquad 4.23$$

In the majority of the models constructed, two types of explanatory variables could be identified: essential and subsidiary variables. Essential variables are the most important explanatory variables which could not be removed from the model for the prediction of a particular variable (e.g. the present value of the response variable and the age difference in dbh and total height prediction models). The subsidiary variables could be removed from the models if they were not statistically significant (e.g. crown dimensions in the total volume prediction model). However, before constructing the basic model structures, the distributions of each variable with the response variables were thoroughly examined using scatter plots in order to determine the correct sign of the parameter, shape of the distribution etc. In addition to scatter plots, descriptive statistics (i.e. arithmetic mean, median, minimum, maximum, lower quartile, upper quartile, variance, standard deviation, standard of the mean, coefficient of variance, skewness, standard error of skewness, kurtosis and standard error of kurtosis) of each variable and the correlation with the response variables were also examined to select the most appropriate explanatory variables.

4.2.3 Determination of top height

As a representative factor of the quality of the site, top height was used in the past because many authors (Clutter *et al.*, 1992; Garcia 1983) have described it as a good indicator of the quality of the site in any forest. For the current study, the following method was used to obtain top height in the sample plot data.

The relationship between height and dbh is exponential type. However, within a short period of time, it should be linear. First, the sample plot data were grouped by five year age-classes and then by general yield class. Then using simple linear regression, parameters were estimated to predict the height from the diameter at breast height of individual trees (equation 4.24).

$$h_i = a + b * dbh_i \qquad 4.24$$

where: dbh_i = diameter at breast height of ith tree, cm

h_i = total height of ith tree, m

a, b = unknown parameters

Using the resulting models, the top height could be estimated if the average dbh of the 100 largest trees per hectare is known.

In 1988, Hamilton explained that irrespective of the stand conditions, it is broadly true in British conditions that within a given height and dbh class, the height-dbh relationship remains same. This theory can easily be applied when the data are partitioned into short periods such as five years. Therefore, the expected result was a family of parallel lines of general yield classes for each age class. This was confirmed for most of the general yield classes in each age class. If the lines were not parallel, the procedure written below was followed to obtain the family of parallel lines.

4.2.3.1 Obtaining a family of parallel lines

(i) Testing for common slope

Two lines at a time were tested using the following procedure.

For both lines the degrees of freedom (df), corrected sums of squares of X, Y and products (Σx^2, Σy^2 Σxy), residual degrees of freedom and residual sum of squares (df_{res}, ss_{res}) were calculated separately using the following formulae:

$$df = (n-1) \qquad 4.25$$

$$\Sigma y^2 = \overset{n}{\Sigma} Y^2 - \frac{(\overset{n}{\Sigma} Y)^2}{n} \qquad 4.26$$

$$\Sigma x^2 = \overset{n}{\Sigma} X^2 - \frac{(\overset{n}{\Sigma} X)^2}{n} \qquad 4.27$$

$$\Sigma xy = \overset{n}{\Sigma}(XY) - \frac{(\overset{n}{\Sigma} X)(\overset{n}{\Sigma} Y)}{n} \qquad 4.28$$

$$ss_{res} = \Sigma y^2 - \frac{(\Sigma xy)^2}{\Sigma x^2} \qquad 4.29$$

where: Y = response variable (total height, m)

X = predictor variable (diameter at breast height, cm)

Since only simple linear regressions were fitted, the residual df for each group is one less than the total df (equation 4.30).

$$df_{res} = df - 1 \qquad 4.30$$

Then by adding the above results together, the pooled values for both equations (both lines) were obtained.

The total values for residuals were obtained by using the following equations.

$$ss_{res(tot)} = (\Sigma y^2)_{pooled} - \frac{(\Sigma xy)^2_{pooled}}{(\Sigma x^2)_{pooled}} \qquad 4.31$$

The mean square values for the residuals were obtained by:

$$ms_{res(pooled)} = \frac{ss_{res(pooled)}}{df_{pooled}} \qquad 4.32$$

$$ms_{res(tot)} = \frac{ss_{res(tot)}}{df_{res(tot)}} \qquad 4.33$$

$$ms_{res(res)} = \frac{(ss_{res-tot} - ss_{res-pooled})}{(df_{res-tot} - df_{res-pooled})} \qquad 4.34$$

The Fisher statistic (F) value for the test of common slopes was obtained by:

$$F_{\alpha, df1, df2} = \frac{ms_{res(res)}}{ms_{res(pooled)}} \qquad 4.35$$

An F-value was calculated for each pair of lines obtained from the regression analyses. The resultant F-values were checked with the theoretical F-values from the table for the appropriate degrees of freedom and at the 0.05 probability level. The majority of estimated slopes were significantly different from each other. Where a line was significantly different, the common slope for all the GYCs (including the significant GYC) was estimated as the mean slope ignoring the significant line.

$$b_{com} = \frac{\Sigma b_{i(ns)}}{n_{ns}} \qquad 4.36$$

where: b_{com} = common slope for the non-significant lines

$b_{i(ns)}$ = slope parameter for non-significant lines

n_{ns} = number of non-significant lines in each age class

(ii) Smoothing the intercept

When the slope of an equation is changed, a re-adjustment of intercept might be needed to obtain precise predictions. Therefore the intercepts were modelled after smoothing with the general yield class to obtain a clear relationship (most of the time non-linear) using the following procedure.

First the height intercept was re-adjusted using the equation 4.37.

$$h_{sm} = h - (b_{com} * dbh) \qquad 4.37$$

The arithmetic mean of the smoothed height was calculated using the equation 4.38.

$$\bar{h}_{sm} = \frac{\Sigma h_{sm}}{n} \qquad 4.38$$

where: dbh = diameter at breast height of individual tree, cm

h_{sm} = smoothed height of individual tree, m

$\bar{h}_{sm}$ = mean smoothed height, m

h = total height of individual tree, m

n = number of trees

This procedure was followed for general yield class in each age class and then the mean smoothed height was regressed against the general yield class for each age class (equation 4.39) to estimate the new intercept for each GYC.

$$\bar{h}_{sm} = f \text{ GYC} \qquad 4.39$$

The best fit (linear or non-linear) was selected following an examination of the residual plots, plots of fitted lines and calculated R^2 values. The resultant parameters of the best fitted model were used to predict the intercept for each general yield class in each age class. Finally, using these intercepts and the common slopes the top heights were calculated (4.40).

$$h_{top} = a_{sm} + b_{com} * \overline{dbh}_{top} \qquad 4.40$$

where:

a_{sm} = new intercept

h_{top} = top height, m

$\overline{dbh}_{top}$ = mean diameter of the 100 thickest trees per ha, cm

(Freese, 1990)

Top height is the average total height of the 100 largest diameter trees per hectare. Assuming a random distribution of such trees, there would, on average, be one top height tree in 0.01 hectare. The sizes of the permanent sample plots established by the Forestry Commission varied. Therefore, the number of top height trees per plot were calculated by the following formula and rounded to the nearest whole number.

$$No.\ of\ trees\ to\ be\ taken = plot\ size\ * 100 \qquad 4.41$$

The resultant number of trees was then used to determine the mean diameter of the 100 trees of largest diameter per hectare.

4.2.4 Partition of the data

4.2.4.1 Thinning types

The size of the surviving trees is influenced by the type of the thinning carried out. It was therefore decided to partition the data by the declared thinning regime. However, after examination of the data intermediate and neutral thinning types were selected for the current work because only these two thinning types contained enough sample plots (19 and 18 respectively out of 49) for model construction.

4.2.4.2 Working and validation data

The most common method of partitioning data for fitting the models and subsequently validating them is 3/4 and 1/4 respectively (Chhetri and Fowler, 1996a; Shifley 1987; West 1981). From each thinning type 1/4 of the sample plots were randomly selected and reserved for the validation (Table 4.1).

Intermediate thinning		Neutral thinning	
Fitting	Validating	Fitting	Validating
1181	1186	1635	1634
1185	1214	1637	1645
1187	1371	1640	1648
1246	1424	1642	1649
1248	1427	1644	1652
1366		1647	
1370		1651	
1372		1636	
1428		1638	
1430		1641	
1519		1643	
1543		1646	
1746		1650	
1749			

Table 4.1: Partition of the sample plots by thinning type and by plot number used for the fitting and validating.

4.2.5 Fitting the equations to data

The final set of growth models was constructed from a series of models predicting certain tree growth variables. They were the models of predicting dbh, total height, timber height, total volume and merchantable volume for any tree and mean basal area, mean dbh and mean total height for the trees removed in thinning. The construction of each model was described in the following procedures.

The GENSTAT statistical programme was selected for the construction of the models because of the robustness of both standard and non-standard non-linear regression (Lane and Payne, 1996; Payne *et al.*, 1993).

Before constructing each model, the distributions of the response variables with the explanatory variables were examined using scatter plots. The basic statistics and correlations of both response and candidate explanatory variables were also studied for possible deviation from the model assumptions.

4.2.5.1 Main crop predictions

(i) Diameter at breast height model

Factors representing the site

It is common to use site index as a variable to represent the site quality in growth and yield modelling. However, this value could be changed by variations of site due to the changes of nutrient and water levels. Therefore, some selected values at each measurement time were tested in the current work.

In addition to top height, three variations were used to represent the quality of the site:

a. h_{top} 4.42

b. $$site_{top,age} = \frac{h_{top(t1)}}{age_{t1}}$$ 4.43

c. $$site_{ba,age} = \frac{G_{t1}}{age_{t1}}$$ 4.44

d. $$site_{ba,top} = \frac{h_{top}}{G}$$ 4.45

where: G = total basal area, m^2ha^{-1}

t_1 = time at the beginning of the simulating period, years

Passage of time

Passage of time is an important factor for the construction of models because the growth of most tree variables has a high correlation with the age of the tree. However, determination of the actual age of some plantations is sometimes difficult due to the lack of data. Therefore, the time difference between the beginning and the end of the simulating period (4.46) was used for the diameter prediction models. This reduces the complexity of the models which might have arisen from the use of the plantation ages at the beginning and the end of the simulating period as two explanatory variables. By using passage of time, it is possible to start with a value of a variable (e.g. dbh) at the present time and use this to predict the future values of that variable irrespective of the current age of the stand.

$$a_{dif} = a_{t+\Delta t} - a_t$$ 4.46

where: a_{dif} = difference between the start and the end of simulating period, years

Transformation of the variables

Transformations were done in this work in order to find the best residual distributions, obtain some parameters equal to unity and sometimes to obtain the normal distribution.

The following transformations were tested in order to obtain the best fitting model while meeting all the assumptions.

a. untransformed y_i, x_i;
b. square root $\sqrt{y_i}$, $\sqrt{x_i}$;
c. squared y^2, x_i^2;
d. logarithmic $\log_{10}(y_i)$, $\log_{10}(x_i)$; and
e. inverse $1/y_i$, $1/x_i$.

The tested explanatory variables (x) were: dbh at time t, site factors (h_{top}, $site_{top,age}$, $site_{ba,age}$, $site_{ba,top}$), age difference and number of trees per hectare. The diameters at the beginning and end of the simulating period were conditioned to be the same because,

if there is little difference in age in the diameter prediction model, then:

$$a_{t2} - a_{t1} = \Delta t \rightarrow 0 \qquad (4.47)$$

Site factor can be ignored because it does not change when the age difference is zero or if one assumes trees are growing on an area where the competition has not started.

Then $$dbh_{t+\Delta t} \equiv dbh_t \qquad (4.48)$$

Conditioning a parameter to unity

When the conditions mentioned in equations 4.47 and 4.48 are achieved, the parameter associated with the diameter at the present time must theoretically not be significantly different from unity. If they were significantly difference these were conditioned to equal one by re-arranging the model using the method described below. This procedure was followed in order to obtain the models which are compatible with the theory used for the structure formulation.

The deviation of parameters from unity was tested using formula 4.49 in GENSTAT (Payne *et al.*, 1993).

$$t_{cal} = \frac{p_{est} - 1}{se} \qquad 4.49$$

where: p_{est} = estimated value for the parameter

se = standard error of the particular parameter

t_{cal} = the calculated t-value

If the calculated t-value was lower than the theoretical t-value at 0.05 probability level at the appropriate degrees of freedom, the parameter was not significantly different from unity.

However, if it was significantly different from unity, it was set to one using the following procedure.

Assuming equation 4.50 was the model fitted to the data with a zero intercept,

$$y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \varepsilon_i \qquad 4.50$$

If x_{li} is the variable of interest, then parameter β_1 should theoretically be equal to one. If β_1 was found to be significantly different from unity, a new observed variable z_i was obtained by equation 4.51.

$$z_i = y_i - x_{1i} \qquad 4.51$$

z_i s were then regressed against the rest of the explanatory variables (4.52).

$$z_i = \beta_2' x_{2i} + \beta_3' x_{3i} + \varepsilon_i \qquad 4.52$$

Finally, the resultant new parameters were substituted in the original equation (4.50) to obtain the adjusted equation shown in 4.53.

$$y_i = x_{1i} + \beta_2^{\prime} x_{2i} + \beta_3^{\prime} x_{3i} + \varepsilon_i \qquad 4.53$$

(Whitaker, pers. comm.)

However, following application of this method, model bias can be increased and a careful examination of the regression fit is therefore essential.

(ii) Total height prediction model

The first explanatory variable in the total height prediction model was the height at the beginning of the simulating period. Two variables were tested in order to represent the quality of the site i.e. h_{top} and $site_{top,age}$. However, the other two variables were ignored assuming there is a higher correlation between the total height and top height than the total height and total basal area. Passage of time was also used as a variable in total height prediction models.

Transformation of the variables

Unlike for the dbh prediction models, the response variable total height at the time t_2 and the first explanatory variable total height at the time t_1 were not transformed for the total height prediction models because conditioning of the associated parameter with h_t were not necessary. However, all the other variables were transformed in the way and for the reasons described in the dbh prediction models.

(iii) Timber height prediction model

A function was built by multiplying the over bark diameter at breast height (m) and the total height (m) of individual trees in order to obtain a single explanatory variable. This model was constructed to predict the timber height at a particular age using the dbh and total height at the same age. In that sense, this model is different from the dbh and total height prediction models. Therefore, age was not used as an explanatory variable. Total height was originally tried as the only explanatory variable, but this was unsuccessful as indicated by bias in the residual distributions. Variables were transformed in the way and for the reasons described in dbh prediction models.

(iv) Total volume prediction

Data partitioning

Data were divided by thinning type, and then by age in order to fit the models to one year at a time. The intention was to estimate the parameters for the selected model, or models, at each age and then to examine the pattern of each parameter with age. The resultant parameters were regressed against plantation age in order to build parameter prediction models. The resultant predicted parameters can be used to predict volume at any time in the future assuming all the explanatory variables at the particular age are known.

As in equation 4.5 (page 72), the variable $g*h$ was selected as the first explanatory variable and was used in conjunction with the following variables:

a. to represent the site, h_{top} and $site_{top,age}$,
b. to represent the competition and the form factor, crown depth (c_h - equation 4.8), crown ratio (c_r - equation 4.9) and crown volume (v_c - equation 3.8),
c. to represent the competition through density of the plantation, total number trees per hectare (N) and total basal area per hectare (G).

(v) Merchantable volume prediction

As in the total volume prediction models above, the intention was to estimate the parameters separately for each age and then to regress these against age in order to build parameter prediction models for each variable. Therefore the constructed models 4.15 and 4.16 (pages 76, 77) were fitted separately to the partitioned data.

4.2.5.2 Prediction of basal area, diameter at breast height and total height in thinned trees

All the models constructed for predicting the thinned tree variables were stand-level models and therefore the mean values were used. Mean values of dbh, basal area and total height were calculated separately for each thinning occasion using the formulae 3.1, 3.3 (page 53) and 3.4 (page 54) respectively for each age.

All three models contain just one explanatory variable which is the same as the response variable but just before the thinning.

4.2.6 Evaluation of the models

A combination of qualitative and quantitative tests were used to determine the bias and the precision of the constructed models.

4.2.6.1 Qualitative tests

The qualitative tests used for model evaluation in this study are:

a. Standardised residual plots.
b. Graphs of standard deviation of the residuals at selected points of fitted values.

4.2.6.2 Quantitative tests

(i) Average model bias

Average model bias was calculated in this study using the following formula:

$$\frac{\Sigma(\hat{y}_i - y_i)}{n} \qquad 4.54$$

(ii) Mean absolute difference

This was calculated using the following formula:

$$\frac{\Sigma|\hat{y}_i - y_i|}{n} \qquad 4.55$$

(iii) Modelling efficiency

Modelling efficiency was calculated using the following formula:

$$EF = 1 - \frac{\Sigma(y_i - \hat{y}_i)^2}{\Sigma(y_i - \bar{y})^2} \qquad 4.56$$

where: n = number of data
y_i = observed value for the ith variable
$\hat{y}_i$ = predicted value for the ith variable
$\bar{y}$ = arithmetic mean value for the observed variables

(Soares *et al.*, 1995)

4.2.7 Determination of the lack of fit

The procedure introduced by Weisburg (1985) was followed to highlight the lack of fit of the models constructed. Weisburg suggested that the error of any mathematical model occurs for two reasons namely, lack of fit and pure error. Pure error occurs because of the different response values for similar explanatory values (i.e. population variance). Lack of fit arises because the model does not fit the trend in the data. Lack of fit may occur by not using enough explanatory variables and using inappropriate variables.

The response values for similar explanatory variables were obtained using the following procedure. An illustration of an imaginary data set of a model which contains three explanatory variables is shown in Table 4.2 in order to facilitate understanding.

Before sorting				After sorting			
Y	X_1	X_2	X_3	Y	X_1	X_2	X_3
	1	5	7		1	4	7
	3	6	8		1	4	7
	3	5	9		1	4	7
	1	4	7		1	4	8
	2	4	9		1	4	9
	3	6	9		1	4	9
	2	4	7		1	5	7
	2	6	7		1	5	7
	1	6	7		1	5	8
	2	4	8		1	5	8
	2	5	7		1	5	9
	1	4	9		1	6	7
	1	5	8		1	6	7
	1	6	7		1	6	7
	1	5	8		2	4	7
	2	5	8		2	4	8
	1	4	7		2	4	8
	3	4	8		2	4	8
	3	6	9		2	4	9
	3	5	7		2	4	9
	2	6	7		2	5	7
	1	5	9		2	5	8
	2	4	9		2	5	8
	3	4	8		2	5	9
	2	6	8		2	6	7
	3	5	9		2	6	7
	3	5	7		2	6	7
	1	6	7		2	6	8
	1	4	9		2	6	9
	1	4	8		3	4	8
	2	4	8		3	4	8
	2	4	8		3	4	8
	2	5	9		3	4	8
	3	4	8		3	4	9
	3	6	8		3	4	9
	2	5	8		3	5	7
	3	4	9		3	5	7
	3	4	9		3	5	7
	1	5	7		3	5	9
	3	4	8		3	5	9
	2	6	9		3	6	8
	2	6	7		3	6	8
	3	5	7		3	6	8
	1	4	7		3	6	9
	3	6	8		3	6	9

Table 4.2: An example of the data distribution of a model which contains three explanatory variables.

First the data were sorted in ascending order, sorting with explanatory variable X_1, then X_2 and finally by X_3.

If the number of data in each group (which has the similar explanatory variables) was less than two, that group was ignored from the calculations. For each group the following characteristics were determined for the response variable.

(i) Average y value:

$$\bar{y} = \Sigma y_i / n \qquad 4.57$$

(ii) Sum of squares:

$$ss = \Sigma(y_i - \bar{y})^2 \qquad 4.58$$

(iii) Degrees of freedom:

$$df = n - 1 \qquad 4.59$$

where: n = no. of data within each group

Then the total degrees of freedom (DF) and total sum of squares (SS) were calculated by summing all the df and ss values respectively. These DF and SS are known as the pure error DF and pure error SS.

The total residual DF and SS of the corresponding model were calculated by equations 4.60 and 4.61 respectively.

$$DF = N - 1 \qquad 4.60$$

$$SS_{res} = \Sigma(y_i - \hat{y}_i)^2 \qquad 4.61$$

where: N = total no. of data in the response variable

The DF and SS for the lack of fit were obtained by subtracting the DF and SS for pure error from the residual DF and SS. Then the mean square (MS) values for lack of fit and pure error were calculated using the following formula

$$MS = SS / DF \qquad 4.62$$

Finally the F-value was calculated by the equation 4.63,

$$F_{\alpha,df1,df2} = MS_{lack\_fit} / MS_{pure\_error} \qquad 4.63$$

The null hypothesis was that there was no lack of fit in the model constructed. If the calculated F-value was lower than the theoretical F-value in the table for $\alpha = 0.05; DF_{lack\_fit}, DF_{pure\_error}$ the model was considered as adequate.

4.2.8 Validation with the reserved data

The reserved data were fitted to the constructed models for the similar thinning type without changing the originally estimated parameters. The distribution of normal residuals with the fitted values was observed. Whenever possible, fitted lines were observed after overlaying on the reserved data used for the validation. However, this was only possible when the fitted line resulted from simple linear or standard non-linear estimations. If residuals were normally distributed without an identifiable pattern, the model was finally selected to use in the field.

4.3 Results

4.3.1 Estimation of top height

Models for each general yield class in each 5 year age class were developed using equation 4.24 ($h_i = a + b * dbh_i$). The resultant parameters a and b are given in tables 4.3, 4.4 and 4.5 below:

Age class (years)	General yield class (GYC)						
	10	12	14	16	18	20	22
16-20	-	-	5.35	-	8.57	-	7.03
21-25	-	4.17	7.59	6.72	8.49	9.56	10.50
26-30	8.93	9.13	10.21	9.41	9.23	10.07	12.49
31-35	9.83	7.68	10.83	10.26	11.18	13.35	14.44
36-40	10.56	10.78	13.22	12.20	16.18	17.53	-
41-45	-	12.49	17.50	13.55	15.11	21.20	-
46-50	14.64	-	18.89	17.76	18.51	20.98	-
51-55	-	-	18.17	16.66	20.61	23.82	-
56-60	-	-	17.04	18.02	19.97	23.84	-
61-65	-	-	20.11	20.56	23.61	-	-
66-70	-	-	25.02	24.64	-	-	-

Table 4.3: Parameter a for h-dbh relationships (intermediate thinning) estimated via linear regression.

Age class (years)	General yield class (GYC)						
	10	12	14	16	18	20	22
16-20	-	-	0.17	-	0.18	-	0.23
21-25	-	0.27	0.17	0.29	0.21	0.17	0.21
26-30	0.14	0.09	0.12	0.23	0.27	0.27	0.25
31-35	0.19	0.21	0.18	0.22	0.26	0.18	0.24
36-40	0.12	0.10	0.16	0.23	0.18	0.16	-
41-45	-	0.11	0.03	0.20	0.23	0.10	-
46-50	0.13	-	0.06	0.17	0.18	0.16	-
51-55	-	-	0.10	0.20	0.17	0.13	-
56-60	-	-	0.22	0.18	0.19	0.14	-
61-65	-	-	0.15	0.15	0.14	-	-
66-70	-	-	0.05	0.10	-	-	-

Table 4.4: Parameter b for h-dbh relationships (intermediate thinning) estimated via linear regression.

Age class (years)	Parameter			
	a		*b*	
	GYC14	GYC16	GYC14	GYC16
16-20	6.05	6.39	0.24	0.22
21-25	8.08	8.23	0.20	0.21
26-30	-	9.49	-	0.19
31-35	8.55	10.52	0.25	0.20
36-40	12.52	12.67	0.18	0.20
41-45	15.04	16.34	0.13	0.13

Table 4.5: Resultant parameters of h-dbh relationships for each age class (neutral thinning).

4.3.1.1 Obtaining sets of parallel lines for each age class

A linear non-parallel family of straight lines resulted for each age class (e.g. Figure 4.2). A set of parallel lines was produced as the second step for each general yield class in each 5 year age class using the procedure described in section 4.2.3.1 (pages 82-85).

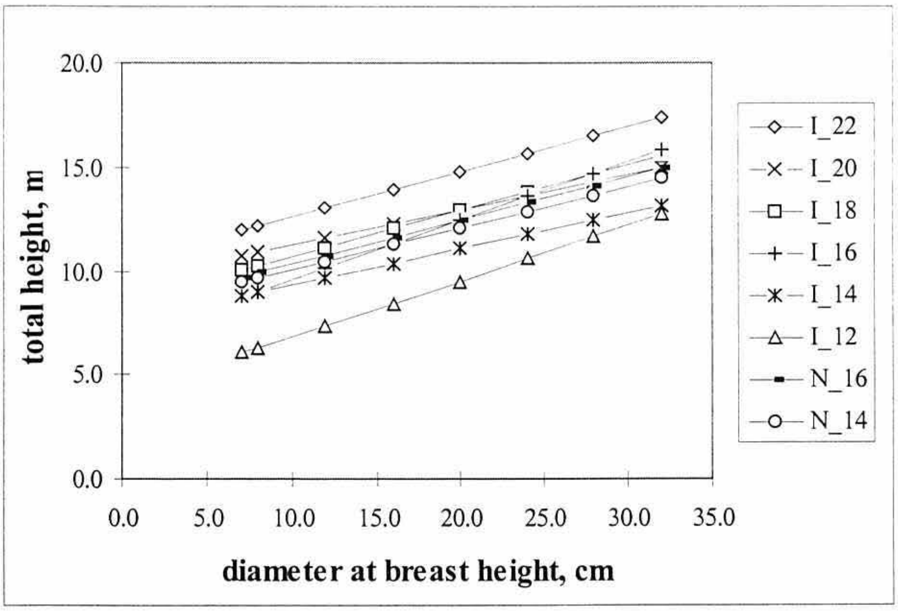


Figure 4.2: Resultant dbh-height relationships before smoothing the intercepts and slopes (age class 21-25).

Five year age classes were selected for this procedure as these produced the most consistent relationships between height and dbh. The results of the F-tests for the common slopes of the general yield classes in each five year age class are given in Appendix 2.1. Most of the slopes were not statistically different from each other. Only the statistically similar lines were selected in order to calculate the mean slope. The calculated mean slopes for each age class are given in table 4.6.

Age class (years)	Common slope
16-20	0.2077
21-25	0.2181
26-30	0.1954
31-35	0.2196
36-40	0.1860
41-45 (I)*	0.1147
(II)	0.2160
46-50	0.1400
51-55	0.1566
56-60	0.1835
61-65	0.1482

* Age class 41-45 (I) - GYC I10, I12, I20, I22, N14, N16: (II) - GYC I16, I18

Table 4.6: Calculated common (mean) slopes for each age class after testing the significance.

Then the mean heights for each age class were re-estimated using the procedure outlined using equations 4.37 and 4.38 (page 84). The resultant values are given in the following table:

Age class, years	General yield class, $m^3ha^{-1}yr^{-1}$								
	Intermediate thinning							Neutral thinning	
	GYC 10	GYC 12	GYC 14	GYC 16	GYC 18	GYC 20	GYC 22	GYC 14	GYC 16
16-20	-	-	4.75	-	7.60	-	7.41	6.41	6.34
21-25	-	-	4.96	6.88	6.85	8.36	8.84	7.79	8.13
26-30	8.04	7.35	8.77	9.09	10.81	11.56	12.16	-	9.33
31-35	9.30	9.30	9.31	10.31	12.17	14.23	13.02	9.30	10.07
36-40	10.62	11.26	11.67	12.23	16.74	16.85	-	12.18	13.05
41-45	-	14.20	15.17	14.51	17.17	18.86	-	14.12	15.15
46-50	14.39	-	16.21	17.88	19.86	21.59	-	-	-
51-55	-	-	17.37	17.88	20.96	22.79	-	-	-
56-60	-	-	18.26	17.99	20.40	21.79	-	-	-
61-65	-	-	20.46	20.61	23.28	-	-	-	-

Table 4.7: Mean smoothed heights for each GYC in each age class after adjusting.

Tables 4.8a and 4.8b indicate the new intercepts for each set of parallel height diameter lines in each five year age class. In these tables the intercepts for all the general yield classes (10-22) are given including those for which raw data did not exist. Thinning type was not taken into account in the estimation of these intercepts because similar GYCs in the two thinning types indicated very similar results.

Age class (years)	Relationship	R^2	GYC	New intercept
16-20	$a_{sm} = a + b * r^{gyc}$	0.938	10	2.302
	(exponential)		12	4.329
			14	5.600
			16	6.398
			18	6.898
			20	7.212
			22	7.409
21-25	$a_{sm} = a + b * r^{gyc}$	0.819	10	1.085
	(exponential)		12	5.117
			14	7.019
			16	7.917
			18	8.341
			20	8.541
			22	8.636
26-30	$a_{sm} = a + c / (1 + \exp(-b * (gyc - m)))$	0.943	10	7.758
	(logistic)		12	7.912
			14	8.351
			16	9.327
			18	10.667
			20	11.664
			22	12.118
31-35	$a_{sm} = a + c / (1 + \exp(-b * (gyc - m)))$	0.910	10	9.296
	(logistic)		12	9.303
			14	9.385
			16	10.108
			18	12.399
			20	13.447
			22	13.614
36-40	$a_{sm} = a + c / (1 + \exp(-b * (gyc - m)))$	0.910	10	11.364
	(logistic)		12	11.364
			14	11.392
			16	12.663
			18	16.648
			20	16.931
			22	16.936

Table 4.8a: Adjusted intercepts with new mean heights for each GYC.

Age class (years)	Relationship	R^2	GYC	New intercept
41-45	(i) $a_{sm} = a + b * gyc$ (linear)	0.978	10	11.319
			12	13.177
	(ii) $a_{sm} = a + b * gyc$ (linear)	1.000	14	15.035
			16	13.140
			18	15.580
			20	20.608
			22	22.466
46-50	$a_{sm} = a + c/(1 + \exp(-b * (gyc - m)))$ (logistic)	0.999	10	14.396
			12	15.046
			14	16.179
			16	17.857
			18	19.828
			20	21.600
			22	22.847
51-55	$a_{sm} = a + c/(1 + \exp(-b * (gyc - m)))$ (logistic)	1.000	10	17.330
			12	17.332
			14	17.366
			16	17.877
			18	20.959
			20	22.786
			22	22.954
56-60	$a_{sm} = a + b * r^{gyc}$ (exponential)	0.843	10	17.223
			12	17.496
			14	17.946
			16	18.687
			18	19.906
			20	21.912
			22	25.215
61-65	$a_{sm} = a + b * r^{gyc}$ (exponential)	1.000	10	20.450
			12	20.451
			14	20.459
			16	20.609
			18	23.284
			20	25.351
			22	27.891

Table 4.8b: Adjusted intercepts with new mean heights for each GYC.

Although the relationships between the smoothed intercept and the general yield class were non-linear, a typical pattern from lowest to highest age class was not observed. For age class 41-45, two lines were estimated as shown in Table 4.8b, because it was impossible to construct one family of lines without introducing bias. Despite careful examination of the raw data and the measurement procedures adopted, this bias could not be fully explained. The reason for this bias is most likely due to some undocumented feature of the thinning practices adopted within some sample plots at this age class. In this study, the purpose of such work was to estimate the top height precisely using equation 4.40 (page 85). However, when the constructed models are used in the field, the user will be required to measure the top height of the particular plantation rather than using the above relationships. The resultant family of parallel lines for age class 21-25 after following the above procedure is given in Figure 4.43.

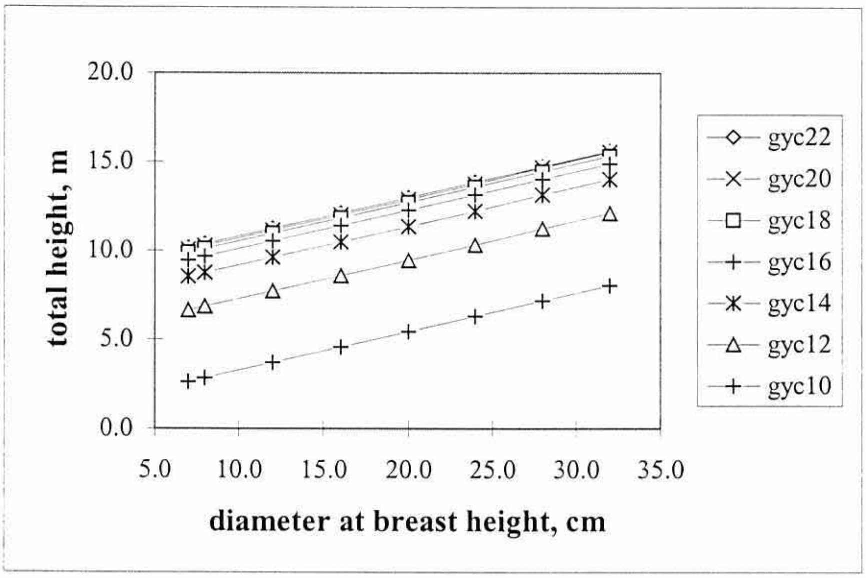


Figure 4.3: The resultant dbh-height relationships after smoothing the intercepts and slopes (age class 21-25).

Finally the top height can be calculated using equation 4.40.

4.3.2 Prediction of tree diameter at breast height

In order to select the best possible explanations of model structures, first the distributions of the selected explanatory variables with the dbh at time $t+\Delta t$ (Y-axes of all the plots in Figure 4.4) were examined using scatter plots. The descriptive statistics of the tested variables and the correlation with the diameter at the time $t+\Delta t$ are shown in Appendix 2.2(i) and Appendix 2.3(i) respectively.

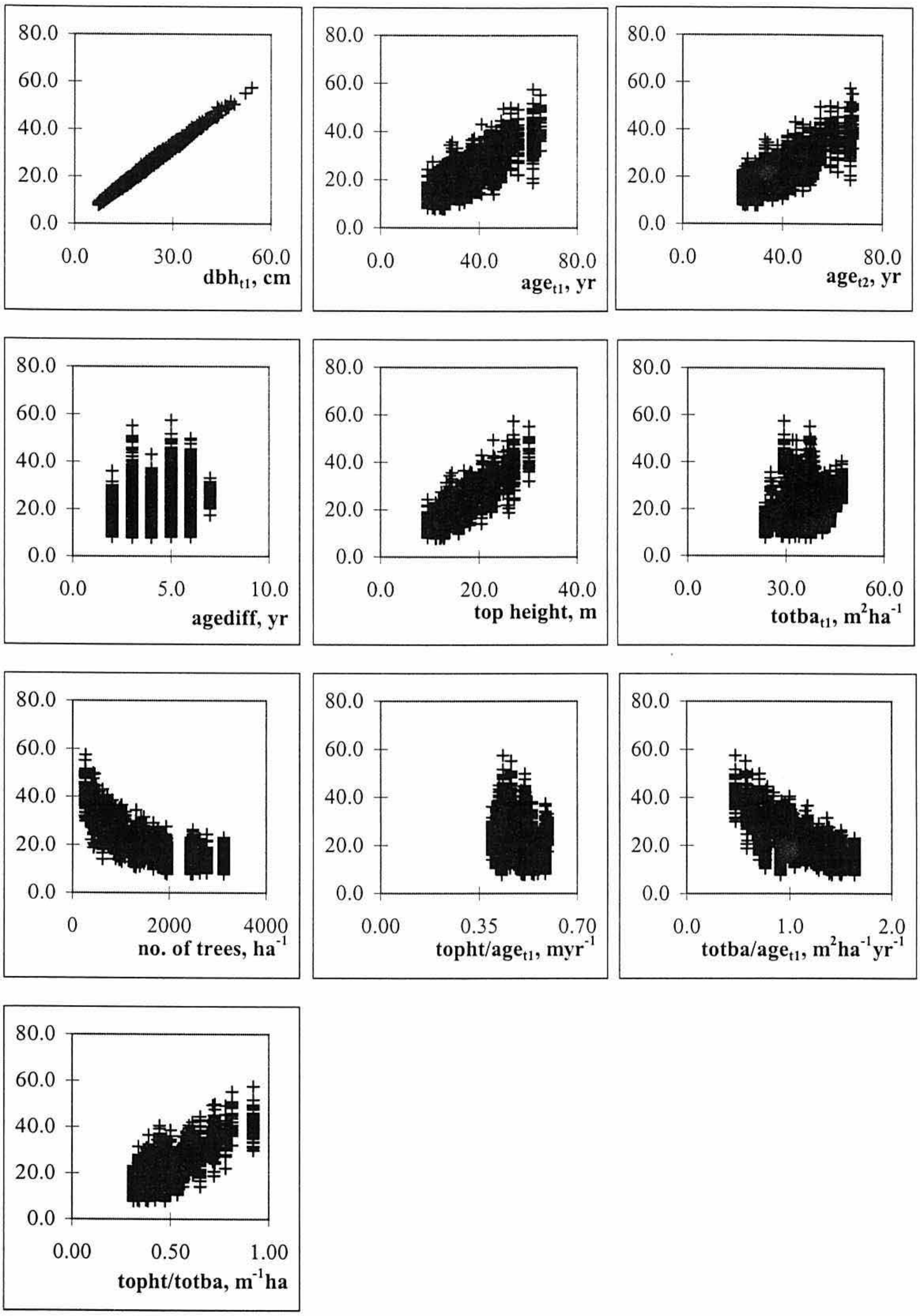


Figure 4.4: Distribution of dbh at the end of the simulating period with the tested explanatory variables (intermediate thinning type).

4.3.2.1 Models developed for the prediction of dbh

The model structure described in section 4.2.1.2 (page 70) is:

$$dbh_{t+\Delta t} = a + b_1 * dbh_t + b_2 * a_{dif} + b_3 * s + b_4 * d \qquad (4.64)$$

4.3.2.2 The best relationships

All the possibilities described on pages 87-89 were tested with the residual plots and by examining the R^2 values. The factors selected to represent the site i.e. ht_{top}, $site_{top,age}$, $site_{ba,age}$ and $site_{ba,top}$ were changed one at a time and the simulating period was repeated. The best and acceptable relationships are listed in Table 4.9. The transformation of $dbh_{t+\Delta t}$ is always similar to the form of dbh_t.

Intermediate thinning		Neutral thinning	
Model	R^2	Model	R^2
$\log dbh_t + site_{ba,age} + \sqrt{a}_{dif}$	0.990	$\sqrt{dbh}_t - \sqrt{site}_{ba,age} + a_{dif}$	0.990
$\sqrt{dbh}_{t1} + \sqrt{site}_{ba,top} + a_{dif}$	0.991	$\sqrt{dbh}_{t1} - site_{ba,age} + a^2_{dif}$	0.990
$\sqrt{dbh}_{t1} + site_{ba,age} + a^2_{dif}$	0.991	$\sqrt{dbh}_{t1} - \sqrt{site}_{ba,age} + \log a_{dif}$	0.990
$\sqrt{dbh}_{t1} + site_{ba,top} + \log a_{dif}$	0.991	$\sqrt{dbh}_{t1} - site_{ba,top} + a^2_{dif}$	0.990
$\sqrt{dbh}_{t1} + \sqrt{site}_{ba,top} + \log a_{dif}$	0.991	$\sqrt{dbh}_{t1} - \log site_{ba,top} + a^2_{dif}$	0.990
$\sqrt{dbh}_{t1} + site^2_{ba,top} + \log a_{dif}$	0.991	$\sqrt{dbh}_{t1} - \sqrt{site}_{ba,top} + \log a_{dif}$	0.990
$\sqrt{dbh}_{t1} + \log site_{ba,top} + \log a_{dif}$	0.991	$\sqrt{dbh}_{t1} + 1 / site_{ba,top} - 1 / a_{dif}$	0.990
$\log dbh_{t1} + site^2_{top,age} + \sqrt{a}_{dif}$	0.991	$\sqrt{dbh}_{t1} - site_{top,age} + a_{dif}$	0.990
$\log dbh_{t1} + \sqrt{site}_{top,age} + a^2_{dif}$	0.992	$\sqrt{dbh}_{t1} - \sqrt{site}_{top,age} + \sqrt{a}_{dif}$	0.990
$\log dbh_{t1} + site_{top,age} + \log a_{dif}$	0.991	$\sqrt{dbh}_{t1} - site^2_{top,age} + a^2_{dif}$	0.990
$\sqrt{dbh}_{t1} + \log site_{top,age} + \sqrt{a}_{dif}$	0.992	$\sqrt{dbh}_{t1} - site_{top,age} + \log a_{dif}$	0.990
$\sqrt{dbh}_{t1} + site^2_{top,age} + a^2_{dif}$	0.991	$\sqrt{dbh}_{t1} - \log site_{top,age} - 1 / a_{dif}$	0.990
		$\log dbh_{t1} - site_{ba,age} + \sqrt{a}_{dif}$	0.990
		$\log dbh_{t1} - site_{ba,top} + \sqrt{a}_{dif}$	0.990
		$\log dbh_{t1} - \log site_{ba,top} + \sqrt{a}_{dif}$	0.990
		$\log dbh_{t1} + 1 / site_{ba,age} + a^2_{dif}$	0.990
		$\log dbh_{t1} - site_{top,agr} + \sqrt{a}_{dif}$	0.989
		$\log dbh_{t1} - 1 / site_{top,age} + a_{dif} - d$	0.989

Table 4.9: The best relationships obtained after fitting the possible equations.

In all the tabulated relationships, the parameter a (intercept) was not significantly different from zero at 0.05 probability level.

According to the theory described by equations 4.47 and 4.48 (page 89) the parameter associated with dbh_t (b_1 in model 4.64) should not theoretically be significantly different from one. However, in all the models, except one (intermediate thinning - $\sqrt{dbh}_{t1} + site_{ba,top} + \log a_{dif}$), the parameter b_1 was significantly different from one, although close. Therefore that parameter was forced manually to one by using the procedure described on pages 90 and 91 (equations 4.49 to 4.53).

Although the data were grouped by thinning type, ideally the basic model structure for both thinning types should be identical except for the parameter values. Four such identical models were found in this work. These models are listed below:

(i) Dbh prediction model *a*

This model is:

$$\sqrt{dbh}_{t+\Delta t} = b_1 * \sqrt{dbh}_t + b_2 * site^2_{top,age} + b_3 * a^2_{dif} \qquad (4.65)$$

Parameters before forcing $b_1 \rightarrow 1$ in this model

Parameter	Intermediate		Neutral	
	estimate	std. error	estimate	std. error
b_1	1.00762	0.00074	1.05184	0.00166
b_2	0.34095	0.00895	-0.26630	0.02040
b_3	0.00400	0.00008	0.00523	0.00014

Parameters after forcing $b_1 \rightarrow 1$ in this model

Parameter	Intermediate		Neutral	
	estimate	std. Error	estimate	std. error
b_2	0.41773	0.00498	0.38444	0.00606
b_3	0.00449	0.00006	0.00759	0.00013

(ii) Dbh prediction model *b*

Dbh prediction model *b* is:

$$\sqrt{dbh}_{t+\Delta t} = c_1 * \sqrt{dbh}_t + c_2 * site_{ba,age} + c_3 * a^2_{dif} \tag{4.66}$$

Parameters before forcing $c_1 \to 1$ *in this model*

Parameter	Intermediate		Neutral	
	estimate	std. error	estimate	std. error
c_1	1.01383	0.00071	1.05047	0.00103
c_2	0.05882	0.00195	-0.05577	0.00268
c_3	0.00382	0.00008	0.00491	0.00013

Parameters after forcing $c_1 \to 1$ *in this model*

Parameter	Intermediate		Neutral	
	estimate	std. error	estimate	std. error
c_2	0.08908	0.00120	0.06362	0.00141
c_3	0.00479	0.00007	0.00901	0.00013

(iii) Dbh prediction model *c*

Dbh prediction model *c* is:

$$\sqrt{dbh}_{t+\Delta t} = d_1 * \sqrt{dbh}_t + d_2 * \sqrt{site}_{ba,top} + d_3 * \log a_{dif} \tag{4.67}$$

Parameters before forcing $d_1 \to 1$ *in this model*

Parameter	Intermediate		Neutral	
	estimate	std. error	estimate	std. error
d_1	1.00157	0.00108	1.04992	0.00158
d_2	0.01855	0.00271	-0.09844	0.00313
d_3	0.28695	0.00674	0.29524	0.00756

Parameters after forcing $d_1 \to 1$ *in this model*

Parameter	Intermediate		Neutral	
	estimate	std. error	estimate	std. error
d_2	0.01855	0.00271	-0.01804	0.00203
d_3	0.28695	0.00674	0.44869	0.00647

(iv) Dbh prediction model *d*

This model is:

$$\log dbh_{t+\Delta t} = e_1 * \log dbh_t + e_2 * site_{ba,age} + e_3 * \sqrt{a}_{dif} \qquad (4.68)$$

Parameters before forcing $e_1 \rightarrow 1$ in this model

Parameter	Intermediate		Neutral	
	estimate	std. error	estimate	std. error
e_1	0.97211	0.00093	1.00527	0.00137
e_2	0.01386	0.00044	-0.01324	0.00060
e_3	0.02945	0.00055	0.02818	0.00079

Parameters after forcing $e_1 \rightarrow 1$ in this model

Parameter	Intermediate		Neutral	
	estimate	std. error	estimate	std. error
e_2	0.00961	0.00044	-0.01206	0.00052
e_3	0.01487	0.00028	0.03088	0.00037

The sign of each parameter must be the same in the models for both thinning types. However, in model *c* (4.67) and *d* (4.68), the parameters associated with the variables representing the quality of the site have different signs i.e. positive in intermediate thinning and negative in neutral thinning. Therefore, both models were ignored and dbh prediction models *a* (4.65) and *b* (4.66) selected for further testing.

4.3.2.3 Evaluation of the diameter prediction models

When the age difference increases, the dbh should also increase making the parameter associated with a_{dif} positive. Dbh growth should also increase with the quality of the site if there is no other limitation for growth. Therefore, the parameter associated with the site should also be positive.

According to Whitaker (pers. comm.) it is impossible to calculate the standard residuals manually after smoothing parameters and, therefore it was decided instead to examine the distribution of normal residuals. The distribution of the normal residuals of dbh prediction model *a* for intermediate thinning and dbh prediction model *b* for both thinning types are given in Figure 4.5 and Appendix 2.4(i) respectively. The normal residuals of both models were very similar and did not indicate a bias for the intermediate thinning type. In the neutral thinning type, there was an indication of over estimation at early ages after smoothing the parameters for both models.

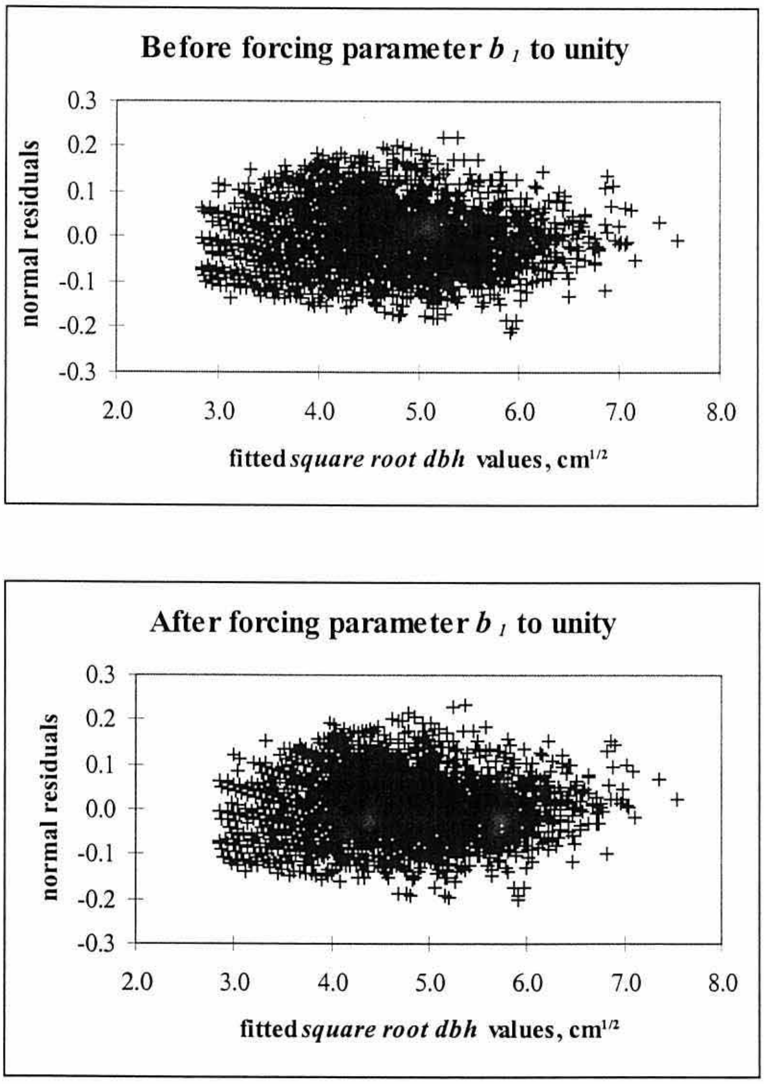


Figure 4.5: The distribution of the normal residuals for *dbh* prediction model *a* (intermediate thinning).

The standard deviation of the residuals was distributed evenly with the fitted values (Figure 4.6) indicating a good fit except for model *b* at the fitted value point six.

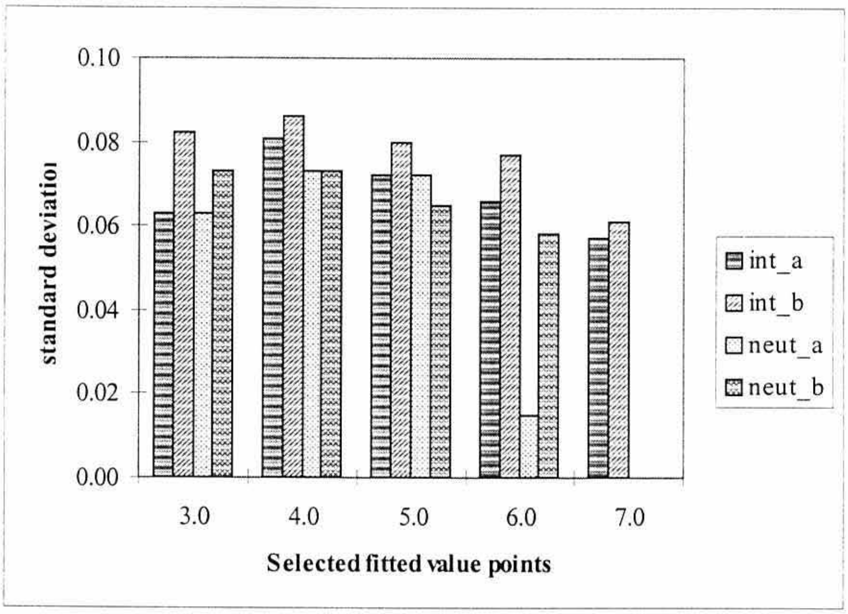


Figure 4.6: Distribution of standard deviation of residuals at selected points of fitted values.

The average model bias and the mean absolute difference for both models were very low and the modelling efficiency was over 0.98 (Table 4.10). After the validation tests with the reserved data (Figure 4.7) it was concluded that all the models were suitable for the field application and also that there was no indication of a lack of fit (Table 4.11). Therefore all four models were selected for further studies.

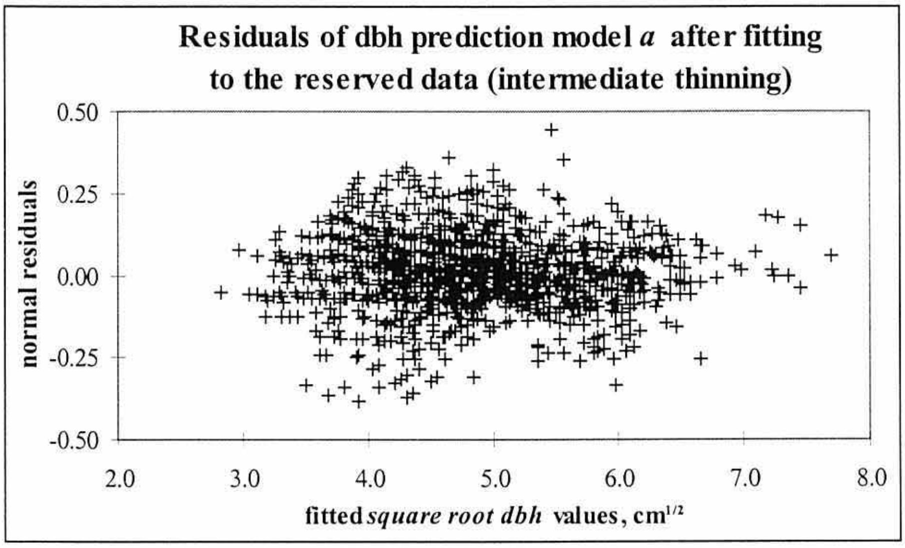
Test	Intermediate thinning		Neutral thinning	
	Model *a*	Model *b*	Model *a*	Model *b*
Average model bias	-0.0020	0.0039	-0.0013	-0.0089
Mean absolute difference	0.0580	0.0580	0.0570	0.0630
Modelling efficiency	0.9920	0.9910	0.9870	0.9840

Table 4.10: Results of the quantitative tests for selected dbh prediction models.

Model	Intermediate thinning No. of data = 5473	Neutral thinning No. of data = 4024
a	0.93	0.91
b	0.90	0.24

None of the F-values were significant at 0.05 probability level.

Table 4.11: Results of lack of fit tests (F-values) for the selected dbh prediction models.



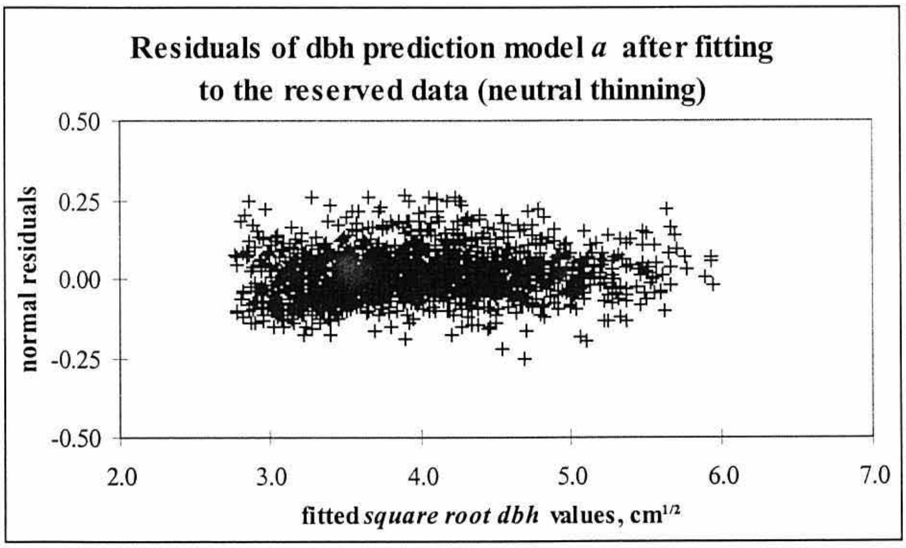


Figure 4.7: Distribution of the residuals after fitting the unchanged models to the data reserved for validation.

4.3.3 Prediction of the total height of individual trees

The distributions of selected explanatory variables with the total height at time $t+\Delta t$ (Y-axes of the graphs in Figure 4.8) were examined in order to determine the correct sign of the parameters. These distributions for the intermediate thinning type are given in Figure 4.8.

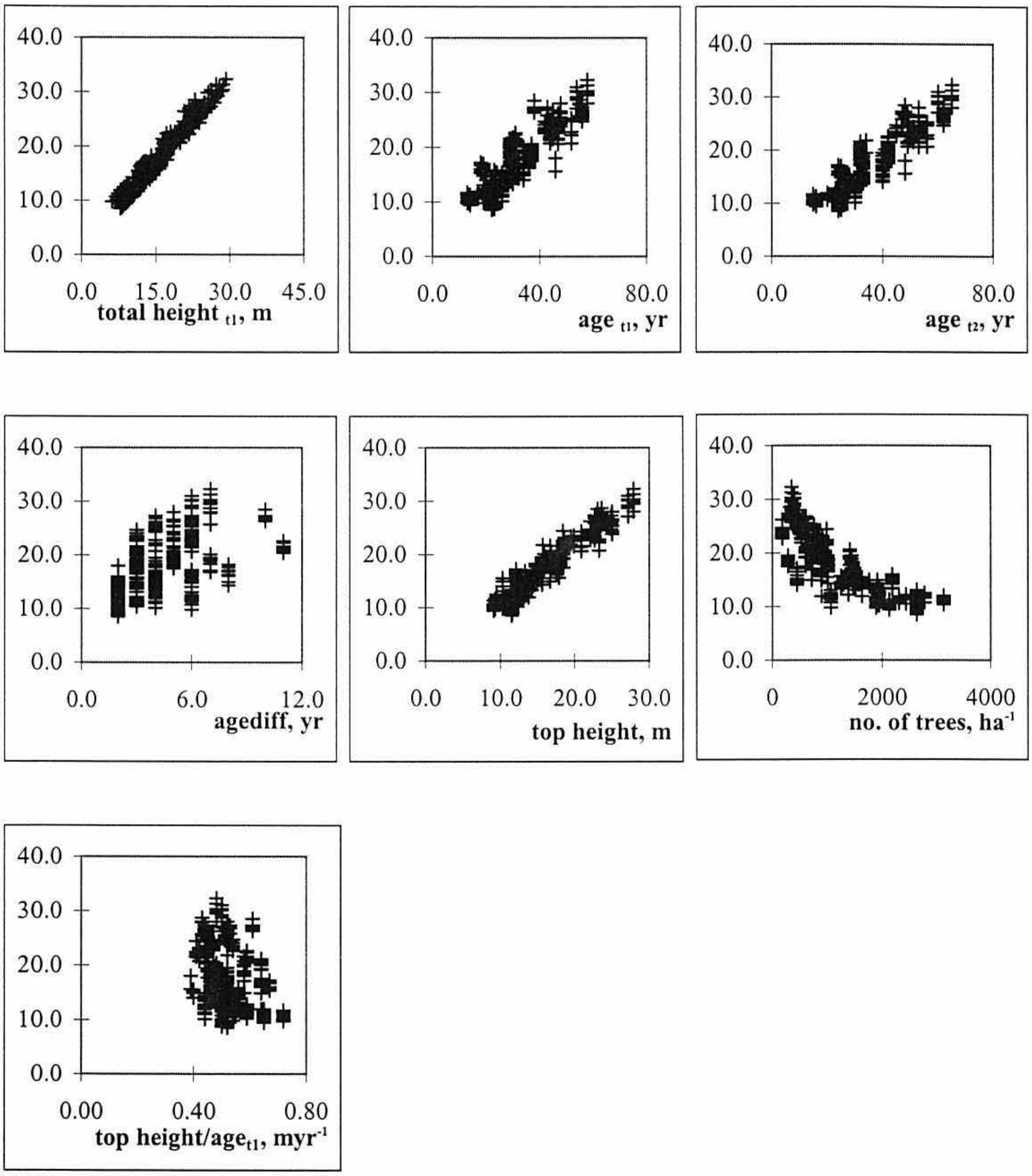


Figure 4.8: Distribution of total height at the end of simulating period with the explanatory variables tested (intermediate thinning).

Descriptive statistics of the Y variables and the selected explanatory variables with the total height at time $t+\Delta t$ are shown in Appendix 2.2(ii). and 2.3(ii) respectively.

4.3.3.1 Models constructed for the prediction of total height

The selected model structure according to section 4.2.1.3 (page 71) is,

$$ht_{t+\Delta t} = a + b_1 * h_t + b_2 * a_{dif} + b_3 * s + b_4 * d \tag{4.69}$$

4.3.3.2 The best relationships

After trying all the possibilities described in page 91, the models listed in Table 4.12 were selected because these were the only models which contained similar structures for both thinning types.

When h_{top} alone was used to represent the quality of the site, it was not statistically significant and the intercept (parameter a in model 4.69) was significantly different from zero. When the total number of trees per hectare was added as an additional predictor variable to the model, it was not statistically significant and also the distribution of standard residuals was not improved. Therefore the total number of trees per hectare was removed from the final equations and $site_{top,age}$ was used to represent the site quality instead of h_{top}.

Intermediate thinning		Neutral thinning	
Model	R^2	Model	R^2
$h_t + site_{top,age} + a^2_{diff}$	0.983	$h_t + site_{top,age} + a^2_{diff}$	0.973
$h_t + \sqrt{site}_{top,age} + a^2_{diff}$	0.983	$ht_t + \sqrt{site}_{top,age} + a^2_{diff}$	0.973

Table 4.12: The best possible relationships obtained for the prediction of total height after fitting the possible equations.

The intercepts of both equations listed above were not significantly different from zero at the 0.05 probability level.

The basic model structure for both diameter at breast height and total height was similar and therefore the theory described in equations 4.47 and 4.48 (page 89) could also be applied to the height prediction model. Therefore, assuming a tree growing on open ground or on a site where tree-tree competition has not started, and where age has not changed, then both explanatory and response heights would be the same (4.70) and therefore the parameter b_1 associated with h_t should not be significantly different from one.

$$h_{t+\Delta t} \equiv h_t \tag{4.70}$$

Both equations listed above were selected for further tests because they fulfilled the above requirements and also forcing parameters to unity was not necessary.

(i) Total height prediction model *a*

This model is:

$$h_{t+\Delta t} = h_t + b_2 * site_{top,age} + b_3 * a_{dif}^2 \tag{4.71}$$

Parameter	Intermediate thinning		Neutral thinning	
	estimate	std. error	estimate	std. error
b_1	1.00301	0.00553	1.00070	0.01100
b_2	2.39100	0.14900	3.55555	0.35300
b_3	0.03337	0.00149	0.02807	0.00403

(ii) Total height prediction model *b*

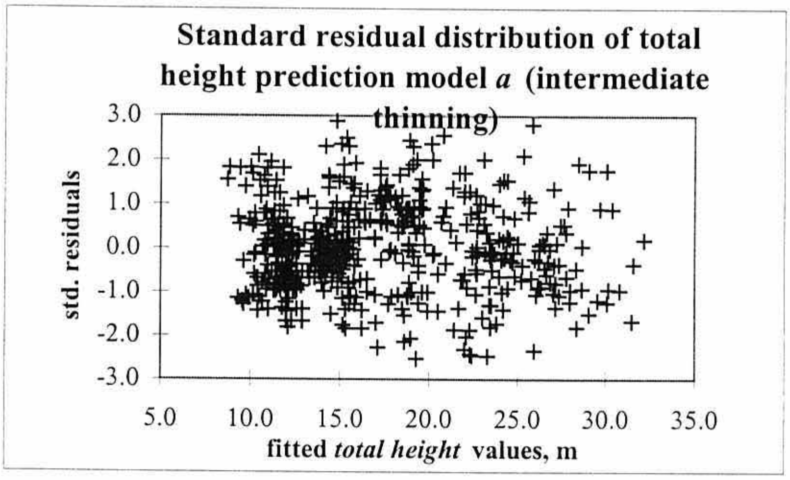
This model is:

$$h_{t+\Delta t} = h_t + c_2 * \sqrt{site}_{top,age} + c_3 * a_{dif}^2 \tag{4.72}$$

Parameter	Intermediate thinning		Neutral thinning	
	estimate	std. error	estimate	std. error
c_1	0.99469	0.00601	0.99310	0.01160
c_2	1.90600	0.12000	2.73400	0.27100
c_3	0.03347	0.00150	0.02759	0.00405

4.3.3.3 Evaluation of the total height prediction models

The distributions of standardised residuals of the total height prediction model *a* and *b* are included in Figure 4.9 and Appendix 2.4(ii) respectively. The standard residual distribution for both models for the intermediate thinning type showed an even distribution highlighting the high predictive ability of the model (Figure 4.9). The standard deviation of the residuals also indicated an even distribution for that thinning type (Figure 4.10). However, in the range of total height 13-16 m there was an indication of over-estimation for neutral thinning and the distribution of the standard deviation at the 15m point of the fitted values had a narrower distribution than those for the other fitted values (Figure 4.10). More data are needed in the above range for neutral thinning type for a proper conclusion of this matter. These were unavailable for the present study.



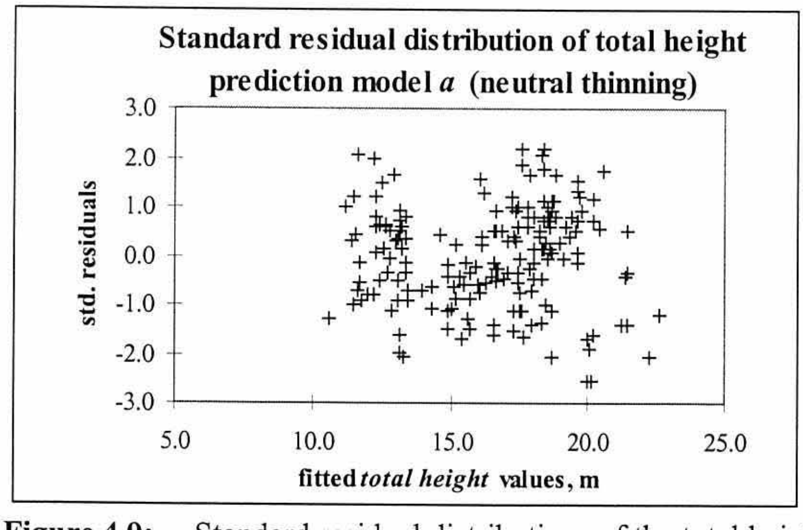
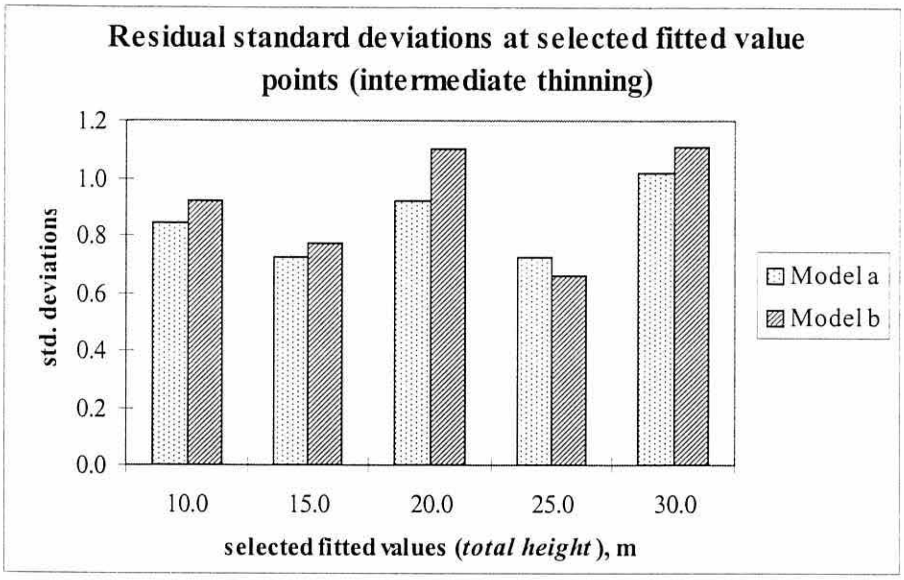


Figure 4.9: Standard residual distributions of the total height prediction model *a* for both thinning types.



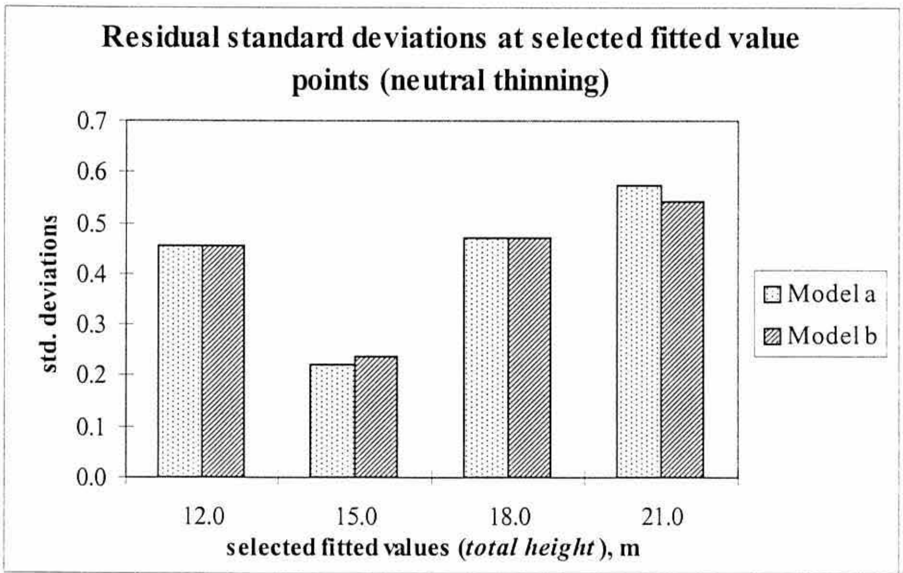


Figure 4.10: Distribution of the residual standard deviations with the fitted values.

Looking at the distributions of the variables in Figure 4.8, age difference should have a positive parameter because the total height increases with age. The parameter associated with the quality of the site should also have a positive sign because under conditions of unrestricted growth, the height increases with site quality (increment of height goes up with both age and quality of the site). Both models indicate very low bias. Modelling efficiencies were over 0.95 (Table 4.13). The results of the lack of fit testing were negative (Table 4.14) and when applied to the reserved data (Figure 4.11), a good residual distribution could be seen. Therefore, both models were selected for further tests (Chapter 6).

Test	Intermediate thinning		Neutral thinning	
	Model *a*	Model *b*	Model *a*	Model *b*
Average model bias	-0.053	0.121	0.076	0.054
Mean absolute difference	0.564	0.503	0.557	0.379
Modelling efficiency	0.982	0.990	0.956	0.981

Table 4.13: Results of the quantitative tests applied for total height prediction models.

Model	Intermediate thinning No. of data = 554	Neutral thinning No. of data = 185
a	0.99	1.31
b	1.00	1.31

None of the F-values were significant at 0.05 probability level.

Table 4.14: Results of the lack of fit test (F-values) of total height prediction models.

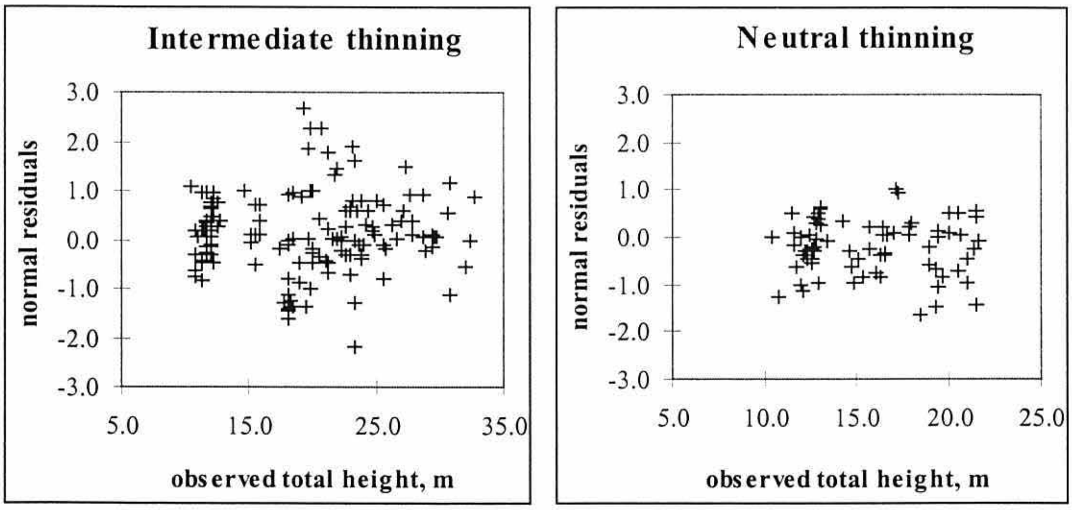


Figure 4.11: Residual distributions after fitting the unchanged model *a* to the reserved data for validation.

4.3.4 Prediction of timber height

Unlike dbh and total height prediction models, the timber height prediction model is a current state prediction model. First the distributions of the selected explanatory variables with timber height (for intermediate thinning - Figure 4.12) were examined to find the basic model structure.

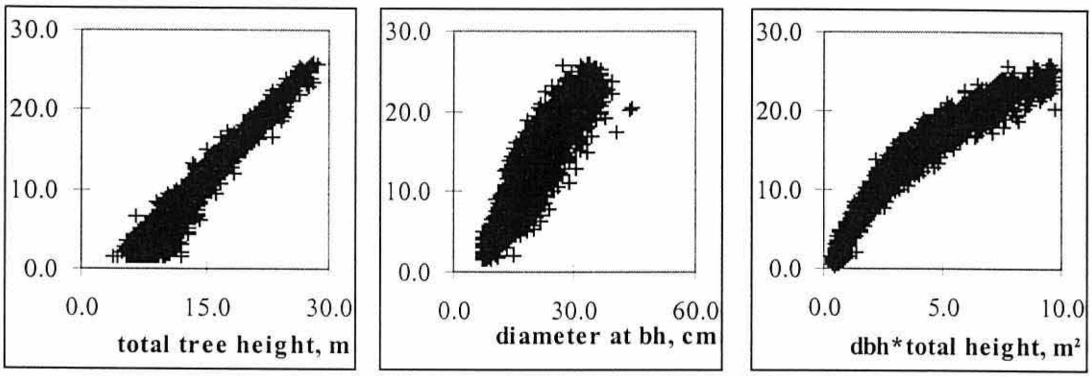


Figure 4.12: Distribution of the timber height (Y-axes) with selected explanatory variables (intermediate thinning).

The descriptive statistics and the correlations of all the variables used for timber height modelling are shown in Appendix 2.2(iii) and 2.3(iii) respectively.

4.3.4.1 Developed models for the prediction of timber height

From the observation of the distribution of variables in Figure 4.12, two types of relationships were identified for further development.

(i) $$h_{tim} = a + b * h \quad (4.73)$$

(ii) $$h_{tim} = a + (dbh * h)^b \quad (4.74)$$

The variable $dbh*h$ could not be replaced by dbh itself because of the heteroscedasticity of the standard residuals with respect to the fitted values. Equation 4.73 is a linear relationship with one explanatory variable (total height). In equation 4.74, timber height is predicted by an exponential function (dbh*total height).

4.3.4.2 The best relationships

All the possible combinations of the explanatory variables were fitted to the data and the standard residual plots and R^2 values were examined. The linear relationship (equation 4.73) was ignored due to the poor fit and the selected basic structure was equation 4.74. Selected best relationships obtained from that basic structure are listed in Table 4.15.

Intermediate thinning		Neutral thinning	
Model	R^2	Model	R^2
$h_{tim} = a + b * r^{(dbh*h)}$	0.967	$h_{tim} = a + b * r^{(dbh*h)}$	0.969
$h_{tim} = a + b * r^{\sqrt{(dbh*h)}}$	0.968	$h_{tim} = a + b * r^{\sqrt{(dbh*h)}}$	0.969
$h_{tim} = a + b * r^{\log(dbh*h)}$	0.967	$h_{tim} = a + b * r^{\log(dbh*h)}$	0.969

a, b and r are parameters.

Table 4.15: The best relationships obtained for the prediction of timber height.

The distribution of the standard residuals of the second model in the above table constructed for intermediate thinning type was poor. Therefore, the following two models were selected for further tests.

(i) Timber height prediction model *a*

This model is:

$$h_{tim} = a_1 + b_1 * r_1^{\sqrt{dbh*h}} \tag{4.75}$$

Parameter	Intermediate thinning		Neutral thinning	
	estimate	std. error	estimate	std. Error
a_1	56.89000	2.03000	29.45700	0.83800
b_1	-65.71000	1.87000	-40.17500	0.52700
r_1	0.79676	0.00822	0.60600	0.01310

(ii) Timber height prediction model *b*

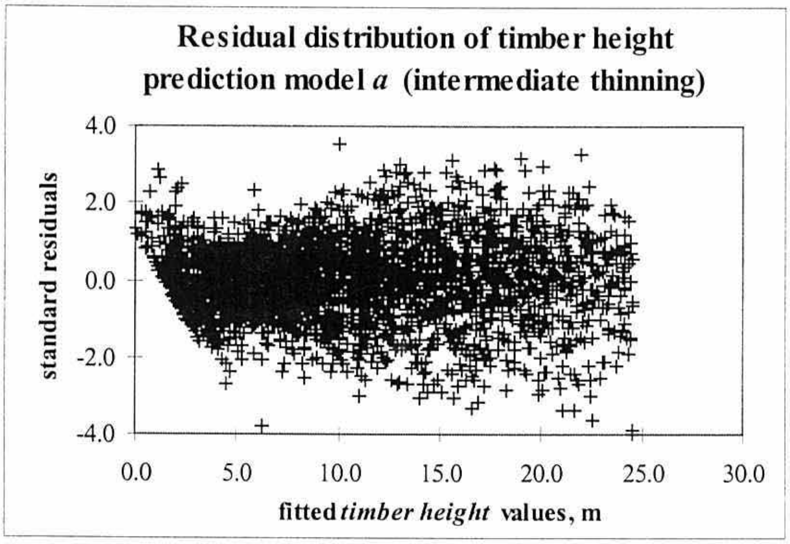
This model is:

$$h_{tim} = a_2 + b_2 * r_2^{\log(dbh*h)} \tag{4.76}$$

Parameter	Intermediate thinning		Neutral thinning	
	estimate	std. error	estimate	std. Error
a_2	-13.90800	0.61400	-33.65000	3.95000
b_2	18.48700	0.60700	38.80000	3.95000
r_2	2.11400	0.03980	1.44100	0.04850

4.3.4.3 Evaluation of the timber height prediction models

Standard residual distributions of the timber height models *a* and *b* are given in the Figure 4.13 and Appendix 2.4(iii) respectively. When the standard residual distributions were examined for both models constructed for the intermediate thinning type, a bias was indicated when the fitted values were lower than 5m. This is because it is impossible to have timber height below zero even if the total height is very low e.g. 2m. Such a distribution was not clearly highlighted in neutral thinning type (Figure 4.13). The distributions of the standard deviation of normal residuals with the fitted values indicated an increase with higher fitted values especially for the intermediate thinning type (Figure 4.14).



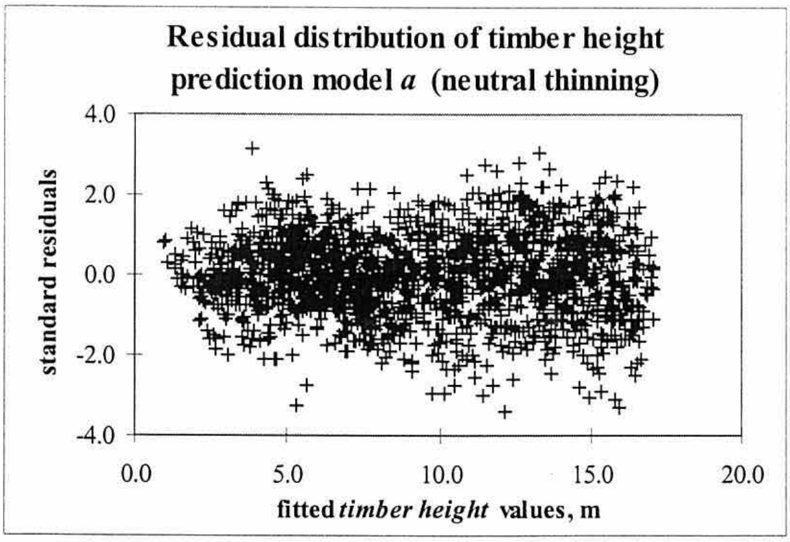


Figure 4.13: Standard residual distributions of timber height prediction model *a*.

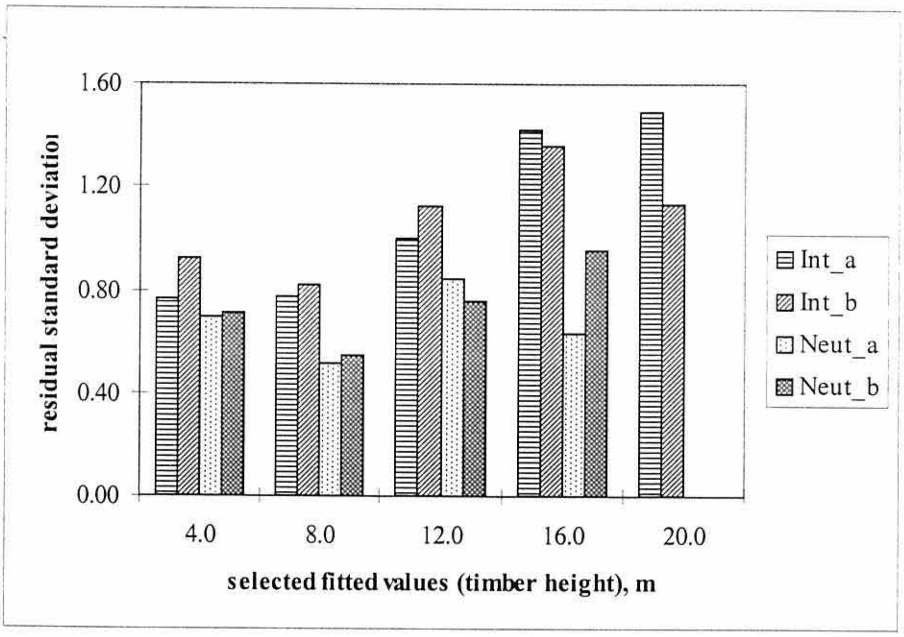


Figure 4.14: Distribution of standard deviation of normal residuals at selected points of fitted values.

However, the calculated values for average model bias and the mean absolute difference were negligible for all the timber height prediction models (Table 4.16). The lack of fit tests suggested that the model fit was adequate (Table 4.17).

Test	Intermediate thinning		Neutral thinning	
	Model *a*	Model *b*	Model *a*	Model *b*
Average model bias	0.0009	0.0019	0.0000	0.0000
Mean absolute difference	0.8513	0.8551	0.5836	0.5868
Modelling efficiency	0.9670	0.9660	0.9690	0.9690

Table 4.16: The results of the quantitative tests applied for the timber height prediction models.

Model	Intermediate thinning No. of data = 3247	Neutral thinning No. of data = 1839
a	0.88	0.79
b	0.97	0.81

None of the F-values were significant at 0.05 probability level.

Table 4.17: Results of the lack of fit tests of timber height prediction models.

When both models were applied to the reserved data for the validation (Figure 4.15), the residual distribution was higher for the intermediate thinning type than it was for the neutral thinning type. This may be due to the higher variation of timber height with total height and dbh observed in Figure 4.12 in the intermediate thinning type. As with diameter growth, the change in form factor and rate of taper in different parts of the bole of a tree depend upon the competition and site factors affection a tree at a particular age (Philip, 1994). In the data obtained for model construction and validation, the sample plots maintained under the intermediate thinning type covered a large range of GYCs and measurement periods, 10-22 and 1920-1995 respectively, while those for the neutral thinning type were 14-16 and 1951-1992. This could be the reason for such a distribution of timber height in the intermediate thinning type.

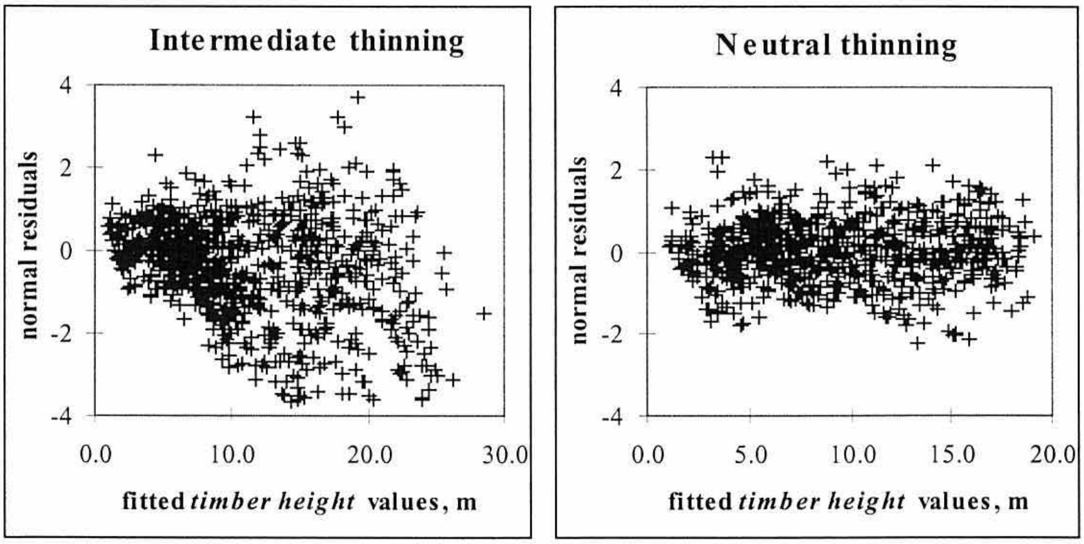


Figure 4.15: Distribution of the normal residuals after fitting the unchanged model *a* to the reserved data for validation.

4.3.5 Prediction of total volume of individual trees

The distributions of the tested explanatory variables with total volume for the intermediate thinning type are given in Figure 4.16.

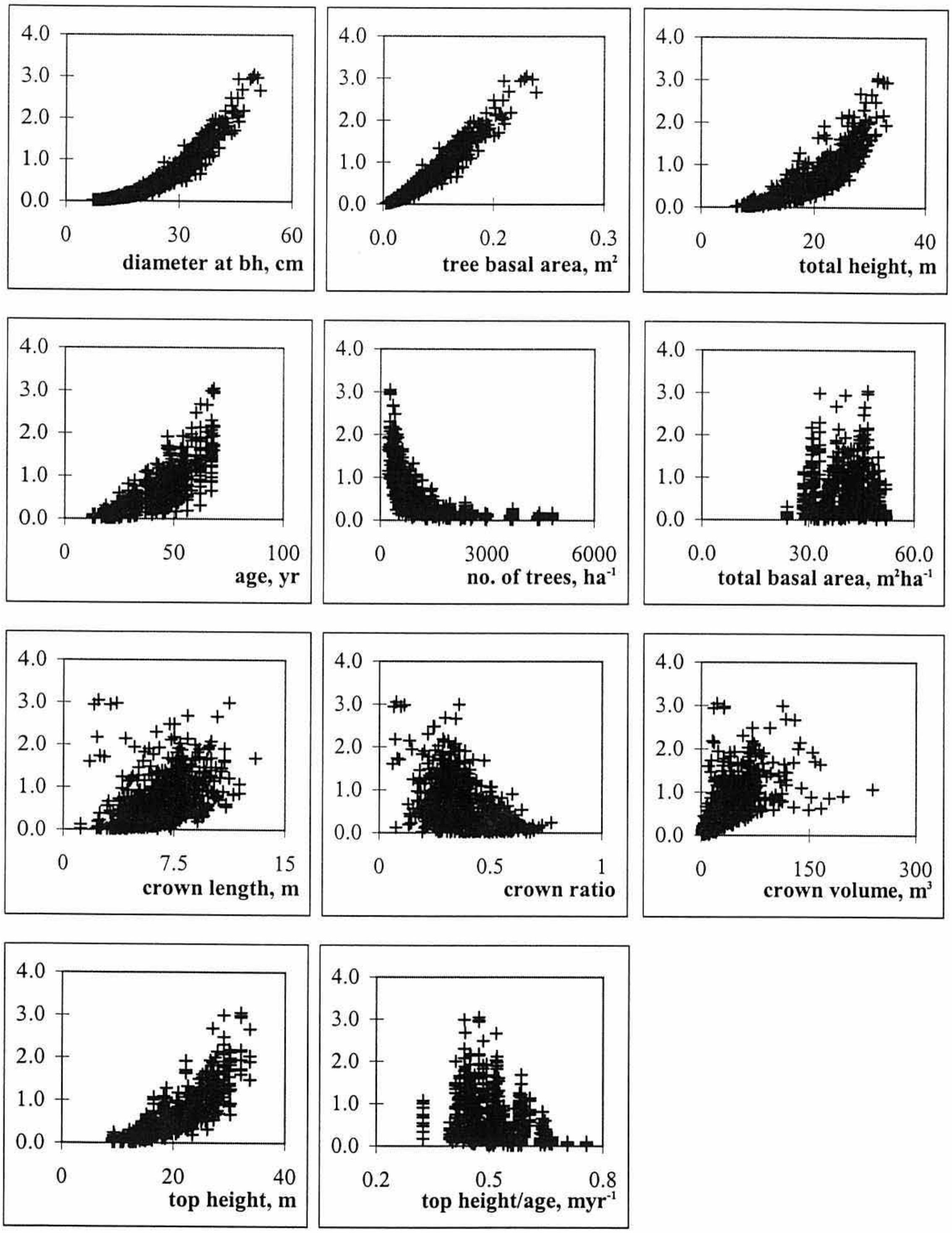


Figure 4.16: Distribution of tested explanatory variables with total volume (*Y*- axes) (intermediate thinning).

The descriptive statistics of the variables used for the modelling of the total tree volume and the correlations of the tested explanatory variables with the total volume are given in Appendices 2.2(iv) and 2.3(iv) respectively.

4.3.5.1 Developed models for total volume prediction

The relationships given in equations 4.7 and 4.11 (pages 74 and 75) were fitted separately to the data for each one-year age class. Model 4.11 was fitted to the data without changing the explanatory variables. Length of the crown, crown ratio and crown volume were not significant and when these variables were added, the standard residual distribution indicated no improvement to model 4.7. The variables h_{top} or $site_{top,age}$ were also not statistically significant when added to the above model. Neither did they improve the distribution of the standardised residuals. The total number of trees or total basal area per hectare which were added to represent the competition were similarly non-significant.

(i) Total volume prediction model *a*

The resultant volume prediction model from the relationships in equation 4.7 is:

$$v = b * \left(\frac{\pi * dbh^2}{40000} \right) * h \qquad 4.77$$

(ii) Total volume prediction model *b*

The linearised Shumacher-Hall model for individual tree volume prediction is:

$$\log v_{tot} = c_0 + c_1 \log(dbh) + c_2 \log(h) \qquad 4.78$$

Theoretically, in model 4.77, if one of the dbh or total tree height values = 0, the other variable has no value either. Then, the total volume → 0, and therefore the intercept should not be statistically significant. Model 4.78 is the linear transformation of model 4.10 (page 75). Therefore the parameter c_0 in model 4.78 equals parameter b_0 in model 4.11 (*a* in model 4.10 is equivalent to *log a* in the linear form which is c_0 in model 4.78). Therefore the theory applied to model 4.77 can be justified for model 4.78 as well.

4.3.5.2 Evaluation of the total volume prediction models

When the total volume prediction model *a* was fitted to the data at each age, the coefficient of determination (R^2) was between 0.972-0.999 for both thinning types. That value varied between 0.891-0.996 for volume prediction model *b*. When the standard residuals were checked, they were distributed without showing any particular pattern at each age (Figure: 4.17). For convenience only the data at age 25 were given in Figure 4.17. The values estimated for the quantitative tests for all ages indicated negligible bias and very high modelling efficiency (Table 4.18). The calculated F-values used to observe the lack of fit (Table 4.19) indicated the models developed for all the ages were adequate. The ages listed in Tables 4.18 and 4.19 were the common ages for both intermediate and neutral thinning types.

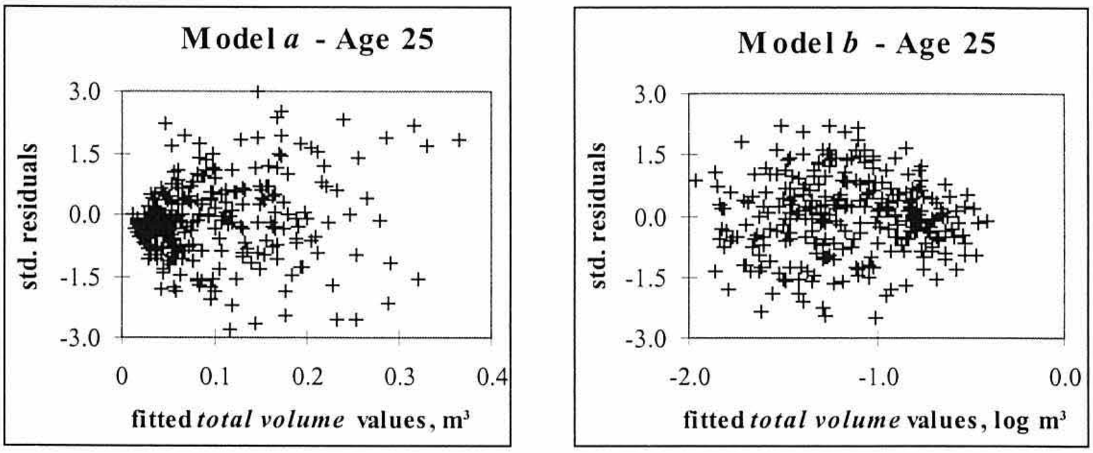


Figure 4.17a: Standard residuals for the intermediate thinning type at age 25 years.

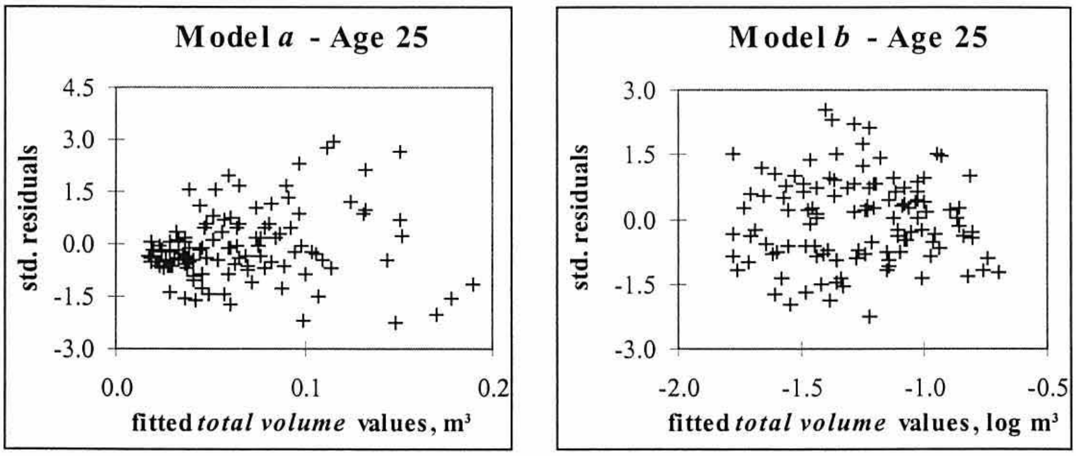


Figure 4.17b: Standard residuals for the neutral thinning type at age 25 years.

Age	Test	Intermediate thinning		Neutral thinning	
		Model *a*	Model *b*	Model *a*	Model *b*
19	Average model bias	-0.0003	-0.0013	-0.0001	0.0005
	Mean absolute difference	0.0035	0.0037	0.0030	0.0030
	Modelling efficiency	0.9920	0.9920	0.9840	0.9840
24	Average model bias	0.0004	0.0024	0.0002	-0.0008
	Mean absolute difference	0.0061	0.0063	0.0044	0.0045
	Modelling efficiency	0.9900	0.9890	0.9890	0.9880
25	Average model bias	0.0008	0.0027	0.0005	-0.0007
	Mean absolute difference	0.0061	0.0064	0.0036	0.0036
	Modelling efficiency	0.9870	0.9860	0.9860	0.9850
26	Average model bias	0.0000	0.0004	0.0000	-0.0036
	Mean absolute difference	0.0042	0.0042	0.0024	0.0038
	Modelling efficiency	0.9970	0.9930	0.9970	0.9930
28	Average model bias	0.0000	0.0050	0.0015	-0.0043
	Mean absolute difference	0.0062	0.0068	0.0023	0.0046
	Modelling efficiency	0.9810	0.9730	0.9990	0.9930
31	Average model bias	-0.0012	0.0049	0.0011	0.0029
	Mean absolute difference	0.1470	0.0150	0.0091	0.0093
	Modelling efficiency	0.9780	0.9780	0.9850	0.9850
36	Average model bias	0.0013	-0.0010	0.0006	0.0112
	Mean absolute difference	0.0042	0.0048	0.0136	0.0158
	Modelling efficiency	0.9750	0.9680	0.9840	0.9770
37	Average model bias	0.0012	-0.0165	0.0001	0.0026
	Mean absolute difference	0.0139	0.2010	0.0165	0.0166
	Modelling efficiency	0.9940	0.9840	0.9860	0.9860
41	Average model bias	-0.0049	-0.0062	-0.0007	0.0196
	Mean absolute difference	0.0211	0.0211	0.0167	0.0230
	Modelling efficiency	0.9630	0.9620	0.9900	0.9800

Table 4.18: Results of the quantitative tests of the total volume prediction models.

Age	Intermediate thinning		Neutral thinning	
	Model *a*	Model *b*	Model *a*	Model *b*
19	0.87	0.73	1.14	0.99
24	0.81	0.58	0.75	1.39
25	0.77	0.63	1.23	0.62
26	1.47	0.99	2.69	0.10
28	1.15	0.90	2.24	0.57
31	1.57	1.80	1.27	1.20
36	1.18	0.23	1.14	1.19
37	2.98	0.52	1.12	1.49
41	1.16	0.95	4.44	1.34

None of the F-values were significant at 0.05 probability level.

Table 4.19: Calculated F-values for the lack of fit tests for total volume prediction models.

The results obtained from the quantitative tests and lack of fit test were confirmed by the validation with the reserved data (Figures 4. 18a and b) proving both the models are adequate for predicting the total volume of individual trees.

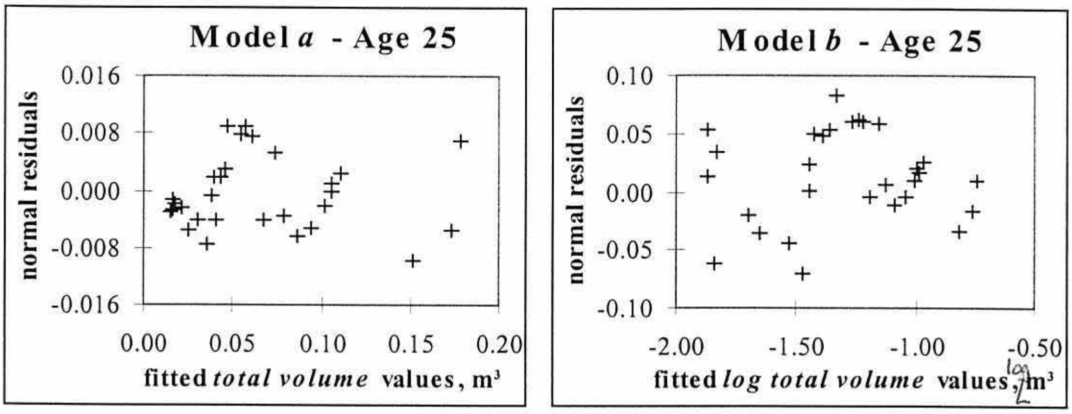


Figure 4.18a: Residual distributions after fitting unchanged volume prediction models to reserved data at age 25 (intermediate thinning type).

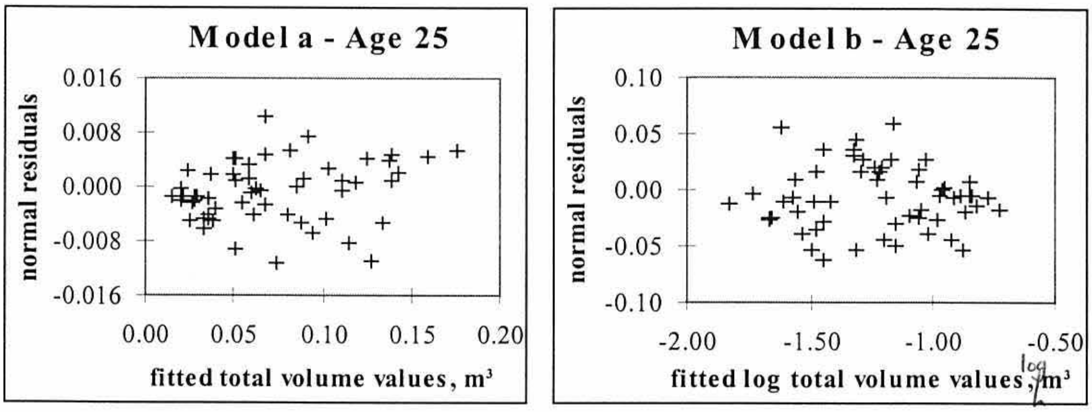


Figure 4.18b: Residual distributions after fitting unchanged volume prediction models to reserved data at age 25 (neutral thinning type).

The reason for trying to construct parameter prediction models was to reduce the complexity of using specific parameters for each age. This work did not succeed because all the estimated parameters for total volume prediction models were distributed with age without any pattern (Figure 4.19). Therefore no regression relationships could be established. It is extremely difficult to use specific parameters for each age in the field and therefore, the possibility of using one set of parameter for all ages will be tested in Chapter 6. The connecting lines of the parameters estimated for the neutral thinning type was interrupted because of the lack of continuous measurements.

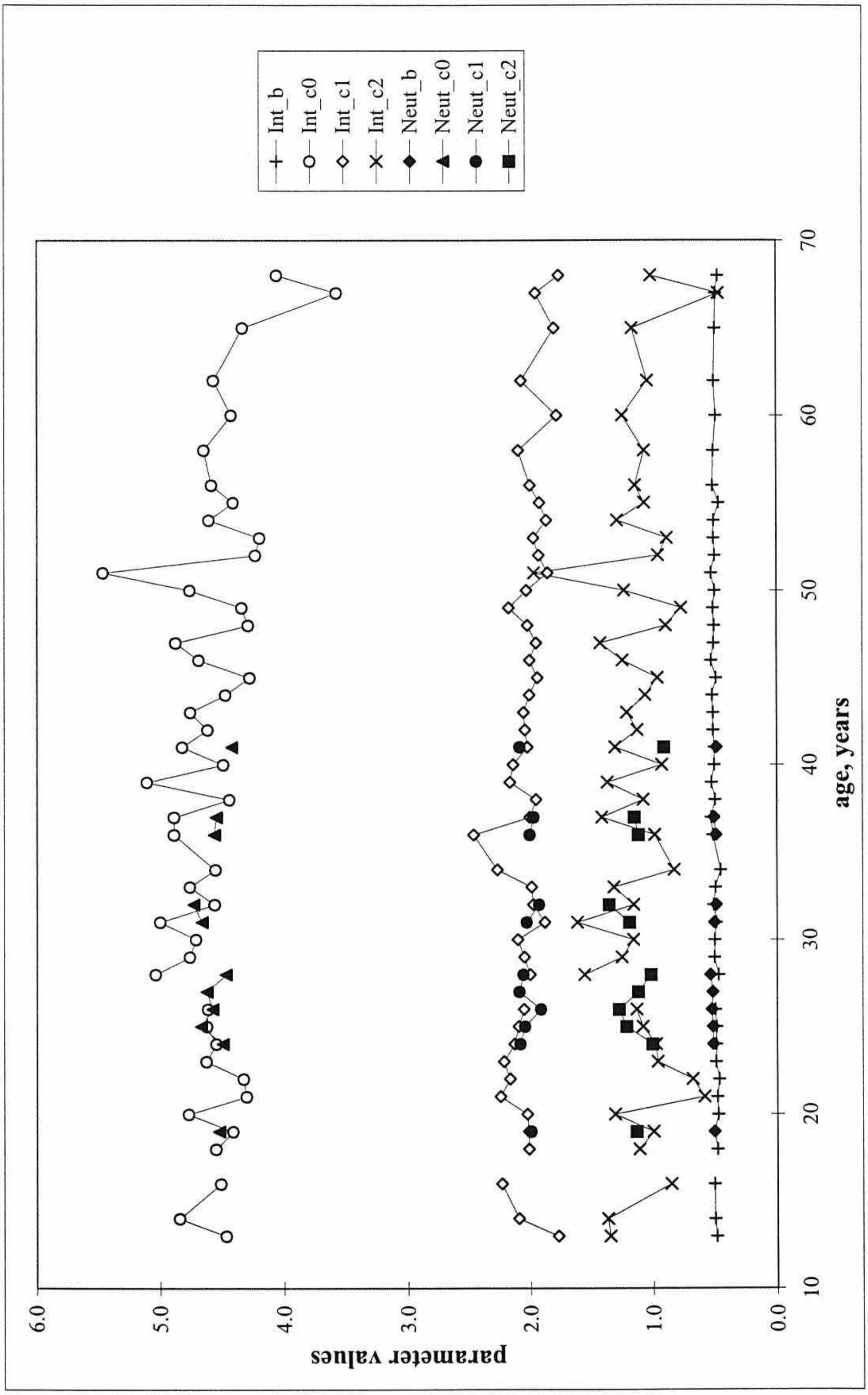


Figure 4.19: Distributions of parameters of the selected models for total volume prediction with age.

4.3.6 Prediction of merchantable volume

The distributions of the tested explanatory variables with merchantable volume for the intermediate thinning type (Y-axes) are given in Figure 4.20. Descriptive statistics and the correlation of the variables used are shown in Appendices 2.2(v) and 2.3(v) respectively.

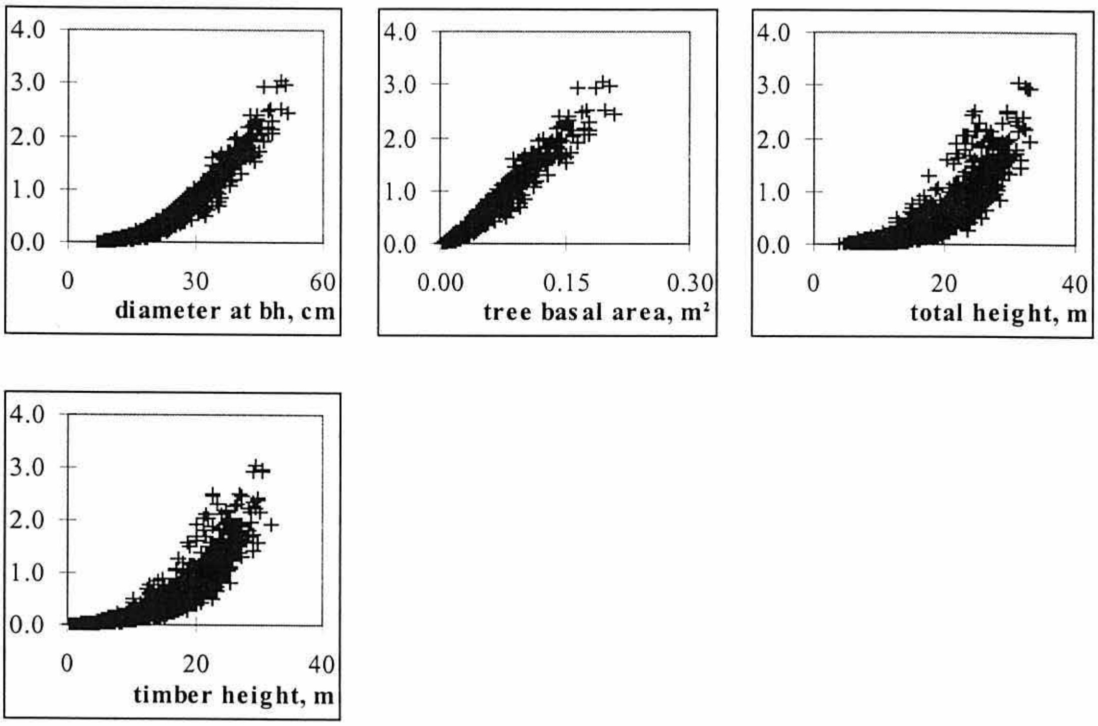


Figure 4.20: Distribution of tested explanatory variables with merchantable volume.

4.3.6.1 Developed models to predict merchantable volume

The relationships outlined in equations 4.15 and 4.16 (pages 76 and 77) were fitted separately to the data at each one year age class.

(i) Merchantable volume prediction model *a*

This is defined as:

$$v_{mer} = b * \left\{ h_{tim} \left(\frac{dbh^2}{10000} + \frac{49.0}{10000} \right) \right\} \qquad 4.79\ (4.15)$$

(ii) Merchantable volume prediction model *b*

This is defined as:

$$v_{mer} = c_0 + c_1 * \left\{ (g * h) - \left(\frac{\pi * 49.0}{40000} * \left(\frac{h - h_{tim}}{3} \right) \right) \right\} \qquad 4.80\ (4.16)$$

4.3.6.2 Evaluation of the merchantable volume prediction models

The R^2 value for the merchantable volume prediction model *a* varied between 0.977-0.998 for both thinning types. The range of R^2 for the model *b* varied between 0.970-0.995. The standard residual distributions for all ages indicated normal distributions (residual distributions at age 25 years - Figures 4.21a and b). The average model bias and mean absolute difference of the models developed for all ages (Table 4.20) were very low allowing the modelling efficiency to be over 0.95. There was no lack of fit (Table 4.21).

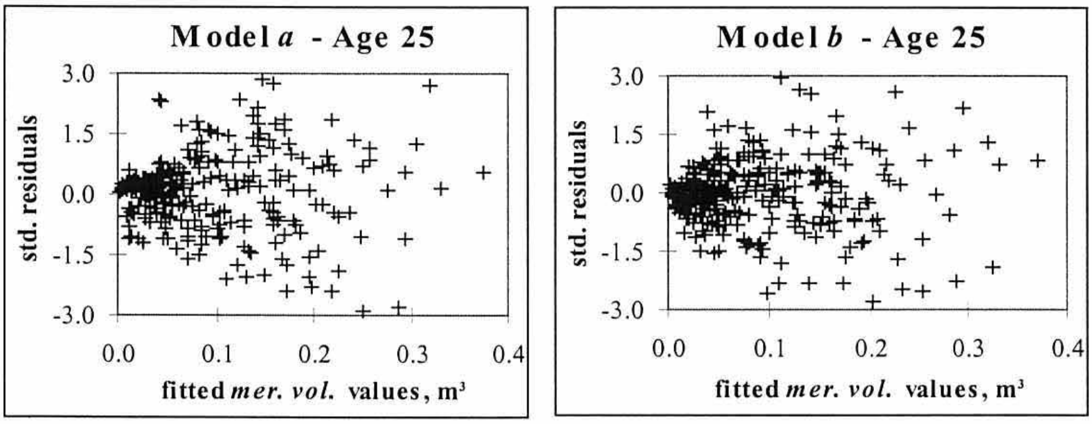


Figure 4.21a: Standard residuals for the intermediate thinning type at age 25 years.

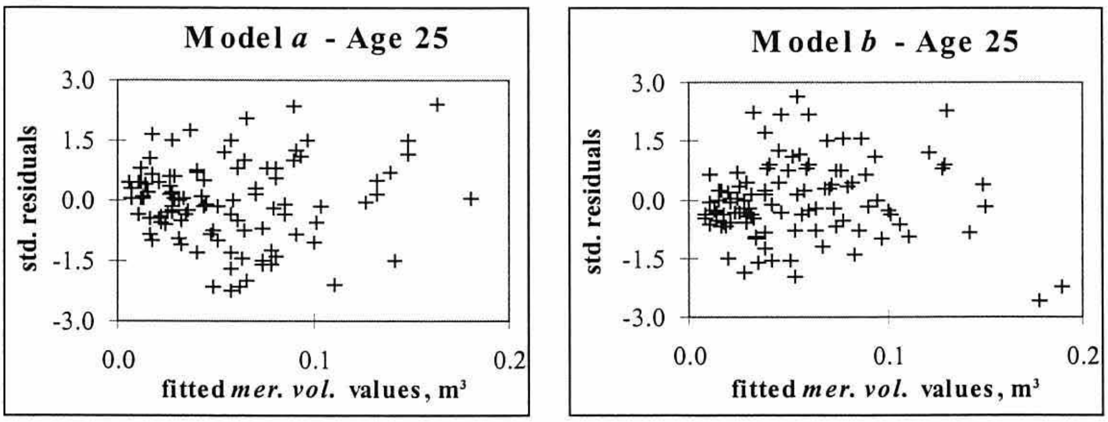


Figure 4.21b: Standard residuals for the neutral thinning type at age 25 years.

Age	Test	Intermediate thinning		Neutral thinning	
		Model *a*	Model *b*	Model *a*	Model *b*
19	Average model bias	-0.0004	0.0004	0.0000	0.0001
	Mean absolute difference	0.0026	0.0032	0.0019	0.0033
	Modelling efficiency	0.9920	0.9870	0.9930	0.9810
24	Average model bias	-0.0008	0.0000	0.0003	0.0000
	Mean absolute difference	0.0071	0.0067	0.0036	0.0051
	Modelling efficiency	0.9870	0.9880	0.9920	0.9860
25	Average model bias	-0.0005	0.0003	0.0001	0.0000
	Mean absolute difference	0.0050	0.0063	0.0012	0.0033
	Modelling efficiency	0.9910	0.9860	0.9980	0.9880
26	Average model bias	-0.0003	0.0001	0.0005	0.0000
	Mean absolute difference	0.0024	0.0048	0.0025	0.0027
	Modelling efficiency	0.9970	0.9880	0.9960	0.9950
28	Average model bias	-0.0002	0.0000	0.0008	0.0000
	Mean absolute difference	0.0026	0.0057	0.0019	0.0014
	Modelling efficiency	0.9960	0.9800	0.9990	0.9990
31	Average model bias	-0.0003	0.0000	0.0009	-0.0003
	Mean absolute difference	0.0106	0.0141	0.0063	0.0088
	Modelling efficiency	0.9870	0.9790	0.9920	0.9860
36	Average model bias	-0.0001	0.0000	0.0007	0.0000
	Mean absolute difference	0.0017	0.0029	0.0121	0.0145
	Modelling efficiency	0.9960	0.9840	0.9840	0.9790
37	Average model bias	0.0009	0.0000	0.0008	0.0000
	Mean absolute difference	0.0147	0.0187	0.0148	0.0189
	Modelling efficiency	0.9970	0.9870	0.9890	0.9890
41	Average model bias	0.0021	0.0000	0.0010	0.0000
	Mean absolute difference	0.0152	0.0218	0.0114	0.0179
	Modelling efficiency	0.9830	0.9610	0.9910	0.9780

Table 4.20: Results of the quantitative tests of merchantable volume prediction models.

Age	Intermediate thinning		Neutral thinning	
	Model *a*	Model *b*	Model *a*	Model *b*
19	1.59	0.99	0.49	1.10
24	1.55	1.11	0.38	1.45
25	1.28	1.16	0.48	0.95
26	0.95	1.18	4.34	2.63
28	0.82	1.72	5.40	1.12
31	1.25	2.20	1.25	0.82
36	0.73	1.13	1.07	0.81
37	0.99	1.49	1.34	1.18
41	9.80	4.66	1.33	1.42

None of the F-values were significant at 0.05 probability level.

Table 4.21: Calculated F-values for the lack of fit tests for merchantable volume prediction models.

The residuals obtained after fitting the models with unchanged parameters to the data reserved for validation indicated a normal distribution. An example is given in Figures 4.22a and b for age 25.

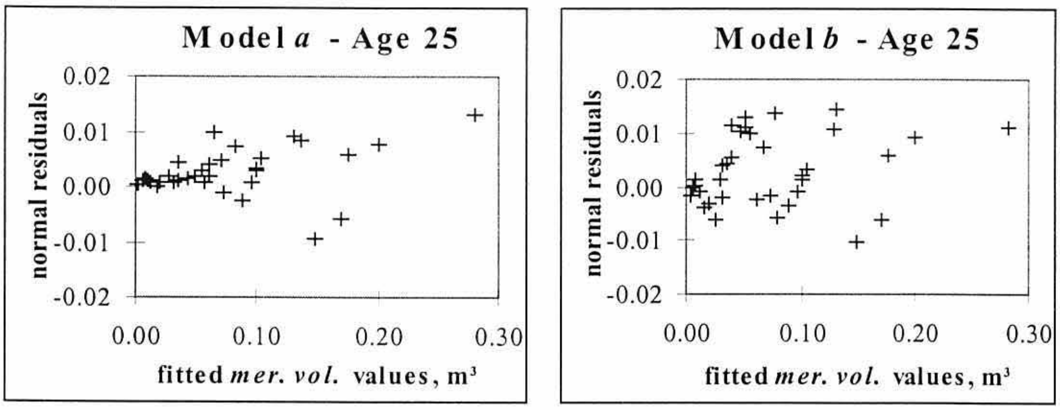


Figure 4.22a: Residual distributions after fitting unchanged volume prediction models to reserved data at age 25 (intermediate thinning type).

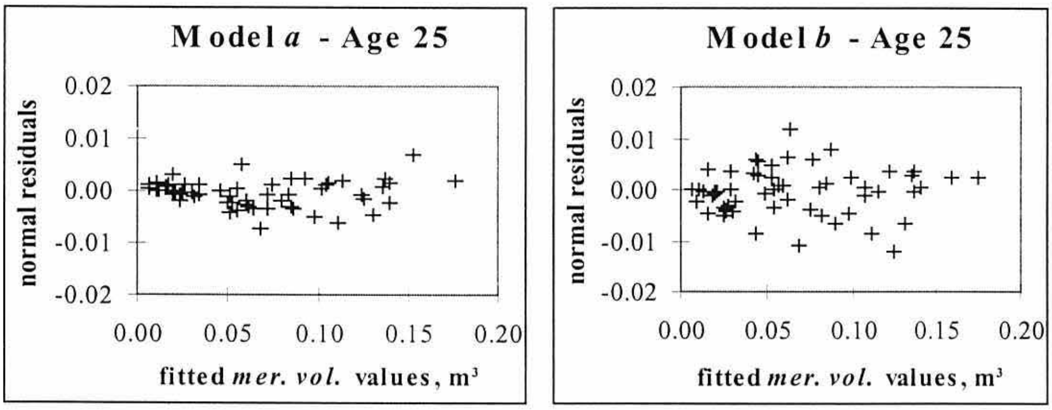


Figure 4.22b: Residual distributions after fitting unchanged volume prediction models to reserved data at age 25 (neutral thinning type).

As in the total volume prediction model, parameter prediction models could not be developed with age because of the lack of any obvious relationship with age (Figure 4.23). In Chapter 6, the possibility of using one set of parameters for all ages will be discussed in order to reduce the complexity.

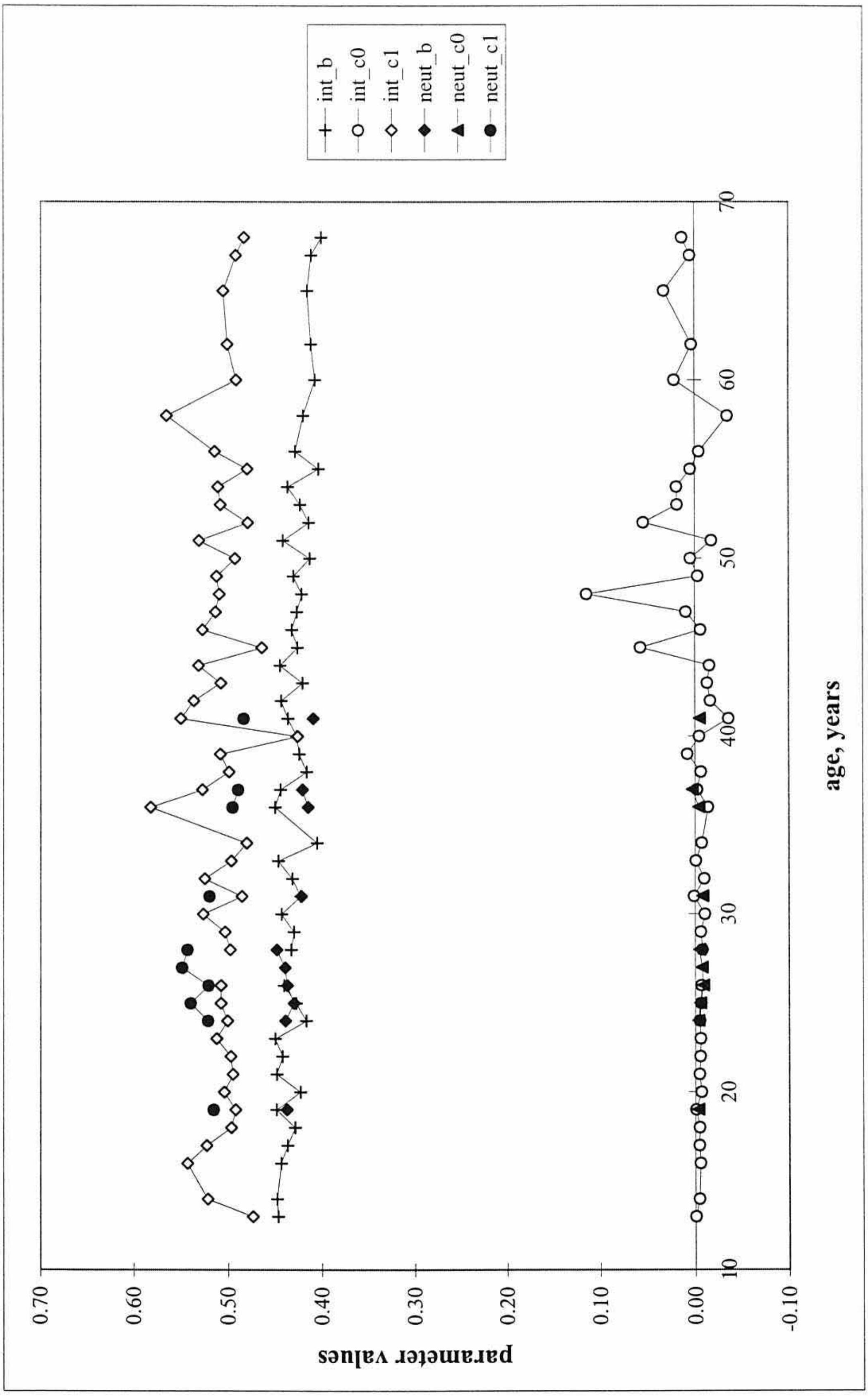


Figure 4.23: Distribution of the estimated parameters of merchantable volume prediction models with age.

4.3.7 Prediction of thinning tree variables

The relationships between response and explanatory variables were very scattered for each model when data for all the neutral thinning intensities were used (Figure 4.24b). Therefore to reduce the bias, only data from an documented intensity equal to or lower than the 300% of marginal thinning intensity were used (Figure 4.24c). However, such a point was not necessary for the intermediate thinning type because the thinning intensity was not as high (Figure 4.24a) as it was for the neutral thinning type. The descriptive statistics and the correlations of the selected variables are shown in Appendices 2.2(vi) and 2.3(vi) respectively.

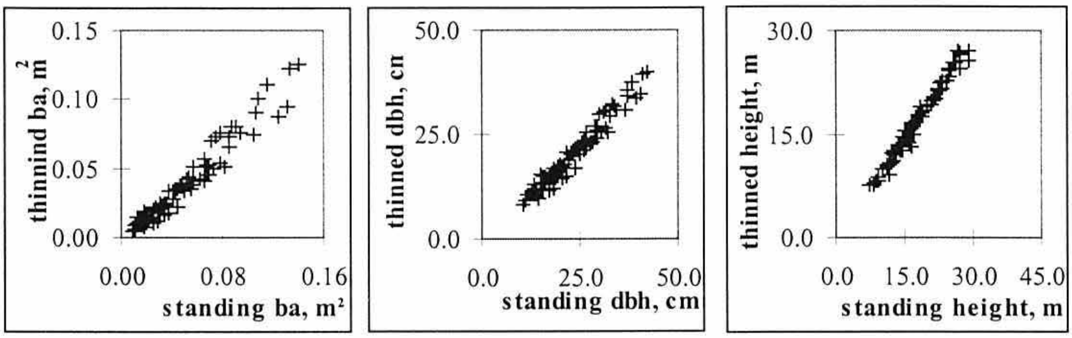


Figure 4.24a: Distributions of the selected variables for intermediate thinning.

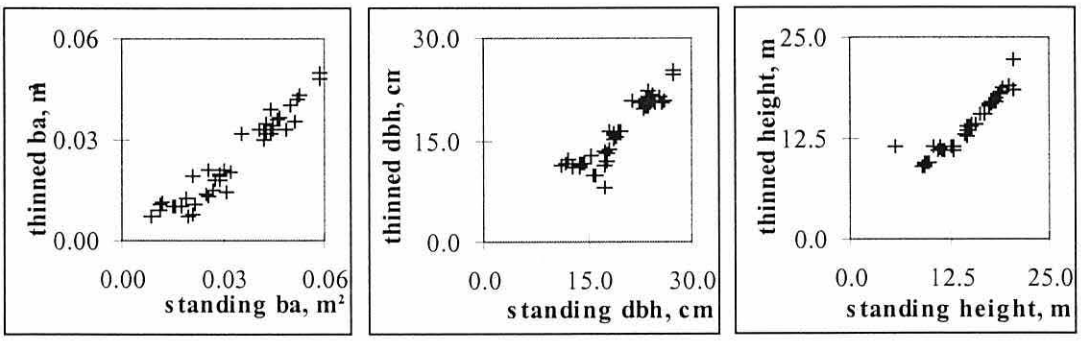


Figure 4.24b: Distributions of the selected variables for the neutral thinning type.

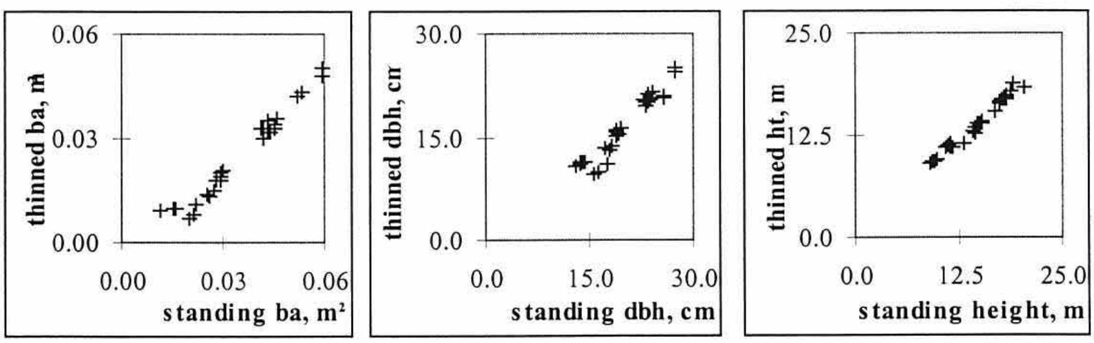


Figure 4.24c: Distributions of variables in neutral thinning after removing the sample plots which had thinning intensity over 300% of marginal intensity.

4.3.7.1 Models for the prediction of thinning variables

Although the initial intention was to develop a linear relationship as described in equations, 4.18, 4.19, and 4.20 (pages 78 and 79) a logistic type curve was also tried as the second model for each variable because the trend of data was believed suitable for such a curve after observing the distributions in Figure 4.24. All the models constructed to predict the mean tree variables removed in thinning are given below:

(i) Basal area prediction

Basal area prediction model a

$$\bar{g}_{th} = a_1 + b * \bar{g}_{bt} \tag{4.81}$$

Parameter	Intermediate thinning			Neutral thinning		
	R^2	estimate	se	R^2	estimate	se
	0.949			0.964		
a_1		-0.0043	0.0012		-0.0071	0.0012
b		0.8614	0.0205		0.9249	0.0334

Basal area prediction model b

$$\bar{g}_{th} = a_2 + c_1 /(1 + \exp(-c_2 * (\bar{g}_{bt} - c_3))) \tag{4.82}$$

Parameter	Intermediate thinning			Neutral thinning		
	R^2	estimate	se	R^2	estimate	se
	0.951			0.979		
a_2		-0.0292	0.0230		0.0031	0.0031
c_1		0.1918	0.0588		0.0564	0.0089
c_2		20.0710	7.0441		81.7000	16.4210
c_3		0.0821	0.0107		0.0418	0.0023

(ii) Diameter prediction

Dbh prediction model a

$$\overline{dbh}_{th} = a_1 + b * \overline{dbh}_{bt} \tag{4.83}$$

Parameter	Intermediate thinning			Neutral thinning		
	R^2	estimate	se	R^2	estimate	se
	0.956			0.929		
a_1		-2.0594	0.5170		-4.0922	1.0900
b		0.9576	0.0212		1.0270	0.0528

Dbh prediction model b

$$\overline{dbh}_{th} = a_2 + c_1/(1+\exp(-c_2 * (\overline{dbh}_{bt} - c_3))) \qquad 4.84$$

Parameter	Intermediate thinning			Neutral thinning		
	R^2	estimate	se	R^2	estimate	se
	0.958			0.944		
a_2		-0.4321	6.6782		9.5240	1.1111
c_1		54.4420	18.1321		15.1723	2.4152
c_2		0.0774	0.0281		0.3580	0.0885
c_3		30.0801	3.2000		20.7272	0.6668

(iii) Total height prediction

Total height prediction model a

$$\overline{h}_{th} = a_1 + b * \overline{h}_{bt} \qquad 4.85$$

Parameter	Intermediate thinning			Neutral thinning		
	R^2	estimate	se	R^2	estimate	se
	0.985			0.987		
a_1		-0.3479	0.2158		1.0274	0.2053
b		0.9694	0.0122		0.8789	0.0143

Total height prediction model b

$$\overline{h}_{th} = a_2 + c_1/(1+\exp(-c_2 * (\overline{h}_{bt} - c_3))) \qquad 4.86$$

Parameter	Intermediate thinning			Neutral thinning		
	R^2	estimate	se	R^2	estimate	se
	0.986			0.988		
a_2		-1.2514	3.4564		3.9300	3.4687
c_1		36.5667	6.7333		22.3456	9.5411
c_2		0.1130	0.0235		0.1697	0.0763
c_3		17.8936	0.7620		16.0641	1.6944

4.3.7.2 Evaluation of the thinning prediction models

In an intermediate thinning the suppressed and dead trees together with competing sub-dominant and dominant trees are removed (Edwards and Christie, 1981). Therefore, the parameter associated with the explanatory variable (standing mean tree size) in linear models, should be lower than one. This condition was fulfilled by the regression analysis of the models built. In neutral thinnings, the trees are selected systematically, which means the size of the trees removed in thinning should be similar to the size of the main crop trees just before thinning. The corresponding parameter should therefore be equal to one. The only slope parameter which was not significantly different from unity was that associated with the dbh prediction model at 0.05 probability level. The same parameter in the basal area prediction model was significantly different from one at 0.05 probability level but, not significant at level 0.1. The reason for the statistical significance in the height model might be the removal of very large number of trees as thinnings which means more suppressed trees were removed thus reducing the slope parameter.

According to the basic theory, if the size of a tree variable is equal to zero, the size of the same variable removed in immediate thinning should not have any value. This was not proved by the linear models and the intercepts were significantly different from zero except in the mean total tree height prediction model of intermediate thinning. The reason for this could be the removal of a large number of trees in the first thinning without considering the documented type of thinning in order to obtain a commercial profit (Jenkins, pers. comm.). Therefore, a valid range for all the models built for the prediction of thinning tree variables is recommended which is after the first thinning until 50 years of plantation age for both thinning types.

The standard residual distribution of linear models constructed for basal area for both thinning types and for dbh for neutral thinning indicated bias (Figure 4.25a and b). The residual distribution of the non-linear models (Appendix 2.4(iv)) for all the variables did not indicate this situation.

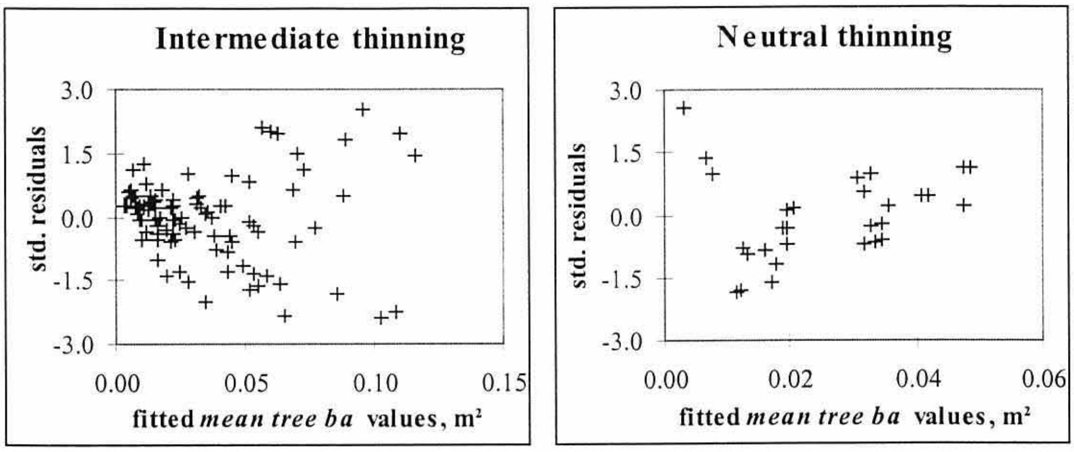


Figure 4.25a: Residuals after fitting the linear model to the data for the prediction of mean basal area of thinned trees.

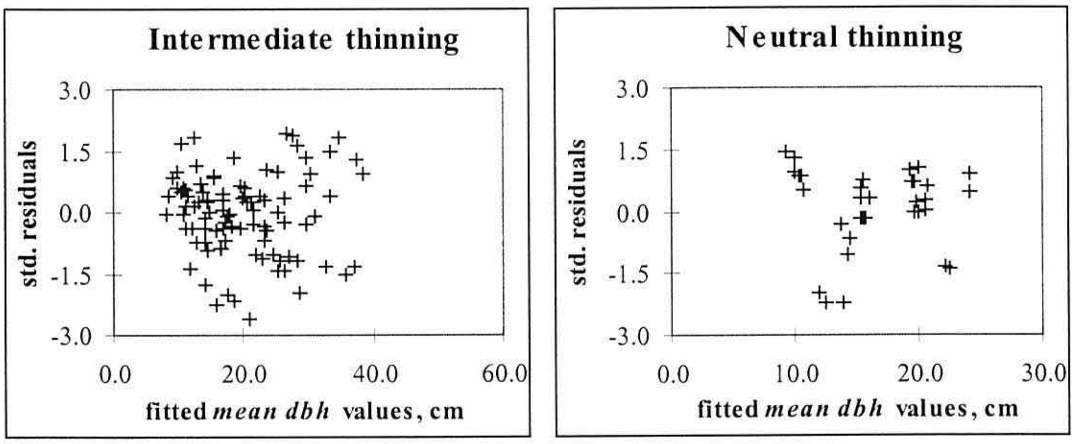


Figure 4.25b: Residuals after fitting the linear model to the data for the prediction of mean diameter at breast height of thinned trees.

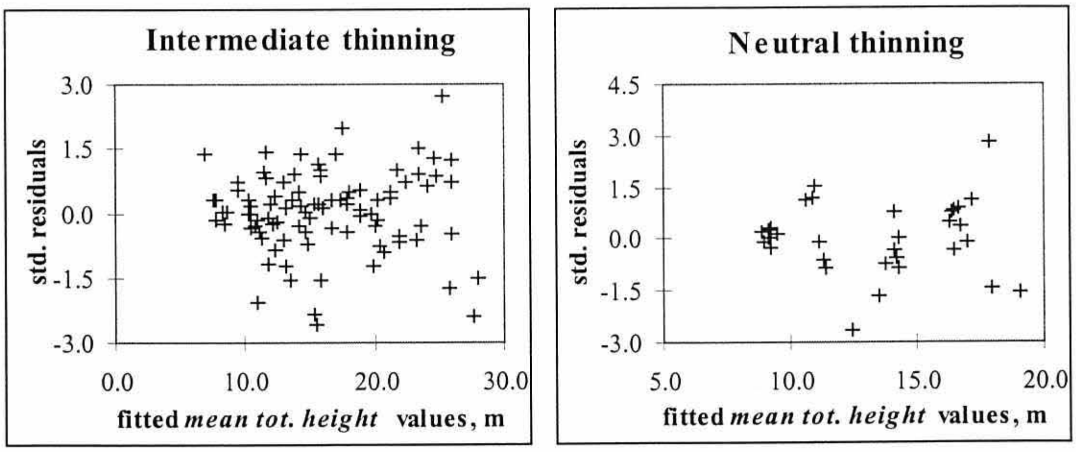


Figure 4.25c: Residuals after fitting the linear model to the data for the prediction of mean height of thinned trees.

Average model bias for all the models was very low. However, the mean absolute differences resultant for the dbh models for both thinning types were relatively high (Table 4.22). Modelling efficiency figures suggest that the accuracy is very high. The results of the lack of fit tests (Table: 4.23) revealed that the fitting procedures for all the models were adequate.

Variable	Test	Intermediate thinning.		Neutral thinning.	
		model *a*	Model *b*	model *a*	model *b*
Basal area	average model bias	0.0000	0.0000	0.0013	0.0013
	mean absolute difference	0.0051	0.0042	0.0022	0.0019
	mod. efficiency	0.9540	0.9550	0.9410	0.9500
diameter at bh	average model bias	-0.0022	-0.0048	-0.0011	-0.0034
	mean absolute difference	1.2753	1.2356	0.9408	0.8279
	mod. efficiency	0.9630	0.9610	0.9330	0.9540
total height	average model bias	0.0000	0.0023	0.0034	0.0043
	mean absolute difference	0.4666	0.4666	0.2712	0.2621
	mod. efficiency	0.9930	0.9920	0.9370	0.9920

Table 4.22: Results of the quantitative tests applied for the thinning prediction models.

Variable	Model	Intermediate thinning	Neutral thinning
basal area	*a*	1.59	1.50
	b	1.52	1.35
diameter at bh	*a*	0.58	1.76
	b	0.54	1.34
total height	*a*	1.70	1.17
	b	1.57	1.08

None of the F-values were significant at 0.05 probability level.

Table 4.23: Results of the lack of fit tests (F-values) for thinning prediction models.

The distributions of the residuals obtained after fitting unchanged linear and non-linear models were normal. However, in this chapter, the lines resulted after fitting the models were drawn on the raw data for validation (Figures 4.26 and 4.27) for an easy comparison. The mean basal area and dbh prediction models indicated little over estimation with the higher fitted values, but more data are needed for a proper conclusion.

Even though the quantitative tests indicated very similar results for linear and non-linear models, the standard residuals indicated bias for the neutral thinning type. For this reason and secondly to obtain the basic model structure for all three variables, non-linear regression models were selected to use in the field.

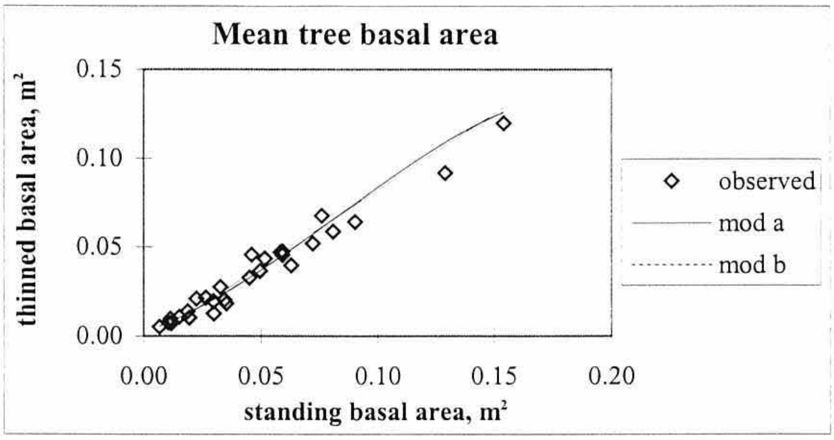


Figure 4.26a: The results after drawing the model predictions for mean basal area on raw data (intermediate thinning type).

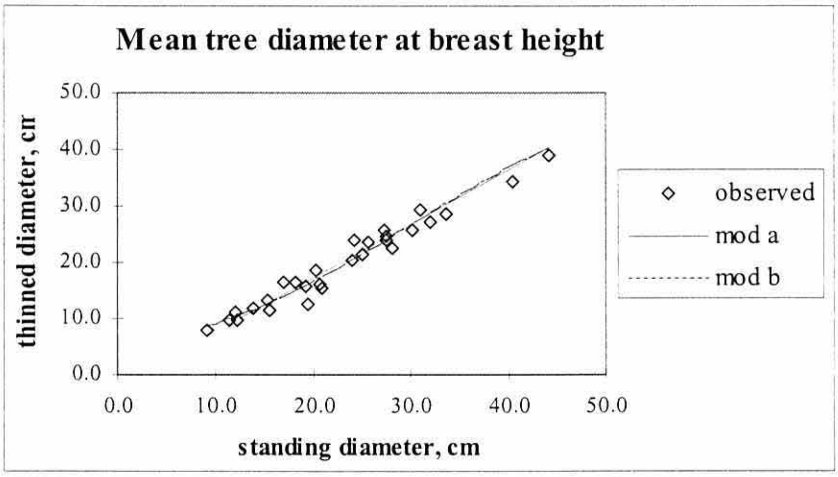


Figure 4.26b: The results after drawing the model predictions for mean dbh on raw data (intermediate thinning type).

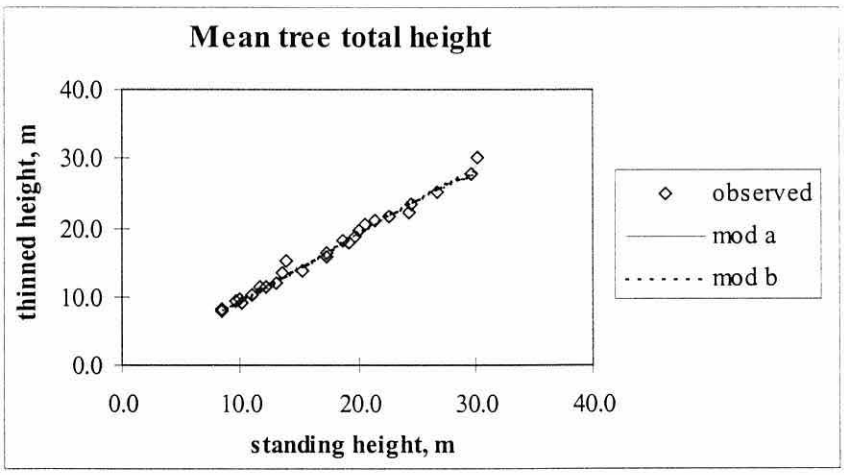


Figure 4.26c: The results after drawing the model predictions for mean total height on raw data (intermediate thinning type).

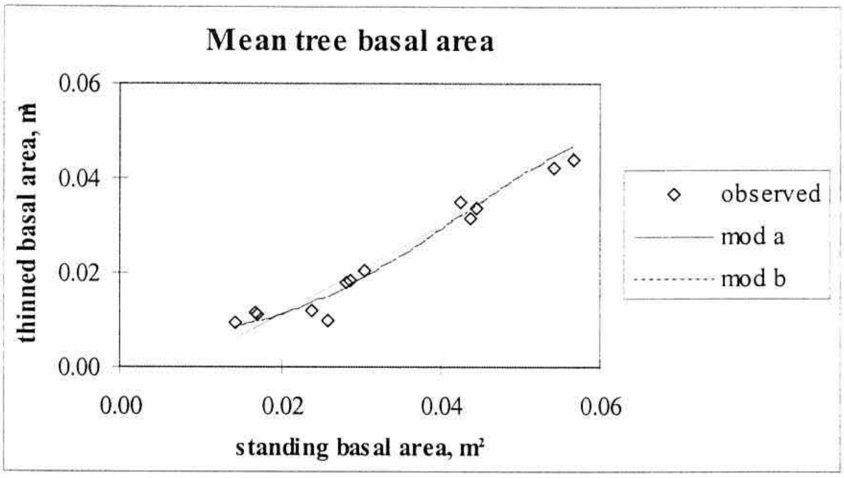


Figure 4.27a: The results after drawing the model predictions for mean basal area on raw data (neutral thinning type).

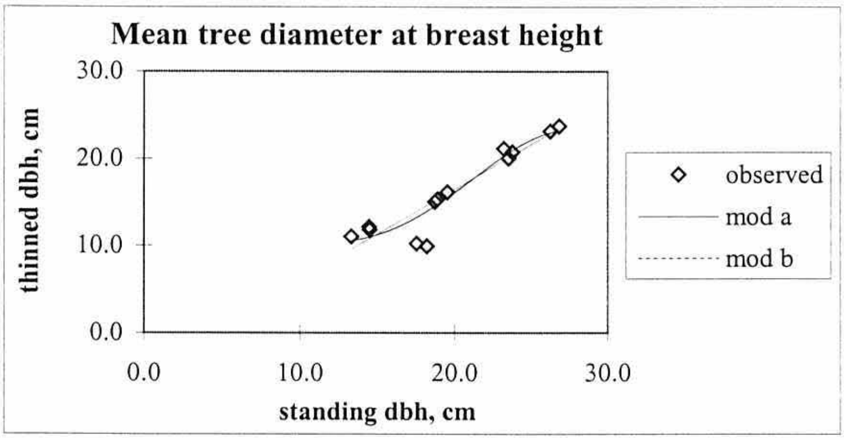


Figure 4.27b: The results after drawing the model predictions for mean dbh on raw data (neutral thinning type).

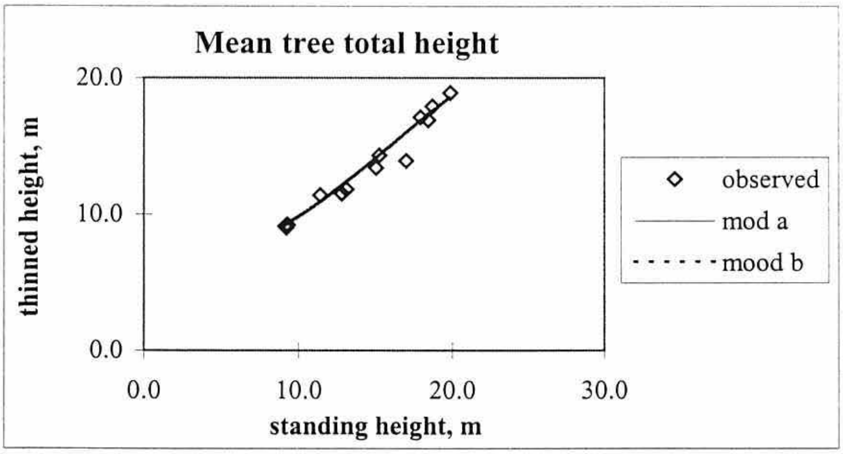


Figure 4.27c: The results after drawing the model predictions for mean total height on raw data (neutral thinning type).

4.4 Discussion

4.4.1 Quantity of data used for model construction

All the models constructed in this chapter had enough data. However, for the construction of some models like dbh in this study a large quantity of data was used (9477 trees from 27 sample plots). This is not an unusual procedure. For a development of site index equations, Elfving and Kiviste (1997) used 156 sample plots and Hasenaur and Monserud (1996) used 5090 plots containing 42479 data items for growth modelling. In 1988 Nystrom and Gemmel collected data on 799 sample plots for model construction and Ritchie and Hann (1997) used data from 105 Douglas fir (*Pseudotsuga menziesii* (Mirb.) Franco) plantations for the evaluation of individual tree disaggregative prediction methods.

4.4.2 Parameter estimation

The most common procedure for estimating parameters is to use only the non-overlapping growth intervals. There are fewer problems with serial correlation of real growth series - derived from either re-measured plots or trees - when the data are arranged in non-overlapping growth intervals (e.g. 5-10, 15-20 etc.) rather than all possible intervals (Borders *et al.*, 1988). Therefore non-overlapping growth intervals were used for the current work.

4.4.3 Variables not included in constructed models

Competition is a major factor determining the size of individual trees and the number of plants in the population (Kimmins, 1997). The change of growth due to competition and the environment is strongly related to plant size (Tang *et al.*, 1997). In a forest stand, there is a definite although not high correlation between variations in stand density and tree parameters (Pukkala, 1994). However, in the present study, total number of trees and total basal area per hectare which were initially included in dbh, total height and total volume prediction models were not statistically significant. All the crown dimensions used were also not statistically significant in the total volume prediction models. When correlations were tested with the response variables, they were relatively high for total trees per hectare but always below 0.2 for total basal area per hectare (Appendix 2.3). The correlations for crown dimensions were not as low as those for basal area although not as high for total tree number (Appendix 2.3). This non-significance might be as an effect of the multi-colinearity which occurred by using many explanatory variables for the volume prediction models. The above values were significant with the correct sign in some models, with the result that some of the more important explanatory variables became non-significant. Monserud and Sterba (1996) wrote that the growth of some tree species more sensitive to the crown ratio e.g. spruce, fir, and Scots pine than many other species. There were some factors such as dbh and total height which were essential for the models constructed in this study. These variables always took precedence over non-essential variables such as crown dimensions and total tree number. Therefore when an essential factor became statistically insignificant due to the addition of a non-essential factor to the model structure, that combination was removed from further studies.

4.4.4 Model predictions

It is very difficult to measure accurately the form factor directly. Therefore in the total volume prediction model *a*, an attempt was made to find the right combination of variables to replace the form factor in order to predict the total volume of individual trees. However, the estimated parameter (parameter *b* in model 4.77) represented the form factor itself and rejected the requirement for all variables except basal area and total height. The average value for that parameter (form factor) was very close to 0.5 suggesting the general shape of Corsican pine trees is an approximation of a paraboloid.

The parameter *b* in the merchantable volume model *a* should theoretically be $\pi/8=0.39$ and the estimated mean value was 0.43. The observed difference could be explained as the error of the particular parameter due to the variation of the main stem of individual trees from the shape of the paraboloid.

Parameter c_0 for the merchantable volume prediction model *b* (model 4.80) was largely statistically insignificant in older plantations (always very close to zero at each age even if it was statistically significant). However, this model was fitted to the data without ignoring the intercept because in early ages that parameter was occasionally statistically significant.

The main objective of this project was to construct models using measured tree variables and age as the explanatory data. For the thinning predictions, single tree models could not be developed unless an advanced graphical system was used to draw the size of trees using computer programs indicating the trees which should be marked for the next thinning. For this reason simple models were constructed using one explanatory variable which usually took tree mean values of the same response variable before thinning.

A separate model for the prediction of total trees per hectare was not built in this work and instead a general procedure was described for the estimation of the number of trees removed in thinning.

4.4.5 Testing of constructed models

For the purpose of evaluation of the constructed models, a set of qualitative and quantitative tests was used. Qualitative tests are easier to understand, especially standard residual plots. These are also helpful for identifying the outliers and observing the distribution pattern of the residuals in the basic model structures.

Because the number of data were very high in diameter, total height and timber height prediction models, bar graphs of the standard deviations of residuals at selected points of fitted values were used to observe the distribution of residual standard deviations. In a good model that distribution should be even. This test was not done for the thinning prediction models and total and merchantable volume prediction models because the number of data were too low. For the total height prediction models, one graph had to be drawn for each thinning type for the observation of residual standard deviance because the distribution of the data was narrower in the neutral thinning type.

Three kinds of quantitative tests were used in the current work as a part of the evaluation of the constructed models. Average model bias is a measure of the expected error when several observations are to be combined by totalling or averaging. The mean absolute difference indicates the average error associated with a single prediction (Soares *et al.*, 1995; Vanclay, 1994). Modelling efficiency provides a simple index of performance on a relative scale, where one indicates a 'perfect fit', and zero reveals that the model is no better than a simple average (Vanclay and Skovsgaard, 1997). The results of these tests are also helpful for the comparison of two or more models developed for the same predictions. Although the average model bias provides an average number, it is a useful test to know the direction of the bias (negative or positive). Modelling efficiency is a better test than the R^2 in the regression results because sometimes R^2 over-estimates the models properties if the number of explanatory variables is high.

However, all the above tests did not give a clear definition of the adequacy or inadequacy of the constructed models. Therefore the test described by Weisburg (1985) was followed to identify the lack of fit. This test uses F-values for the appropriate degrees of freedom and therefore it is a good indicator for this purpose.

The importance of validation with reserved data was discussed in the literature review. Instead of the normalised residual graphs, the fitted lines were drawn on the observed reserved data for the models built for thinning predictions. This was done with the intention of showing the distribution of the observed values along the fitted lines because the number of data was relatively low and interpretation was made possible because the models contained only one explanatory variable.

Plots of standardised and normal residuals created from the validation data suggested that the bias of all the models constructed in this study was negligible (there was however, an indication of little bias in timber height models developed for the intermediate thinning type at the early ages). Quantitative tests confirmed that the mean absolute difference and the average model bias were very low for all the models. The test followed for lack of fit proved the process of fitting was adequate for all the models and the bias was not statistically significant. Therefore all the models were taken forward for testing of parameters to construct one unified model for both thinning types.

CHAPTER 5: RE-CALIBRATION OF THE SELECTED MODELS

5.1 Introduction

Although Sri Lanka has a considerable area of man-made single species plantation forests, the lack of growth and yield models is a disadvantage for planning and marketing. As described in earlier chapters, re-measured sample plot data are needed for sound modelling. A lack of such data means models developed for other species in foreign countries might be re-calibrated for use in Sri Lanka. Therefore the models constructed in chapter 4 for *Pinus nigra* can be used for radiata pine (*Pinus radiata* D. Don) and Caribian pine (*Pinus caribaea* Mor.) in Sri Lanka. Re-calibration involves the re-estimation of model parameters and is a necessary procedure because two entirely different criteria can be found when adopting models from outside, i.e. different climatic zones and different species. Models may also be considered for use with entirely different genera; teak (*Tectona grandis* Linn. F.) and mahogany (*Swietenia macrophylla* King). For this reason, two types of re-calibration should be practised (i) re-calibration of models for same genus and (ii) re-calibration of the models for the different genera.

Selected models were re-calibrated in the present chapter using the same sample plot data used for model construction in Chapter 4 in order to fulfil two requirements; i.e. to gain experience of the difficulties of adapting models from different geographical regions, and secondly to compare the accuracy of the models constructed in Chapter 4 with re-calibrated existing models.

In 1997, Knowe *et al.* successfully re-calibrated models for red alder (*Alnus rubra* Bong.) plantations which were originally developed for pine and other conifer species by Hester *et al.*, 1989; Pienaar and Harrison, 1986; and Wykoff, 1990. Ottorini *et al.* (1996) tried to transfer a model initially developed for Douglas fir (*Pseudotsuga menziesii* (Mirb.) Franco) to common ash (*Fraxinus excelsior* L.). Therefore, inter-genera transformation of models is not a strange or uncommon procedure.

5.2 Considerations for the selection of existing models

The following factors were considered carefully before selecting models for re-calibration:

(i) All the models should contain regression equations

The procedure used to build new models for the current work is based on regression analysis. Therefore similar types of models were used for comparison without selecting models in other forms i.e. graphs, computer software etc.

(ii) Models developed outside of Great Britain

This would help to observe the effect of different geographical regions on these growth or yield models.

(iii) Models developed for species other than *Pinus nigra*

Using such models for the re-calibration, difficulties encountered when transforming the models can be understood and the experience may be applicable when transforming the models built for the current work for the selected tree species grown in Sri Lanka.

(iv) Models must be used widely

A higher number of tests will have been done on models which are widely used in the forestry community. Such models have also been developed or used by a number of experienced modellers leaving much less room for bias. Using such models, the predictive ability of the new set of models built in Chapter 4 can be easily observed.

(v) Models developed after the mid 1980s

Model development has benefited from advances in technology. The most commonly used models have been built or developed recently using modern statistical techniques and computer software.

(vi) Each model should contain at least two sub-models

The set of models constructed for this work contains many parts. Therefore similar models were used for re-calibration.

(vii) Empirical models should not contain guessed parameters

It is possible to guess some of the parameters in process based models (Makela, 1997; Sievanen, 1993; Sievanen and Burk, 1993). However, none of the estimated parameters were guessed in this work and therefore all the selected empirical models contained only estimated parameters.

(viii) The parameters and the explanatory variables should be explainable theoretically

The sign of each parameter should be logical and the combination and the relationship of the variables in the selected models must be explainable.

5.3 Methods applied for estimation of new parameters in re-calibration

5.3.1 Partition of the data

For the construction of a new set of growth and yield models in Chapter 4, the data were divided up according to thinning type. The same data partitions as used in Chapter 4 were used for re-calibration of the above models and for validating them (Table 4.1). However, different variables were needed for some of the selected models. The methods of gathering and preparing such data are described with the specific model when the results are discussed.

5.3.2 Evaluation of the re-calibrated models

A careful study of the processes of construction was done before selecting the existing models. However, after re-calibrating, it was still necessary to evaluate the models to know their suitability for the new geographical regions. Therefore, some of the tests used to evaluate the models newly constructed in Chapter 4 were used for the same purpose with the re-calibrated models; they are considered below.

5.3.2.1 Qualitative tests

Examination of standard residual distributions of residuals was selected.

The majority of the selected models predict stand level variables and therefore the number of data used for re-calibrating and validating was relatively low. Because the distribution of the residuals could be observed easily, graphs of the standard deviations of the residuals distributed with the fitted values were not necessary.

5.3.2.2 Quantitative tests

The decided quantitative tests for the evaluation of models in this Chapter are:

a. average model bias (equation 4.54 - page 93),
b. mean absolute difference (equation 4.55 - page 94),
c. modelling efficiency (equation 4.56 - page 94).

5.3.2.3 Validation with the reserved data

The reserved data were fitted to the re-calibrated models for each thinning type without changing the newly estimated parameters. The distribution of the normal residuals with the fitted values was then observed.

5.3.3 Fitting equations

Both linear and non-linear equations were fitted using the statistical program GENSTAT. Separate programs were written to obtain the parameters for the non-linear models (Appendices 3.1-3.7). The basic regression results (R^2, standard residual plots and standard errors of the parameters) were then used to test the bias of models and the significance of parameters. If one or more parameter was not significant, re-parameterization was done by ignoring each one or two at a time, following the same tests, so as to obtain the best and simplest model.

5.4 Re-estimation of the parameters for selected models

5.4.1 Models constructed by Pienaar and Harrison (1989)

Pienaar and Harrison (1989) constructed a set of models for *Pinus elliotti* Englem. (slash pine) in Zululand in South Africa for both thinned and unthinned plantations. The model contained compatible prediction (prediction of current growth) and projection (prediction of future growth) equations for total basal area and total volume per hectare.

5.4.1.1 Basal area prediction model

The model constructed by Pienaar and Harrison for the prediction of total basal area is:

$$\ln G = b_0 + b_1 * \left(\frac{1}{A}\right) + b_2 * (\ln N) + b_3 * (\ln h_{dom}) + b_4 * \left(\frac{\ln N}{A}\right) + b_5 * \left(\frac{\ln h_{dom}}{A}\right) + b_6 * \left[\frac{N_t}{N_a}\left(\frac{A_t}{A}\right)^{b7}\right] \quad 5.1$$

where:

A = plantation age, years

A_t = plantation age at last thinning, years

$\ln G$ = natural logarithms of basal area $m^2 ha^{-1}$

h_{dom} = average dominant height, m

N = number of surviving trees, ha^{-1}

N_a = trees remaining after last thinning ha^{-1}

N_t = trees removed in last thinning ha^{-1}

b_1-b_7 = unknown parameters

Total basal area, number of standing trees and thinned trees per hectare were calculated separately for stand and thinned trees using program 2 included in Appendix 1.9. Data for each age were used. For total basal area (G) and total number of trees (N) in the model, both standing and thinned trees at each age were summed because the model predicts the total basal area per hectare at any required age. For each plot, the first data set was ignored because the trees removed in thinning at the previous age (N_t) could not be calculated for those data. Top height was obtained using the height-diameter relationships developed in Chapter 4, and was used instead of dominant height because there is not a significant difference between these two heights (Philip, 1994).

The initial parameters estimated by Pienaar and Harrison (1989) are included in Table 5.1. This model (5.1) has both linear and non-linear parts and all the parameters were estimated in one step using the program presented in Appendix 3.1. Some parameters were statistically insignificant. As the second step, these parameters were ignored and the model was re-parameterized observing R^2 values and the plots of standard residuals using an extended program of the type presented in Appendix 3.1.

Parameter	Unthinned plantations	Thinned plantations
b_0	-0.6512	0.1432
b_1	-25.0905	1.1054
b_2	0.2255	0.0097
b_3	0.9789	0.0351
b_4	3.0660	0.1202
b_5	0.8636	0.2308
b_6	-0.1378	0.0073
b_7	2.2955	0.1966

Table 5.1: Initial parameters estimated by Pienaar and Harrison (1989) for the basal area prediction model.

Fitting the model 5.1, seven possible structures were identified for the intermediate thinning type and eight equations for neutral thinning type. All the estimated R^2 values for intermediate and neutral thinning types were between 0.674-0.678 and 0.967-0.971 respectively. However, only two models were identified which fulfilled the requirement that the selected model or models should be similar in the structure for both thinning types.

(i) Basal area prediction model *a*

The estimated parameters for the basal area projection model *a* (5.1 - with all parameters) are:

Parameter	Intermediate thinning		Neutral thinning	
	Estimate	Standard error	Estimate	Standard error
b_0	-1.5300	1.3200	4.6200	1.0700
b_1	80.7000	45.9000	-155.2000	26.6000
b_2	0.4513	0.1050	0.1239	0.0860
b_3	0.9121	0.2320	-0.2350	0.1770
b_4	-3.8100	3.6100	13.0700	2.0400
b_5	-24.5300	8.4600	7.8600	4.3000
b_6	-0.3634	0.1710	-0.0307	0.0160
b_7	3.7500	3.2900	-0.8600	1.2900

(ii) Basal area prediction model *b*

The equation without parameter b_7:

$$\ln G = b_0 + b_1 * \left(\frac{1}{A}\right) + b_2 * (\ln N) + b_3 * (\ln h_{dom}) + b_4 * \left(\frac{\ln N}{A}\right) + b_5 * \left(\frac{\ln h_{dom}}{A}\right) + b_6 * \left[\frac{N_t}{N_a}\left(\frac{A_t}{A}\right)\right] \qquad 5.2$$

Estimated parameters for the above model are:

Parameter	Intermediate thinning		Neutral thinning	
	Estimate	Standard error	Estimate	Standard error
b_0	-1.4580	1.3150	3.6860	0.9600
b_1	80.1200	45.7200	-124.7000	22.4000
b_2	0.4506	0.10430	0.1818	0.0820
b_3	0.8908	0.22950	-0.0570	0.1560
b_4	-3.8130	3.6050	10.8100	1.9800
b_5	-24.3540	8.4330	3.5900	3.8300
b_6	-0.2500	0.07540	-0.0618	0.0090

Evaluation of the selected models

The distributions of the standard residuals for models *a* and *b* are included in Figure 5.1 and Appendix 3.9(i) respectively. Both models displayed similar results in all the tests applied. The average bias was zero for both thinning types (Table 5.2). However, mean absolute difference was relatively high in the intermediate thinning type while the modelling efficiency (Table 5.2) and R^2 were low. This can be explained by the standard residuals (Figure 5.1) and the normal residuals obtained by validating with the reserved data (Figure 5.2). Standard residual distribution was not even with the fitted data in the intermediate thinning type and the validation results indicated bias. The reason may be the similar distribution pattern of A_t in neutral thinning and more scattered distribution in intermediate thinning.

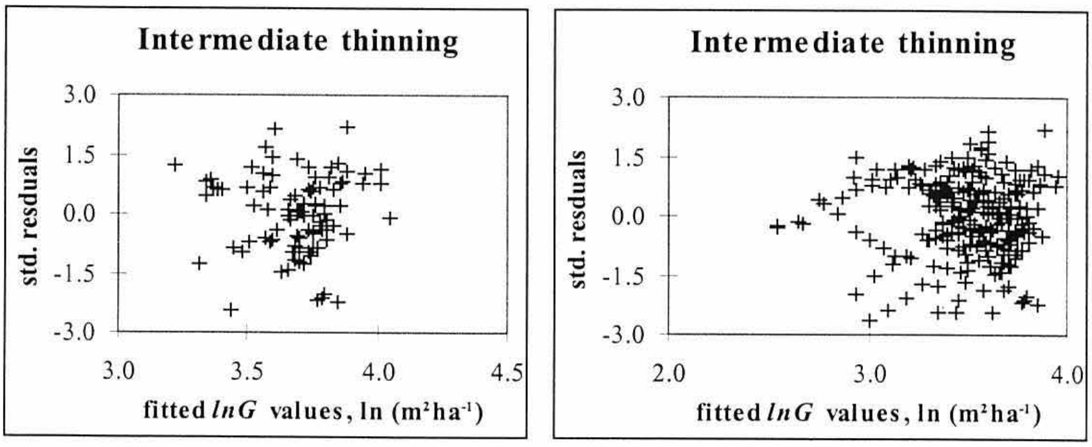


Figure 5.1: Standard residual distribution of the selected basal area prediction model *a*.

Test	Intermediate thinning		Neutral thinning	
	Model *a*	Model *b*	Model *a*	Model *b*
Average model bias	0.0000	0.0000	-0.0001	0.0000
Mean absolute difference	0.0850	0.0844	0.0345	0.0347
Modelling efficiency	0.7000	0.6980	0.9720	0.9720

Table 5.2: Results of the quantitative tests applied for the selected basal area prediction models.

The intention in the previous chapter was to construct models with less complexity. The same principle was used in this chapter and the model *b* (without the non-linear parameter) was selected for further examination.

The parameters b_0, b_3, b_4 and b_7 indicated different signs and different magnitudes for both models (equations 5.1 and 5.2) for the two thinning types used. The magnitude and the sign of the initial parameters estimated by the authors for thinned and unthinned plantations were also different. Considering this situation, it can be assumed that the reason for the differences mentioned above is due to the sensitivity of the original model to the different thinning types.

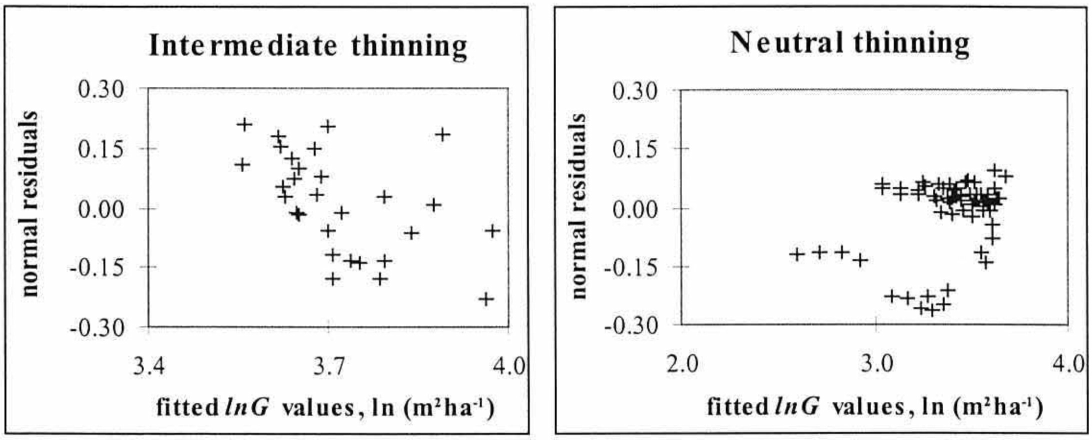


Figure 5.2: Distribution of normal residuals after fitting the unchanged basal area prediction model *a* to reserved data for validation.

5.4.1.2 Basal area projection model

The model constructed for the projection of basal area by Pienaar and Harrison (1989) is:

$$\ln G_2 = \ln G_1 + b_1 * \left(\frac{1}{A_2} - \frac{1}{A_1}\right) + b_2 * (\ln N_2 - \ln N_1) + b_3 * (\ln h_{dom(2)} - \ln h_{dom(1)})$$
$$+ b_4 * \left(\frac{\ln N_2}{A_2} - \frac{\ln N_1}{A_1}\right) + b_5 * \left(\frac{\ln h_{dom(2)}}{A_2} - \frac{\ln h_{dom(1)}}{A_1}\right)$$
$$+ b_6 * \left(\frac{N_t}{N_2}\right) * \left[\left(\frac{A_t}{A_2}\right)^{b7} - \left(\frac{A_t}{A_1}\right)^{b7}\right] \qquad 5.3$$

where: G_1 = basal area at age A_1, $m^2 ha^{-1}$

G_2 = basal area at age A_2, $m^2 ha^{-1}$

$h_{dom(1)}$ = average dominant height at age A_1, m

$h_{dom(2)}$ = average dominant height at age A_2, m

N_1 = number of surviving trees at age A_1, ha^{-1}

N_2 = number of surviving trees at age A_2, ha^{-1}

The data required were gathered using the same methods as for the basal area prediction method. In the sample plot data measured by the Forestry Commission, the number of surviving trees between two near measurements are similar in number indicating the absence of mortality. Therefore, N_1 was used instead of $N_2 - N_1 = 0$, assuming the N_1 can reduce the effects which could have emerged due to the geographical changes between two countries.

Both linear and non-linear parts of the model were fitted in one step. The parameters estimated by Pienaar and Harrison (1989) for the original basal area projection model are listed in Table 5.3.

Parameter	Unthinned plantations	Thinned plantations
b_0	-0.6512	0.1432
b_1	-25.0905	1.1054
b_2	0.2255	0.0097
b_3	0.9789	0.0351
b_4	3.0660	0.1202
b_5	0.8636	0.2308
b_6	-0.1378	0.0073
b_7	2.2955	0.1966

Table 5.3: Initial parameters estimated by Pienaar and Harrison (1989) for the basal area projection model.

In the original model, there was no associated parameter with the variable $\ln G_1$. In other words, this parameter was not significantly different from unity. When the model was used for the intermediate thinning type, this parameter was not significantly different from one. However, it was statistically significantly different from one when the model was fitted to the neutral thinnings. Therefore, two stage fitting was used by forcing the relevant parameter to one manually, as described in the section 4.2.5.1 with equations 4.49 - 4.53 (pages 90 - 91). All the possible models were observed with and without the non-significant parameters using the program written in Appendix 3.2.

Three appropriate models were identified for the intermediate thinning type, i.e. with all parameters, without parameter b_7, without parameters b_6,b_7. Four such models were identified for the neutral thinning type, i.e. as for intermediate thinning and without parameters b_3, b_6 and b_7. R^2 values were 0.890-0.912 and 0.880-0.883 for intermediate and neutral thinning respectively At the first attempt of fitting, the non-linear parameter became insignificant for the neutral thinning type. When the model was fitted without that parameter using linear regression, the parameter associated with lnG_1 was not statistically significant from one. Therefore, R^2 could be estimated. To be common to both thinning types, the following two models were selected for further studies.

(i) Basal area projection model *a*

The model without parameter b_7:

$$\ln G_2 = \ln G_1 + b_1 * \left(\frac{1}{A_2} - \frac{1}{A_1}\right) + b_2 * (N_1) + b_3 * (\ln h_{dom(2)} - \ln h_{dom(1)})$$
$$+ b_4 * \left(\frac{\ln N_2}{A_2} - \frac{\ln N_1}{A_1}\right) + b_5 * \left(\frac{\ln h_{dom(2)}}{A_2} - \frac{\ln h_{dom(1)}}{A_1}\right) + b_6 * \left(\frac{N_t}{N_a}\right) \qquad 5.4$$

The estimated parameters for the above model are given below:

Parameter	Intermediate thinning		Neutral thinning	
	Estimate	Standard error	Estimate	Standard error
lnG_1	1.0450	0.0360	0.8888	0.0960
b_1	-104.8000	44.5000	-95.000	124.0000
b_2	-0.0217	0.0190	0.0965	0.0450
b_3	0.0238	0.2610	0.8480	0.7270
b_4	17.5900	4.4600	16.7000	11.6000
b_5	-6.4700	7.3800	-28.3000	18.0000
b_6	0.1202	0.3940	0.1380	0.2390

(ii) Basal area projection model *b*

The model without parameters b_6 and b_7:

$$\ln G_2 = \ln G_1 + b_1 * \left(\frac{1}{A_2} - \frac{1}{A_1} \right) + b_2 * (\ln N_1) + b_3 * (\ln h_{dom(2)} - \ln h_{dom(1)})$$
$$+ b_4 * \left(\frac{\ln N_2}{A_2} - \frac{\ln N_1}{A_1} \right) + b_5 * \left(\frac{\ln h_{dom(2)}}{A_2} - \frac{\ln h_{dom(1)}}{A_1} \right) \quad 5.5$$

Estimated parameters for the basal area projection model *b* are:

Parameter	Intermediate thinning		Neutral thinning	
	Estimate	Standard error	Estimate	Standard error
LnG_1	1.0350	0.0360	0.9054	0.0910
b_1	-146.8400	73.7500	-45.5000	89.2000
b_2	-0.0105	0.1990	0.0879	0.0420
b_3	0.1011	0.1730	1.0480	0.6350
b_4	14.3380	5.3430	11.7300	7.8200
b_5	5.1700	16.4900	-32.6000	16.2000

Evaluation of the basal area projection models

The standard residual distributions for model *a* and model *b* are given in Figure 5.3 and Appendix 3.9(ii) respectively. The standard residual distributions for all the selected models were similar although there were some outliers in the data fitted for the intermediate thinning. However, as an overall conclusion, the data were over-estimated for both thinning types (Figure 5.3).

In the neutral thinning type there was an indication of the bias with the validation data (Figure 5.4). However, the distribution range of normal residuals in the neutral thinning type is much lower than that of intermediate thinning.

The quantitative tests showed that the estimated values for average model bias, mean absolute difference and the modelling efficiency for the two selected models were similar for the neutral thinning type (Table 5.4). However, these values were better for model *a* than model *b* for intermediate thinning and therefore model *a* (5.4) was selected for further studies for both thinning types.

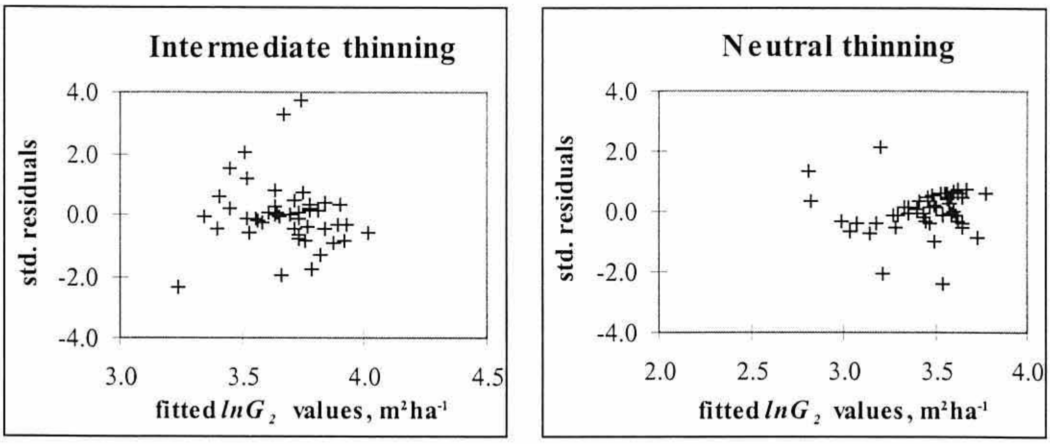


Figure 5.3: Standard residual distribution of the selected basal area projection model *a*.

Test	Intermediate thinning		Neutral thinning	
	Model *a*	Model *b*	Model *a*	Model *b*
Average model bias	-0.0003	-0.0007	0.0001	0.0000
Mean absolute difference	0.0304	0.0318	0.0430	0.0435
Modelling efficiency	0.9240	0.9080	0.8950	0.8960

Table 5.4: Quantitative test results applied for the selected basal area projection models.

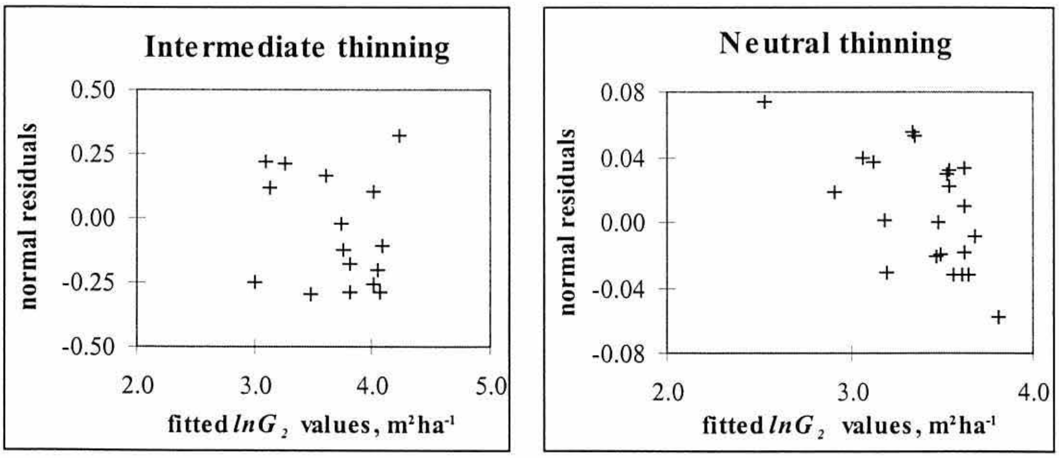


Figure 5.4: Normal residual distribution of the basal area projection model *a* after validating with reserved data.

5.4.1.3 Stand volume prediction model

The initial model constructed by Pienaar and Harrison (1989) for the prediction of total stand volume is:

$$\ln V = a_0 + a_1 * (\ln N) + a_2 * (\ln h_{dom}) + a_3 * (\ln G) \qquad 5.6$$

where: V = total stand volume m^3ha^{-1}

a_0-a_3 = unknown parameters

In this model, the present volume is estimated; therefore time difference is not used. All the parameters in this model are linear, so multiple linear regression was used for the parameter estimation. The initial parameters estimated by the authors are shown in Table 5.3.

Parameter	Unthinned plantations	Thinned plantations
a_0	-1.2333	0.0625
a_1	0.0190	0.0081
a_2	1.1899	0.0224
a_3	0.8655	0.0162

Table 5.5: Parameters estimated initially for the total volume prediction model by Pienaar and Harrison (1989).

Three better fits were identified for the thinning types in this study, i.e. with all the parameters and without parameter a_0 for intermediate thinning and with all the parameters for neutral thinning. R^2 values for the intermediate thinning type were 0.806 and 0.804 for the original model and the model without parameter a_0 respectively. The corresponding value was 0.716 for the neutral thinning type. However, one model was common for the both thinning types and therefore that model (5.7) was taken forward for further tests.

$$\ln V = a_0 + a_1 * (\ln N) + a_2 * (\ln h_{dom}) + a_3 * (\ln G) \qquad 5.7$$

The estimated parameters for the above total volume prediction model is:

Parameter	Intermediate thinning		Neutral thinning	
	Estimate	Standard error	Estimate	Standard error
a_0	1.0840	0.7690	6.0850	0.7360
a_1	-0.3187	0.0640	-0.6218	0.0760
a_2	0.5320	0.1480	-0.7500	0.1640
a_3	1.4100	0.1460	1.6350	0.1270

Evaluation of stand volume prediction model

The distributions of standard residuals were reasonable (Figure 5.5) although the values estimated for R^2 and modelling efficiency (Table 5.6) were relatively low for both intermediate and neutral thinning types. The average model bias was a negative value for both thinning types (Table 5.6) indicating an over-estimation. The mean absolute differences were relatively low.

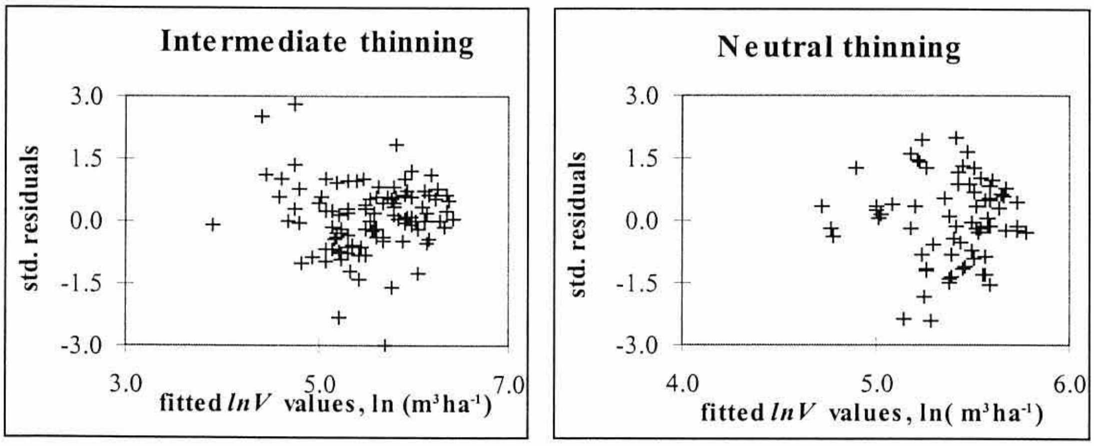


Figure 5.5: Standard residual distributions of the selected total volume prediction model.

Test	Intermediate thinning	Neutral thinning
Average model bias	-0.0189	-0.0001
Mean absolute difference	0.1384	0.1167
Modelling efficiency	0.8720	0.7930

Table 5.6: Quantitative results obtained from the selected volume prediction model.

The normal residuals resultant after fitting of the validation data (Figure 5.6) did not indicate a very good fit. A dramatic difference could be identified for the parameters estimated separately for two thinning types i.e. the magnitudes of the values estimated for a_0 i.e. 1.08 for intermediate thinning and 6.08 for neutral thinning. Pienaar and Harrison (1989) estimated different magnitudes and signs for the initial parameters for thinned and unthinned plantations. This could be due to the sensitivity of the model parameters for different growth rates, and may be the reason for parameter differences examined after re-calibrating.

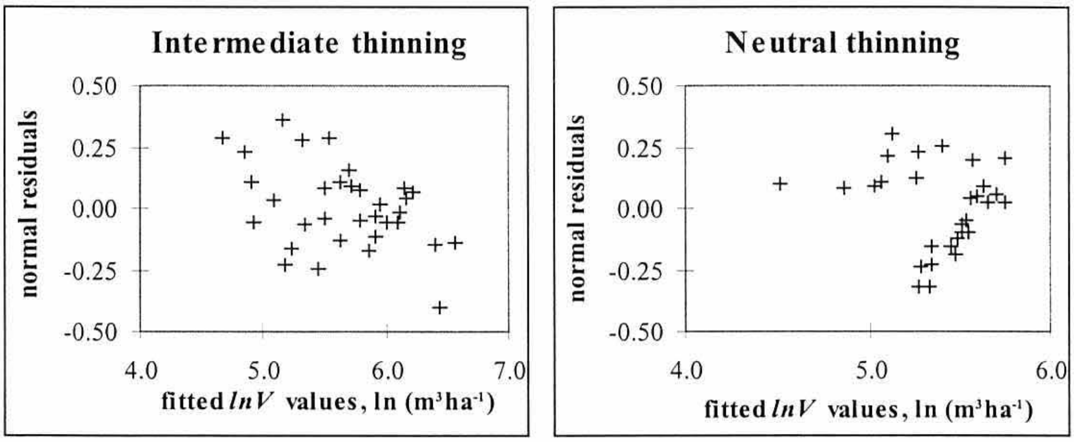


Figure 5.6: Residual distribution with the fitted values of the total volume prediction model to the reserved data for validation.

5.4.1.4 Stand volume projection model

The model constructed for volume projection by Pienaar and Harrison (1989) is:

$$\ln V_2 = \ln V_1 + a_1 * (\ln N_2 - \ln N_1) + a_2 * (\ln h_{dom(2)} - \ln h_{dom(1)}) + a_3 * (\ln G_2 - \ln G_1) \quad 5.8$$

As described in section 5.4.1.1, in the sample plot data $\ln N_1 = \ln N_2$ allowing the model to be changed to:

$$\ln V_2 = \ln V_1 + a_1 * \ln N_1 + a_2 * (\ln h_{dom(2)} - \ln h_{dom(1)}) + a_3 * (\ln G_2 - \ln G_1) \quad 5.9$$

The form of the above model is linear and therefore, multiple linear regression was used to estimate the parameters for Corsican pine plantations. However, the parameter associated with the variable $\ln V_1$ was significantly different from unity in neutral thinning and therefore the two stage fit was used as described in section 4.2.5.1 (pages 90-91). Parameters estimated by Pienaar and Harrison for the initial model for slash pine are given in table 5.7.

Parameter	Unthinned plantations	Thinned plantations
a_0	-1.2333	0.0625
a_1	0.0190	0.0081
a_2	1.1899	0.0224
a_3	0.8655	0.0162

Table 5.7: Parameters estimated for volume projection model by Pienaar and Harrison (1989) for slash pine.

After fitting all possible equations to the data, the two best possible models i.e. with all parameters and without parameter a_2 were identified for the intermediate thinning type. The three best models for the neutral thinning type were those with all parameters, without parameter a_1 and without parameter a_2. R^2 values were 0.917 and 0.919 respectively for the two models in intermediate thinning. However, R^2 values were not estimated for the models identified for neutral thinning because of the parameter estimation done by two stage fitting. The following two models, the forms of which were common to both intermediate and neutral thinning types, were selected for further tests.

(i) Stand volume prediction model *a*

The equation with all the parameters (5.9). Estimated parameters are:

Parameter	Intermediate thinning		Neutral thinning	
	Estimate	Standard error	Estimate	Standard error
$\ln V_1$	0.9639	0.0190	1.0000	*
a_1	0.0410	0.0170	-0.0234	0.0190
a_2	0.0420	0.1280	0.9691	0.7130
a_3	0.6580	0.1590	1.0400	0.3920

* Standard error was not estimated for the parameter forced manually to be one.

(ii) Stand volume projection model *b*

The model without parameter a_2.

$$\ln V_2 = \ln V_1 + a_1 * \ln N_1 + a_3 * (\ln G_2 - \ln G_1) \qquad 5.10$$

Estimated parameters for the selected total volume projection model *b* are:

Parameter	Intermediate thinning		Neutral thinning	
	Estimate	Standard error	Estimate	Standard error
$\ln V_1$	0.9649	0.0190	1.0000	*
a_1	0.0403	0.0170	0.0095	0.0158
a_3	0.6700	0.1530	1.2650	0.3530

* Standard error was not estimated for the parameter forced manually to be one.

Evaluation of stand volume projection models

Distributions of normal residuals of models *a* and *b* are given in Figure 5.7 and Appendix 3.9(iii) respectively. Normal residuals were calculated for the neutral thinning type as due to the two-stage fitting process it was not possible to calculate standard residuals. The distribution of normal residuals of model *b* for neutral thinning indicated a very poor fit and therefore model *b* was not tested further.

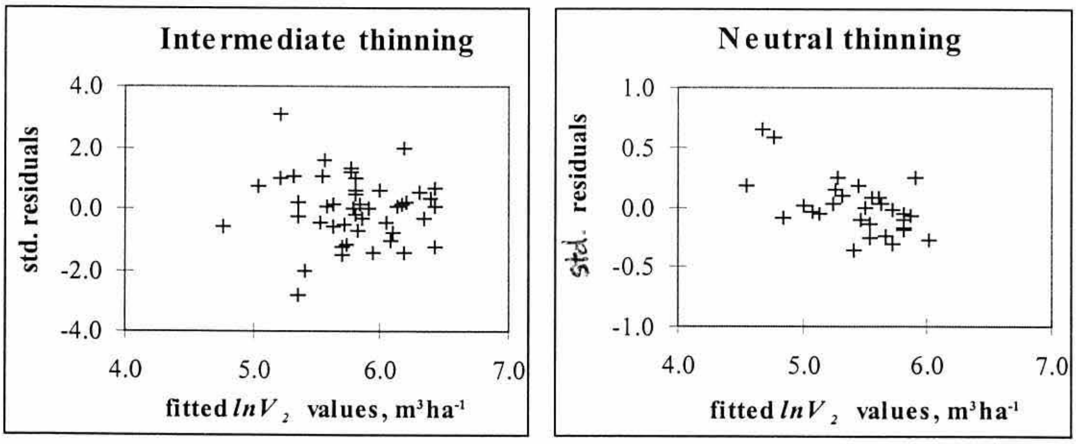


Figure 5.7: Standard residual distribution of the selected volume projection model *a*.

The modelling efficiency calculated for model *a* was relatively low for the neutral thinning type even though the average bias and mean absolute difference were low (Table 5.8). The validation results (Figure 5.8) showed very poor fit for neutral thinning. The reason could be due to three reasons, i.e. (i) forcing of one parameter; (ii) maintenance of a higher thinning intensity; or (iii) lack of another explanatory variable.

Test	Intermediate thinning	Neutral thinning
Average model bias	-0.0006	0.0002
Mean absolute difference	0.0849	0.1682
Modelling efficiency	0.9230	0.7740

Table 5.8: Results of the quantitative tests applied for the selected stand volume projection model.

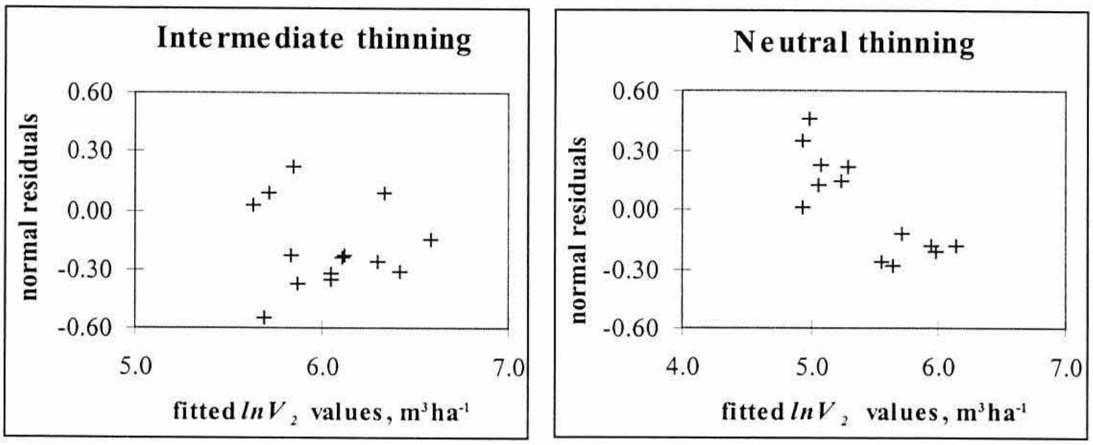


Figure 5.8: Normal residuals after fitting volume projection model *a* to the reserved data.

5.4.2 Models developed by Soares *et al.* (1995)

A model called PBRAVO was originally constructed by Pascoa (1990) for *Pinus pinaster* Ait. (maritime pine) in Portugal. Soares *et al.* (1995) adapted one version called Leiria which was developed for the National Forest of Leiria. The initial PBRAVO model has two sub-models: 'early growth model' which is used for the trees up to age 15 years (before the first thinning) and the 'main sub-model' which is for plantations older than 15 years. The authors used the second sub-model for older plantations. This part of the model contained both individual tree prediction (i.e. individual tree total height and total volume prediction models) and stand sub-models (i.e. prediction models for total basal area and number of surviving trees after thinning).

5.4.2.1 Total height of individual trees

The model developed by Soares *et al.* to predict the total height is:

$$h = a_0 * h_{dom}^{a_1} * G^{a_2} * N^{a_3} e^{(a_4/A - a_5/dbh)} \qquad 5.11$$

where:
A = stand age, years
dbh = diameter at breast height of individual trees, cm
G = stand basal area, m^2, ha^{-1}
h = total height of individual trees, m
h_{dom} = dominant height, m
N = stem number, ha^{-1}
a_0-a_5 = unknown parameters

Top height was used instead of the dominant height. This model could easily be transformed to a linear form but only the original model was used avoiding any transformation bias.

A program was written (Appendix 3.3) for estimating the parameters in this model. When statistically insignificant parameters were obtained after fitting the model in the first step, it was re-parameterized, ignoring the insignificant parameters one or two at a time. Parameters estimated for maritime pine by Soares *et al.* (1995) authors are shown in table 5.9.

Parameter	Estimation
a_0	1.8910
a_1	0.8907
a_2	-0.1467
a_3	0.0755
a_4	2.0010
a_5	11.9600

Table 5.9: Estimated parameters of total height prediction model for maritime pine by Soares *et al.* (1995).

Three possible models were identified for both thinning types. The model with all the parameters and a model without parameter a_2 were common for both thinning types and were selected for further tests. The rejected models were one without parameters a_2 and a_3 for intermediate thinning and one without parameters a_2 and a_4 for neutral thinning.

The R^2 values were 0.952 for all the models identified for intermediate thinning and varied from 0.926 to 0.948 for neutral thinning.

(i) Total height prediction model *a*

The estimated parameters for the selected model *a* (model 5.11 - with all the parameters) are given in the table below:

Parameter	Intermediate thinning		Neutral thinning	
	Estimate	Standard error	Estimate	Standard error
A_0	1.2080	0.1470	4.8600	1.7600
a_1	0.9401	0.0380	0.7092	0.1010
a_2	-0.0067	0.0280	0.0544	0.0290
a_3	0.0202	0.0140	-0.0992	0.0280
a_4	-28.200	11.9000	66.3000	34.3000
a_5	3.4070	0.6380	8.1900	1.3500

(ii) Total height prediction model *b*

The model without parameter a_2 associated with total basal area per hectare (*G*) is:

$$h = a_0 * h_{dom}^{a_1} * N^{a_3} e^{(a_4/A - a_5/dbh)} \quad 5.12$$

The estimated parameters for the above model are:

Parameter	Intermediate thinning		Neutral thinning	
	Estimate	Standard error	Estimate	Standard error
a_0	1.2200	0.1390	3.8600	1.4800
a_1	0.9347	0.0260	0.7746	0.0860
a_3	0.0174	0.0080	-0.0750	0.0190
a_4	-27.500	11.6000	66.0000	34.4000
a_5	3.4060	0.6370	8.2100	1.3500

Evaluation of the selected models

The standard residual distributions for model *a* (Figure 5.9) and model *b* (Appendix 3.9(iv)) indicated unbiased distribution for both intermediate and neutral thinning types.

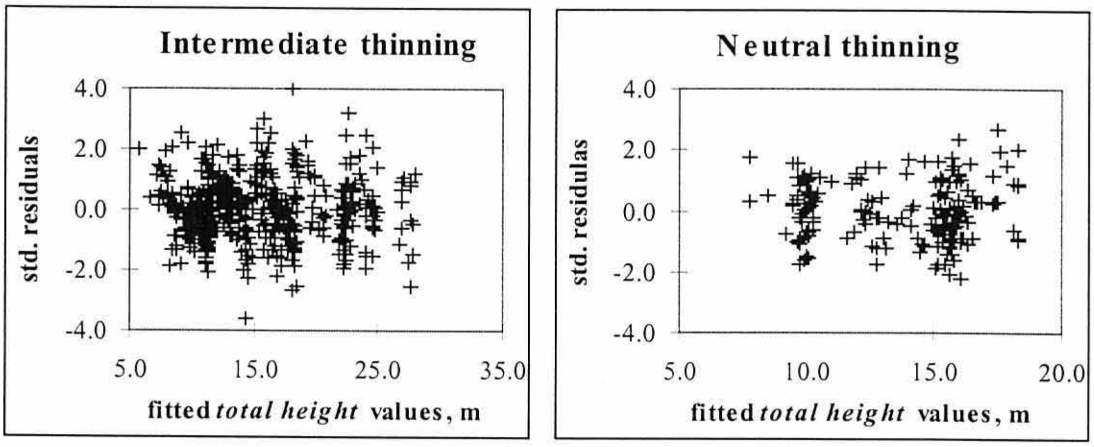


Figure 5.9: Standard residual distribution of the selected total height prediction model *a*.

The values estimated from the quantitative tests indicated both very low average bias and very low mean absolute difference. The modelling efficiencies were always higher than 0.94 for all the models (Table 5.10). The normal residuals obtained after fitting the unchanged models to raw data indicated a normal distribution (Figure 5.10).

Test	Intermediate thinning		Neutral thinning	
	Model *a*	Model *b*	Model *a*	Model *b*
Average model bias	-0.0008	-0.0008	-0.0011	-0.0013
Mean absolute difference	0.8541	0.8666	0.5560	0.5463
Modelling efficiency	0.9520	0.9520	0.9430	0.9420

Table 5.10: Results of quantitative tests obtained from the selected total height prediction models.

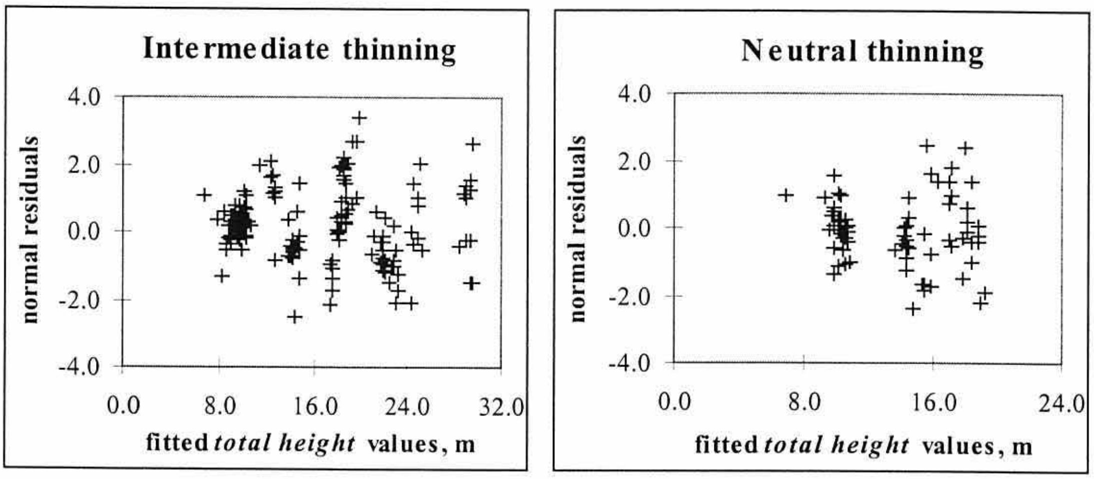


Figure 5.10: Normal residual distributions after fitting the total height prediction model a to reserved data.

The estimated parameters for the initial model by Soares *et al.* (1995) carried positive signs except parameter a_2 which is associated with the total basal area per hectare. However, after re-calibration, some differences were found for the two thinning types, i.e. parameters a_2, a_3 and a_4 indicated different signs; magnitudes were different for all parameters except parameter a_1. Therefore, to reduce the complexity, model b in which parameter a_2 was not included (model 5.12) was selected for further tests. Normally the total height of individual trees should increase with the number of trees (N) per unit area (McClain *et al.*, 1994) allowing the associated parameter to be positive . This rule was broken by estimated parameter a_3 for neutral thinning. The initial parameter a_4 estimated for maritime pine (Table 5.9) associated with the inverse age carried a positive sign. After re-calibrating, it was positive for neutral thinning and negative for intermediate thinning. Theoretically, when the value of the inverse age increases, the total height decreases because height increases with the age. Two reasons can be assumed for the above difference. This problem could have emerged because of the high sensitivity of the model to the data collected under different conditions. The thinning intensity for the neutral thinning type was very high in the data used: sometimes over 300% of the marginal thinning intensity indicating a tendency towards an exploitation thinning regime. Under such high level of thinning intensities, the trees which had grown fast in the plantation could have been removed leaving many trees which were not growing as fast. Therefore, as an average, the height after the thinning could be similar or less than the height of the plantation before thinning, causing the associated parameter to be negative.

5.4.2.2 Total volume of individual trees

The individual tree total volume prediction model developed by Soares *et al.* (1995) is:

$$v = \left(\frac{\pi * dbh^2 * h}{40000}\right) * b_0 e^{(b_1/h + b_2/dbh)} \qquad 5.13$$

where: v = total volume of the individual trees, m^3

b_0, b_1, b_2 = unknown parameters

This model could have been used for all the individual trees gathered from the sample plot data without concerning the age. However, to be compatible with the volume prediction models constructed for the current work in Chapter 4, the same data sets grouped by one year age class were used for the re-calibration of this model.

A program (Appendix 3.4) was written to estimate all the parameters in one step. In all the age classes used for both thinnings, parameters b_1 and b_2 was not statistically significant and the distribution of the standard residuals was not changed when these two parameters were removed from the analysis. Therefore, the selected model used for both thinning types is described below:

$$v = b_0 * \left(\frac{\pi * dbh^2 * h}{40000}\right) \qquad 5.14$$

Parameters estimated for maritime pine by Soares *et al.* are given in Table 5.11.

Parameter	Estimation
b_0	0.336
b_1	0.940
b_2	3.790

Table 5.11: Parameters estimated by Soares *et al.* (1995) for the total volume prediction model for maritime pine.

Evaluation of the total volume prediction model

The estimated parameters for different ages were similar (Appendix 3.8). The possibility of using one parameter for all ages will be tested in Chapter 6. For convenience, only the standard residual distribution at age 25 is shown in this chapter (Figure 5.11). When observed, the residual distributions at all ages were even, without showing bias. This was confirmed by the quantitative tests (Table 5.12). In these the average model bias was positive or negative but was always very low. Mean absolute difference was low and modelling efficiency was high for all ages for both thinning types. In table 5.12, only the test results of common ages for intermediate and neutral thinning types were outlined. The normal residuals generated from the validation data indicated a reasonable fit (Figure 5.12).

When tested, it was observed that both non-linear parameters (b_1 and b_2) of the original model (5.13) were not statistically significant, leaving the model in a linear form. In the original paper, the authors applied this model to the data of a wide range of ages. In this study, it was narrowed to one year age classes and this may be the reason for the non-significance of the above two parameters.

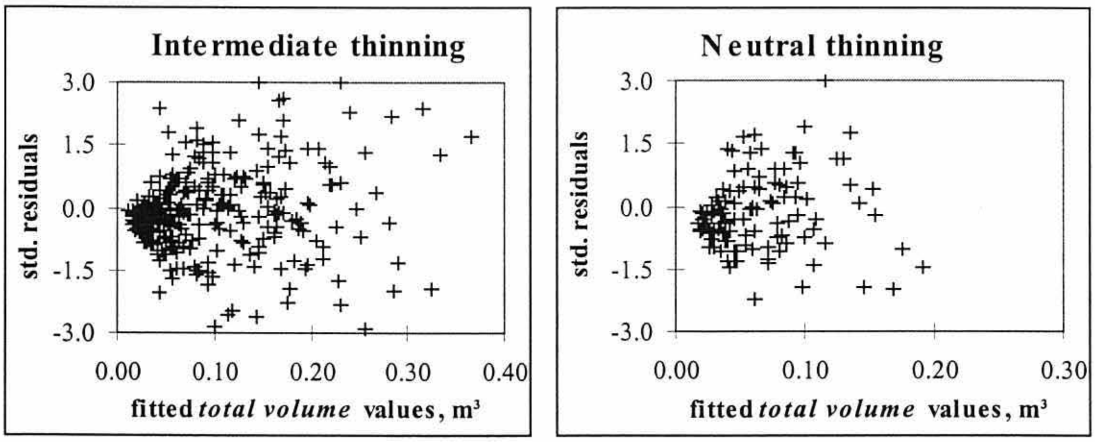


Figure 5.11: Standard residual distribution at age 25 for total volume prediction model after re-calibrating the initial model developed by Soares *et al.* (1995).

Age	Test	Intermediate thinning	Neutral thinning
19	Average model bias	-0.0013	-0.0009
	Mean absolute difference	0.0037	0.0031
	Modelling efficiency	0.9920	0.9870
24	Average model bias	0.0024	-0.0006
	Mean absolute difference	0.0063	0.0056
	Modelling efficiency	0.9890	0.9880
25	Average model bias	0.0027	0.0012
	Mean absolute difference	0.0064	0.0057
	Modelling efficiency	0.9860	0.9870
26	Average model bias	0.0004	-0.0005
	Mean absolute difference	0.0042	0.0043
	Modelling efficiency	0.9930	0.9920
28	Average model bias	0.0050	0.0036
	Mean absolute difference	0.0068	0.0067
	Modelling efficiency	0.9730	0.9770
31	Average model bias	0.0049	-0.0005
	Mean absolute difference	0.0150	0.0097
	Modelling efficiency	0.9760	0.9850
36	Average model bias	-0.0010	0.0039
	Mean absolute difference	0.0048	0.0133
	Modelling efficiency	0.9690	0.9860
37	Average model bias	-0.0165	-0.0088
	Mean absolute difference	0.0201	0.0180
	Modelling efficiency	0.9850	0.9850
41	Average model bias	-0.0062	0.0048
	Mean absolute difference	0.0211	0.0189
	Modelling efficiency	0.9620	0.9840

Table 5.12: Quantitative test results of the total volume prediction model after re-calibrating the initial model developed by Soares *et al.* (1995).

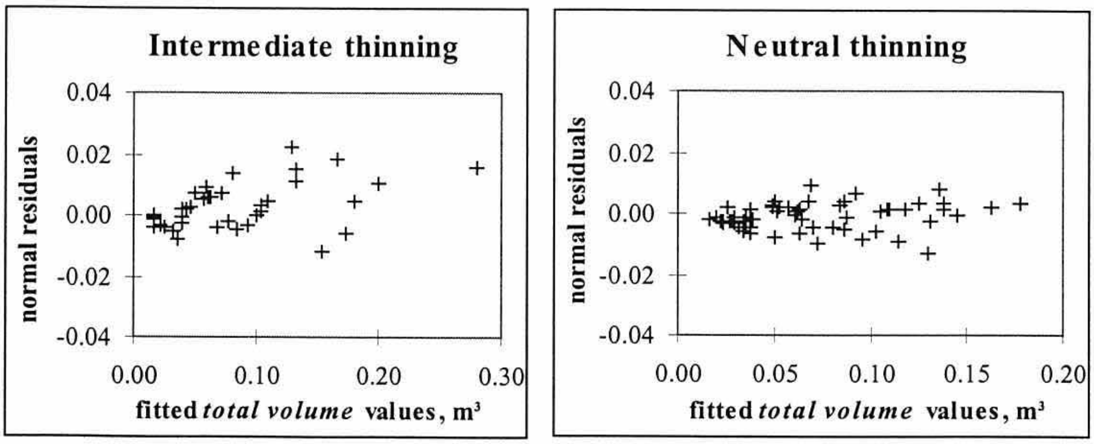


Figure 5.12: Normal residuals generated by fitting the volume prediction model to the reserved data at age 25.

5.4.2.3 Prediction of total basal area

The whole stand level model developed by Soares *et al.*(1995) to predict the total basal area is:

$$G_2 = G_1^{A_1/A_2} e^{(1-A_1/A_2)\ (c_1+c_2*h_{dom})} \qquad 5.15$$

where: A_1 = plantation age at the beginning of the simulating period, years

A_2 = plantation age at the end of the simulating period, years

G_1 = total stand basal area at time A_1, m^2ha^{-1}

G_2 = total stand basal area at time A_2, m^2ha^{-1}

c_1, c_2 = unknown parameters

The present basal area per hectare was calculated using only the main crop trees and the basal area predicted by the model was calculated using both main crop and thinned trees. Top height was used instead of dominant height for the reasons outlined earlier.

The initial values of parameters c_1 and c_2 estimated by Soares *et al.* (1995) were 4.1780 and 0.0390 respectively. The original model was fitted to the data of the two thinning types separately using a GENSTAT program written to estimate the non linear parameters (Appendix 3.5). For both thinning types the parameter c_2 was not statistically significant. However, when the re-parameterization was done ignoring this parameter, the variance of the *y* variate (G_2) was exceeded by the residual variance, indicating a very poor fit. Therefore, the model with all the parameters (5.15) was selected. The parameter values for the two thinning types are given in the table below:

Parameter	Intermediate thinning			Neutral thinning		
	R^2	Estimate	std. Error	R^2	estimate	std. error
	0.907			0.894		
c_1		4.7440	0.181		4.4590	0.440
c_2		0.0140	0.011		0.0336	0.032

Evaluation of the selected models

The estimated parameters for both thinning types were positive. Parameter c_2 for intermediate thinning was lower than the same estimated parameter both for neutral thinning and in the initial model. The distributions of the standard residuals with the fitted values were even (Figure 5.13). R^2 was reasonably high and modelling efficiency was 0.9 for both intermediate and neutral thinning types (Table 5.13). Re-calibrated models for both thinning types indicated negative bias. However, the validation results indicated little bias in intermediate thinning (Figure 5.14). The number of data used in validation was very low for both thinning types, and therefore, a proper conclusion could not be attained from the validation.

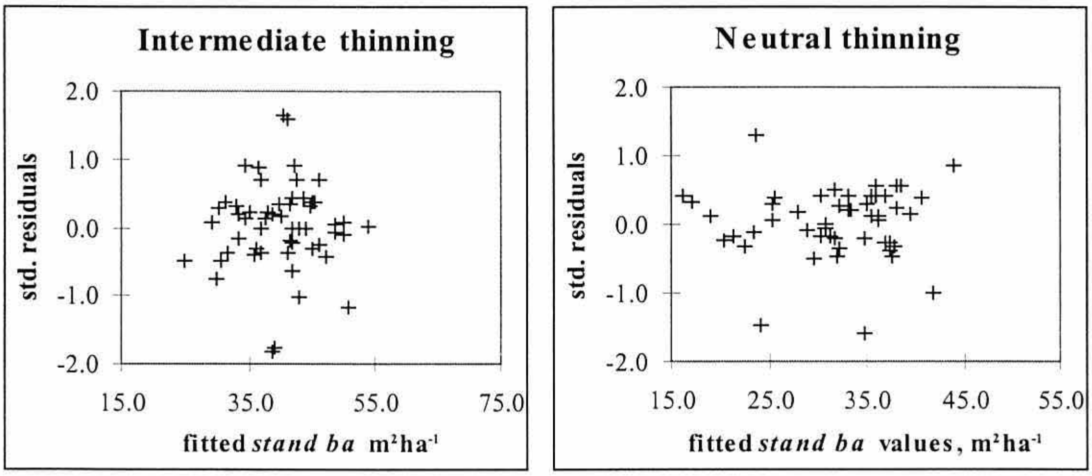
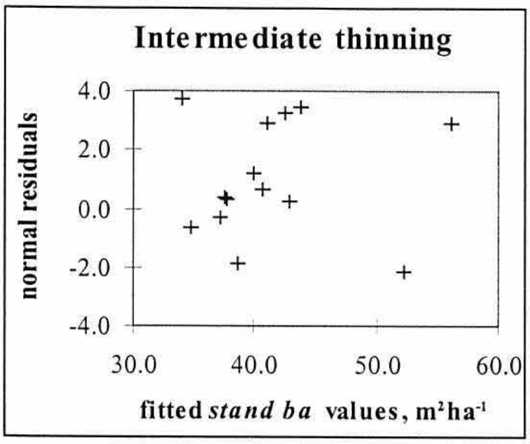


Figure 5.13: Standard residual distributions of the total basal area prediction model after re-calibrating the initial model developed by Soares *et al.* (1995).

Test	Intermediate thinning	Neutral thinning
Average model bias	-0.0782	-0.0561
Mean absolute difference	1.0879	1.0984
Modelling efficiency	0.9090	0.8960

Table 5.13: Results of the quantitative tests applied for the resulted basal area prediction model.



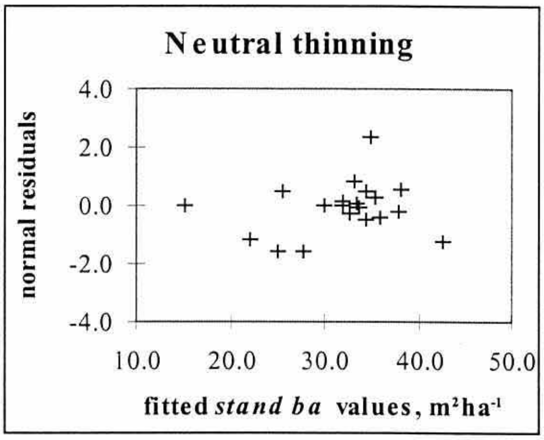


Figure 5.14: Residual distribution of the basal area prediction model with validation data.

5.4.2.4 Prediction of number of remaining trees after thinning

The number of trees remaining after thinning is predicted by:

$$N_r = N_{bt}[1-(1-G_r / G_{bt})^{f_1}]^{f_2} \qquad 5.16$$

where:
G_{bt} = stand basal area just before thinning, m^2ha^{-1}
G_r = stand basal area remaining after thinning, m^2ha^{-1}
N_{bt} = stem number just before thinning, ha^{-1}
N_r = stem number remaining after thinning, ha^{-1}
f_1, f_2 = unknown parameters

The numbers of trees in the main crop after thinning, and removed as thinnings, were calculated using program 2 (Figure 3.8). In the original model, the basal area remaining after thinning (G_r) is defined by the user. However, for re-calibration, this value was calculated using sub-routine 2 (Figure 3.4) because thinned tree data could be obtained from the Forestry Commission sample plot measurements.

Soares *et al.* (1995) estimated the values of parameters f_1 and f_2 to be 0.7151 and 0.8206 respectively for maritime pine. The non-linear equation was fitted to the data using a GENSTAT program (Appendix 3.6). For both thinning types, parameters f_1 and f_2 were statistically significant and therefore the unchanged model structure was selected. The estimated parameters are given in the table below:

Parameter	Intermediate thinning			Neutral thinning		
	R^2	Estimate	std. Error	R^2	estimate	std. error
	0.988			0.963		
f_1		0.6919	0.0586		0.7887	0.0637
f_2		0.8077	0.0990		1.1750	0.1220

Evaluation of the selected model

Although the R^2 values for both the intermediate and neutral thinnings were high, the standard error of the model was also high, i.e. 54.0 and 59.8 respectively. The estimated parameters after re-calibration were similar to those in the original model. However, both models indicated bias when the standard residual distributions were examined (Figure 5.15). The worst fit occurred with the neutral thinning type . The quantitative results showed very high negative bias and high mean absolute differences (Table 5.14). The modelling efficiency was also low. All these tests indicate the poor fit of these models to the Forestry Commission Corsican pine data, and this was confirmed by the validation procedure (Figure 5.16). This model might be developed originally for a different thinning type from the two thinning types used in this study, and this may be the reason for the bias generated by re-calibration. Because of its obvious unsuitability to the current study, this model was removed from any further tests.

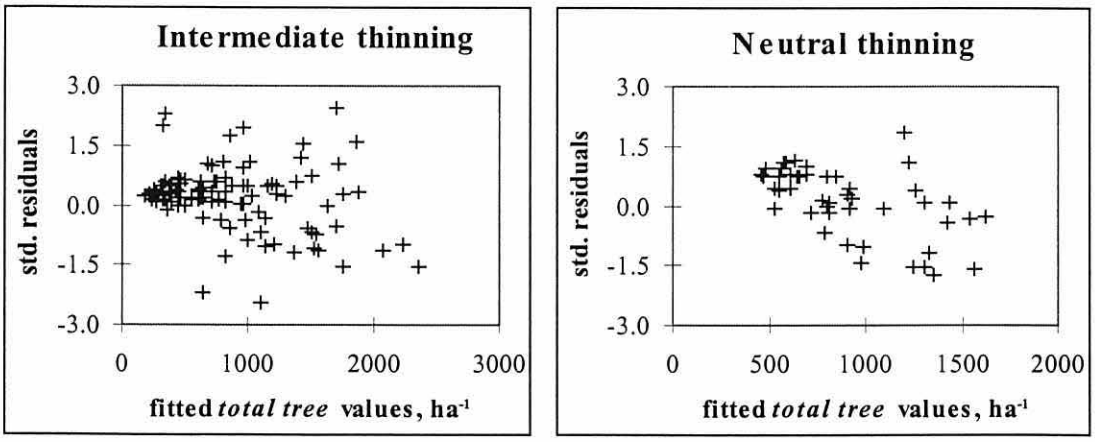


Figure 5.15: Standard residual distributions of the tree prediction model after thinning, after re-calibrating.

Test	Intermediate thinning	Neutral thinning
Average model bias	-8.7741	-10.6747
Mean absolute difference	37.4128	45.4172
Modelling efficiency	0.7880	0.6640

Table 5.14: Quantitative test results for the prediction model of the number of trees removed in thinning.

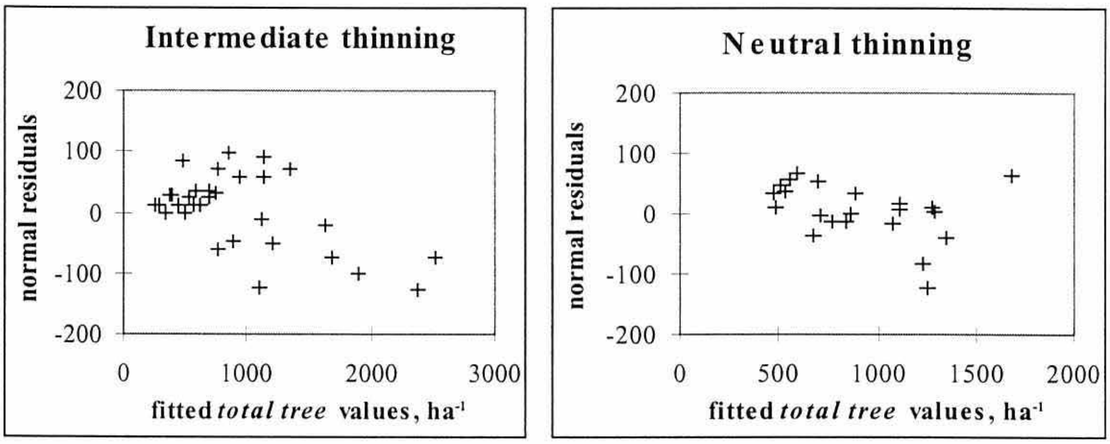


Figure 5.16: Residual distribution of the remaining tree number prediction model after thinning, with reserved data.

5.4.3 Models built by West and Mattay (1993)

This set of models was built by West and Mattay (1993) for the prediction of the growth of six eucalyptus species, i.e. *Eucalyptus delegatensis* R. Baker, *E. diversicolor* F. Muell. (Karri), *E. grandis* Hill ex Maiden, *E. obliqua* L'Her, *E. piluaris* Smith (Blackbutt) and *E. regnans* F.Muell. in Australia. Although the models were developed for six Eucalyptus species, they can be applied only for even-aged monoculture plantations.

5.4.3.1 Prediction of total tree height

This sub-model was developed originally to obtain top height as an average of measured individual trees using the mean diameter values although it was named as a total height prediction model.

$$h = 1.3 + dbh / (p + q * dbh) \qquad 5.17$$

where: dbh = diameter at breast height, cm
h = total height of individual trees, m
p, q = unknown parameters

The initial parameters were not given in the paper by West and Mattay (1993). The same data set generated for the construction of total height prediction models in Chapter 4 was used for the re-calibration of the model 5.17.

The GENSTAT program outlined in Appendix 3.7 was used to estimate the parameters of the above non-linear model. Parameter q was not significant for intermediate thinning. However, when re-parameterization was done ignoring this parameter, the variance of the y variate (h) was exceeded by the residual variance indicating a poor fit. Therefore, the original model given in 5.17 was selected. The estimated parameters for this model are given below:

Parameter	Intermediate thinning			Neutral thinning		
	R^2	estimate	std. error	R^2	estimate	std. error
	0.763			0.706		
p		1.8201	0.048		1.5836	0.084
q		-0.0008	0.002		0.0096	0.003

Evaluation of the selected model

The distributions of the standard residuals indicated slight over-estimation for both thinning types (Figure 5.17). The values estimated for both R^2 and modelling efficiency were low for the two thinning types (Table 5.15) However, the average model bias for both thinning types was low (Table 5.15). The distributions of normal residuals generated after fitting the validating data (Figure 5.18) indicated bias.

The estimated parameters p for intermediate and neutral thinning types were similar but, parameter q was negative for intermediate thinning and positive for neutral thinning. The magnitudes of parameter q were significantly different for two thinning types.

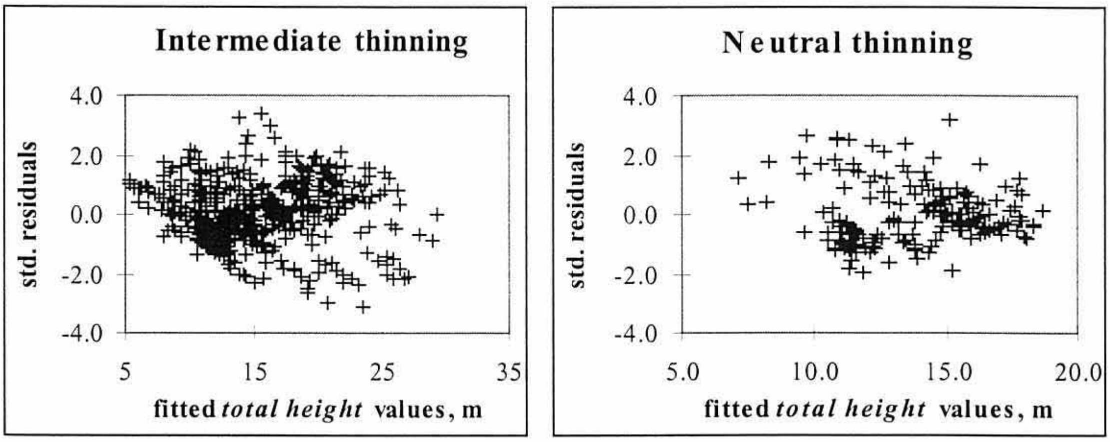


Figure 5.17: Distributions of standard residuals of the total height prediction model.

Test	Intermediate thinning	Neutral thinning
Average model bias	-0.0081	-0.0075
Mean absolute difference	1.9570	1.2165
Modelling efficiency	0.7640	0.7080

Table 5.15: Quantitative test results for the re-calibrated model initially built by West and Mattay (1993) to predict the total tree height.

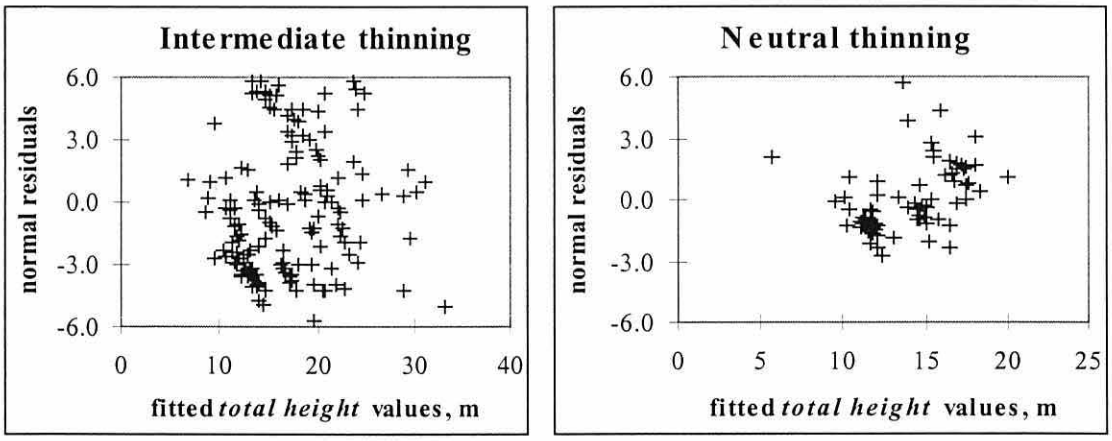


Figure 5.18: Residual distribution of total height prediction model with the reserved data for validation.

Being originally developed for the eucalyptus plantations, when applied to *Pinus nigra* the model may contain calibration errors without adding one or more new explanatory variables.

5.4.3.2 Stand volume (under bark)

Two models were developed for estimating stand volume. Equation 5.18 is used for fully stocked, high quality stands and equation 5.19 estimates the under bark volume per hectare of the rest of the stands. However, there was not a clear definition of the measurement of the quality of the stands in the test data. Considering the model errors which could occur when re-calibrating this model to Corsican pine for the prediction of over bark volume instead of under bark volume, the second equation (5.19) was selected.

$$\ln V_{ub} = b_1 + b_2 * \frac{1}{A} + b_3 * S \tag{5.18}$$

$$\ln V_{ub} = b_1 + b_2 * \frac{1}{A} + b_3 * S + b_4 * D_s \tag{5.19}$$

where:

A = stand age, years
D_s = stand density, stems ha^{-1}
S = site index (top height at age 20 years), m $year^{-1}$
V_{ub} = under bark stem volume, m^3 ha^{-1}

It is common to use a variable such as stand basal area to represent stand density as in equation 5.19 (West and Mattay, 1993). Parameters initially estimated for the six eucalyptus species by West and Mattay in 1993 are shown in the Table 5.16.

Eucalyptus species	Parameter		
	b_1	b_2	b_3
E. delegatensis	4.46	-27.5	0.1080
E. diversicolor	5.16	-23.4	0.0505
E. grandis	3.61	-28.4	0.0930
E. obliqua	4.64	-31.1	0.0915
E. piluaris	2.75	-42.5	0.1479
E. regnans	3.93	-32.2	0.1146

Table 5.16: Parameters estimated for the total under bark volume prediction for eucalyptus species by West and Mattay (1993) using the model 5.18.

Model 5.19 was re-calibrated for over bark volume per hectare instead of the under bark volume using multiple linear regression. In the model built by West and Mattay (1993), the quality of the site was represented by site index 20 (top height at age 20) of the *Eucalyptus* species. Because the usual rotation age of *Pinus nigra* varies between 45-80 (Hart, 1994) two site indices, i.e. at 20 (to be compatible with the original model) and 40 years and top height at each age were used in this study to examine the possibility of replacing site index with top height at a particular age. Top height values were calculated using the equation 4.40 in Chapter 4.

(i) Stand volume prediction model *a*

The model constructed by West and Mattay to predict the stand volume for poor quality stands is:

$$\ln V = b_1 + b_2 * \frac{1}{A} + b_3 * SI_{20} + b_4 * D_s \qquad 5.20$$

where: SI_{20} = top height (m) at the age of 20, years

The estimated parameters for the above model are given in the table below:

Parameter	Intermediate thinning			Neutral thinning		
	R^2	estimate	std. error	R^2	estimate	std. error
	0.836			0.669		
b_1		5.1260	0.261		5.1680	0.376
b_2		-38.3000	2.480		-31.1900	3.700
b_3		0.0811	0.016		0.0729	0.035
b_4		0.0200	0.004		0.0134	0.004

(ii) Stand volume prediction model *b*

The re-structured model with top height at age 40 as the site index is:

$$\ln V = b_1 + b_2 * \frac{1}{A} + b_3 * SI_{40} + b_4 * D_s \qquad 5.21$$

where: SI_{40} = top height (m) at the age of 40, years

The estimated parameters for the above model are:

Parameter	Intermediate thinning			Neutral thinning		
	R^2	estimate	std. error	R^2	estimate	std. error
	0.821			0.641		
b_1		5.1460	0.288		5.7850	0.635
b_2		-36.1000	2.480		-30.2400	3.870
b_3		0.0382	0.009		0.0017	0.034
b_4		0.0196	0.004		0.0169	0.005

(iii) Stand volume prediction model *c*

The model with the top height at each age instead of site index of the original model is:

$$\ln V = b_1 + b_2 * \frac{1}{A} + b_3 * h_{top} + b_4 * D_s \qquad 5.22$$

where: h_{top} = top height (m) at the age A, years

The estimated parameters are:

Parameter	Intermediate thinning			Neutral thinning		
	R^2	estimate	std. error	R^2	estimate	std. error
	0.892			0.733		
b_1		4.2200	0.267		2.2080	0.899
b_2		-12.6200	3.180		22.2000	13.200
b_3		0.0454	0.005		0.1202	0.029
b_4		0.0239	0.003		0.0153	0.004

For the intermediate thinning type, all the parameters in all three models were statistically significant. However, for the neutral thinning type, parameter b_3 in model *b* (5.21) and parameter b_2 in model *c* (5.22) was not statistically significant. However, to be compatible for both thinning types the unchanged models (5.20, 5.21 and 5.22) were selected.

Parameter b_2 in model c estimated for the neutral thinning type was positive. Stand volume per unit area should increase with the plantation age if the number of trees per unit area is more or less constant. Therefore, parameter b_2 which was associated with inverse age should have been negative. However, in the neutral thinning sample plots used for re-calibrating, thinning intensity and therefore the thinning yield is very high, sometimes more than 300% of the marginal thinning intensity. This explains the reason for the positive parameter associated with the inverse age because after thinning, the remaining number of trees is very much less than it was before thinning, leaving less total volume on the ground. In order to reduce the complexity of parameter sets with different signs for different thinning types, model c was removed from further studies. When the quality of the site increases, stand volume should also increase and the same phenomenon should happen when the stand basal area increases. All the parameters estimated for models a and b followed this pattern.

All the models indicated a good distribution of standard residuals for both thinning types (Figure 5.19) although the R^2 and modelling efficiency values were low (Table 5.17). Average model bias was zero and mean absolute difference was lower than 0.5 for all the models (Table 5.17). However, the normal residuals generated after fitting the unchanged models to the reserved data indicated bias for the neutral thinning type (Figure 5.20).

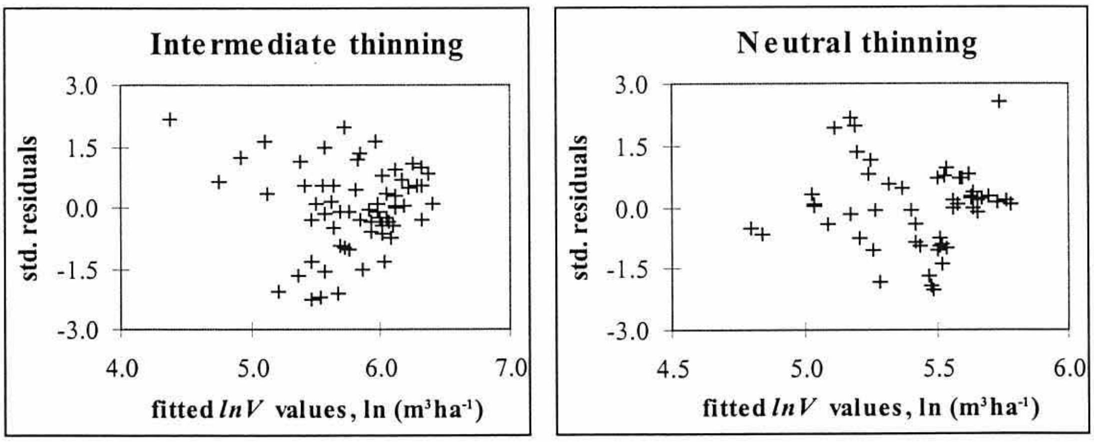


Figure 5.19a: Standard residual distributions of stand volume prediction model a.

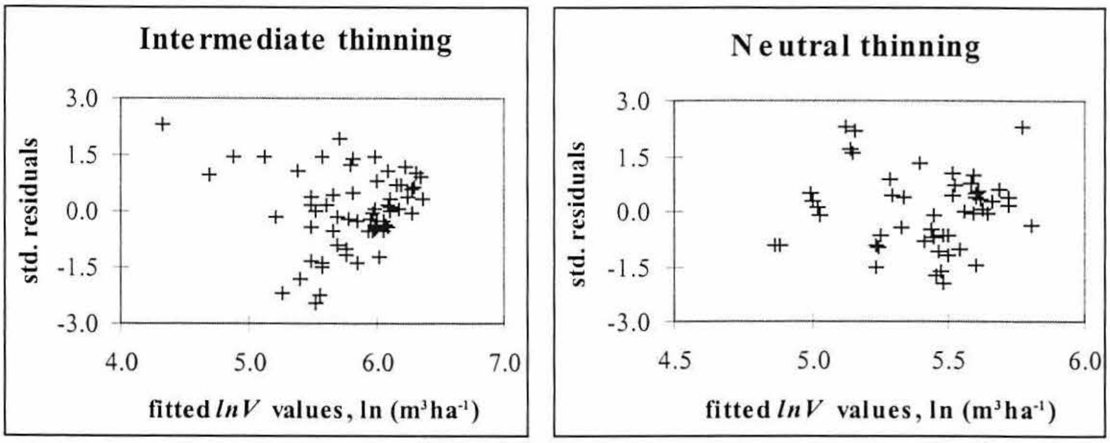


Figure 5.19b: Standard residual distributions of stand volume prediction model *b*.

Test	Intermediate thinning		Neutral thinning	
	Model *a*	Model *b*	Model *a*	Model *b*
Average model bias	0.0000	0.0000	0.0001	0.0000
Mean absolute difference	0.1344	0.1401	0.1190	0.1318
Modelling efficiency	0.8440	0.8440	0.6880	0.6620

Table 5.17: Results of the quantitative tests applied for the selected stand volume prediction models.

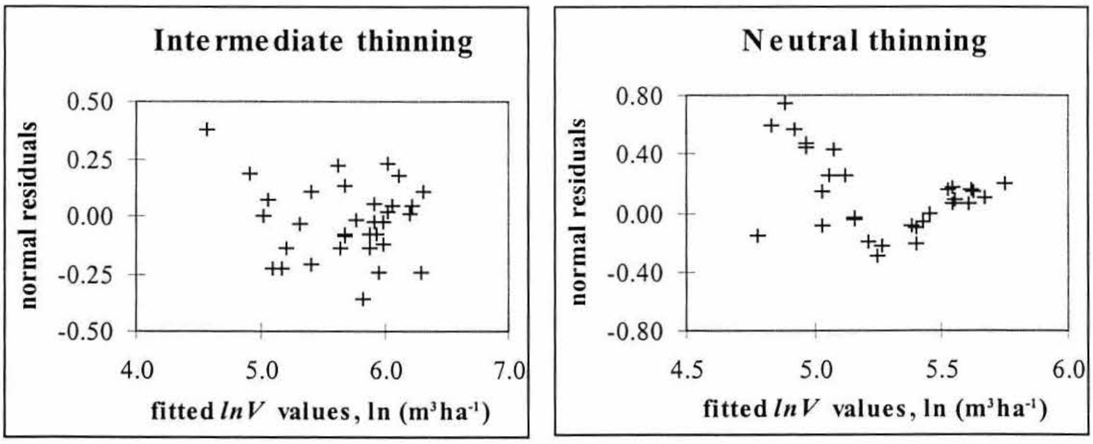


Figure 5.20a: Distribution of the residuals after fitting the volume prediction model *a* to the reserved data.

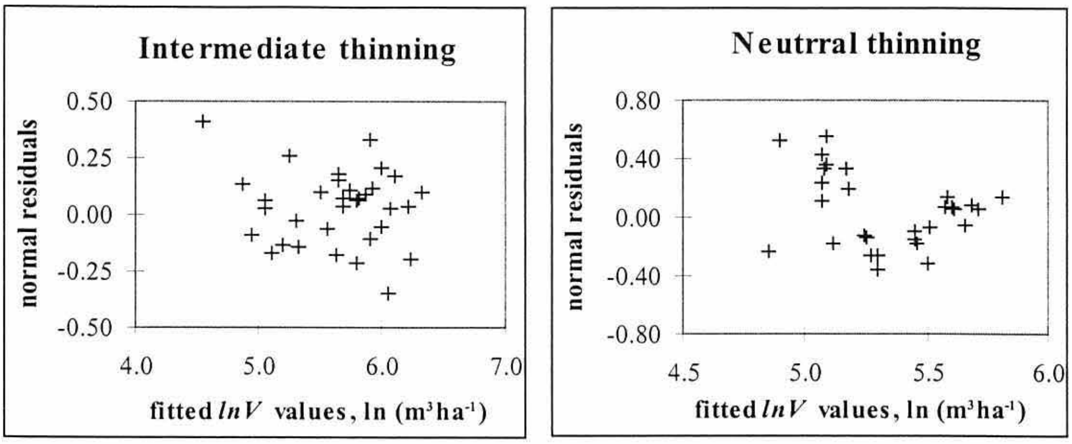


Figure 5.20b: Distribution of the residuals after fitting the volume prediction model *b* to the reserved data.

Model *b*, in which the site index at age 40 was included, was selected for further tests in this study even though it showed lower values for R^2 and modelling efficiency than model *a* for neutral thinning. The reason for not selecting the model *a* (one with the site index at age 20) was that at age 20, some pine plantations have only just passed the age of first thinning, and at this stage, the number of removed trees could be very high leaving much space for the growing trees. Also the thinning type may not be regular at this stage (Jenkins, pers. comm.). Therefore, reliable top height values may not have been measured at age 20 and taking the site index at age 40 indicates a more solid reference point about the site.

5.5 Discussion on re-calibration of selected models

Most of the re-calibrated models in this chapter do not predict the same variables as predicted by the newly constructed models in Chapter 4. The difference is that these re-calibrated models predict stand-level variables while the new growth and yield models of Chapter 4 predict tree-level variables. However, a comparison can be made after predicting the individual tree variables using the new models, and calculating the particular variables for a unit area. It was difficult to find empirical models which predict the tree-level variables from past work which would fulfil the requirements described in section 5.2.

There are many difficulties to be faced when a non-linear model is transformed to its linear form, i.e. (i) the non-normal distribution of the new error term; and (ii) inaccurate re-estimation of the precision of some of the original parameters (Kassab, 1987). Therefore all the non-linear models were fitted to the data without changing the non-linear form. Fitting non-standard non-linear models could not be done using standard GENSTAT algorithms. Therefore, GENSTAT programs were newly written for this work.

5.5.1 Testing the predictive ability of models

For the constructed new models in Chapter 4, lack of fit was tested using the procedure described by Weisburg (1985). All the selected models for the re-calibration in this chapter were constructed or developed by well-experienced modellers for wide use and therefore, lack of fit was unlikely to be an issue for these models. Because of this reason, it was not tested for the selected models. However, quantitative tests were used to indicate bias and also to compare the predictive ability of one model for the two thinning types or to compare two or more models which predict the same variable in one thinning type.

5.5.2 Estimation of parameters

Many of the models re-calibrated in this chapter contained parameters with different signs from the original structures. This situation can be statistically explained, but theoretically may not be correct. The main reason for such a difference may be because the original models were adapted from three different countries having entirely different climates from Great Britain.

The qualitative and quantitative results, together with the R^2 values, indicated good results for all the models except the basal area prediction model developed by Pienaar and Harrison (1989) re-calibrated for the intermediate thinning type. As emphasised earlier in this chapter, the rate of removal of trees in each thinning was very high for the plots which were maintained under a neutral thinning, sometimes over three times the marginal thinning intensity. This could dramatically affect the rate of growth of the remaining trees in plantations.

From the beginning of the current study, the intention was to construct the best and the simplest models to predict the particular variables. The same objective was applied to the selected models for re-calibration. Therefore, whenever parameters were found statistically non-significant after re-calibrating, all the possible variations of models were tested with the qualitative and quantitative tests and R^2 in order to find the simplest model. However sometimes the parameters which were statistically not significant were included in some models because either bias resulted if they were removed or the non-significant parameters for one thinning type were statistically significant for the other.

CHAPTER 6: TESTING FOR COMMON PARAMETERS FOR NEUTRAL AND INTERMEDIATE THINNING TYPES

6.1 Introduction

As stated earlier, the data obtained from the Forestry Commission in Great Britain contained only enough Corsican pine data for analysis of intermediate and neutral thinning types. However, estimating separate parameters for the separate thinning types sometimes creates major problems for the user, especially when converting the plantation or estate from one thinning regime to another. To overcome this disadvantage, the possibility of using one set of parameters for each model for both thinning types was tested. Although all the models selected from past work were developed as common models for thinned plantations (e.g. Pienaar and Harrison, 1989; Soares *et al.*, 1995; West and Mattay, 1993), the same data partition was used in all cases so as to be compatible with the newly constructed models in Chapter 4. Given that all the re-calibrated models were originally designed for all thinning types (thinning types were not mentioned in the original papers), it was felt entirely appropriate to attempt re-parameterization on separate data sets.

The two thinning types used for the building and re-calibrating of the models contain different numbers of data sets. The documented methods for testing the possibility of constructing a common model out of similar regression equations containing different parameter sets are very few. Therefore a simple t-test was identified to use for this purpose using the normal residuals. Another method was developed by McRoberts (1988) but this requires that the number of data for both sets should be similar and therefore this test was not used for the current work. For the two-sample t-test, the number of data in the two samples do not necessarily have to be similar.

6.2 Methods

6.2.1 Testing of the significance of the parameters of volume prediction models for two thinning types

For the newly constructed total volume and merchantable volume prediction models (Chapter 4) and the re-calibrated total volume prediction model developed by Soares *et al.* (1995), the parameters were estimated for each age. Therefore, before pooling the data for both thinning types, the significance of the difference of the parameters at each age for the two thinning types was tested using the following procedure.

A two-sample t-test was done for the parameters from each model using the procedure described below:

$$t = \frac{(\bar{x}_{in} - \bar{x}_{neut})}{\sqrt{\dfrac{s^2(n_{in} + n_{neut})}{(n_{in})(n_{neut})}}} \qquad 6.1$$

where:

$\bar{x}_{in}$ = arithmetic mean of the parameters estimated for intermediate thinning

$\bar{x}_{neut}$ = arithmetic mean of the parameters estimated for neutral thinning

n_{in} = number of data of intermediate thinning

n_{neut} = number of data of neutral thinning

s^2 = variance for the pooled data

(Freese, 1990)

The variance for the pooled data was calculated using the following method:

$$SS_{in} = \Sigma x_{in}^2 - \frac{(\Sigma x)^2}{n_{in}} \qquad 6.2$$

$$SS_{neut} = \Sigma x_{neut}^2 - \frac{(\Sigma x)^2}{n_{neut}} \qquad 6.3$$

$$s^2 = \frac{SS_{in} + SS_{neut}}{(n_{in} - 1) + (n_{neut} - 1)} \qquad 6.4$$

where: SS_{in} = corrected sum of squares of parameters of intermediate thinning
SS_{neut} = corrected sum of squares of parameters of neutral thinning

(Freese, 1990)

The calculated t-value for the degrees of freedom $(n_{in} - 1) + (n_{neut} - 1)$ was compared with the tabulated t-value at 0.05 probability level. The null hypothesis was that there was no significant difference between the parameters calculated for intermediate and neutral thinning types. If the calculated t-value was lower than the tabulated value (if the null hypothesis was accepted) it was confirmed that the parameters estimated (for each age class) were not significantly different for the two thinning types. Then the data at each age were pooled separately for the two thinning types in order to estimate the new parameters common for all ages for separate thinning types. Finally the tests described below were done to examine the validity of adopting one set of common parameters for both thinning types.

6.2.2 Testing of the common parameter values

6.2.2.1 Significance of the normal residuals

First, the selected models identified in Chapter 4 and Chapter 5 were fitted to the pooled data for both thinning types to obtain a new set of parameters. When a parameter was required not to be significantly different from one, the possibility was tested using equation 4.49 (page 90). If that particular parameter was significant, it was manually forced to be one using the procedure described in equations 4.50 - 4.53 (page 90-91).

The resultant models, with common parameters, were then compared with the data for intermediate and neutral thinning types. For this comparison, the normal residuals for each thinning type were calculated using the following equation:

$$\varepsilon_i = y_i - \hat{y}_i \qquad 6.5$$

where: ε_i = error of the individual observation

y_i = observed ith value

$\hat{y}_i$ = predicted ith value from the model

Finally the two sample t-test (6.1) was done for the residuals of both thinning types using the null hypothesis that there was no significant difference between the normal residuals when the model from pooled data was fitted to the thinning types separately.

6.2.2.2 Distribution of normal residuals versus fitted values

Even if all the residuals were negatively or positively biased for both thinning types, they might still be statistically non-significant when the t-test is applied. Therefore, when the null hypothesis was accepted, the common model was fitted separately to the two thinning types and then the distribution of the normal residuals with the fitted values was observed to visually identify any bias.

6.3 Results

6.3.1 Models newly constructed for this work

6.3.1.1 Significance of the parameters in volume prediction models for the intermediate and neutral thinning types

Results of the two-sample t-test described in section 6.2.1 for the volume prediction models constructed in Chapter 4 are given in Table 6.1. All the parameters in the total volume prediction models *a* and *b* and the merchantable volume model *a* and *b* were not statistically significant for the intermediate and neutral thinning types (Table 6.1).

Model	Parameter	Calculated t-value	Degrees of freedom	Significance
Total volume				
Model *a*	b	1.93	55	NS
Model *b*	c_0	0.27	55	NS
	c_1	0.02	55	NS
	c_2	0.10	55	NS
Merchantable volume				
Model *a*	b	0.10	55	NS
Model *b*	c_0	0.87	55	NS
	c_1	1.31	55	NS

None of the t-values were significant at 0.05 probability level.

Table 6.1: Calculated t-values for each parameter in volume prediction models.

6.3.1.2 Calculated t-values for residuals

The resultant t-values for the residuals of both thinning types after fitting the common parameters are given in Table 6.2 below.

Model	Calculated t-value	Degrees of freedom	Significance
Diameter at breast height			
Model *a*	10.05	7475	*
Model *b*	14.46	7475	*
Total height			
Model *a*	06.22	754	*
Model *b*	05.95	754	*
Timber height			
Model *a*	07.28	4084	*
Model *b*	05.84	4084	*
Total volume			
Model *a*	01.55	4072	NS
Model *b*	10.60	4072	*
Merchantable volume			
Model *a*	06.38	4072	*
Model *b*	00.44	4072	NS

* significant at 0.05 probability level.

Table 6.2: Calculated t-values for the residuals obtained after fitting the models contained common parameters to both thinning types.

Only model a of the total volume prediction model and model b of the merchantable volume prediction model indicated the possibility of using common sets of parameters for both thinning types (Table 6.2). Therefore, only these two models were selected for the residual tests.

Parameter b in total volume prediction model a represents the form factor. Having a value of 0.50 indicated the shape of individual Corsican pine tree stems approximates a paraboloid. Because of the non-significance of the difference in this parameter, it can be assumed that this is more stable in the total volume prediction model a than the parameters in model b. The merchantable volume prediction model b was a derivation of the total volume model a. This could be the reason for the greater stability of parameters c_0 and c_1 in that model than in the merchantable volume prediction model a. The less stability of the merchantable volume prediction model a could be due to the application of the assumption that the tree stem of Corsican pine is a paraboloid, instead of an approximation of a paraboloid.

6.3.1.3 Estimated new common parameters for all ages for the selected models

(i) Total volume prediction model *a*

The model constructed for predicting the total volume of individual trees in Chapter 4 is:

$$v = b * \left(\frac{\pi * dbh^2}{40000} \right) * h \qquad 6.6$$

The newly estimated common parameter for all the ages for the above model is:

Parameter	Estimate	Standard error	R^2
b	0.5040	0.0004	0.995

(ii) Merchantable volume prediction model *b*

The merchantable volume prediction model *b* for individual trees constructed in Chapter 4 is:

$$v_{mer} = c_0 + c_1 * \left\{(g * h) - \left(\frac{\pi * 49.0}{40000} * \left(\frac{h - h_{mer}}{3}\right)\right)\right\} \qquad 6.7$$

Common parameters estimated for the two thinning types covering all ages are:

Parameters	Estimate	Standard error	R^2
			0.995
c_0	-0.0038	0.0004	
c_1	0.5061	0.0005	

After selecting the total volume model *a* and merchantable volume prediction model *b*, these two models were fitted to the data separately, but with the common set of parameters, in order to observe the distribution of the residuals. The results are shown in Figures 6.1 and 6.2.

When comparing the normal residuals obtained after fitting the model with common parameters, the residual distributions were more scattered for the intermediate thinning type for both total and merchantable volume prediction models (Figure 6.1 and Figure 6.2). If the distribution is considered without the magnitude of the residuals taken into account, total and merchantable volume models fitted to the two thinning types indicated a reasonable fit. The use of both models is made more convenient by having one set of parameters instead of one set for each age or thinning type. Therefore, from all the volume prediction models newly constructed in Chapter 4, total volume prediction model *a* and merchantable volume prediction model *b* were shown to be the most appropriate for use in the field.

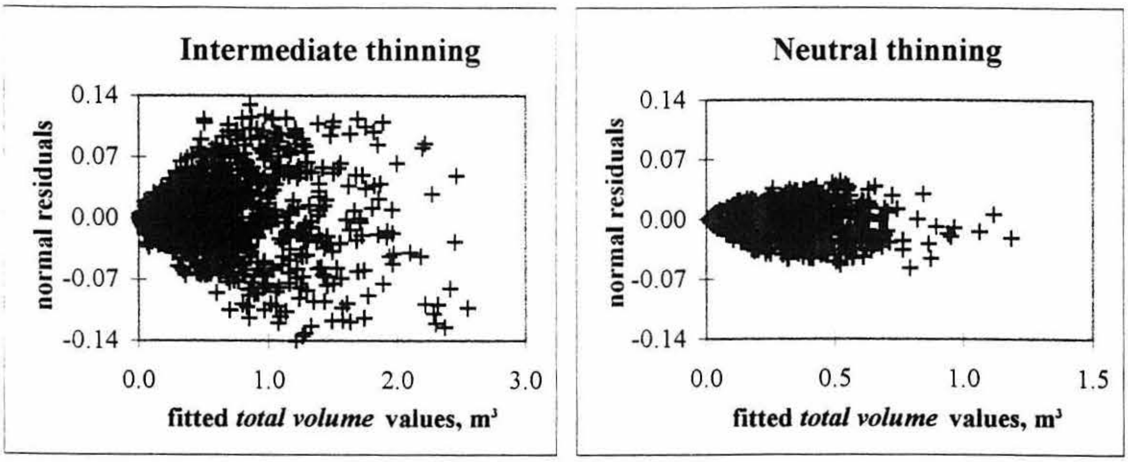


Figure 6.1: Normal residual distribution of the newly constructed total volume prediction model *a* after fitting with the common parameter.

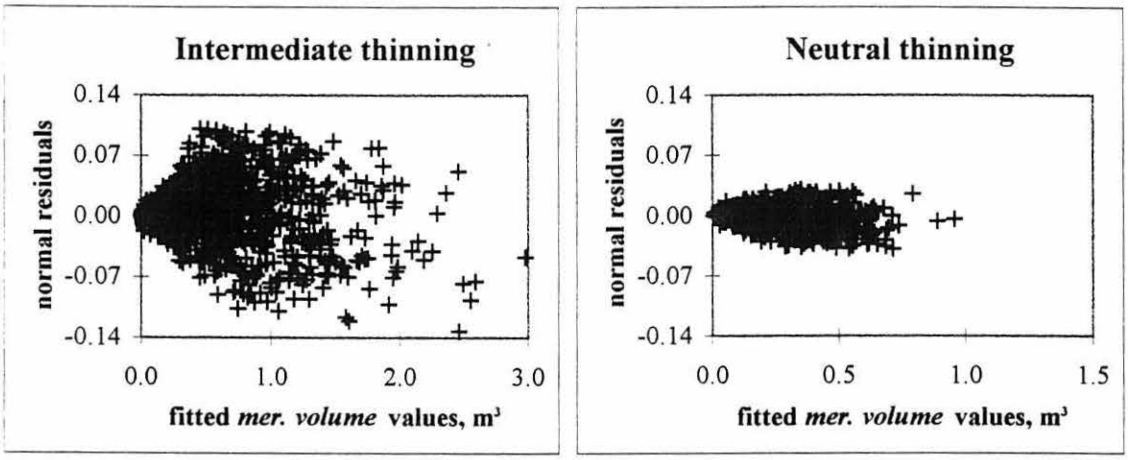


Figure 6.2: Normal residual distribution of the newly constructed merchantable volume prediction model *b* after fitting with the common parameters.

6.3.2 Re-calibrated models

6.3.2.1 Models developed by Pienaar and Harrison (1989)

These authors developed compatible equations for both prediction and projection of total basal area and total volume. However, only the differences between parameters of the basal area projection model and the total volume prediction model were not statistically significant for both thinning types in this study (Table 6.3).

Model	Calculated t-value	Degrees of freedom	Significance
Basal area prediction Model *b*	5.12	269	*
Basal area projection Model *a*	0.10	94	NS
Volume prediction	0.83	175	NS
Volume projection Model *a*	2.66	72	*

* significant at 0.05 probability level.

Table 6.3: Results of the two-sample t-tests of the re-calibrated models initially constructed by Pienaar and Harrison (1989).

(i) Basal area projection model

The selected basal area projection model after the re-calibration in Chapter 5 is:

$$\ln G_2 = \ln G_1 + b_1 * \left(\frac{1}{A_2} - \frac{1}{A_1}\right) + b_2 * (\ln N_1) + b_3 * (\ln h_{dom(2)} - \ln h_{dom(1)})$$
$$+ b_4 * \left(\frac{\ln N_1}{A_2} - \frac{\ln N_1}{A_1}\right) + b_5 * \left(\frac{\ln h_{dom(2)}}{A_2} - \frac{\ln h_{dom(1)}}{A_1}\right) + b_6 * \left(\frac{N_t}{N_a}\right) \quad 6.8$$

Newly estimated common parameters for the above model are:

Parameters	Estimate	Standard error	R^2
			0.911
b_1	-129.9000	46.6000	
b_2	0.0175	0.0209	
b_3	0.2100	0.2770	
b_4	15.7200	4.6900	
b_5	-6.7000	7.5800	
b_6	0.0006	0.0140	

Distribution of the normal residuals with the fitted values

The distribution of the normal residuals estimated after fitting the model with common parameters to both thinning types separately, indicated little over-estimation (Figure 6.3). However, this model did not work well even with separate parameter sets for intermediate and neutral thinnings in Chapter 5. Therefore, it was decided to use the common parameter set estimated in this chapter. The different parameter sets estimated for the same model in chapter 5 and in this chapter indicated different magnitudes providing an idea of the high sensitivity of the model to different data sets.

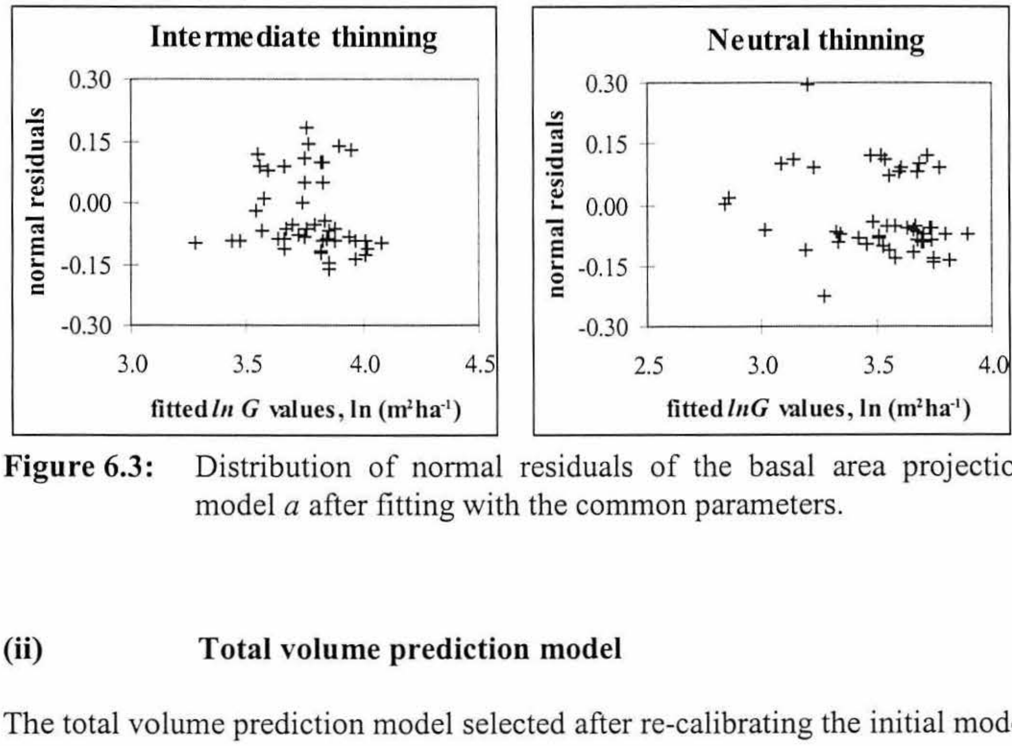


Figure 6.3: Distribution of normal residuals of the basal area projection model a after fitting with the common parameters.

(ii) Total volume prediction model

The total volume prediction model selected after re-calibrating the initial model constructed by Pienaar and Harrison (1989) is:

$$\ln V = a_0 + a_1 * (\ln N) + a_2 * (\ln h_{dom}) + a_3 * (\ln G) \qquad 6.9$$

The estimated common parameters for both thinning types for the above model are given below:

Parameters	Estimate	Standard error	R^2
			0.733
a_0	2.2510	0.6160	
a_1	-0.3685	0.0558	
a_2	0.1880	0.1270	
a_3	1.4680	0.1060	

Distribution of the normal residuals with the fitted values

The distribution of the normal residuals obtained after fitting the model with common parameters separately to the two thinning types (Figure 6.4) indicated similar results to those obtained after fitting the model with different parameters for two thinning types in the previous chapter (Figure 5.5). Therefore, the new common set of parameters was selected for use.

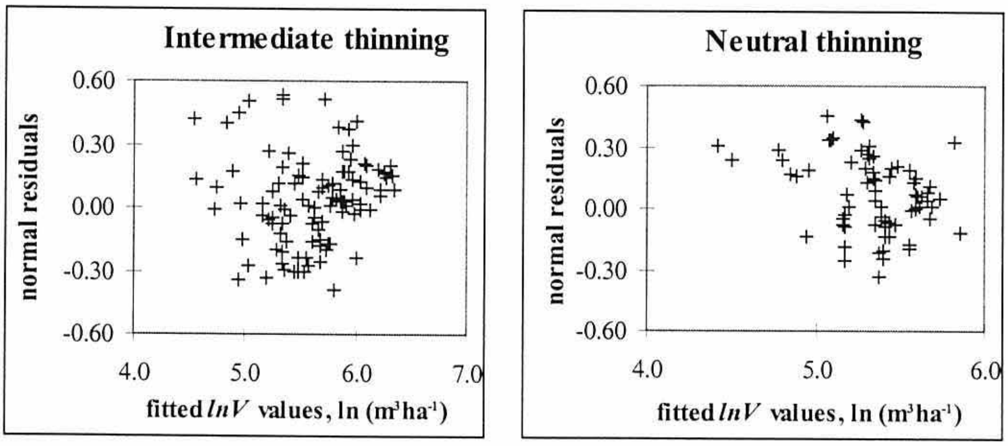


Figure 6.4: Normal residuals of total volume prediction model for both thinning types after fitting with common parameters.

6.3.2.2 Models developed by Soares *et al.*(1995)

(i) Significance of the parameter in the volume prediction model for the intermediate and neutral thinning types

As for the newly constructed and developed total and merchantable volume prediction models, the parameter b_0 in the total volume prediction model developed by Soares *et al.* (1995) was not statistically significant for both thinning types when the two sample t-test was applied (Table 6.4).

Parameter	Calculated t value	Degrees of freedom	Significance
b_0	1.55	55	NS

Table 6.4: The calculated t-value for the parameters estimated for each age for the intermediate and neutral thinning types.

(ii) Calculated t-values for the residuals

The difference of the parameters in the model selected for the prediction of total tree height were statistically significant while the other two models were not (Table 6.5). Therefore only the total volume prediction and total basal area prediction model were taken forward to test the distribution of residuals.

Model	Calculated t value	Degrees of freedom	Significance
Total height Model *b*	2.67	754	*
Total volume	1.55	4072	NS
Total basal area	0.18	94	NS

* significant at 0.05 probability level

Table 6.5: Results of the two sample t-test applied for the models developed by Soares *et al.* (1995).

(iii) Total volume prediction model

The selected model for the prediction of individual tree volume after re-calibrating the initial model developed by Soares *et al.* (1995) is:

$$v = b_0 * \left(\frac{\pi * dbh^2 * h}{40000} \right) \qquad 6.10$$

The common parameter for both thinning types for all ages for the above model is:

Parameter	Estimate	Standard error	R^2
b_0	0.5041	0.0004	0.995

Distribution of the normal residuals with the fitted values

The distributions of the normal residuals calculated for the intermediate thinning was more scattered than for the neutral thinning (Figure 6.5). However, comparing Figure 6.5 with the results obtained from the same model when fitted separately using separate parameters for each age in Chapter 5, the common parameter was believed to be adequate to predict the total volume of individual trees at any age if the diameter at breast height and total height is known.

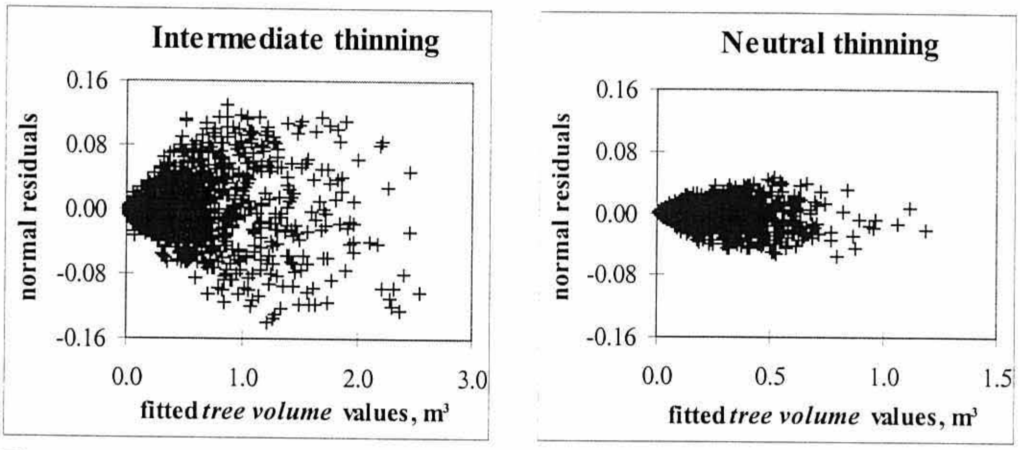


Figure 6.5: Residual distribution of the volume prediction model with the common parameter.

(iv) Total basal area prediction model

The selected model in Chapter 5 to predict the total basal area is:

$$G_2 = G_1^{A_1/A_2} e^{(1-A_1/A_2)\,(c_1+c_2 * h_{dom})} \qquad 6.11$$

Estimated common parameters for the above model are:

Parameter	Estimate	Standard error	R^2
			0.895
c_1	4.6920	0.1630	
c_2	0.0170	0.0106	

Distribution of the normal residuals with the fitted values

The distributions of the residuals observed in Figure 6.6 were very similar to the distributions observed in Figure 5.13 in Chapter 5 when fitted with the separate parameters. The magnitudes of parameters c_1 and c_2 were also very similar when estimated separately for the two thinning types and estimated for the pooled data. Therefore, the newly estimated common parameters in this chapter were selected for use.

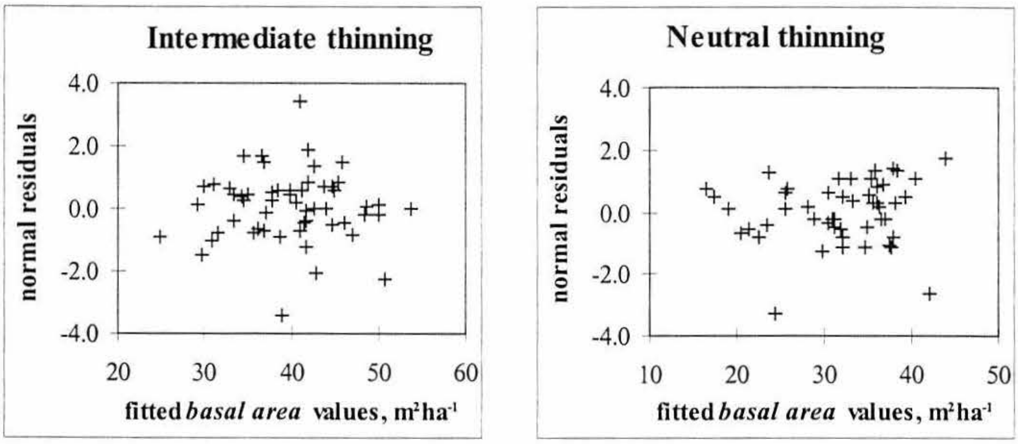


Figure 6.6: Distribution of normal residuals after fitting the basal area prediction model with the common parameters.

6.3.2.3 Models developed by West and Mattay (1993)

Differences of the parameters of both the total tree height prediction model and the derivation of the total volume prediction model built by West and Mattay (1993) were statistically non-significant for intermediate and neutral thinning types thus indicating the robustness of the parameters (Table 6.6).

Model	Calculated t value	Significance
Total height	0.32	NS
Total volume		
Model *b*	0.36	NS

None of the models were significant at 0.05 probability level

Table 6.6: Calculated t-values for the models built by West and Mattay (1993).

(i) Total height prediction model

The resulting model after re-calibration in Chapter 5 is:

$$h = 1.3 + dbh / (p + q * dbh) \qquad 6.12$$

The estimated common parameters for the above model are:

Parameter	Estimate	Standard error	R^2
			0.760
p	1.8103	0.0399	
q	-0.0003	0.0013	

Distribution of the normal residuals with the fitted values

Very similar residual distributions were obtained when this model was fitted with separate parameters (Figure 5.17) and common parameters (Figure 6.7) to intermediate and neutral thinning types. Therefore, the new set of parameters was selected for use from this point on.

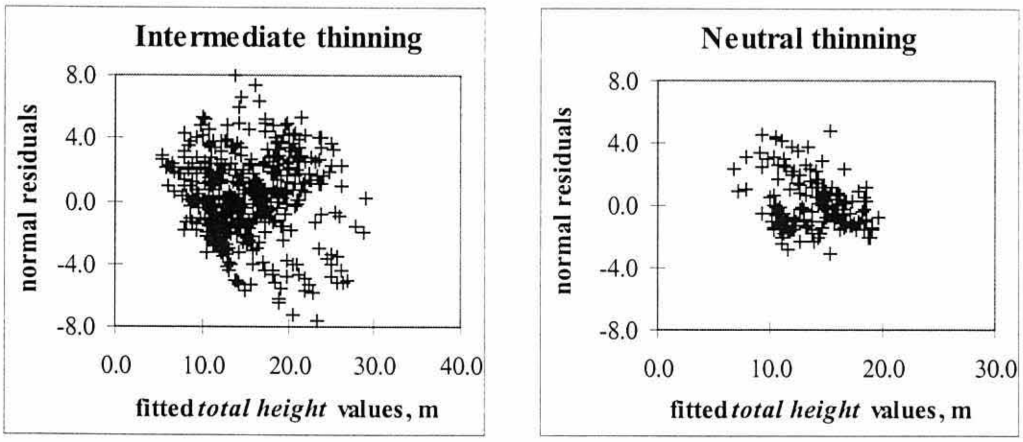


Figure 6.7: Normal residuals of the total height prediction model when fitted to the data with common parameters.

(ii) Total volume prediction model *b*

The selected total stand volume prediction model in Chapter 5 is:

$$\ln V = b_1 + b_2 * \frac{1}{A} + b_3 * SI_{40} + b_4 * D_s \qquad 6.13$$

The newly estimated common parameters in this chapter for the above model are:

Parameter	Estimate	Stand error	R^2
			0.819
B_1	5.2250	0.2010	
b_2	-34.9500	1.9500	
b_3	0.0344	0.0079	
b_4	0.0180	0.0027	

Distribution of the normal residuals with the fitted values

The distribution of the residuals for the both thinning types when the model was fitted with separate sets of parameters (Figure 5.19b) and common set of parameters (Figure 6.8) were very similar. This indicates that there is no harm in using the common parameters for intermediate and neutral thinning types and also indicates the robustness of the model for different data types.

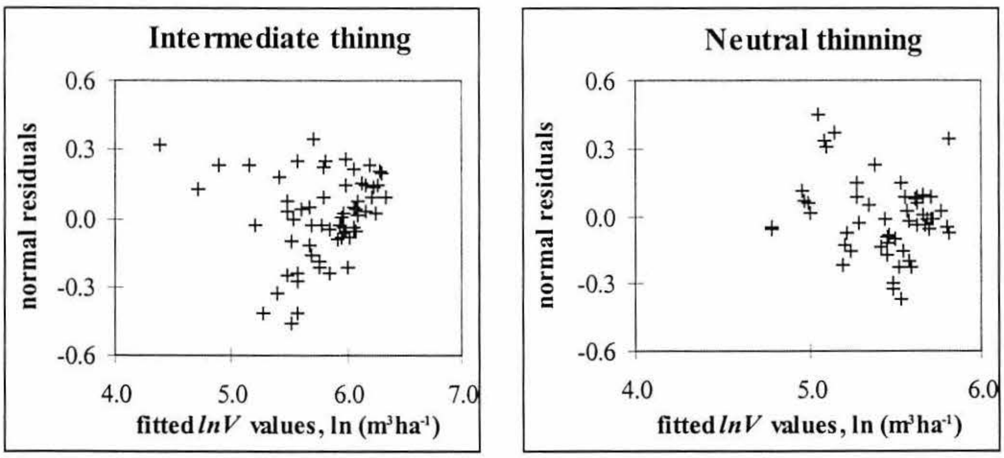


Figure 6.8: Distribution of normal residuals of the total volume prediction model with common paramters.

6.4 Conclusions for the testing of common parameters for the intermediate and neutral thinning types

6.4.1 Newly constructed models

According to the tests used in this chapter, only the total volume prediction model *a* (6.6) and the merchantable volume prediction model *b* (6.7) present the possibility of using the same set of parameters for intermediate and neutral thinning types. Therefore, these two models were selected for use in future work. The other two models developed for volume predictions i.e. total volume prediction model *b* and merchantable volume prediction model *a* were rejected.

6.4.2 Re-calibrated models

The basal area projection model and total volume prediction model developed by Pienaar and Harrison (1989), the total volume prediction model and total basal area prediction model developed by Soares *et al.* (1995) and both total height and the volume prediction models developed by West and Mattay (1993) were possible to use with common sets of parameters for the intermediate and neutral thinning types. Therefore these common models were selected for future use. The other re-calibrated and selected models which indicated significantly different parameters for intermediate and neutral thinning types are taken forward with separate parameter sets.

6.5 Discussion

It can be argued that as a test to observe the possibility of using one set of parameters, the examination of separate graphs of the residual distribution of two populations after fitting the common model is enough. But this test only indicates a visual impact of the possibility. It was necessary to define some sort of quantitative test. With the intention of fulfilling this requirement, the two sample t-test was used. There is a disadvantage in using only the two sample t-test for the residuals because, even if the residuals are very high, or unevenly distributed within one sample, the result could be statistically non-significant. However, this sort of trend can be easily identified using the residual plots. Therefore both tests were used. Using such tests, not only the possibility for common models, but also a sensitivity of the parameters of each model to different populations can be studied to some extent.

The models constructed in this study to predict the mean sizes of the trees removed in thinning were not tested for the possibility of common parameters because that set of models is clearly dependent on the thinning type.

When the confidence intervals were checked, these were shown to have different ranges for the similar parameters in similar models of the two thinning types except in the two models developed for the prediction of total height of individual trees in Chapter 4. Basically the result of Chapter 6 indicates that most models constructed for this study contain some parameters which have a high sensitivity to different thinning types.

CHAPTER 7: COMPARISON OF THE MODEL PREDICTIONS

7.1 Introduction

The overall performances of the models constructed and re-calibrated were indicated by the tests applied for those models in Chapters 4, 5 and 6. The number of individual data used for the above tests was very high, sometimes over 6000 and the sample data varied in general yield class, initial planting density and site quality. However, when the selected models are applied to a site in the field which has one general yield class and one planting density, the above tests could be brought into question because, if two data sets are biased with opposing signs and similar magnitudes, the test result can indicate a highly accurate model. To avoid such circumstances, one sample plot from each thinning type was selected and the predictive ability of the models compared directly with the observed data. This also allowed comparison of models newly constructed in Chapter 4 with the re-calibrated models in Chapter 5.

However, all the re-calibrated models could not be tested because not all of these models predicted the same variables predicted by newly constructed models, such as timber height and merchantable volume. In such situations, only the predictions of the newly constructed models in Chapter 4 were compared with the observed values.

7.2 Methods used for comparison of model predictions

7.2.1 Selection of sample plots

Only two sample plots were tested in order to reduce the amount of the present study. The sample plots reserved for validation were used in order to obtain independent results because these were not involved in the construction or re-calibration of any model. One out of five sample plots from each thinning type i.e. plot number 1186 for the intermediate thinning type and plot number 1648 for the neutral thinning were randomly selected for the comparison tests.

7.2.2 Comparison of dbh and total height predictions

Dbh and total height models predict future values using variables such as top height and age difference together with the present value of the corresponding variable. Using the first data set in the selected sample plot, the values were predicted at the second measurement time. The predicted data set was used as the second set of diameter and total height at the beginning of the second simulation period and the next set of dbh values were predicted. This procedure was continued until the final data set was obtained at the last measurement. The arithmetic mean values at each time were then calculated using formula 7.1. The total heights of individual trees were also predicted using the re-calibrated models developed by Soares *et al.* (1995), and West and Mattay (1993), and the respective mean values were calculated. Finally all the mean values were compared with the mean observed values at each measurement. There are no re-calibrated models for the prediction of dbh. Therefore predicted dbh values were derived using only the newly constructed models in Chapter 4 to compare with the observed data.

Arithmetic mean of a tree variable is defined as:

$$\bar{y} = \frac{\Sigma y_i}{n} \qquad 7.1$$

where: n = number of data at each measurement

y_i = value of the response variable of ith tree

$\bar{y}$ = mean value of the response variable at each measurement

7.2.3 Comparison of timber height predictions

The timber height prediction models constructed in Chapter 4 are growth prediction models. Timber height was predicted using the total height and dbh at each measurement. The mean observed and predicted timber heights at each measurement were then calculated using formula 7.1. There are no re-calibrated models for the prediction of timber height. Therefore, predicted timber height values derived from only the newly constructed models *a* and *b* were compared with the observed values using line graphs.

7.2.4 Comparison of total volume, merchantable volume and total basal area

There are three re-calibrated models which predict total basal area per hectare i.e. the basal area prediction and projection models developed by Pienaar and Harrison (1989) and the basal area prediction model developed by Soares *et al.* (1995). The newly constructed models do not predict directly the total basal area per hectare. However, a comparison was done with the observed data after calculating the basal area per individual tree from newly constructed dbh prediction models (equation 4.3) and then calculating the value per hectare using the following formula:

$$Y = \frac{\Sigma y_i}{n} * N \qquad 7.2$$

where:
n = number of trees measured
N = number of trees per hectare
y_i = value of the ith tree
Y = total value per hectare

Newly constructed total and merchantable volume models and the total volume model constructed by Soares *et al.* (1995) predict individual tree values. These predictions were converted to per hectare values using formula 7.2 and compared with the observed values and the two other re-calibrated volume prediction and projection models constructed by Pienaar and Harrison (1989).

Observed total basal area and total volume were gathered by the same methods described in Chapter 5. The same procedure was used to calculate the merchantable volume per hectare. For the basal area and volume projection models developed by Pienaar and Harrison (1989) and the basal area prediction model developed by Soares *et al.* (1995), the values at the beginning of the current simulating period were the values predicted by the models for the previous simulating period.

7.3 Results of comparison of model predictions

7.3.1 Diameter at breast height

For the intermediate thinning type, the newly constructed model *b* indicated the closest predictions to the observed data (Figure 7.1). For the neutral thinning type (Appendix 4.1(i)) the newly constructed model *a* was better, but the predictions were very similar. However, taking the results of both thinning types into consideration, model *b* was selected for use in the field because it indicated closer predictions to the observed data in intermediate thinning.

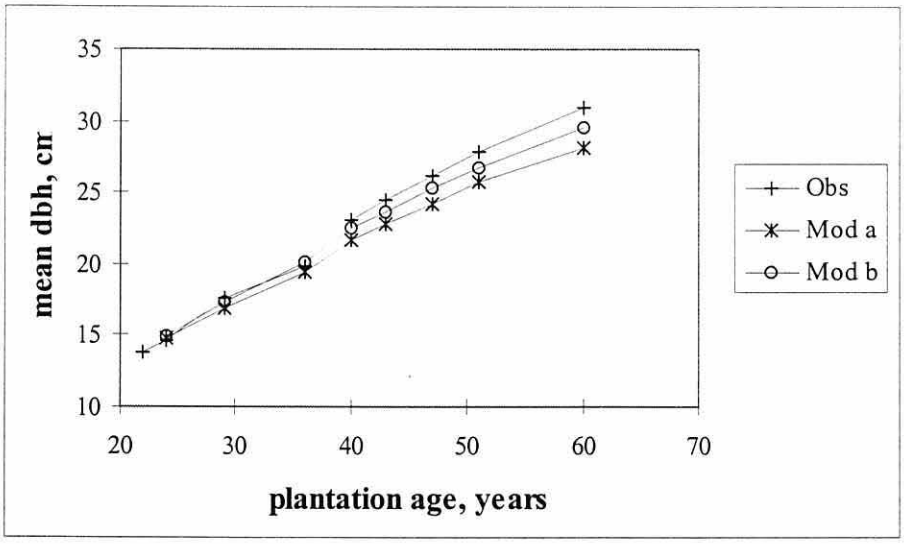


Figure 7.1: Comparison of the dbh predictions with the observed values for intermediate thinning.

7.3.2 Total height

All the tested models predicted total height reasonably well for both the intermediate (Figure 7.2) and neutral thinning types (Appendix 4.1(ii)) until age 40. After age 40 in intermediate thinning, both re-calibrated models constructed by Soares *et al.* (1995) and West and Mattay (1993) started to disperse away from the observed data. However, there were no data available after age 40 in the neutral thinning, so further conclusions cannot be drawn. For both thinning types, the newly constructed total height prediction model *a* and *b* indicated better results than the re-calibrated models. Of the two new models, height prediction model *a* was selected for field use due to its closer fit to the observed data.

Figure 7.2: Comparison of mean total heights predicted by new and re-calibrated models with observed values for intermediate thinning.

7.3.3 Timber height

There are no re-calibrated models for the prediction of timber height. Both the newly constructed timber height models indicated very similar predictions to the observed data (intermediate thinning - Figure 7.3 and neutral thinning - Appendix 4.1(iii)). Therefore, the parameter in both models associated with *dbh*h*, which determines the shape of the curve, was considered (section 4.3.4.2 - page 119). In model *a*, it was less than one while in model *b* it was greater than one. Finally model *a* was selected for use in the field because when the parameter mentioned above is less than one, the function becomes asymptotic, which is compatible with biological realities.

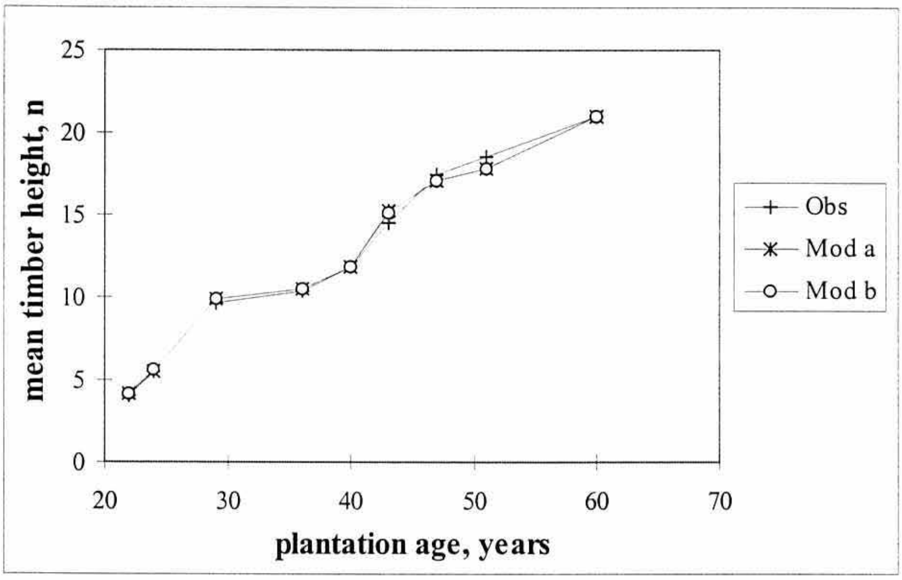


Figure 7.3: Results of comparison of mean timber height values for the intermediate thinning type.

7.3.4 Total volume

Total volume per hectare was calculated from the individual tree prediction models for comparison with the observed data. The distributions of the predictions obtained from the total volume prediction models newly constructed in Chapter 4 and by Soares *et al.* (1995) were very similar to those of observed values for intermediate (Figure 7.4) and neutral thinning (Appendix 4.1(iv)). The models developed by Pienaar and Harrison (1989) and West and Mattay (1993) did not clearly indicate the reduction of total volume in intermediate thinning due to the removal of trees (Figure 7.4). In the neutral thinning type there was a steep decrease in the volume from age 19 to 25 years (Appendix 4.4(iv)). This decrease is due to the removal of a very large number of trees from the plots in the early stages in the neutral thinning type in order to obtain a commercial profit. Unlike for the intermediate thinning type, all the tested models indicated the volume reductions in neutral thinning. However, the best models were the models constructed newly for this study and the model constructed by Soares *et al.* in 1995 which initially predicted the volume of individual trees using the total height and dbh.

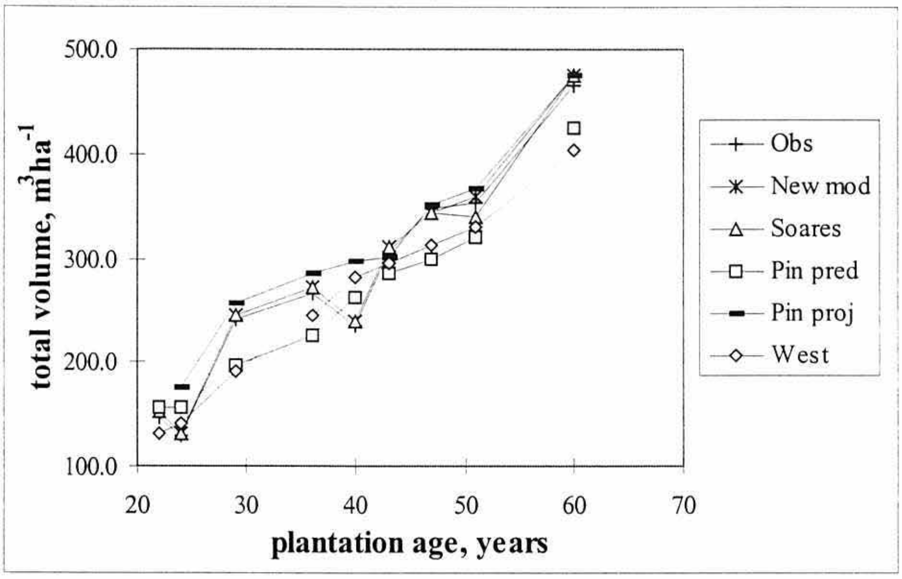


Figure 7.4: Results of the comparisons of total volume predictions with the observed values for intermediate thinning.

7.3.5 Merchantable volume

The newly constructed merchantable volume prediction model indicated very similar predictions to the observed values for the intermediate (Figure 7.5) and neutral (Appendix 4.1(v)) thinning types. There were no re-calibrated models available for the prediction of merchantable volume.

7.3.6 Total basal area

Total basal area per hectare was calculated from newly constructed dbh prediction models in order to compare with the predictions from the re-calibrated models. All the models predicted the total basal area within 3m^2 of the observed values for the neutral thinning type (Appendix 4.1(vi)). However, the predictions were more scattered for the intermediate thinning type (Figure 7.6). The worst predictions for the intermediate thinning type came from the models developed by Pienaar and Harrison (1989). The newly constructed dbh prediction model b was selected to predict the total basal area due to the reasons described in Chapter 7.3.1.

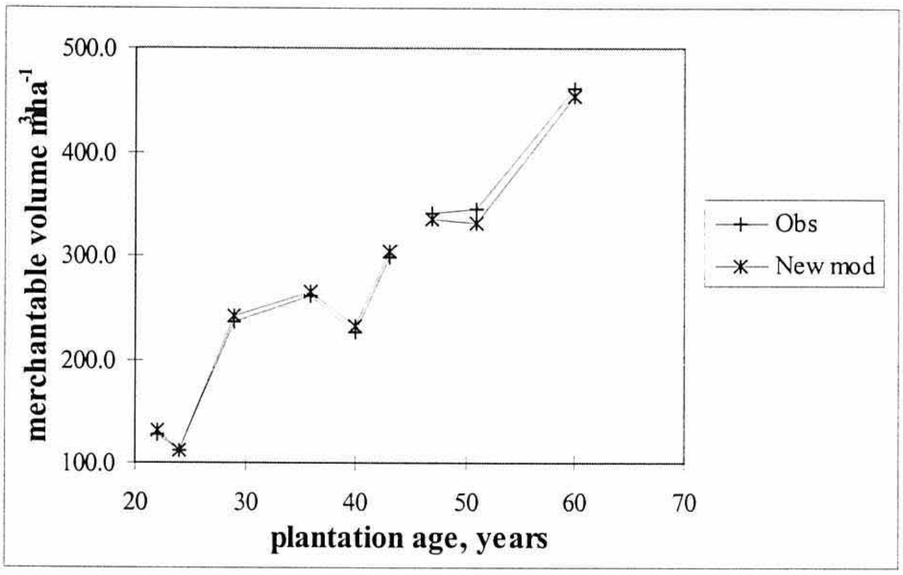


Figure 7.5: Comparison of merchantable volume predictions for intermediate thinning.

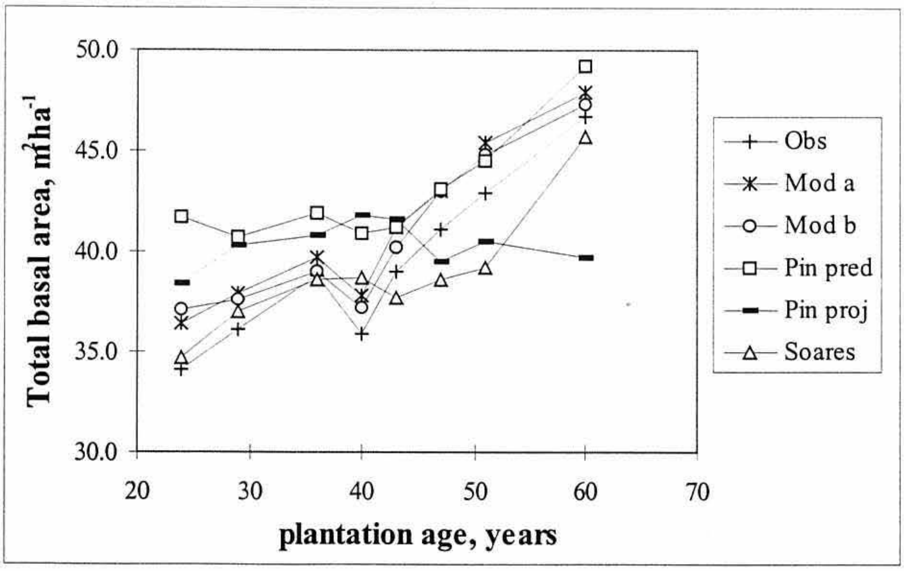


Figure 7.6: Comparison of the total basal area predictions with the observed values for intermediate thinning.

7.4 Discussion concerning comparison of model predictions

In this chapter model comparisons were done using only one sample plot form each thinning type in order to reduce the amount of the thesis. However, the predictions of all the models were tested with all the sample plots reserved for validation, and the examined results were very similar to the results of the two sample plots included in this chapter.

For the construction and re-calibration of the growth models in Chapters 4 and 5 non-overlapping growth intervals were used in order to minimise the correlation of the variables. However, for the model comparisons tested in this chapter, data at every possible measurement cycle were fitted to the models to obtain a higher number of data points. The reason was that a more precise comparison could be carried out with a higher number of data points.

The observed mean values of dbh, total height and timber height were not smoothly distributed with respect to age. When trees are removed in thinning, the competition is reduced and this can increase the growth rate of the remaining individuals. Also, well-grown trees can be removed according to the preference of the forest manager leaving smaller trees on the ground. It is obvious that the removal of trees as thinnings causes dramatic changes in total stand volume, merchantable volume and total basal area per hectare.

All the re-calibrated models indicated a greater dispersion for the tested sample plots with the exception of the total volume prediction model developed by Soares *et al.* (1995). These dispersions may be due to the adoption of the models from different geographical regions without adding new functions or variables. All the tested newly constructed models performed well suggesting confidence in their future field use.

CHAPTER 8: GENERAL DISCUSSION

8.1 Construction of models

The accuracy requirements of forest growth and yield models vary from user to user and may also depend on the levels at which the input variables are set. For a certain set of values for the input variables, the response variable may be large and the acceptable error may also be relatively large. However, for another set of variables the response may be small and the acceptable error may also be relatively small (Reynolds and Chung, 1986).

Sometimes models are constructed to predict only one variable e.g. individual tree volume or stand volume per unit area, following many analyses at each stage to obtain the most precise model (e.g. the work of Gertner, 1987; Mowrer and Frayer, 1986). However, sometimes, a set of growth and/or yield models are constructed or developed for the prediction of many variables using basic statistical analysis. The methods adopted are dependent on the requirements of the modeller or end-user. In this study, a combination of these two procedures was followed to construct a set of precise growth models. A similar procedure was followed by Soares *et al.* (1995) in order to further develop a set of growth models originally constructed by Pascoa (1990).

As described in Chapter 2, process-based models are very much still in the development stage for forest yield and growth predictions, largely due to the difficulties of obtaining some of the measurements, such as the maintenance respiration of stem sap wood, and the senescence rate of fine roots (e.g. Sievanen, 1993; Sievanen and Burk, 1993). Therefore, empirical models still play a major role in forestry. The data obtained from the British Forestry Commission lacked measurements which would be useful for process-based modelling. However, the result of Chapter 4 in the present study indicated that empirical growth models can be constructed to obtain highly precise predictions.

For a good model, the requirements of the accuracy from the starting point of data measurement through to the model fitting are a very important feature. The initial phase of the current study was to examine and filter the data as required for each model from the large number of data sets obtained. For these reasons, some complex computer programmes were written using the FORTRAN language as described in Chapter 3. Throughout the construction and use of these programmes, the results were checked from time to time via manual calculation in order to highlight all possible errors or mistakes.

Stepwise regression is often used by modellers to estimate the parameters for given sets of variables, and to highlight the best combination of variables (Vanclay, 1994). However, for the present study, most of the explanatory variables were selected as essential on the basis of a biological knowledge of forest growth. Therefore, the basic model structures were built before estimating the parameters and fitting the data. So, instead of using stepwise regression, all possibilities were tested, changing one variable at a time and using all possible transformations. This procedure is slower than stepwise regression, but better models can be obtained which are both statistically and biologically compatible.

8.1.1 Prediction of top height

The sample plot data could be grouped into many categories on the basis of thinning type, thinning intensity, plantation age and general yield class. However, for top height prediction, five-year age classes were adopted after testing all the possible partitions. Some modellers did not use such age-wise partition for construction of height prediction models in order to obtain the top height (e.g. Renolls, 1995; Wang and Payandeh, 1995; West and Mattay, 1993). Partition of data into five-year age classes reduced the complexity of the top height modelling procedure in this study. One set of parameters for each age class resulted from this procedure. If the top height prediction model had been constructed for field use, it would have been complicated for the average end-user. However, the reason behind this modelling procedure was to obtain a precise estimate of top height for use in the modelling procedures for other predictions, not for field use.

8.1.2 Prediction of growth variables for main crop trees

Generally, modellers have used data covering many thinning regimes in order to construct growth and yield models (e.g. Pienaar and Harrison, 1989; Soares *et al.*, 1995; Wenk, 1994). In such work, the sensitivity of parameters is difficult to identify unless a detailed validation procedure is carried out. However for the present study, it was decided to partition the sample plot data into sub-sets to obtain better predictions models. In the data used for model construction (27 sample plots), the clearest and most effective possible partition was by thinning type. If the data were divided by general yield class, parameters would need to be estimated for each yield class, and this could confuse the model user. The modelling process would also be more complicated such as the construction of parameter prediction models. Therefore, after partitioning the data only by thinning type, parameters which could be used without knowledge of the general yield class were estimated. Finally the possibility of one model for both thinning types for each particular variable was tested to reduce the ultimate complexity of the model test. However, this aim was unsuccessful on many occasions as described in Chapter 7.

In forestry modelling it is a common procedure to use some assumptions (e.g. the work of Makela, 1988; Sievanen, 1993). The most complex assumptions are made in process-based modelling and further tests are required to test the validity of those (Sievanen, 1993; Thornley, 1991). However, there are occasionally some simple assumptions made which are apparently not tested further (e.g. Soares *et al.*, 1995 on mortality) There were many assumptions used in this study, but the accuracy of these was not tested statistically due to their simplicity. Most of the assumptions (e.g. that the shape of a Corsican pine tree crown is conical; that photosynthetic rate is dependent on tree crown size; that there is no natural mortality if thinning is carried out) were made for the total volume prediction model *a* (Chapter 4). However, these assumptions were ultimately not needed because the variables added as a result of making these were not statistically significant in that model.

All the explanatory variables were selected carefully after observing the correlations and the distributions with the response variables and most of these indicated a good correlation. Stand occupancy is an abstract, multi-dimensional concept used to describe the state of a stand of trees relative to the resource capital of the site. While technically feasible to quantify, the many dimensions of resource consumption negate its use as a practical measure of stand occupancy (Dean and Baldwin, 1996). Consequently, foresters (e.g. Nystrom and Gemmel, 1988; Tang *et al.*, 1994) tend to use indirect measures, such as density indices, to quantify the stand occupancy. However, when total number of stems or total basal area per hectare was tested as subsidiary variables to represent the competition, these variables were found to be not statistically significant. This may be due to the combination of the selected essential variables for the constructed models. However, some modellers (Pienaar and Harrison, 1989; Soares *et al.*, 1995) used total tree number and total basal area per hectare successfully in their models.

For the construction of the new models described in Chapter 4, four transformations were used i.e. logarithmic, square, square root and reciprocal. These transformations can be biologically explained and have been used in the past by various modellers (e.g. Nystrom and Gemmel, 1988; Pienaar and Harrison, 1989; West and Mattay, 1993). Other possible transformations, such as arcsine were not used in the current study because of the incompatibility with biological reality.

All the finally selected newly constructed models are satisfactory in form, meeting both statistical and biological assumptions. For any kind of regression model one should first observe the R^2 value although it is not a very good indicator of model performance (Draper and Smith, 1981) and also the distribution of the residuals before using other tests. These initial tests indicated the high performance of all the newly constructed models. All the models had low bias and a high modelling efficiency over 0.9. The models re-calibrated in Chapter 5 did not indicate such an accuracy except the model developed by Soares *et al.* (1995) to predict the individual tree volume. The signs of estimated parameters were all compatible with the possible biological explanations. The main reason for this could be the careful formulation of the basic model structures before estimating the parameters. However, there is an indication of bias in the dbh prediction models for the neutral thinning type. This could be due to re-adjusting the parameter associated with dbh at time t_1. In such cases, the parameters associated with the other explanatory variables in the model may have to be adjusted thus changing the slope and intercept slightly, and this can cause bias.

8.1.3 Prediction of variables removed in thinning

Models constructed for the prediction of mean variables for trees removed in thinning were simple and only one explanatory variable was used for each model, the response variable but just before thinning. This set of models indicate the relationships of the thinned and main crop trees for separate thinning types after first thinning. As Hart (1994) described, thinning type is highly related to the size of trees in the stand and therefore the prediction models of distribution of these variables were not constructed. In the present study, the number of trees removed in thinning or standing trees after thinning, can be estimated using the procedure described in section 4.2.2.6 and therefore prediction models of tree distribution were not constructed. However, the general procedure developed for the prediction of number of trees removed in thinning may be biased if non-natural mortality occurred due to fire, wind throw etc. To overcome this problem, some modellers (Jenkins, pers. comm.; Vanclay, 1994) emphasised the requirements of more stochastic type models.

8.2 Re-calibration of models

It was interesting to find that the re-calibrated models in Chapter 5 still needed much work such as adding more variables or functions no matter how well the parameters were re-estimated. Re-calibration is a procedure which should be done very carefully no matter how well the models were constructed in their original locations. This was proved as necessary in the present study. Even if the theory and the formulation of the basic model structures are accurate and acceptable, Alder (1978) found some model parts may require modifications, such as the development of local growth functions within the existing framework, to improve accuracy.

8.3 Observation of model performance

Both qualitative and quantitative tests were used in this study for two reasons i.e. to observe the fit of the model to the data and to compare the performance of models for each data set. Tests such as average model bias and modelling efficiency, can also be expressed as percentages, but this was not done in this study because, for comparison, a proportion in the range 0-1 was equivalent and the significance of lack of fit of the models was tested by a specific test. There were other quantitative tests which can be used for comparison of model performances such as fractional variance, root mean square error, index of agreement and alternative index of agreement (Chhetri and Fowler, 1996a; Janssen and Heuberger, 1995). However, following the work done by Soares *et al.* (1995) on maritime pine, the three quantitative tests used in Chapters 4 and 5 were believed robust enough to compare the ability of the models to predict the same variable as well as model performance.

As stated earlier, the data for 49 sample plots in Great Britain obtained from the Forestry Commission, allowed only the modelling of variables for intermediate and neutral thinning types. There were a few sample plots which were maintained under other thinning regimes such as very low, crown, exploitation and also unthinned. For the originally estimated parameters for the total volume and basal area prediction and projection models developed by Pienaar and Harrison (1989), entirely different magnitudes and signs could be observed for thinned and unthinned plantations. In the present study, it was assumed that there is no mortality if thinning is carried out. This assumption was proved to be right by inspection of the Forestry Commission data. However, this assumption is wrong for unthinned plantations because of the inevitable self-thinning. The results may have been very different, as well as interesting, if it had been possible to estimate the parameters for unthinned sample plots.

All three existing models were re-calibrated using the data obtained from the Forestry Commission. The sensitivity of parameters of the newly constructed models can be tested if these models are re-calibrated to a different geographical location. This will be done as the next step by re-calibrating them to the pine plantations grown in Sri Lanka.

CHAPTER 9: GENERAL CONCLUSION

9.1 Conclusions drawn from the present study

The conclusions drawn from the present studies are:

(i) All the models selected in Chapter 4 for the prediction of main crop and thinned tree variables indicated a reasonable distribution of the residuals with the fitted values, a negligible bias and very high modelling efficiency. Therefore all the selected models appear highly satisfactory for future use in the field.

(ii) The factors used for representing site quality were different in each of the finally selected dbh and total height prediction models of individual trees. The factor *total basal area/plantation age* was more suitable for the dbh prediction models while *top height/age* was best for the total height prediction models. Even though the initial attempt was to represent the site quality using only top height related functions, *total basal area/plantation age* was selected for the dbh prediction model, assuming total basal area can represent competition and site quality in different plantations, if the planting density is the same.

(iii) The dbh and total height prediction models of individual trees are the only models constructed in this work to predict future growth. The other models predict the current growth using dbh and total height. All the factors added to represent site quality and competition were either not statistically significant or did not improve the models whenever tested for the prediction models of current growth. However, predicting the future growth of tree variables using the current growth models is not difficult, because these models use total height and dbh as explanatory variables and these can be predicted by the dbh and total height prediction models.

(iv) The maximum age difference used for dbh and total height prediction models was 10 years. However, the recommended maximum projection length for these two models is 7-8 years in order to reduce the bias which could be introduced with a change of growth rates within a long period of growth such as 10 -15 years.

(v) The re-calibrated models did not produce better results than the new set of models when tested for bias and modelling efficiency However, the models developed by Soares *et al.* (1995) and West and Mattay (1993) indicated better predictions than the set of models developed by Pienaar and Harrison (1989). The reason could be that Pienaar and Harrison (1989) developed compatible prediction and projection stand level models and the re-calibrating was probably not good enough without estimating new functions.

(vi) When all the models were tested for the possibility of using one set of parameters instead of separate sets for intermediate and neutral thinning types, some of the models confirmed the possibility while some produced negative results. If the estimated parameters were robust they would be less sensitive to the different data sets. Only two newly constructed models i.e. total volume prediction model *a* and the merchantable volume prediction model *b* indicated the possibility of using common parameters for both thinning types. Among the re-calibrated models, the basal area projection and total volume prediction models developed by Pienaar and Harrison (1989), the total volume prediction model of individual trees and the total basal area prediction model developed by Soares *et al.* (1995) and both the total height and total volume prediction models constructed by West and Mattay (1993) indicated the possibility of using one set of parameters for intermediate thinning and neutral thinning types.

(vii) The direct comparisons of the model predictions with the observed values were done for a sample plot for each thinning type to examine further the model behaviour for sub-sets of the populations. Results again confirmed that the best models were the new models constructed for the current work. Some of the re-calibrated models, such as the volume prediction model developed by Soares *et al.* (1995) provided good results while some models were poor e.g. the basal area projection model developed by Pienaar and Harrison (1989).

(viii) There are many empirical models found in plantation forestry, which have complex equations, such as non-standard non-linear relationships, to predict the same variable predicted by the constructed growth models in this study; e.g. Pienaar and Harrison (1989); Soares *et al.* (1995); Wenk (1994). However, this study proved that if the assumptions and relationships are correct, most of the time linear relationships can be used for growth prediction, which are as accurate as any other kind of models.

9.2 Selected models for the prediction of main crop tree variables

The newly constructed models selected for final use in the field for the main crop trees are listed below together with the estimated parameters.

(i) Diameter at breast height

Intermediate thinning

$$\sqrt{dbh}_{t+\Delta t} = \sqrt{dbh}_{t} + 0.0891 * site_{ba,age} + 0.0048 * a_{dif}^{2} \qquad 9.1$$

Neutral thinning

$$\sqrt{dbh}_{t+\Delta t} = \sqrt{dbh}_{t} + 0.0636 * site_{ba,age} + 0.0090 * a_{dif}^{2} \qquad 9.2$$

(ii) Total height

Intermediate thinning

$$h_{t+\Delta t} = h_{t} + 2.3910 * site_{top,age} + 0.0334 * a_{dif}^{2} \qquad 9.3$$

Neutral thinning

$$h_{t+\Delta t} = h_{t} + 3.5556 * site_{top,age} + 0.0281 * a_{dif}^{2} \qquad 9.4$$

(iii) Timber height

Intermediate thinning

$$h_{tim} = 56.8900 - 65.7100 * 0.7968^{\sqrt{dbh*h}} \qquad 9.5$$

Neutral thinning

$$h_{tim} = 29.4570 - 40.1750 * 0.6060^{\sqrt{dbh*h}} \qquad 9.6$$

*In the timber height prediction models dbh should be in metres (m).

(iv) Total volume

$$v = 0.5040 * \left(\frac{\pi * dbh^2}{40000} \right) * h \qquad 9.7$$

(v) Merchantable volume

$$v_{mer} = -0.0038 + 0.5061 * \left\{ \left(\left(\frac{\pi * dbh^2}{40000} \right) \right) * h - \left(\frac{\pi * 49.0}{40000} * \left(\frac{h - h_{tim}}{3} \right) \right) \right\} \qquad 9.8$$

9.3 Prediction of mean variables for trees removed in thinning

The sample plots used for the construction of models to predict the thinned tree variables highlighted a different first thinning from the documented thinning regime with a very high yield. Therefore, a valid range for all the models built for the prediction of thinning tree variables is recommended which starts after the first thinning and runs for 50 years of plantation life for both thinning types.

(i) Basal area

Intermediate thinning

$$\overline{g}_{th} = -0.0292 + 0.1918 / (1 + \exp(-20.0710 * (\overline{g}_{bt} - 0.0821))) \qquad 9.9$$

Neutral thinning

$$\overline{g}_{th} = 0.0031 + 0.0564/(1+\exp(-81.7000*(\overline{g}_{bt} - 0.0418))) \qquad 9.10$$

(ii) Diameter at breast height

Intermediate thinning

$$\overline{dbh}_{th} = -0.4321 + 54.4420/(1+\exp(-0.0774*(\overline{dbh}_{bt} - 30.8010))) \qquad 9.11$$

Neutral thinning

$$\overline{dbh}_{th} = 9.5240 + 15.1723/(1+\exp(-0.3580*(\overline{dbh}_{bt} - 20.7272))) \qquad 9.12$$

(iii) Total height

Intermediate thinning

$$\overline{h}_{th} = -1.2514 + 36.5667/(1+\exp(-0.1130*(\overline{h}_{bt} - 17.8396))) \qquad 9.13$$

Neutral thinning

$$\overline{h}_{th} = 3.9300 + 22.3456/(1+\exp(-0.1697*(\overline{h}_{bt} - 16.0641))) \qquad 9.14$$

CHAPTER 10: RECOMMENDATIONS FOR FUTURE RESEARCH

A new set of growth models was constructed in this study to predict the individual tree growth of Corsican pine for intermediate and neutral thinning types. When considering the possibility of using one set of parameters for both thinning types, only a total volume and a merchantable volume prediction models confirmed this as a possibility. It would be useful to convert the parameters of the other models to be common for many thinning types such as low, intermediate, neutral, and crown. However, it was not possible to estimate the parameters for the thinning types other than the intermediate and neutral thinning types in this study due to the lack of data; therefore more data might be collected for future work.

Mathematical models have recently become a primary source of information about future stand dynamics (Leary, 1997). Efficient forest management entails the use of forest growth modelling systems which can predict stand growth and yield as well as provide diameter distribution and individual tree growth information. Generally such a system is composed of whole stand growth models, diameter distribution models and individual tree growth models (Zhang *et al.*, 1997). Therefore, if new models are constructed to predict the distribution of dbh and total height in conjunction with the constructed models in the present study, it will help forest managers to understand more about the future growth of plantations and to identify the trees removed in the next thinning according to the desirable thinning type.

In some countries, some even-aged and/or mixed species plantations are being considered for replacement by mixed-aged stands. This shift away from classical even-aged forest management renders existing yield tables inappropriate. For uneven-aged, mixed species stands we need to develop stand growth models that operate at the individual tree level. There are only a few single tree growth models for uneven aged mixed stands (Monserud and Sterba, 1996). Further work on this might be useful, perhaps by improving the models constructed for the present study. However, Corsican pine is a strong light demander and not well suited to growing in mixed stands (Mayhead, pers. comm.).

It is now possible to consider including climatic data as input variables in growth model equations, due to the availability of excellent long term average weather statistics obtained from an intensive grid of weather stations (Woollans *et al.*, 1997). It would be possible to change the new set of models by adding functions derived from such data or changing some of the existing model variables or functions to such new functions in order to obtain stochastic predictions.

In this study, the future growth of timber height, total height and merchantable volume are predicted by using the results of dbh and total volume prediction models. Process errors might have occurred as the predictions of one model are used for another model as explanatory variables to obtain a new set of predictions (Kangas, 1996). Therefore a continuous validation process will be necessary when the new models are applied in the field in order to minimise model errors.

Experience gained in this study demonstrated the difficulty of re-calibration of models originally developed for different species and different geographical locations. Therefore, if the models constructed for the present study are re-calibrated for different locations, it is strongly advised that the robustness of some functions, such as site factors in the dbh and total height prediction models, are considered carefully for the new conditions. However, other models will not cause this problem of re-evaluating the functions because these only contain individual tree variables such as dbh, basal area and total height.

In this study, two models were initially constructed for each variable and, after following many tests, one model was selected in Chapter 7. The removed set of models in that Chapter is worthy of re-calibration with the finally selected models, because the variables or functions in these models may be more favourable for the new localities.

Finally, it will be useful to make yield tables for selected plantations from the newly constructed models so that forest managers can easily predict the growth and yield of these plantations. A yield table constructed for a Corsican pine plantation using the newly built models in Chapter 4 is given in Appendix 5.1. This yield table is compatible with British Forestry Commission yield tables.

Throughout this study, many newly constructed models were rejected despite a very good fit, in order to find the best set of growth models. Following the difficulties experienced when re-calibrating the selected, well-developed models, even the finally selected new models could indicate bias when applied to the plantations in Sri Lanka. Therefore, the models removed from the study in Chapters 6 and 7 also remain as possible models for use in the future.

LITERATURE REFERENCES

Adlard, P.G., 1995. Myth and reality in growth estimation. *For. Ecol. Manage.* 71: 171-176.

Adman, P., 1984. *FORTRAN 77 for non-scientists.* Chartwell-Brat (Publishing and Training) Ltd., Kent, UK.

Aitkin, M., Anderson, D., Francis, B. and Hinde, J., 1989. *Statistical modelling in GLIM.* Oxford Statistical Science Series 4. Oxford Sci. Publ., Clarendon Press, Oxford.

Alder, D., 1978. A forecasting model for conifer plantations in the tropical highlands of eastern Africa. *In* J. Fries, T.E., Burkhart and T.A. Max (eds) Growth models for long term forecasting of timber yields. Proc. IUFRO. School of Forestry and Wildlife Resources, Virginia Polytechnic Institute and State university, Blacksbburg, USA. pp 1-13.

Alder, D., 1980. Forest volume estimation and yield prediction. vol. 2. FAO Forestry paper 22/2. Food and Agriculture Organisation of the United Nations, Rome.

Amateis, R.L. and McDill, M.E., 1993. Fitting discrete time dynamic models having any time interval. *For. Sci.* 39: 499-519.

Ashcroft, J., Eldrige, R.H., Paulson, R.W. and Wilson., G.A., 1986. *Programming with FORTRAN 77.* Collins Professionals and Technical Books, London.

Assman, E., 1970. *The principles of forest yield studies.* Pergamon Press, Oxford.

Avery, T.E. and Burkhart, H.E., 1994. *Forest measurements*, 4th ed. McGraw Hill, New York.

Ayhan, H.O., 1978. A simulation model of stand growth of Sitka spruce in north Wales. Ph.D. thesis. Univ. of Wales Bangor, UK.

Balfour, A. and Marwick, D.H., 1986. *Programming in standard FORTRAN 77.* Heinemann Educational Books Ltd., London.

Bates, G.M. and Watts, D.G., 1988. *Non-linear regression and its applications.* John Wiley and Sons., New York.

Biging, G.S. and Gill, S.J., 1997. Stochastic models for conifer tree crown profiles. *For. Sci.* 43: 25-34.

Borders, B.E., Baily, R.L. and Clutter, M.L., 1988. Forest growth models: parameter estimation using real growth series. *In* A.R. Ek, S.R. Shifley and T.E. Burk (eds) Forest growth modelling and prediction, vol. 2. Proc. of IUFRO conf., 23-27 Aug. 1987, Minneapolis, Minnesota. USDA For. Serv. Gen. Tech. Rep. NC-120, pp 660-667.

Box, G.E.P., Hunter, W.G. and Hunter, J.S., 1978. *Statistics for experiments: an introduction to design, data analysis, and model building.* John Wiley & Sons, USA.

Brown, T.N. and Kulasiri, D., 1996. Validating models for complex, stochastic, biological systems. *Ecol. Modelling.* 86: 129-134.

Bruce, D. and Wensel, L.C., 1988. Modelling forest growth, approaches, definitions and problems. *In* A.R. Ek, S.R. Shifley and T.E. Burk (eds) Forest growth modelling and prediction, vol. 1. Proc. of IUFRO conf., 23-27 Aug. 1987, Minneapolis, Minnesota. USDA For. Serv. Gen. Tech. Rep. NC-120, pp 1-8.

Buckman, R.E., 1962. Growth and yield of red pine. USDA for. Serv., Tech. Bull. 1272.

Buchman, R.G. and Shifley, S.R, 1983. Guide to evaluating forest growth projection systems. *J. For.* 81: 232-25.

Burkhart, H.E. and Tennent, R.., 1977. Site index equations for radiata pine in New Zealand. *N. Z. J. For. Sci.* 7: 408-416.

Carmean, W.H., 1975. Forest site quality evaluation in the United States. Adv. Agron. 27: 209-267.

Chen, C.W., Gomez, L.E., Fox, C.A., Goldstein, R.A. and Lucier, A.A, 1988. A tree growth model with multiple stress. *In* A.R. Ek, S.R. Shifley and T.E. Burk (eds) Forest growth modelling and prediction, vol. 1. Proc. of IUFRO conf., 23-27 Aug. 1987, Minneapolis, Minnesota. USDA For. Serv. Gen. Tech. Rep. NC-120, pp 270-277.

Chhetri, D.B.K. and Fowler, G.W., 1996a. Estimating diameter at breast height and basal diameter of trees from stump measurements in Nepal's lower temperate broad-leaved forests. *For Ecol. Manage.* 81: 75-84.

Chhetri, D.B.K. and Fowler, G.W., 1996b. Prediction models for estimating total heights of trees from diameter at breast height measurements in Nepal's lower temperate broad-leaved forests. *For. Ecol. Manage.* 84: 177-186.

Christie, J.M., 1994. Provisional yield tables for poplar in Britain. For. Comm. Tech. Pap. 6, For. Comm., Edinburgh.

Cailliez, F., 1980. Forest volume estimation and yield prediction, vol. 1. FAO Forestry paper 22/1. Food and Agriculture Organization of the United Nations, Rome.

Clutter, J.L., 1963. Compatible yield and growth models for loblolly pine. *For. Sci.* 9: 354:-371.

Clutter, J.L., and Allison, B.J., 1974. Growth and yield models for *Pinus radiata* in New Zealand. *In* J.L. Clutter, J.C. Fortson, L.V. Pienaar, G.H. Brister and R.L. Bailey 1992. *Timber management: a quantitative approach.* Kreiger publishing Co., Florida.

Clutter, J.L., Fortson, J.C., Pienaar, L.V., Brister, G.H. and Bailey, R.L., 1992. *Timber management: a quantitative approach.* Kreiger publishing Co., Florida.

Daniels, R.F., Burkhart, H.E. and Clason, T.R., 1986. A comparison of competition measures for predicting growth of loblolly pine trees. *Can. J. For. Res.* 16: 1230-1237.

Daniels, R.F., Burkhart, H.E. and Strub, M.R., 1979a. Yield estimates for loblolly pine plantations. *J. For.* 77:581-583.

Daniels, T.W., Helmer, T.J. and Baher, F.S., 1979b. *The principles of silviculture*, 2nd ed. McGrow-Hill, New York.

Davis, L.S. and Johnson, K.N., 1987. *Forest management*, 3rd ed. McGrow-Hill, New York.

Dean T.J. and Baldwin, Jr. V.C., 1996. The relationship between Reineke's stand density index and physical stem mechanics. *For. Ecol. Manage.* 81: 25-34.

DeAngelis, D.L. and Gross, L.J., 1992. *Individual based models and approaches in ecology: populations, communities and ecosystems.* Chapman and Hall, New York.

Deleuze, C., Herve, J., Colin, F. and Ribeyrolles, L., 1996. Modelling crown shape of *Picea abies* spacing effects. *Can. J. For. Res.* 27: 1957-1960.

Dewar, R.C., 1993. A mathematical analysis of self-thinning in terms of the carbon balance of trees. *Ann. of Bot.* 71: 147-159.

Dixon, R.K., Meldhal., R.S., Ruark., G.A. and Warren, W.G., 1990. *Process modelling for forest growth responses to environmental stress.* Timber Press, Portland.

Doucet, F. and Sloep, B., 1992. *Mathematical modeling in the life sciences.* Ellis Horwood Ltd., England.

Draper, N.R. and Smith, H., 1981. *Applied regression analysis*, 2nd ed. Wiley and Sons, New York.

Duerr, W.A. and Gevorkiantz, S.R., 1938. Growth prediction and site determination in uneven-aged timber stands. *J. Agric. Res.* 56:81-98. *In* J.K. Vanclay, 1994. *Modelling forest growth and yield: Applications to mixed tropical forests.* CAB International, UK.

Edwards, P.N., 1976. Forestry Commission sample plot code. For. Comm., UK.

Edwards, P.N. and Christie, J.M., 1981. Yield models for forest management. For. Comm. Booklet 48. HMSO, London.

Elfving, B. and Kiviste, A., 1997. Construction of site index equations for *Pinus sylvestris* L. using permanent plot data in Sweden. *For Ecol. Manage.* 98: 125-134.

Ek, A.R and Dudek, A. 1982. Development of individual tree based stand growth simulators: Progress and Applications. Staff paper series 20, Dept. of For. Res., Univ. of Minnesota.

Eriksson, H., Johansson, U. and Kiviste, A., 1997. A site index model for pure and mixed stands of *Betula pendula* and *Betula pubescens* in Sweden. *Scand. J. For. Res.* 12: 149-156.

Evans, J., 1996. *Plantation forestry in the tropics.* 2nd ed. Oxford Univ. Press, New York.

Figueiredo-Filho, A., Borders, B.E. and Hitch, K.L., 1996. Taper equations for *Pinus teada* plantations in southern Brazil. *For. Ecol. Manage.* 83: 39-46.

Ford-Robertson, F.C., 1983. *Terminology of forest science, technology practice and products.* Soc. of American Foresters, Washington DC.

Fraser, A.I, 1980. *A manual on the management of plantation forests*. International Forest Science consultancy, Penicuik, Midlothian.

Freese, F., 1990. *Statistics for land managers*. Paeony Press, Edingburg.

Garcia, O., 1983. A Stochastic differential equation for the height growth of forest stands. *Biometrics* 39: 1059-1072.

Garcia, O., 1984. New class of growth models for even-aged stands: *Pinus radiata* in Golden Down forest. *N. Z. J. of For. Sci.* 14: 65-88.

Gary, H.R., 1931. Yield table - Pinus radiata. Forestry and Timber Bureau Bull. No. 1, Canberra. *In* N.B. Lewis, A. Keeves and J.W. Leech, 1976. Yield regulation in south Australian *Pinus radiata* plantations. Bull. No. 23. Woods and Forests Dept., south Australia

Gertner, G., 1987. Approximating precision in simulation projections: an efficient alternative to Monte Carlo methods. *For. Sci.* 33: 230-239.

Gertner, G., 1991. Prediction bias and response surface curvature. *For. Sci.* 37: 755-765.

Gilchrist, W., 1984. *Statistical modelling*. John Wiley and Sons, New York.

Ginn, S.E., Seiler, J.R., Cazel, B.H. and Creh, R.E., 1991. Physiological and growth responses of eight year old loblolly pine stands to thinning. *For. Sci.*, 37: 1030-1040.

Hamilton, G.J., 1980. Line thinning. For. Comm. leaflet 77. For. Comm. UK.

Hamilton, G.J., 1988. *Forest mensuration hand book*. For. Comm. Booklet 39. HMSO. London.

Hammond, R.H., Rogers, W.B. and Crittenden, J.B., 1988. *Introduction to FORTRAN 77 and the personal computer*. McGraw-Hill, Singapore.

Harrison, W.C. and Daniels, R.F., 1988. A new biomathematical model for growth and yield of loblolly pine plantations. *In* A.R. Ek, S.R. Shifley and T.E. Burk (eds) Forest growth modelling and prediction, vol. 1. Proc. of IUFRO conf., 23-27 Aug. 1987, Minneapolis, Minnesota. USDA For. Serv. Gen. Tech. Rep. NC-120, pp 293-304.

Hart, C., 1994. *Practical forestry for the agent an surveyor*, 3rd ed. Alan Sutton Publishing Inc., United Kingdom.

Hasenaur, H. and Monserud, R.A., 1996. A crown ratio model for Austrian forests. *For. Ecol. Manage.* 84: 49-60.

Henderson-Sellers, B., 1996. Towards a conceptual model of randomness. *Ecol. Modelling.* 85: 303-308.

Hester, A.S., Hann, D.W. and Larson, D.R., 1989. *ORGANON: south west Oregon growth and yield models user manual*, Version 2.0. Forestry Research Laboratory, College of Forestry, Oregon State university, Cervalli, OR.

Houllier, F., Leban, J. and Colin, F., 1995. Linking growth modelling to timber quality assessment for Norway spruce. *For. Ecol. Manage.* 74: 91-102.

Hughes, C.E., Pfleeger, C.P. and Rose, L.L., 1978. *Advanced Programming techniques: a second course in programming using FORTRAN.* John Wiley & Sons Inc., USA.

Hummel, F.C., Locke, G.M.L., Jeffers, J.N.R. and Christie, J.M., 1959. Code of sample plot procedure. For. Comm. Bull. No. 31. For. Comm., UK.

Janssen, P.H.M. and Heuberger, P.S.C., 1995. Calibration of process- oriented models. *Ecol. Modelling* 83: 55-66.

Jenkins, T.A.R., Personal Communications. Sch. of Agric. and For. Sci., Univ. of Wales Bangor, UK.

Johnstone, W.D., 1976. Variable density yield tables for natural stands of lodgepole pine in Alberta. For. Tech. Rep. 20. Ottawa.

Kangas, A., 1996. On the bias and variance in tree volume predictions due to model and measurement errors. *Scand. J. For. Res.* 11: 281-290.

Kassab, J.Y., 1987. *Experimental design and statistical analysis*. Statistical Advisory Unit, Univ. Wales Bangor, UK.

Killian, W., 1984. Site classification and mapping: principles and trends. Symp. Site productivity of forest growing plantations. IUFRO Proc. South Africa. 1: 51-68.

Kimmins, J.P., 1997. *Forest ecology: a foundation for sustainable management*, 2nd ed. Prentice Hall Inc., New Jersey.

Kimmins, J.P., Scoullar, K.A., Comeau, P.G., Kurz, W.A., Appes, M.J. and Chatarpaul, L., 1988. FORCYTE-11: an example of the simulation approach to predicting the consequences for production, yield economics, soil fertility, nutrient and organic matter reserves and energy efficiency of alternative crop production systems. *In* A.R. Ek, S.R. Shifley and T.E. Burk (eds) Forest growth modelling and prediction, vol. 1. Proc. of IUFRO conf., 23-27 Aug. 1987, Minneapolis, Minnesota. USDA For. Serv. Gen. Tech. Rep. NC-120, pp 305-314.

Kimmins, J.P., Comeau, P.G. and Kurz, W., 1990. Modelling the interactions between moisture and nutrients in the control of forest growth. *For. Ecol. Manage.* 30: 361-379.

Knowe, S.A., Ahrens, G.R. and DeBell, D.S., 1997. Comparison of diameter distribution prediction, stand-table projection and individual tree growth modelling approaches for young red alder plantations. *For. Ecol. Manage.* 98: 49-60.

Korzukhin, M.D., Ter-Mikaelin, M.T. and Wagner, R.G., 1996. Process versus empirical models: which approach for forest ecosystem management. *Can. J. For. Res.* 26: 879-887.

Kozlowski, T.T., 1971. *Growth and development of trees. Vol. I: seed germination, ontogeny and shoot growth.* Academic press. New York.

Kuuluvainen, T., 1991. Long term development of needle mass, radiation interception and stem wood production in naturally regenerated *Pinus sylvestris* stands in Empetrum Vaccinium site types in the northern boreal zone in Finland: an analysis based on an empirical study and simulation. *For. Ecol. Manage.* 46: 103-122.

Larson, P.R. and Isebrands, J.G., 1972. The relationship between leaf production and wood weight in first year sprouts of two *Populus* clones. *Can. J. For. Res.* 2: 98-104.

Lane, P.W., Payne, R.W., 1996. *GENSTAT for windows: an introductory course.* Stat. Dept. Rothamsted, Hertz, UK.

Leary, R.A., 1997. Testing models of unthinned red pine plantation dynamics using a modified Bakuzis matrix of stand properties. *Ecol. Modelling.* 98: 35-46.

Lewis, N.B., Keeves, A. and Leech, J.W., 1976. Yield regulation in south Australia *Pinus radiata* plantations. Woods and Forests Dept., South Australia. Bull. No. 23.

Liu, J. and Ashton, P.S., 1995. Individual-based simulation models for forest succession and management. *For. Ecol. Manage.* 73: 157-175.

Mackey, B.G., McKenney, D.W., Yang, Y., McMahon, J.P. and Hutchinson, M.F., 1996. Site regions revisited: a climatic analysis of Hills' site regions for the province of Ontario using a parametric method. *Can. J. For. Res.* 26: 333-354.

MacKinney, A.L., Schumacher, F.X. and Chaiken, L.E., 1937. Construction of yield tables for non-normal loblolly pine stands. *J. Agric. Res.* 54: 531-545.

Magnussen, S. and Penner, M., 1996. Recovering the time trends in dominant height from stem analysis. *Can. J. For. Res.* 26: 9-22.

Maguire, D.A. and Hann, D.W., 1989. The relationship between gross crown dimensions and sap wood area at crown base in Douglas-fir. *Can. J. For. Res.* 19: 557-565.

Makela, A., 1988. Parameter estimation and testing of a process-based stand growth model using Monte Carlo techniques. *In* A.R. Ek, S.R. Shifley and T.E. Burk (eds) Forest growth modelling and prediction, vol. 1. Proc. of IUFRO conf., 23-27 Aug. 1987, Minneapolis, Minnesota. USDA For. Serv. Gen. Tech. Rep. NC-120, pp 315-322.

Makela, A., 1997. A carbon balance model growth and self-pruning in trees based on silvicultural relationships. *For. Sci.* 43: 7-24.

Makela, A. and Hari, P. 1986. Stand growth model based on carbon uptake and allocation in individual trees. *Ecol. Modelling* 33: 205-229.

Martin, G.L. and Ek, A.R., 1984. A comparison of competition measures and growth models for predicting red pine diameter and height growth. *For. Sci.* 30: 731-743.

Mayer, D.G. and Butler, D.G., 1993. Statistical validation. *Ecol. Modelling* 68: 21-32.

Mayhead G.J., Personal Communications. Sch. of Agric. and For. Sci., Univ. of Wales Bangor, UK.

McClain, K.M., Morris, D.M., Hills, S.C. and Buse, L.J., 1994. The effects of initial spacing on growth and crown development for planted northern conifers: 37 year results. *For. Chron.*, 70: 174-182.

McRoberts, R.E., 1988. Comparing regression curves. *In*. A.R. Ek, S.R. Shifley and T.E. Burk (eds) Forest growth modelling and prediction, vol. 2. Proc. of IUFRO conf., 23-27 Aug. 1987, Minneapolis, Minnesota. USDA For. Serv. Gen. Tech. Rep. NC-120, pp 762-769.

Mitchell, K.J., 1975. Dynamics and simulated yield of Douglas-fir. *For. Sci. Monog.* 17: 39.

Mitchell, K.J., 1988. SYLVER: Modelling the impact of silviculture on yield, lumber value, and economic return. *For. Chron.* 64: 127-131.

Monro, D.A., 1983. *FORTRAN 77*. Edward Arnold Ltd., London.

Monserud, R.A. and Sterba, H., 1996. A basal area increment model for individual trees growing in even and uneven-aged forest stands in Austria. *For. Ecol. Manage.* 80: 57-80.

Mowrer, H.T., 1989. The effect of forest simulation model complexity on estimate precision. *In* H.E., Burkhart, M. Rauscher and K. Johann, (eds). Artificial intelligence and growth models for forest management decisions. Proc. IUFRO Meeting, Vienna, 18-22 Sept. 1989, VPI & SU, Blacksburg VA. Publ. FWS-1-89, pp 100-109.

Mowrer, H.T. and Frayer, W.E., 1986. Variance propagation in growth and yield projections. *Can. J. For. Sci.* 16: 1196-1200.

Newberry, J.D. and Stage, A.R., 1988. Validating forest growth models: procedures defined by resource decisions. *In* A.R. Ek, S.R. Shifley and T.E. Burk (eds) Forest growth modelling and prediction, vol. 2. Proc. of IUFRO conf., 23-27 Aug. 1987, Minneapolis, Minnesota. USDA For. Serv. Gen. Tech. Rep. NC-120, pp 786-793.

Nigh, G.D. and Sit, V., 1996. Validation of forest height-age models. *Can. J. For. Res.* 26: 810-818.

Nystrom, K. and Gemmel, P., 1988. Models for predicting height and diameter of individual trees in young *Picea abies* (L.) Krast. stands. *Scand. J. For. Res.* 3: 213-228.

Oderwald, R.G. and Hans, R.P., 1993. Corroborating models within model properties. *For. Ecol. Mange.* 62: 271-283.

Ottorini, J., Goff, N.L. and Cluzeau, C., 1996. Relationships between crown dimensions and stem development in *Fraxinus excelsior*. *Can. J. For. Res.* 26: 394-401.

Pascoa, F., 1990. Using forest inventory data to build growth and yield stand models. *In* L.C. Wensel and G.S. Biging (Eds) Forest simulation systems, Proc. of IUFRO Conf., Bull 1927, Division of Agric. and Natural resources, Univ. of California, pp 279-286.

Payne, R.W., Lane, P.W., Digby, P.G.N., Harding, S.A., Leech, P.K., Morgan, G.W., Todd, A.D., Thompson, R., Wilson, G.T., Welham, S.J., and White, R.P., 1993. *GENSTAT 5 release 3 reference manual.* Oxford Univ. Press, UK.

Peterson, H., 1997. Functions for predicting crown height of *Pinus sylvestris* and *Picea abies* in Sweden. *Scand. J. For. Res.* 12: 179-188.

Peterson, J.A., Seiler, J.R., Nowak, J., Ginn, S.E. and Kreh, R.E., 1997. Growth and physiological responses of young loblolly pine stands to thinning. *For. Sci.* 43: 529-534.

Philip, M.S., 1994. *Measuring trees and forests*. 2nd ed. CAB International, UK.

Pienaar, L.V., 1989 A stand table projection approach to yield prediction in plantations. *S. African For. J.* 149: 44-47.

Pienaar, L.V. and Harrison, W.M., 1989. Simultaneous growth and yield prediction equations for *Pinus elliotti* plantations in Zululand. *S. African for. J.* 149: 48-53.

Pienaar, L.V. and Shiver, B.D., 1986. Basal area prediction and projection equations for pine plantations. *For. Sci.* 32: 623-633.

Price, C., Personal Communications. Sch. of Agric. and For. Sci., Univ. of Wales Bangor, UK

Pukkala, P., Kolstrom, T. and Miina, J., 1994. A method for predicting tree dimensions in Scots pine and Norway spruce. *For. Ecol. Manage.* 65: 123-134.

Ratkowsky, D.A., 1983. *Non-linear regression modeling*. Marcel Dekker, New York.

Rennie, P.J., 1963. Methods of assessing of site capacity. *Comm. For. Res.*, 42: 306-317.

Renolls, K., 1995. Forest height growth modelling. *For. Ecol. Manage.* 71: 217-225.

Reynolds, M.R. and Chung, J., 1986. Regression methodology for estimating model prediction error. *Can. J. For. Res.* 16: 931-938.

Ritchie, M.W. and Hann, D.W., 1997. Evaluation of individual tree and disaggregative prediction methods for Douglas-fir stands in western Oregon. *Can. J. For. Res.* 27: 207-216.

Rollinson, T.J.D., 1985. *Thinning control.* For. Comm. Booklet 54. HMSO, London.

Seber, G.A.F. and Wild, C.J., 1989. *Non-linear regression.* John Wiley and Sons. New York.

Shelburne, V.B. and Hedden, R.L., 1996. Effect of stem weight, dominant class and site quality on sap wood permeability in loblolly pine (*Pinus teada* L.) *For. Ecol. Manage.* 83: 163-169.

Shonau, A.P.G., 1988. Problems in using vegetation or soil classification in determining forest site quality. *In* D.W. Cole and S.P. Gessel. (eds). Forest site evaluation and long term productivity. Institute of Forest Resources, No. 63. Univ. Washington press, London. pp. 3-11

Shonau, A.P.G. and Purnel, R.C., 1988. Site evaluation for exotic trees in South Africa. *In* D.W. Cole and S.P. Gessel (eds). Forest site evaluation and long term productivity. Contribution/institute of Forest Resources, No. 63. Univ. Washington press, London. pp. 44-53.

Short, E.A. and Burkhart, H.E., 1992. Predicting crown height increment for thinned and unthinned loblolly pine plantations. *For. Sci.*, 38: 594-610.

Shugart, H.H., 1984. *A theory of forest dynamics.* Springer Verleg, New York.

Sievanen, R., 1993. A process-based model for the dimensional growth of even-aged stands. *Scand. J. For. Res.* 8: 28-48.

Sievanen, R., Burk, T.E. and Ek, A.R., 1988. Parameter estimation of a photosynthesis-based growth model. *In* A.R. Ek, S.R. Shifley and T.E. Burk (eds) Forest growth modelling and prediction, vol. 1. Proc. of IUFRO conf., 23-27 Aug. 1987, Minneapolis, Minnesota. USDA For. Serv. Gen. Tech. Rep. NC-120, pp 345-352.

Sievanen, R. and Burk, T.E., 1993. Adjusting a process-based model for varying site conditions through parameter estimation. *Can. J. For. Res.* 23:1837-1854.

Sievanen, R. and Burk, T.E., 1994. Fitting process-based models with stand growth data: problems and experiences. *For. Ecol. Manage.* 69: 145-156.

Smith, D.M., 1986. *The practice of silviculture.* Wiley, N.Y.

Spurr, S.H. and Barnes, B.V., 1980. *Forest ecology*, 3rd ed. Wiley, New York.

Soares, P., Tome, M., Skovsgaard, J.P. and Vanclay, J.K., 1995. Evaluating a growth model for forest management using continuous forest inventory data. *For. Ecol. Manage.* 71: 251-266.

Sprinz, P.T. and Burkhart, H.E., 1987. Relationships between tree crown, stem and stand characteristics in unthinned loblolly pine plantations. *Can. J. For. Res.* 17: 534-538.

Stage, A.R., 1973. Prognosis model for stand development. USDA For. Serv., Res. Pap. INT-137. *In* J.K. Vanclay, 1994. *Modelling forest growth and prediction.* CAB international, UK.

Tajchman, S.J. and Waint, V. 1983. Topography and biomass characteristics of a forest catchment in the northern Appalachians. *For. Ecol. Manage.* 5: 55-69 .

Tang, S., Meng, C.S., Meng, F. and Wang, Y.H., 1994. A growth and self-thinning model for pure even-aged stands: theory and applications. *For. Ecol. Manage.* 70: 67-73.

Tang, S., Wang, Y., Zhang, L. and Meng, C., 1997. A distribution independent approach to predicting stand diameter distribution. *For. Sci.* 43: 491-500.

Thornley, J.H.M., 1991. A transport resistant model for forest growth and partitioning. *Ann. of Bot.* 68:2 11-226.

Trousdell, K.B., Beck, D.E. and Lloyd, F.T., 1974. Site index for loblolly pine in the Atlantic Coastal Plain of the Carolinas and Virginia. USDA For. Serv., Res. Pap. SE-115.

Turnbull, K.J., 1963. Population dynamics in mixed forest stands. A systems of mathematical models of mixed stand growth and structure. Ph.D. Thesis, Univ. of Washington, Seattle, Washington.

Valentine, H.T., 1985. Tree-growth models: derivations employing the pipe-model theory. *J. Theor. Biol.* 117: 579-585.

Valentine, H.T., Baldwin, V.C., Gregoire, T.G. and Burkhart, T.E., 1994. Surrogate model for foliar dry matter in loblolly pine. *For. Sci.* 40: 476-585.

Vanclay, J.K., 1988. PLATIPUS physiology: Design of a plantation growth and conversion simulator incorporating silviculture and wood quality *In* A.R. Ek, S.R. Shifley and T.E. Burk (eds) Forest growth modelling and prediction, vol. 2. Proc. of IUFRO conf., 23-27 Aug. 1987, Minneapolis, Minnesota. USDA For. Serv. Gen. Tech. Rep. NC-120, pp 998-1005.

Vanclay, J.K., 1991. Compatible deterministic and stochastic predictions by probabilistic modelling of individual trees. *For. Sci.* 37: 1656-1663.

Vanclay, J.K., 1994. *Modelling forest growth and yield: Applications to mixed tropical forests*. CAB International, UK.

Vanclay, J.K. and Skovsgaard, J.P., 1997. Evaluating forest growth models. *Ecol. Modelling* 98: 1-12.

Voet, H. and Mohren, G.M.J., 1994. An uncertainty analysis of the process-based growth model FORGO. *For. Ecol. Manage.* 69: 157-166.

Voit, E.O. and Sands, P.J., 1996. Modelling forest growth: 1. canonical approach. *Ecol. Modelling* 86: 51-71.

Vuokila, Y., 1965. Functions for variable density yield tables of pine based on temporary sample plots. *Commun. Inst. For. Fenn.* 60: 1-86.

Wang, G.G. and Klinka, K., 1996. Use of synotopic variables in predicting white spruce site index. *For. Ecol. Manage.* 80: 95-105.

Wang, Y. and Payandeh, B., 1995. A base-age invariant site index model for aspen stands in north central Ontario. *For. Ecol. Manage.* 72: 207-211.

Weisburg, S., 1985. *Applied linear regression*, 2nd ed. John Wiley & Sons Inc., USA.

Wenk, G., 1994. A yield prediction model for pure and mixed stands. *For. Ecol. Manage.* 69:259-268.

West, P.W., 1981. Simulation of diameter growth and mortality in re-growth eucalypt forest of southern Tasmania. *For. Sci.* 27:603-616.

West, P.W., 1987. A model for biomass growth of individual trees in forest monoculture. *Ann. of Bot.* 60: 571-577.

West, P.W., 1993. Model of above ground assimilate partitioning and growth of individual trees in even-aged forest monoculture. *J. Theor. Biol.* 161: 369-394.

West, P.W. and Mattay, J.P., 1993. Yield prediction models and comparative growth rates for six eucalypt species. *Aust. For.* 56: 211-225.

Whitaker, C.J., Personal communications, Dept. of Stat., Univ. of Wales Bangor, UK.

Woollans, R.C., Snowdon, P. and Mitchell, N.D., 1997. Augmenting empirical stand projection equations with edaphic and climatic variables. *For. Ecol. Manage.* 98: 267-275.

Wykoff, W.R., 1990. A basal area increment model for individual conifers in the northern Rocky Mountains. *For. Sci.* 36: 1077-1104.

Young, H.E., Robbins, W.C. and Wilson, S., 1967. Errors in volume determination of primary forest products. A preparation for the meeting of IUFRO meeting, 4-9, Sep. 1967, Munich, Germany.

Zhang, S., Amateis, R.L. and Burkhart, H.E., 1997. Constraining individual tree diameter increment and survival models for loblolly pine plantations. *For. Sci.* 43: 414-423.

Zhang, Y., Reed, D.D., Cattelino, P.J., Gale, M.R., Jones, E.A., Liechty, H.O. and Mroz, G.D., 1994. A process based growth model for young red pine. *For. Ecol. Manage.* 69: 21-40.

Zuuring, H.R., Arney, J.D. and Milner, K.S., 1988. Generic graphical analyses for tree growth model invalidation *In* A.R. Ek, S.R. Shifley and T.E. Burk (eds) Forest growth modelling and prediction, vol. 2. Proc. of IUFRO conf., 23-27 Aug. 1987, Minneapolis, Minnesota. USDA For. Serv. Gen. Tech. Rep. NC-120, pp 828-834.

Appendix 1.1: Description of the Forestry Commission sample plot measurement data (plot1149)

1149 194907 1 H

1	121	222	2	222	186	3	222	162	4	131	210	5	111	263
6	223	133	7	111	214	8	221	186	9	131	251	10	221	230
11	222	146	12	121	234	13	222	194	14	222	162	15	121	214
16	121	243	17	121	222	18	221	202	19	121	210	20	111	247
21	111	202	22	221	234	23	222	178	24	121	255	25	222	182
26	222	170	27	121	214	28	221	194	29	111	234	30	212	146
31	212	178	32	313	-93	33	111	222	34	222	141	35	222	-166
36	212	150	37	121	222	38	221	210	39	112	210	40	111	226
41	121	230	42	222	178	43	111	251	44	121	243	45	121	263
46	222	174	47	222	166	48	232	202	49	222	178	50	121	210
51	111	214	52	313	125	53	111	190	54	212	170	55	122	158
56	121	210	57	121	226	58	222	182	59	211	222	60	222	162
61	111	206	62	223	141	63	111	251	64	121	226	65	221	194
66	132	198	67	111	222	68	213	-129	69	132	210	70	122	182
71	121	222	72	222	174	73	121	295	74	232	182	75	121	243
76	121	206	77	121	206	78	222	178	79	121	230	80	222	150
81	121	202	82	222	174	83	121	-202	84	212	170	85	121	218
86	222	137	87	111	239	88	121	271	89	221	194	90	121	206
91	222	190	92	111	190	93	222	210	94	121	279	95	121	214
96	322	121	97	211	222	98	111	251	99	221	186	100	221	194
101	121	190	102	121	247	103	121	259	104	111	226	105	111	226
106	122	190	107	222	-162	108	111	190	109	121	275	110	223	129
111	121	202	112	221	190	113	121	190	114	121	194	115	111	210
116	221	182	117	222	178	118	211	239	119	121	287	120	222	182
121	121	210	122	221	158	123	111	194	124	111	186	125	111	194
126	121	202	127	211	170	129	121	218	130	111	222	131	112	186
132	212	198	133	212	162	134	211	198	135	111	230	136	212	166
137	131	311	138	323	105	139	212	178	140	221	170	141	121	222
142	222	170	143	111	194	144	211	170	145	111	222	146	111	214
147	212	186	148	212	182	149	121	174	150	212	125	151	111	194
152	111	271	153	111	190	154	111	247	155	111	259	156	122	210
157	211	210	158	121	239	159	112	158	160	211	226	161	111	218
162	111	259	163	111	259	164	212	202	165	121	222	166	121	210
167	212	166	168	222	182	169	111	295	170	232	174	171	222	129
172	111	186	173	111	279	174	232	194	175	111	239	176	121	206
177	123	146	178	122	174	179	212	162	180	121	230	181	213	133
182	111	222	183	111	218	184	131	287	185	223	146	186	111	251
187	212	125	188	111	243	189	222	150	190	121	230	191	223	113
192	122	194	193	111	202	194	222	166	195	211	178	196	111	174
197	121	279	198	111	214	199	111	214	200	111	198	201	111	206
202	221	218	203	212	170	204	112	210	205	121	186	206	323	113
207	111	174	208	111	198	209	232	186	210	111	210	211	213	150
212	111	287	213	212	182	214	222	141	215	111	222	216	121	186
217	111	251	218	121	271	219	221	194	220	121	247	221	222	186
222	121	202	223	131	202	224	111	247	225	121	230	226	121	251
227	222	158	228	222	154	229	122	214	230	212	190	231	121	222
232	111	287	233	212	137	234	232	137	235	222	141	236	111	154
237	121	267	238	222	158	239	212	129	240	212	117	241	121	198
242	222	162	243	111	186	244	211	174	245	221	190	246	231	166
247	121	190	248	221	178	249	111	226	250	222	170	251	222	158
253	121	255	254	121	251	255	222	158	256	221	194	257	221	194
258	233	198	259	221	-198	260	111	279	261	121	239	262	111	218
263	222	166	264	121	230	265	121	218	266	212	178	267	211	162
268	121	247	269	111	202	270	112	218	271	122	247	272	121	234
273	222	-117	274	222	154	275	111	198	276	222	182	277	111	190
278	111	210	279	211	174	280	212	166	281	111	226	282	121	255
283	212	158	284	222	174	285	111	210	286	212	125	287	212	125
288	111	226	289	121	234	290	121	234	291	222	174	292	111	218
293	111	255	294	111	263	295	111	234	296	121	230	297	223	125
298	222	194	299	111	263	300	111	226	301	111	243	302	122	154
303	121	178	304	222	158	305	111	259	306	111	271	307	222	121
308	222	162	309	111	247	310	112	182	311	121	214	312	121	206
313	121	186	314	122	170	315	121	214	316	122	194	317	111	291
318	222	170	319	212	194	320	232	-133	321	121	218	322	211	166
323	222	158	324	122	162	325	212	141	326	222	150	327	313	105
328	112	178	329	222	141	330	221	162						

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1149 194907  2                                                                    H
  19  210    131  104     58  62   0      3
  30 210  10      30 186   8      44 137   6
  64  226    133  106     47  56   0      3
  30 226  10      30 194   8      46 129   4
  65  194    125  100     47  52   0      3
  30 190  12      30 166  10      40 117   6
 107 -162    122   92     61  69   0      3
  30 154   8      30 125   4      32  97   4
 197  279    140  117     49  64   0      4
  30 283  14      30 251  12      30 186   8      27 113   6
 232  287    145  119     52  64   0      4
  30 283  16      30 263  10      30 210   8      29 129   6
 244  174    122   86     40  49   0      3
  30 174  12      30 133   8      26 105   4
 268  247    137  111     56  67   0      4
  30 247  12      30 214   8      30 158   6      21 113   6

1149 194907  3                                                                    H
  32  -93     96  38                     1
  38  89   4
  35 -166    130  98                     3
  30 162  10      30 146   6      38 105   4
  68 -129    125  78                     3
  30 125   8      30 101   4      18  85   4
  83 -202    136 107                     3
  30 202  12      30 170   6      47 117   4
 107 -162    122  92                     3
  30 154   8      30 125   4      32  97   4
 259 -198    134 104                     3
  30 194  10      30 162   6      44 121   4
 273 -117     96  50                     2
  30 113   8      20  85   4
 320 -133    113  68                     2
  30 137   6      38  89   4

1149 194907  5                                                                    H
  5   263 136    19   210 131    20   247 143    40   226 139    55   158 123
 64   226 133    65   194 125    73   295 157    76   206 143    77   206 123
 88   271 145    91   190 107    92   190 142   101   190 130   107   162 122
119   287 139   137   311 142   169   295 139   173   279 133   184   287 148
197   279 140   207   174 126   212   287 128   220   247 134   227   158 125
232   287 145   237   267 143   238   158 116   244   174 122   260   279 151
268   247 137   281   226 143   284   174 117   309   247 126   312   206 125
317   291 148
```

Appendix 1.2: Formulae other than Huber's formula suitable for volume calculation of individual trees.

Smalian's formula

$$v = \frac{\pi L\left(d_1^2 + d_2^2\right)}{8}$$

Newton's formula

$$v = \frac{\pi L\left(d_1^2 + 4d_m^2 + d_2^2\right)}{24}$$

Where,

d_1 = diameter of the base of log, m

d_m = diameter at mid-length of log, m

d_2 = diameter at top of log, m

L = log length, m

v = volume of log, m^3

(Philip, 1994)

Appendix 1.3: Programme 1 written to read the sample plot data

```
C       *** PROGRAMME 1 TO READ FOR. COMM. DATA ***

C       *** Written by S.M.C.U.P. Subasinghe ***

C       ********** SUBROUTINE NAMES **********
C       *** (To write the different types of measurements)

        EXTERNAL TYPE1
        EXTERNAL TYPE2
        EXTERNAL TYPE3
        EXTERNAL TYPE4
        EXTERNAL TYPE5
        EXTERNAL TYPE8

        CHARACTER*19  filename2
        CHARACTER*60 H

        OPEN(UNIT=15,FILE='NEW-PLOTS/filename2.dat',STATUS='OLD')
        OPEN(UNIT=10,FILE='result.dat',STATUS='UNKNOWN')
        OPEN(UNIT=11,FILE='dmbat_1.dat',STATUS='UNKNOWN')
        OPEN(UNIT=12,FILE='dmbat_2.dat',STATUS='UNKNOWN')

     21 READ(15,22,END=777)filename2
     22 FORMAT(A9)

        filename2='NEW-PLOTS/'//filename2
        OPEN(UNIT=5,FILE=filename2,STATUS='OLD')
        PRINT*,'Opened file '//filename2

     10 READ(5,11,ERR=888,END=999)I1,I2,I3,A
     11 FORMAT(I4,I8,I8,59X,A1)

C       ****** To read the measurement type---1
C       ****** (Diameter measurements - always present)

        IF(A.EQ.'H'.AND.I3.EQ.1)WRITE(11,12)I1,I2,I3
     12 FORMAT(/3I8)
        IF(A.EQ.'H'.AND.I3.EQ.1)WRITE(12,2)I1,I2,I3
      2 FORMAT(/3I8)

        IF(A.EQ.'H'.AND.I3.EQ.1)CALL TYPE1

C       ****** To read the measurement type---2
C       ****** (Volume measurements after thinning)

        IF(A.EQ.'H'.AND.I3.EQ.2)WRITE(10,13)I1,I2
     13 FORMAT(2I10)
        IF(A.EQ.'H'.AND.I3.EQ.2)CALL TYPE2

C       ****** To read the measurement type---3
C       ****** (Thinning measurements - all or sample)

        IF(A.EQ.'H'.AND.I3.EQ.3)WRITE(10,14)I1,I2
     14 FORMAT(2I10)
        IF(A.EQ.'H'.AND.I3.EQ.3)CALL TYPE3

C       ****** To read the measurement type---5
C       ****** (height measurements of sample trees)

        IF(A.EQ.'H'.AND.I3.EQ.5)WRITE(10,15)I1,I2,I3
     15 FORMAT(3I10)
        IF(A.EQ.'H'.AND.I3.EQ.5)CALL TYPE5

        GO TO 10

        STOP

    999 CLOSE (UNIT=5)

        GO TO 21

    888 PRINT*,'Error at this point in the main programme.'
    777 PRINT*,'End of file - filename2.dat'

        END
C       *** END OF PROGRAMME 1 ***
```

Appendix 1.4: Sub-routine 1

```
      SUBROUTINE TYPE1

C     *** Written by S.M.C.U.P. Subasinghe ***

C     ****** Reads the measurement type 1 from the For. Com. data
C     and separates the main crop and trees marked for thinning ***

      CHARACTER *80 STRING

  100 READ(5,'(A80)',ERR=888,END=999)STRING
      IF(STRING(1:10).NE.'          ')THEN

        BACKSPACE 5

  110   READ(5,15,ERR=888,END=999)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     +  I11,I12,I13,I14,I15
   15   FORMAT(I4,I4,I5,I6,I4,I5,I6,I4,I5,I6,I4,I5,I6,I4,I5)

        IF(I3.LT.0)WRITE(12,20)I1,I3
        IF(I3.GT.0)WRITE(11,25)I1,I3

   20   FORMAT(2I5)
   25   FORMAT(2I5)

        IF(I6.LT.0)WRITE(12,30)I4,I6
        IF(I6.GT.0)WRITE(11,35)I4,I6

   30   FORMAT(2I5)
   35   FORMAT(2I5)

        IF(I9.LT.0)WRITE(12,40)I7,I9
        IF(I9.GT.0)WRITE(11,45)I7,I9

   40   FORMAT(2I5)
   45   FORMAT(2I5)

        IF(I12.LT.0)WRITE(12,50)I10,I12
        IF(I12.GT.0)WRITE(11,55)I10,I12

   50   FORMAT(2I5)
   55   FORMAT(2I5)

        IF(I15.LT.0)WRITE(12,60)I13,I15
        IF(I15.GT.0)WRITE(11,65)I13,I15

   60   FORMAT(2I5)
   65   FORMAT(2I5)


        GO TO 100
      ENDIF

      RETURN
      STOP

  888 PRINT*,'Error at this point in subroutine TYPE1.'
      PRINT*,'Last data read were: '
      PRINT*,STRING
      PRINT*,I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I11,I12,13,14,15
  999 PRINT*,'Subroutine TYPE1 finished the run successfully.'

      END

C     ****** End of subroutine TYPE1 ******
```

Appendix 1.5: Sub-routine 2

```
      SUBROUTINE TYPE2

C     *** Written by S.M.C.U.P. Subasinghe ***
C     *** Calculates the tot. vol. of individual trees without
C     forked trees, basal area, total height and total volume per plot ***

      CHARACTER *80 STRING
      PARAMETER(PI=3.14159265,X=4.0*10.0**7.0)

      VOLUME=0.0

  110 READ(5,'(A80)',ERR=888,END=999)STRING
      IF(STRING(1:10).NE.'          ')THEN

        BACKSPACE 5

  105   READ(5,25,ERR=888,END=999)I1,I2,I3,I4,I5,I6,I7,I8
   25   FORMAT(I4,I5,I6,I5,I6,I4,I4,I6)

        Y=I3
        W=I2
        B=PI*(W**2.0)/(4.0*(10.0**6.0))
        H=I3/10.0

C       *** For volume measurements, trees are divided into sections.

C       *** If the sections are less or equal to 5,
        IF(I8.GT.0.AND.I8.LE.5)GO TO 101

C       *** If the sections are less or equal to 10,
        IF(I8.GE.6.AND.I8.LE.10)GO TO 102

C       *** If the sections are less or equal to 15,
        IF(I8.GE.11.AND.I8.LE.15)GO TO 103

C       *** If the sections are less or equal to 20,
        IF(I8.GE.16.AND.I8.LE.20)GO TO 104

        BACKSPACE 5

C       *** This reads and writes upto 5 sections

  101   READ(5,35,ERR=555,END=666)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     +  I11,I12,I13,I14,I15
   35   FORMAT(3I4,I7,2I4,I7,2I4,I7,2I4,I7,2I4)

        A1=(PI*I1*I2**2.0)/X
        A2=(PI*I4*I5**2.0)/X
        A3=(PI*I7*I8**2.0)/X
        A4=(PI*I10*I11**2.0)/X
        A5=(PI*I13*I14**2.0)/X

        T1=(I1+I4+I7+I10+I13)

        PP1=(Y-T1)
        IF(PP1.LT.0.0)GO TO 110
        IF(PP1.GE.0.0)GO TO 201

  201   P1=PI*(Y-T1)*(7.0**2.0)/(12.0*(10.0**5.0))
        VOL1=A1+A2+A3+A4+A5+P1

        WRITE(11,40)VOL1,B,H
   40   FORMAT(F25.4,F10.3,F10.1)

        VOLUME=VOLUME+VOL1

        GO TO 110

C       *** This reads and writes upto 10 sections

  102   VOL_2=0.0
        T2=0.0

        DO 7 L=1,2
```

```
      READ(5,45,ERR=555,END=666)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     +     I11,I12,I13,I14,I15
 45   FORMAT(3I4,I7,2I4,I7,2I4,I7,2I4,I7,2I4)

      B1=(PI*I1*I2**2.0)/X
      B2=(PI*I4*I5**2.0)/X
      B3=(PI*I7*I8**2.0)/X
      B4=(PI*I10*I11**2.0)/X
      B5=(PI*I13*I14**2.0)/X

      V2=(B1+B2+B3+B4+B5)
      T_2=(I1+I4+I7+I10+I13)

      VOL_2=VOL_2+V2
      T2=T2+T_2

 7    CONTINUE

      PP2=Y-T2

      IF(PP2.LT.0.0)GO TO 110
      IF(PP2.GE.0.0)GO TO 202

 202  P2=PI*(Y-T2)*(7.0**2.0)/(12.0*(10.0**5.0))
      VOL2=VOL_2+P2

      WRITE(11,50)VOL2,B,H
 50   FORMAT(F25.4,F10.3,F10.1)

      VOLUME=VOLUME+VOL2

      GO TO 110

C     *** This reads and writes upto 15 sections

 103  VOL_3=0.0
      T3=0.0

      DO 8 M=1,3

      READ(5,55,ERR=555,END=666)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     +     I11,I12,I13,I14,I15
 55   FORMAT(3I4,I7,2I4,I7,2I4,I7,2I4,I7,2I4)

      C1=(PI*I1*I2**2.0)/X
      C2=(PI*I4*I5**2.0)/X
      C3=(PI*I7*I8**2.0)/X
      C4=(PI*I10*I11**2.0)/X
      C5=(PI*I13*I14**2.0)/X

      V3=(C1+C2+C3+C4+C5)
      T_3=(I1+I4+I7+I10+I13)

      VOL_3=VOL_3+V3
      T3=T3+T_3

 8    CONTINUE

      PP3=Y-T3

      IF(PP3.LT.0.0)GO TO 110
      IF(PP3.GE.0.0)GO TO 203

 203  P3=PI*(Y-T3)*(7.0**2.0)/(12.0*(10.0**5.0))
      VOL3=VOL_3+P3

      WRITE(11,60)VOL3,B,H
 60   FORMAT(F25.4,F10.3,F10.1)

      VOLUME=VOLUME+VOL3

      GO TO 110

C     *** This reads and writes upto 20 sections

 104  VOL_4=0.0
      T4=0.0

      DO 9 N=1,4
```

```
      READ(5,65,ERR=555,END=666)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     + I11,I12,I13,I14,I15
  65  FORMAT(3I4,I7,2I4,I7,2I4,I7,2I4,I7,2I4)

      D1=(PI*I1*I2**2.0)/X
      D2=(PI*I4*I5**2.0)/X
      D3=(PI*I7*I8**2.0)/X
      D4=(PI*I10*I11**2.0)/X
      D5=(PI*I13*I14**2.0)/X

      V4=(D1+D2+D3+D4+D5)
      T_4=(I1+I4+I7+I10+I13)

      VOL_4=VOL_4+V4
      T4=T4+T_4

   9  CONTINUE

      PP4=Y-T4

      IF(PP4.LT.0.0)GO TO 110
      IF(PP4.GE.0.0)GO TO 204

 204  P4=PI*(Y-T4)*(7.0**2.0)/(12.0*(10.0**5.0))
      VOL4=VOL_4+P4

      WRITE(11,70)VOL4,B,H
  70  FORMAT(F25.4,F10.3,F10.1)

      VOLUME=VOLUME+VOL4

      GO TO 110

     ENDIF

     WRITE(11,75)VOLUME
  75 FORMAT('              TOTAL VOLUME = 'F10.4' (m^3)'/)

     RETURN
     STOP

 555 PRINT*,'Error at this point'
     PRINT*,I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I11,I12,I13,I14,I15
     PRINT*,STRING

 666 PRINT*,'This is the end'
 888 PRINT*,'Error at this point in subroutine TYPE2.'
     PRINT*,'Last data read were'
     PRINT*,I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I11,I12,I13,I14,I15
     PRINT*,STRING
 999 PRINT*,'Subroutine TYPE2 finished the run successfully.'

     END

C    ****** End of subroutine TYPE2 ******
```

Appendix 1.6: Sub-routine 3

```
      SUBROUTINE TYPE2

C     *** Written by S.M.C.U.P. Subasinghe ***
C     *** Calculates the merchantable volume without forked trees, basal
C     area, total height, timber height, and total mer. vol. per plot.

      CHARACTER *80 STRING
      PARAMETER(PI=3.14159265,X=4.0*10.0**7.0)

      VOLUME=0.0

  110 READ(5,'(A80)',ERR=888,END=999)STRING
      IF(STRING(1:10).NE.'          ')THEN
        BACKSPACE 5

  105   READ(5,25,ERR=888,END=999)I1,I2,I3,I4,I5,I6,I7,I8
   25   FORMAT(I4,I5,I6,I5,I6,I4,I4,I6)

        Y=I3
        W=I2
        B=PI*(W**2.0)/(4.0*(10.0**6.0))
        H=I3/10.0

C       *** For volume measurements, trees are divided into sections.

C       *** If the sections are less or equal to 5,
        IF(I8.GT.0.AND.I8.LE.5)GO TO 101

C       *** If the sections are less or equal to 10,
        IF(I8.GE.6.AND.I8.LE.10)GO TO 102

C       *** If the sections are less or equal to 15,
        IF(I8.GE.11.AND.I8.LE.15)GO TO 103

C       *** If the sections are less or equal to 20,
        IF(I8.EQ.16.AND.I8.LE.20)GO TO 104

        BACKSPACE 5

C       *** This reads and writes upto 5 sections

  101   READ(5,35,ERR=555,END=666)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     +  I11,I12,I13,I14,I15
   35   FORMAT(3I4,I7,2I4,I7,2I4,I7,2I4,I7,2I4)

        A1=(PI*I1*I2**2.0)/X
        A2=(PI*I4*I5**2.0)/X
        A3=(PI*I7*I8**2.0)/X
        A4=(PI*I10*I11**2.0)/X
        A5=(PI*I13*I14**2.0)/X

        T1=(I1+I4+I7+I10+I13)
        PP1=Y-T1

        IF(PP1.LT.0.0)GO TO 110
        IF(PP2.GE.0.0)GO TO 201

  201   VOL1=A1+A2+A3+A4+A5
        TIM_HT1=T1/10.0

        WRITE(11,40)VOL1,B,H,TIM_HT1
   40   FORMAT(F15.4,F10.3,2F10.1)

        VOLUME=VOLUME+VOL1

        GO TO 110

C       *** This reads and writes upto 10 sections

  102   VOL2=0.0
        T2=0.0

        DO 7 L=1,2
```

```
      READ(5,45,ERR=555,END=666)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     +  I11,I12,I13,I14,I15
 45   FORMAT(3I4,I7,2I4,I7,2I4,I7,2I4,I7,2I4)

      B1=(PI*I1*I2**2.0)/X
      B2=(PI*I4*I5**2.0)/X
      B3=(PI*I7*I8**2.0)/X
      B4=(PI*I10*I11**2.0)/X
      B5=(PI*I13*I14**2.0)/X

      V2=(B1+B2+B3+B4+B5)
      T_2=(I1+I4+I7+I10+I13)

      VOL2=VOL2+V2
      T2=T2+T_2

  7   CONTINUE

      TIM_HT2=T2/10.0
      PP2=Y-T2

      IF(PP2.LT.0.0)GO TO 110
      IF(PP2.GE.0.0)GO TO 202

 202  WRITE(11,50)VOL2,B,H,TIM_HT2
  50  FORMAT(F15.4,F10.3,2F10.1)

      VOLUME=VOLUME+VOL2

      GO TO 110

C     *** This reads and writes upto 15 sections

 103  VOL3=0.0
      T3=0.0

      DO 8 M=1,3

      READ(5,55,ERR=555,END=666)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     +  I11,I12,I13,I14,I15
  55  FORMAT(3I4,I7,2I4,I7,2I4,I7,2I4,I7,2I4)

      C1=(PI*I1*I2**2.0)/X
      C2=(PI*I4*I5**2.0)/X
      C3=(PI*I7*I8**2.0)/X
      C4=(PI*I10*I11**2.0)/X
      C5=(PI*I13*I14**2.0)/X

      V3=(C1+C2+C3+C4+C5)
      T_3=(I1+I4+I7+I10+I13)

      VOL3=VOL3+V3
      T3=T3+T_3

  8   CONTINUE

      TIM_HT3=T3/10.0
      PP3=Y-T3

      IF(PP3.LT.0.0)GO TO 110
      IF(PP3.GE.0.0)GO TO 203

 203  WRITE(11,60)VOL3,B,H,TIM_HT3
  60  FORMAT(F15.4,F10.3,2F10.1)

      VOLUME=VOLUME+VOL3

      GO TO 110

C     *** This reads and writes upto 20 sections

 104  VOL4=0.0
      T4=0.0

      DO 9 N=1,4

      READ(5,65,ERR=555,END=666)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     +  I11,I12,I13,I14,I15
  65  FORMAT(3I4,I7,2I4,I7,2I4,I7,2I4,I7,2I4)
```

```
      D1=(PI*I1*I2**2.0)/X
      D2=(PI*I4*I5**2.0)/X
      D3=(PI*I7*I8**2.0)/X
      D4=(PI*I10*I11**2.0)/X
      D5=(PI*I13*I14**2.0)/X

      V4=(D1+D2+D3+D4+D5)
      T_4=(I1+I4+I7+I10+I13)

      VOL4=VOL4+V4
      T4=T4+T_4

 9    CONTINUE

      TIM_HT4=T4/10.0
      PP4=Y-T4

      IF(PP4.LT.0.0)GO TO 110
      IF(PP4.GE.0.0)GO TO 204

204   WRITE(11,70)VOL4,B,H,TIM_HT4
 70   FORMAT(F15.4,F10.3,2F10.1)

      VOLUME=VOLUME+VOL4

      GO TO 110

    ENDIF

    WRITE(11,75)VOLUME
 75 FORMAT('              TOTAL VOLUME = 'F10.4' (m^3)'/)

    RETURN
    STOP

555 PRINT*,'Error at this point'
    PRINT*,I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I11,I12,I13,I14,I15
    PRINT*,STRING
666 PRINT*,'This is the end'
888 PRINT*,'Error at this point in subroutine TYPE2.'
    PRINT*,'Last data read were'
    PRINT*,I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I11,I12,I13,I14,I15
    PRINT*,STRING
999 PRINT*,'Subroutine TYPE2 finished the run successfully.'

    END

C   ****** End of subroutine TYPE2 ******
```

Appendix 1.7: Sub-routine 4

```
      SUBROUTINE TYPE5

C     *** Written by S.M.C.U.P. Subasinghe ***
C     *** Calculates the total height, basal area and
C     total height*basal area of individual trees from
C     measurement type 5 ***

      CHARACTER *80 STRING
      PARAMETER(PI=3.14159265,X=4.0*(10.0**6.0))

  140 READ(5,'(A80)',ERR=888,END=999)STRING
      IF(STRING(1:10).NE.'          ')THEN
        BACKSPACE 5.
        READ(5,15,END=888,ERR=999)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     +  I11,I12,I13,I14,I15
   15   FORMAT(I4,I5,I4,I6,I5,I4,I6,I5,I4,I6,I5,I4,I6,I5,I4)

        D1=I2
        A1=I3
        H1=A1/10.0
        B1=PI*(D1**2.0)/X
        H1B1=H1*B1
        WRITE(11,20)H1,B1,H1B1
   20   FORMAT(F6.2,F18.3,F18.3)

        D2=I5
        A2=I6
        H2=A2/10.0
        B2=PI*(D2**2.0)/X
        H2B2=H2*B2
        WRITE(11,25)H2,B2,H2B2
   25   FORMAT(F6.2,F18.3,F18.3)

        D3=I8
        A3=I9
        H3=A3/10.0
        B3=PI*(D3**2.0)/X
        H3B3=H3*B3
        WRITE(11,30)H3,B3,H3B3
   30   FORMAT(F6.2,F18.3,F18.3)

        D4=I11
        A4=I12
        H4=A4/10.0
        B4=PI*(D4**2.0)/X
        H4B4=H4*B4
        WRITE(11,35)H4,B4,H4B4
   35   FORMAT(F6.2,F18.3,F18.3)

        D5=I14
        A5=I15
        H5=A5/10.0
        B5=PI*(D5**2.0)/X
        H5B5=H5*B5
        WRITE(11,40)H5,B5,H5B5
   40   FORMAT(F6.2,F18.3,F18.3)

        GO TO 140

      ENDIF

      RETURN
      STOP

  888 PRINT*,'Error at this point in subroutine TYPE5.'
  999 PRINT*,'Subroutine TYPE5 finished the run successfully.'

      END

C     ****** End of subroutine TYPE5 ******
```

Appendix 1.8: Sub-routine 5

```
      SUBROUTINE TYPE2

C     *** written by S.M.C.U.P. Subasinghe ***
C     *** Calculates the upper and lower crown heights, crown diameter,
C     crown volume, total height, dbh and basal area of individual trees
C     from measurement type 2 ***

      CHARACTER *80 STRING
      PARAMETER(PI=3.14159265,X=4.0*10.0**7.0)

  110 READ(5,'(A80)',ERR=888,END=999)STRING
      IF(STRING(1:10).NE.'          ')THEN
        BACKSPACE 5

  105   READ(5,25,ERR=888,END=999)I1,I2,I3,I4,I5,I6,I7,I8
   25   FORMAT(I4,I5,I6,I5,I6,I4,I4,I6)

        W=I2
        Y=I3
        CL=I5
        CU=I6
        BA=PI*(W**2.0)/(4.0*(10.0**6.0))
        DM=I2/10.0
        HT=I3/10.0
        CV=PI*((I7/10.0)**2.0)*(HT-(I6/10.0))/12.0

C       *** For volume measurements, trees are divided into sections.

C       *** If the sections are less or equal to 5,
        IF(I8.GT.0.AND.I8.LE.5)GO TO 101

C       *** If the sections are less or equal to 10,
        IF(I8.GE.6.AND.I8.LE.10)GO TO 102

C       *** If the sections are less or equal to 15,
        IF(I8.GE.11.AND.I8.LE.15)GO TO 103

C       *** If the sections are less or equal to 20,
        IF(I8.GE.16.AND.I8.LE.20)GO TO 104
        BACKSPACE 5

C       *** This reads and writes upto 5 sections
  101   READ(5,35,ERR=555,END=666)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     +  I11,I12,I13,I14,I15
   35   FORMAT(3I4,I7,2I4,I7,2I4,I7,2I4,I7,2I4)

        T1=(I1+I4+I7+I10+I13)
        PP1=(Y-T1)

        IF(PP1.LT.0.0)GO TO 110
        IF(PP1.GE.0.0)GO TO 201

  201   IF(CV.LE.0.0)GO TO 110
        IF(CV.GT.0.0)GO TO 301

  301   WRITE(11,40)CL,CU,DM,HT,BA,CV
   40   FORMAT(F30.1,3F7.1,F8.3,F10.3)

        GO TO 110

C       *** This reads and writes upto 10 sections
  102   T2=0.0

        DO 7 L=1,2

        READ(5,45,ERR=555,END=666)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     +  I11,I12,I13,I14,I15
   45   FORMAT(3I4,I7,2I4,I7,2I4,I7,2I4,I7,2I4)

        T_2=(I1+I4+I7+I10+I13)
        T2=T2+T_2

    7   CONTINUE

        PP2=(Y-T2)
```

```
      IF(PP2.LT.0.0)GO TO 110
      IF(PP2.GE.0.0)GO TO 202

 202  IF(CV.LE.0.0)GO TO 110
      IF(CV.GT.0.0)GO TO 302

 302  WRITE(11,50)CL,CU,DM,HT,BA,CV
  50  FORMAT(F30.1,3F7.1,F8.3,F10.3)

      GO TO 110

C     *** This reads and writes upto 15 sections
 103  T3=0.0

      DO 8 M=1,3

      READ(5,55,ERR=555,END=666)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     + I11,I12,I13,I14,I15
  55  FORMAT(3I4,I7,2I4,I7,2I4,I7,2I4,I7,2I4)

      T_3=(I1+I4+I7+I10+I13)
      T3=T3+T_3

   8  CONTINUE

      PP3=(Y-T3)

      IF(PP3.LT.0.0)GO TO 110
      IF(PP3.GE.0.0)GO TO 203

 203  IF(CV.LE.0.0)GO TO 110
      IF(CV.GT.0.0)GO TO 303

 303  WRITE(11,60)CL,CU,DM,HT,BA,CV
  60  FORMAT(F30.1,3F7.1,F8.3,F10.3)

      GO TO 110

C     *** This reads and writes upto 20 sections
 104  T4=0.0

      DO 9 N=1,4

      READ(5,65,ERR=555,END=666)I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,
     + I11,I12,I13,I14,I15
  65  FORMAT(3I4,I7,2I4,I7,2I4,I7,2I4,I7,2I4)

      T_4=(I1+I4+I7+I10+I13)
      T4=T4+T_4

   9  CONTINUE

      PP4=(Y-T4)

      IF(PP4.LT.0.0)GO TO 110
      IF(PP4.GE.0.0)GO TO 204

 204  IF(CV.LE.0.0)GO TO 110
      IF(CV.GT.0.0)GO TO 304

 304  WRITE(11,70)CL,CU,DM,HT,BA,CV
  70  FORMAT(F30.1,3F7.1,F8.3,F10.3)

      GO TO 110

    ENDIF

    RETURN
    STOP

 555 PRINT*,'Error at this point'
     PRINT*,I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I11,I12,I13,I14,I15
     PRINT*,STRING
 666 PRINT*,'This is the end'
 888 PRINT*,'Error at this point in subroutine TYPE2.'
     PRINT*,'Last data read were'
     PRINT*,I1,I2,I3,I4,I5,I6,I7,I8,I9,I10,I11,I12,I13,I14,I15
     PRINT*,STRING
 999 PRINT*,'Subroutine TYPE2 finished the run successfully.'

     END
C    ****** End of subroutine TYPE2 *****
```

Appendix 1.9: Programme 2

```
C       *** PROGRAMME 2 FOR DBH MEASUREMENTS ***

C       *** Written by S.M.C.U.P. Subasinghe ***
C       *** Calculates the basal area total numbers of trees total
C       squared dimeter and writes tree number and dbh (mm) ***

        EXTERNAL DIAMET

        OPEN(UNIT=11,FILE='dmbat_2.dat',STATUS='OLD')
        OPEN(UNIT=16,FILE='test.dat',STATUS='UNKNOWN')

      5 READ(11,10,ERR=888,END=999)I1,I2,I3
     10 FORMAT(3I8)

        WRITE(16,15)I1,I2,I3
     15 FORMAT(/3I10)

        CALL DIAMET

        GOTO 5

        STOP

    888 PRINT*,'ERROR AT THIS POINT IN THE MAIN PROGRAMME'
        PRINT*,'LAST DATA IN THE MAIN PROGRAMME READ WERE'
        PRINT*,I1,I2,I3
    999 CLOSE (UNIT=11)

        END


        SUBROUTINE DIAMET

        CHARACTER *80 STRING

        TOTSQDM=0.0
        TOTBA=0.0
        N=0

    100 READ(11,'(A80)',ERR=888,END=999)STRING
        IF(STRING(1:10).NE.'          ')THEN
        BACKSPACE 11

     13   READ(11,15,ERR=888,END=999)I1,I2
     15   FORMAT(2I5)

          count=count+1
          BACKSPACE 13

          DM=I2/10.0
          SQDM=DM**2.0
          BA=(3.14159265*(I2**2.0))/(4.0*(10.0**6.0))
          N=N+1
          TOTSQDM=TOTSQDM+SQDM
          TOTBA=TOTBA+BA

          WRITE(16,20)I1,DM,SQDM,BA
     20   FORMAT(I10,3F10.3)

          GO TO 100

        ENDIF

        WRITE(16,25)N,TOTSQDM,TOTBA
     25 FORMAT(I4,2F20.3)

        RETURN
        STOP

    888 PRINT*,'ERROR AT THIS POINT'
        PRINT*,I1,I2
        PRINT*,STRING
        PRINT*,'LAST DATA READ WERE'
    999 PRINT*,END OF DATA

        END
```

Appendix 1.10: Summary of the sample plot 1149

	main crop trees after thinning									thinning trees								
m_year	no. tree,	dm, cm	ht, m	basal area, m^2		total vol., m^3		merch. vol., m^3		no tree,	dm, cm	ht, m	basal area, m^2		total vol., m^3		merch. vol., m^3	
		mean	mean	mean	total,	mean	total,	mean	total,		mean	mean	mean	total	mean	total	mean	total
yr/mnth	$plot^{-1}$				$plot^{-1}$		$plot^{-1}$		$plot^{-1}$	$plot^{-1}$				$plot^{-1}$		$plot^{-1}$		$plot^{-1}$
1942/02	329	15.0	9.7	0.018	5.807	0.048	15.812	0.044	14.577	1045	11.0	9.6	0.010	9.998	0.000	0.236	0.001	0.753
1942/11	329	15.8	10.1	0.020	6.423	0.074	24.190	0.070	22.958									
1943/12	329	16.4	10.6	0.021	6.919	0.094	30.935	0.090	29.707									
1945/04	328	17.4	11.3	0.024	7.775	0.130	42.769	0.127	41.550									
1945/02	328	17.9	11.5	0.025	8.271	0.151	49.514	0.147	48.299									
1949/07	320	20.4	13.7	0.033	10.424	0.251	80.330	0.247	79.160	8	15.4	12.5	0.019	0.150	0.113	0.901	0.108	0.865
1952/05	286	22.0	14.5	0.038	10.881	0.325	93.072	0.322	92.037	34	17.9	13.1	0.025	0.859	0.192	6.543	0.188	6.392
1960/04	157	27.3	17.7	0.059	9.196	0.605	94.922	0.601	94.375	129	22.3	16.4	0.039	5.053	0.363	46.791	0.359	46.258
1963/12	130	30.0	19.1	0.071	9.185	0.769	99.956	0.765	99.513	27	25.8	18.0	0.052	1.410	0.523	14.109	0.519	14.005
1968/11	109	33.8	21.5	0.090	9.800	1.031	112.352	1.027	111.994	21	29.7	20.9	0.069	1.455	0.731	15.361	0.728	15.289
1971/12	89	36.1	22.3	0.102	9.099	1.198	106.658	1.195	106.373	20	33.8	22.0	0.090	1.792	0.980	19.605	0.977	19.545
1976/04	89	38.3	23.0	0.115	10.270	1.377	122.583	1.374	122.306									

planting year 1920
general yield class 14
thinning type exploitation
plot size 0.3642 ha

Appendix 2.1: Resultant F-values for the common slopes of dbh and total height relationships for each age class

* - significant at the probability level 0.1
** - significant at the probability level 0.05

Age class 16-20		Age class 21-25		Age class 26-30	
combination	F-value	combination	F-value	combination	F-value
I14-I18	3.15	I12-I14	26.55*	I10-I12	3.51
I14-I22	3.71	I12-I16	2.01	I10-I14	0.38
I14-N14	2.84	I12-I18	3.20	I10-I16	11.19*
I14-N16	2.78	I12-I20	3.65	I10-I18	13.16*
I18-I22	3.79	I12-I22	2.79	I10-I20	6.63**
I18-N14	3.75	I12-N14	3.40	I10-I22	0.05
I18-N16	2.51	I12-N16	2.49	I10-N16	2.62
I22-N14	0.17	I14-I16	27.52*	I12-I14	2.84
I22-N16	1.51	I14-I18	7.06*	I12-I16	1.05
N14-N22	2.10	I14-I20	0.28	I12-I18	1.62
		I14-I22	5.94	I12-I20	3.61
		I14-N14	27.67*	I12-I22	3.22
		I14-N16	25.45*	I12-N16	3.51
		I16-I18	0.32	I14-I16	7.09*
		I16-I20	2.32	I14-I18	12.15*
		I16-I22	1.85	I14-20	16.48*
		I16-N14	1.93	I14-I22	13.36*
		I16-N16	1.22	I14-N16	6.87**
		N14-N16	0.75	I16-I18	2.15
				I16-20	1.30
				I16-I22	3.04
				I16-N16	2.12
				I18-I20	1.30
				I18-I22	3.04
				I18-N16	1.12

Age class 31-35		Age class 36-40		Age class 41-45	
combination	F-value	combination	F-value	combination	F-value
I10-I12	2.84	I10-I12	1.29	I12-I14	3.03
I10-I14	3.79	I10-I14	2.76	I12-I16	4.29**
I10-I16	0.56	I10-I16	1.08	I12-I18	5.76**
I10-I18	1.50	I10-I18	0.76	I12-I20	0.13
I10-I20	0.08	I10-I20	0.14	I12-N14	0.31
I10-I22	0.67	I10-I22	0.44	I12-N16	0.16
I10-N14	2.42	I10-N14	0.82	I14-I16	13.69*
I10-N16	0.10	I10-N16		I14-I18	16.50*
I12-I14	3.56			I14-20	3.25
I12-I16	1.58			I14-N14	1.03
I12-I18	2.48			I14-N16	2.58
I12-I20	9.85*			I16-I18	0.75
I12-I22	1.60			I16-I20	7.86*
I12-N14	2.84			I16-N14	13.74*
I12-N16	3.08			I16-N16	8.22*
I14-I16	8.93*			N14-N16	0.05
I14-I18	1.18				
I14-20	5.56*				
I14-I22	3.85				
I14-N14	3.70				
I14-N16	4.99				
N14-N16	5.54				

Age class 46-50		Age class 51-55		Age class 56-60	
combination	F-value	combination	F-value	combination	F-value
I10-I14	3.29	I14-I16	3.66	I14-I16	0.57
I10-I16	1.34	I14-I18	2.41	I14-I18	0.08
I10-I18	0.78	I14-I20	0.44	I14-I20	1.44
I10-I20	0.28	I16-I18	1.92		
I14-I16	12.19*	I16-I20	2.70		
		I18-I20	0.72		

Age class 61-65	
combination	F-value
I14-I16	0.05
I14-I18	0.01
I16-I18	0.08

Appendix 2.2: Descriptive statistics of the variables used for construction of models

(i) Modelling of diameter at breast height

a. Intermediate thinning

Characteristic	dbh_t, cm	$dbh_{t+\Delta}$, cm	age_t, yr	$age_{t+\Delta t}$, yr	age_{dif}, yr	h_{top}, m	total ba (G), m^2ha^{-1}	total tree (N), ha^{-1}	h_{top}/age myr^{-1}	G/age, m^2ha^{-1} yr^{-1}	h_{top}/G, $m^{-1}ha$
mean	19.00	20.90	29.80	33.90	4.10	15.00	33.20	1512.70	0.52	1.22	0.46
median	17.80	19.40	28.00	32.00	4.00	13.80	33.10	1472.00	0.51	1.23	0.42
minimum	7.30	7.70	13.00	16.00	2.00	9.20	17.50	184.00	0.40	0.47	0.29
maximum	54.20	57.40	65.00	68.00	10.00	30.20	46.90	3128.00	0.76	2.21	1.01
lower quartile	13.70	15.40	22.00	26.00	3.00	12.00	29.10	939.00	0.46	0.92	0.37
upper quartile	23.30	25.00	37.00	42.00	6.00	18.00	37.50	1933.00	0.56	1.41	0.54
variance	50.10	56.80	115.30	126.90	2.00	19.00	33.90	514789.50	0.01	0.14	0.02
stand deviation	7.10	7.50	10.70	11.26	1.40	4.40	5.80	717.50	0.08	0.38	0.13
se. of mean	0.10	0.10	0.14	0.15	0.02	0.06	0.08	9.70	0.00	0.01	0.00
coeff. of var.	37.2	36.10	36.00	33.20	34.10	29.90	17.50	47.40	15.00	30.64	27.41
skewness	0.95	0.93	0.94	0.85	0.50	1.10	-0.06	0.03	0.74	0.44	1.31
se. of skewness	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
kurtosis	0.95	0.93	0.56	0.35	0.03	0.57	-0.28	-0.51	0.26	-0.04	1.79
se. of kurtosis	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07

b. Neutral thinning

Characteristics	dbh_t, cm	$dbh_{t+\Delta}$, cm	age_t, yr	$age_{t+\Delta}$, yr	age_{dif}, yr	h_{top}, m	total ba (G), m^2ha^{-1}	total tree (N), ha^{-1}	h_{top}/age, myr^{-1}	G/age, $m^2ha^{-1}yr^{-1}$	h_{top}/G, $m^{-1}ha$
mean	14.70	16.10	23.20	26.10	3.00	12.20	26.90	1794.60	0.53	1.19	0.47
median	14.10	15.50	22.00	25.00	3.00	12.30	27.20	1565.00	0.53	1.129	0.46
minimum	7.00	7.10	19.00	21.00	2.00	9.00	11.40	623.00	0.47	0.60	0.30
maximum	33.00	37.20	26.00	41.00	8.00	18.30	39.70	3055.00	0.56	1.81	0.79
lower quartile	11.30	12.20	19.00	22.00	2.00	10.10	23.90	1246.00	0.51	0.95	0.40
upper quartile	17.50	19.30	25.00	29.00	5.00	13.00	31.90	2316.00	0.55	1.37	0.56
variance	20.90	27.70	17.50	24.60	1.30	3.90	34.90	499100.00	0.00	0.01	0.01
stand deviation	4.60	5.30	4.20	5.00	1.10	2.00	5.90	706.50	0.02	0.31	0.11
se. of mean	0.07	0.08	0.07	0.08	0.02	0.03	0.09	11.10	0.00	0.01	0.00
coeff. of var.	31.00	52.50	18.10	19.00	37.90	16.30	22.00	39.40	4.20	26.22	23.46
skewness	0.72	0.75	1.00	1.13	1.40	0.08	-0.36	0.39	-0.75	0.55	0.41
se. of skewness	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
kurtosis	0.42	0.50	0.59	0.71	2.40	0.46	-0.25	-0.94	0.40	-0.57	-0.04
se. of kurtosis	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08

(ii) Modelling of total height

a. Intermediate thinning

Characteristics	h_t, m	$h_{t+\Delta t}$, m	age_t, yr	$age_{t+\Delta t}$, yr	age_{dif}, yr	h_{top}/age, myr^{-1}	total tree (N), ha^{-1}	h_{top}/age, myr^{-1}
mean	15.01	17.02	31.13	32.30	4.17	15.74	1343.35	0.52
median	13.70	15050	29.00	32.00	4.00	14.20	1341.00	0.52
minimum	6.20	8050	13.00	15.00	2.00	9.10	184.00	0.39
maximum	29.30	32.30	58.00	65.00	10.00	27.90	3128.00	0.72
lower quartile	10.70	12.20	22.00	25.00	2.00	12.30	719.00	0.47
upper quartile	18.30	20.70	42.00	45.00	6.00	18.50	1933.00	0.58
variance	25.94	29.44	122.51	148.28	4.18	21.61	559705.93	0.01
stand deviation	5.09	5.43	11.07	12.18	2.04	4.65	748.13	0.07
se. of mean	0.22	0.23	0.47	0.52	0.09	0.20	31.78	0.00
coeff. of var.	33.93	31.88	35.56	34.50	49.05	29.53	55.69	13.16
skewness	0.66	0.63	0.67	0.63	0.98	0.78	0.44	0.57
se. of skewness	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
kurtosis	-0.48	-0.59	-0.45	-0.57	0.97	-0.33	-0.67	-0.11
se. of kurtosis	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21

b. Neutral thinning

Characteristics	h_t, m	$h_{t+\Delta t}$, m	age_t, yr	$age_{t+\Delta t}$, yr	age_{dif}, yr	h_{top}, m	total tree (N), ha^{-1}	h_{top}/age, myr^{-1}
mean	13.69	16.42	27.33	32.82	5.48	14.09	1227.36	0.52
median	14.50	17.00	31.00	36.00	5.00	15.80	912.00	0.52
minimum	8.00	10.00	19.00	24.00	3.00	9.40	593.00	0.49
maximum	20.10	22.10	36.00	41.00	7.00	18.40	3055.00	0.55
lower quartile	10.70	13.55	19.00	24.00	3.00	10.20	815.00	0.51
upper quartile	15.80	18.75	36.00	37.00	6.00	16.20	1474.00	0.54
variance	8.16	8.24	34.46	35.85	0.58	8.05	42252.34	0.00
stand deviation	2.86	2.87	5.87	5.99	0.76	2.84	649.81	0.02
se. of mean	0.21	0.21	0.43	0.44	0.06	0.21	47.77	0.00
coeff. of var.	20.81	17.48	21.48	18.24	13.86	20.16	52.94	3.27
skewness	-0.15	-0.23	-0.36	-0.51	0.29	-0.41	1.49	0.03
se. of skewness	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18
kurtosis	-1.02	-1.08	-1.33	-1.27	0.86	-1.33	1.20	-0.90
se. of kurtosis	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35

(iii) Modelling of timber height

a. Intermediate thinning

Characteristics	h_{tim}, m	h, m	dbh, cm	dbh*h, m^2
mean	10.32	14.11	17.23	2.70
median	9.20	13.00	15.80	1.92
minimum	1.40	3.80	7.20	0.31
maximum	26.00	28.50	44.70	9.75
lower quartile	5.40	9.95	11.30	1.13
upper quartile	14.70	17.90	21.80	3.72
variance	36.39	27.29	49.59	4.44
stand deviation	6.03	5.22	7.04	2.11
se. of mean	0.11	0.09	0.12	0.04
coeff. of var.	58.45	37.01	90.88	78.02
skewness	0.52	0.64	0.76	1.30
se. of skewness	0.04	0.04	0.04	0.04
kurtosis	-0.70	-0.53	-0.12	0.99
se. of kurtosis	0.09	0.09	0.09	0.08

b. Neutral thinning

Characteristics	h_{tim}, m	h, m	dbh, cm	dbh*h, m^2
mean	9.28	12.66	16.04	2.20
median	8.70	12.20	15.20	1.77
minimum	1.30	5.90	7.00	0.47
maximum	18.00	21.00	30.00	5.54
lower quartile	6.00	9.50	11.60	1.11
upper quartile	13.10	15.90	20.25	3.18
variance	17.77	12.37	30.41	1.67
stand deviation	4.21	3.52	5.51	1.29
se. of mean	0.09	0.08	0.13	0.03
coeff. of var.	45.41	27.79	34.37	58.8
skewness	0.12	0.26	0.45	0.71
se. of skewness	0.06	0.06	0.06	0.06
kurtosis	-1.13	-1.21	-0.71	-0.65
se. of kurtosis	0.11	0.11	0.11	0.11

(iv) Modelling total volume

a. Intermediate thinning

Characteristics	v_t, m^3	dbh, cm	ba (g), m^2	h, m	age, yr	total tree (N), ha^{-1}	total ba (G), $m^2 ha^{-1}$	h_{top}, m	h_{top}/age, myr^{-1}	crn. dpth (c_h), m	crn. ratio (c_r)	crn. vol (c_{vol}), m^3
mean	0.58	24.62	0.05	17.85	38.49	1320.00	40.50	19.00	0.51	6.38	0.38	35.14
median	0.42	23.95	0.05	17.40	39.00	945.00	41.40	18.20	0.50	6.30	0.40	27.60
minimum	0.01	7.30	0.00	6.20	13.00	235.00	23.91	9.20	0.32	1.20	0.10	1.30
maximum	3.05	51.50	0.29	33.10	68.00	4834.00	52.01	33.70	0.76	13.00	0.80	154.60
low. quartile	0.17	18.20	0.03	12.80	25.0	558.00	35.45	13.40	0.46	5.10	0.30	13.90
upp. quartile	0.84	30.70	0.07	22.60	48.50	1572.00	45.77	23.30	0.46	7.60	0.50	47.60
variance	0.30	76.10	0.00	37.12	203.34	1148358.70	42.17	34.35	0.57	3.46	0.01	784.07
stand dev.	0.54	8.72	0.04	6.09	14.26	1071.60	6.49	5.86	0.00	1.86	0.13	28.00
se. of mean	0.02	0.31	0.00	0.21	0.51	38.10	0.23	0.21	0.07	0.07	0.00	1.10
coeff. of var.	93.36	35.43	68.35	34.13	37.04	81.20	16.03	30.84	0.00	29.15	33.01	79.75
skewness	1.54	0.36	1.15	0.19	0.26	1.50	-0.33	0.34	15.47	0.27	0.21	1.44
se. of skew	0.09	0.09	0.09	0.09	0.09	0.10	0.09	0.09	0.68	0.09	0.09	0.09
kurtosis	2.67	-0.32	1.33	-0.88	-0.85	1.50	-0.74	-0.85	0.08	0.08	-0.25	2.20
se. of kurt.	0.17	0.17	0.17	0.17	0.17	0.20	0.17	0.17	0.17	0.17	0.17	0.19

b. Neutral thinning

Characteristics	v_t, m^3	dbh, cm	ba (g), m^2	h, m	age, yr	total tree (N), ha^{-1}	total ba (G), $m^2 ha^{-1}$	h_{top}, m	h_{top}/age, $m yr^{-1}$	crn. dpth (c_h), m	crn. ratio, (c_r)	crn. vol, (c_{vol}) m^3
mean	0.29	20.50	0.04	14.84	31.65	1277.90	34.49	15.967	0.506	6.616	0.446	25.717
median	0.25	20.50	0.03	14.90	31.00	1038.00	36.29	16.000	0.510	6.400	0.450	18.300
minimum	0.01	7.70	0.01	6.60	24.00	593.00	18.37	11.700	0.470	1.400	0.120	0.210
maximum	1.12	36.80	0.12	22.10	41.00	3055.00	45.70	20.700	0.530	12.800	0.740	183.000
low. quartile	0.14	16.30	0.02	12.25	24.00	815.00	32.32	12.700	0.490	5.400	0.390	10.7555
upp. quartile	0.41	24.60	0.05	17.30	36.00	1502.00	37.66	18.300	0.520	7.800	0.510	32.015
variance	0.04	34.88	0.00	9.95	32.58	397780.50	36.37	7.344	0.000	3.486	0.007	567.725
stand dev.	0.20	5.91	0.02	3.15	5.71	630.70	6.03	2.710	0.016	1.867	0.085	23.827
se. of mean	0.01	0.28	0.00	0.15	0.28	30.30	0.29	0.130	0.001	0.090	0.004	1.146
coeff. of var.	68.87	28.80	54.87	21.26	18.03	49.30	17.49	16.973	3.117	28.222	19.038	92.649
skewness	1.05	0.10	0.77	-0.02	-0.04	1.50	-0.85	-0.058	-0.452	0.470	-0.034	2.420
se. of skew	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.117	0.117	0.117	0.117	0.117
kurtosis	0.94	-0.40	0.35	-0.79	-0.16	1.71	0.44	-1.157	-0.550	0.423	0.888	8.190
se. of kurt.	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.234	0.234	0.234	0.234	0.234

(v) Modelling of merchantable volume

a. Intermediate thinning

Characteristics	v_{mer}, m^3	dbh, cm	basal area (g), m^2	h, m	h_{tim}, m
mean	0.28	17.78	0.03	14.32	10.56
median	0.12	15.80	0.02	13.10	9.30
minimum	0.00	7.00	0.00	3.80	1.30
maximum	3.05	51.50	0.21	33.10	31.80
lower quartile	0.04	11.30	0.01	9.90	5.30
upper quartile	0.35	22.30	0.04	18.30	15.10
variance	0.16	67.09	0.00	32.88	43.70
stand deviation	0.40	8.19	0.03	5.73	6.61
se. of mean	0.01	0.14	0.00	0.10	0.11
coeff. of var.	140.97	46.08	96.22	39.83	62.51
skewness	2.58	1.09	2.07	0.75	0.60
se. of skewness	0.04	0.04	0.04	0.04	0.04
kurtosis	8.01	0.76	5.03	-0.24	-0.47
se. of kurtosis	0.08	0.08	0.08	0.08	0.08

b. Neutral thinning

Characteristics	v_{mer}, m^3	dbh, cm	basal area (g), m^2	h, m	h_{tim}, m
mean	0.17	16.24	0.02	12.75	9.38
median	0.10	15.30	0.02	12.30	8.80
minimum	0.00	7.0	0.00	5.70	1.30
maximum	1.16	38.40	0.12	21.70	18.50
lower quartile	0.05	11.60	0.01	9.50	6.00
upper quartile	0.26	20.50	0.03	16.00	13.20
variance	0.03	33.35	0.00	12.95	18.44
stand deviation	0.16	5.78	0.02	3.60	4.30
se. of mean	0.00	0.13	0.00	0.08	0.10
coeff. of var.	97.72	35.56	70.67	28.24	45.84
skewness	1.51	0.56	1.25	0.27	0.11
se. of skewness	0.06	0.06	0.06	0.06	0.06
kurtosis	2.62	-0.37	1.62	-1.18	-1.11
se. of kurtosis	0.11	0.11	0.11	0.11	0.11

(vi) Modelling of the tree variables removed in thinnings

a. Intermediate thinning

Characteristics	mean basal area, m^2		mean dbh, cm		mean total h, m	
	stand	thinned	stand	thinned	stand	thinned
mean	0.05	0.04	23.09	20.05	16.89	16.03
median	0.04	0.03	21.30	17.70	16.10	15.00
minimum	0.01	0.01	10.70	8.10	7.40	7.60
maximum	0.14	0.13	42.20	39.80	29.90	27.0
lower quartile	0.02	0.02	17.00	13.65	12.30	11.80
upper quartile	0.07	0.05	29.25	25.40	21.25	20.00
variance	0.00	0.00	62.62	60.04	30.87	29.45
stand deviation	0.03	0.03	7.91	7.75	5.56	5.43
se. of mean	0.00	0.00	0.81	0.80	0.56	0.55
coeff. of var.	67.66	76.73	34.27	38.64	32.90	33.86
skewness	0.96	1.28	0.52	0.66	0.36	0.37
se. of skewness	0.25	0.25	0.25	0.25	0.24	0.25
kurtosis	0.19	1.05	-0.61	-0.43	-0.81	-0.89
se. of kurtosis	0.49	0.49	0.49	0.49	0.49	0.49

b. Neutral thinning

Characteristic	mean basal area, m^2		mean dbh, cm		mean total h, m	
	stand	thinned	stand	thinned	stand	thinned
mean	0.04	0.03	20.48	16.10	13.88	13.32
median	0.03	0.02	19.60	16.40	14.50	13.20
minimum	0.01	0.01	13.00	9.70	9.00	9.00
maximum	0.06	0.05	27.40	25.10	20.50	18.80
lower quartile	0.02	0.01	17.40	11.50	9.55	4.60
upper quartile	0.05	0.04	23.90	20.80	17.70	16.70
variance	0.00	0.00	19.55	21.62	13.33	10.69
stand deviation	0.01	0.01	4.42	4.65	3.65	3.27
se. of mean	0.00	0.00	0.74	0.78	0.59	0.53
coeff. of var.	38.62	54.26	21.59	27.35	26.31	24.71
skewness	0.35	0.05	-0.12	-0.03	0.05	0.16
se. of skewness	0.39	0.39	0.39	0.37	0.38	0.38
kurtosis	-0.80	-0.84	-1.16	-1.28	-1.43	-1.41
se. of kurtosis	0.79	0.79	0.79	0.79	0.75	0.75

Appendix 2.3: Correlations of the tested explanatory variables with the response variables of the constructed models

(i) Prediction model of diameter at breast height at time $t+\Delta t$

Variable	Intermediate thinning	Neutral thinning
dbh_t, cm	0.994	0.992
age_t, yr	0.743	0.683
$age_{t+\Delta t}$, yr	0.746	0.696
difference of age, yr	0.303	0.526
top height (h_{top}), m	0.806	0.647
total basal area (G), $m^2 ha^{-1}$	0.280	0.102
total tree (N), ha^{-1}	-0.754	-0.517
h_{top}/age_t, $m yr^{-1}$	-0.239	-0.364
G/age_t, $m^2 ha^{-1} yr^{-1}$	-0.560	-0.336
h_{top}/G, $m^{-1} ha$	0.671	0.270

(ii) Prediction model of total height at time $t+\Delta t$

Characteristics	Intermediate thinning	Neutral thinning
h_t, m	0.983	0.983
age_t, yr	0.907	0.923
$age_{t+\Delta t}$, yr	0.921	0.922
difference of age, yr	0.580	0.132
top height (h_{top}), m	0.957	0.927
total no. of trees (N), ha^{-1}	-0.807	-0.740
h_{top}/age_t, $m yr^{-1}$	-0.332	-0.699

(iii) Timber height prediction model

Variable	Intermediate thinning	Neutral thinning
total height (h), m	0.978	0.971
dbh, cm	0.924	0.927
h*dbh, m^2	0.954	0.959

(iv) Total volume prediction model

Characteristics	Intermediate thinning	Neutral thinning
dbh, cm	0.930	0.957
basal area (g), m^2	0.970	0.983
total height (h), m	0.872	0.862
age_t, yr	0.810	0.751
total no. of trees (N), ha^{-1}	-0.608	-0.557
total basal area (G), m^2ha^{-1}	0.030	0.267
top height (h_{top}), m	0.844	0.747
h_{top}/age_t, myr^{-1}	-0.328	-0.329
crown depth (c_h), m	0.419	0.759
crown ratio (c_r)	-0.482	0.113
crown volume (c_{vol}), m^3	0.578	0.820

(v) Merchantable volume prediction model

Variable	Intermediate thinning	Neutral thinning
dbh, cm	0.925	0.956
basal area (g), m^2	0.977	0.986
total height (h), m	0.855	0.868
timber height (h_{tim}), m	0.855	0.888

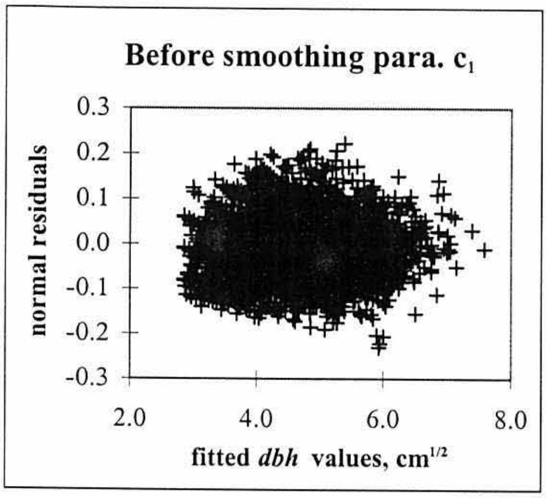
(vi) Prediction models of tree variables removed in thinning

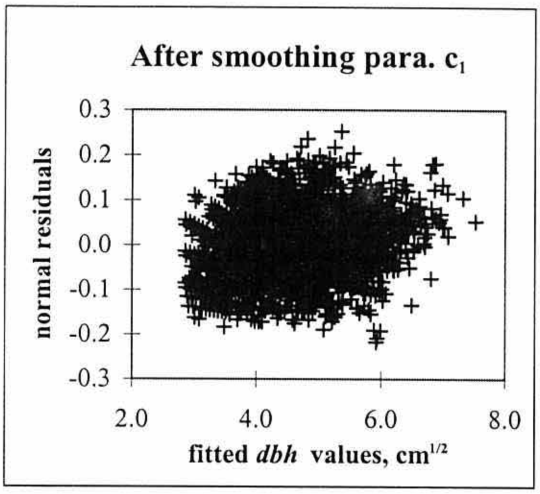
Variable	Intermediate tinning			Neutral thinning		
	$\overline{g}_{bt}$, m^2	$\overline{dbh}_{st}$, cm	$\overline{h}_{bt}$, m	$\overline{g}_{bt}$, m^2	$\overline{dbh}_{st}$, cm	$\overline{h}_{bt}$, m
$\overline{g}_{th}$, m^2	**0.968**	0.948	0.209	**0.954**	0.370	-0.009
$\overline{dbh}_{th}$, cm	0.971	**0.978**	0.229	0.435	**0.968**	0.096
$\overline{h}_{th}$, m	0.388	0.278	**0.993**	-0.031	0.144	**0.993**

Appendix 2.4: Distribution of residuals of the selected models

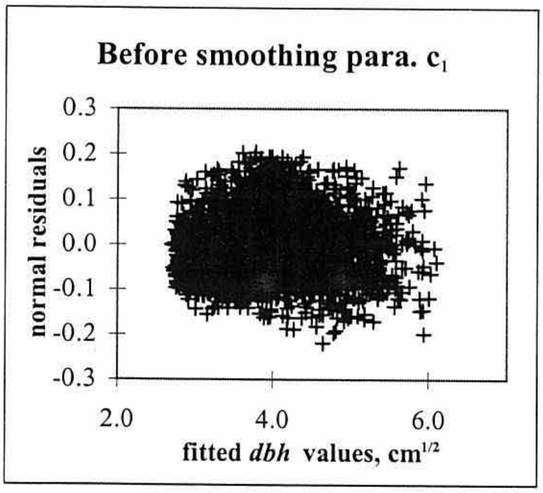
(i) Diameter prediction model *b*

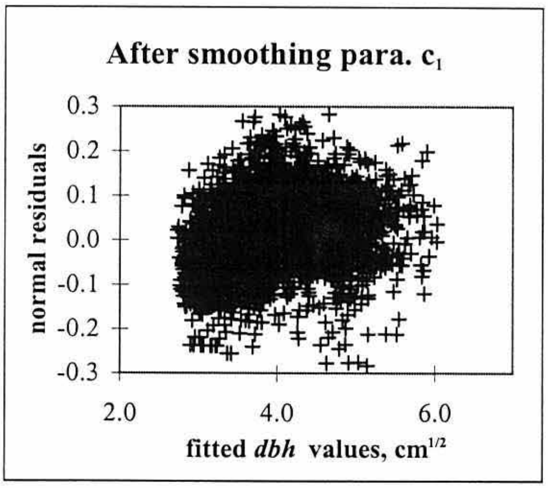
(a) Intermediate thinning



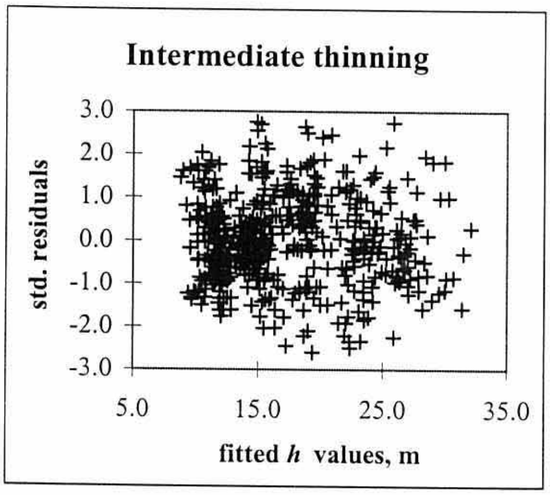


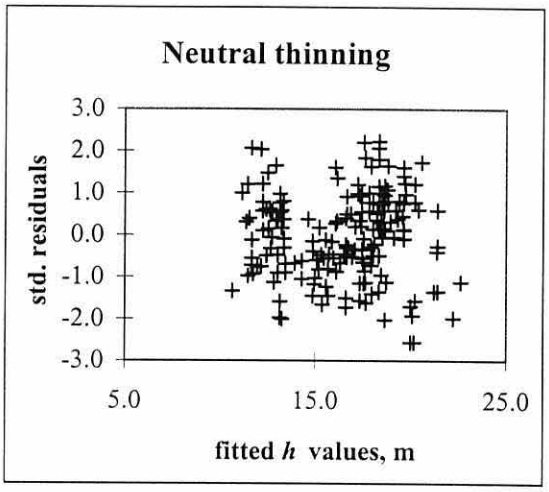
(b) Neutral thinning



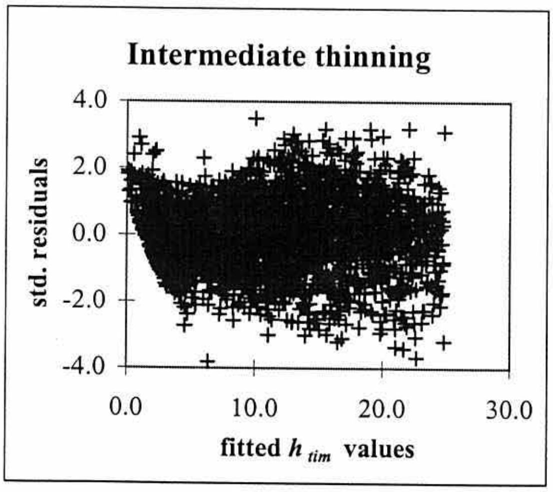


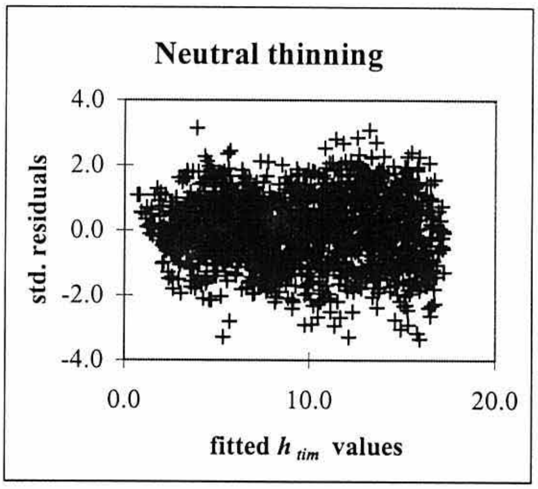
(ii) Total height prediction model *b*



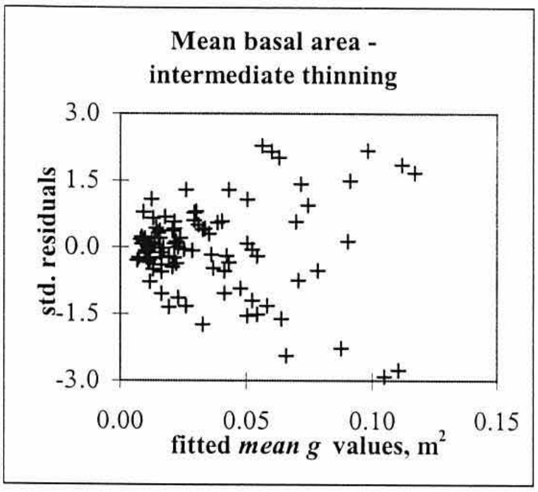


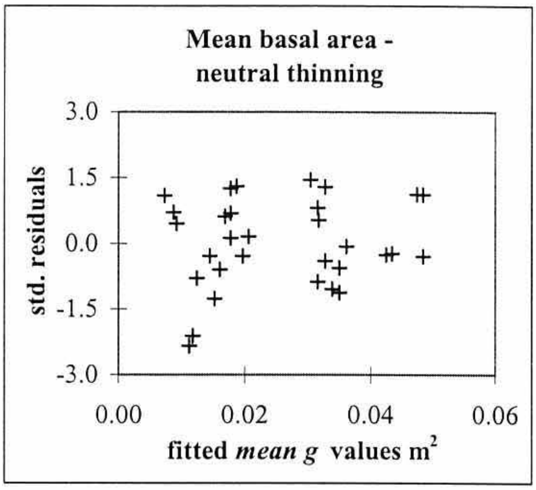
(iii) Timber height prediction model *b*

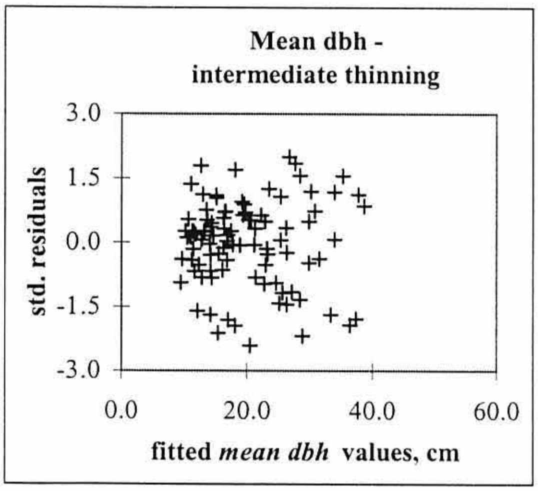


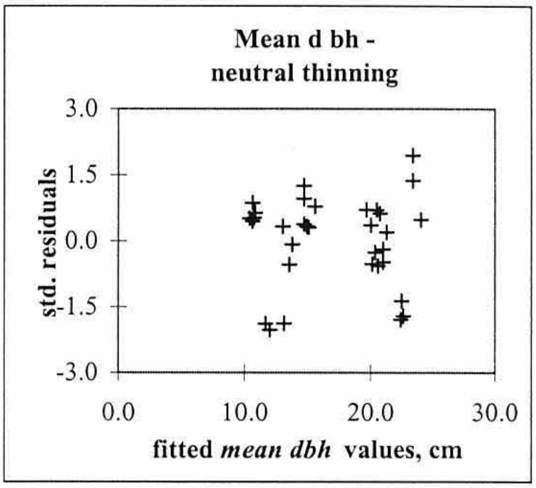


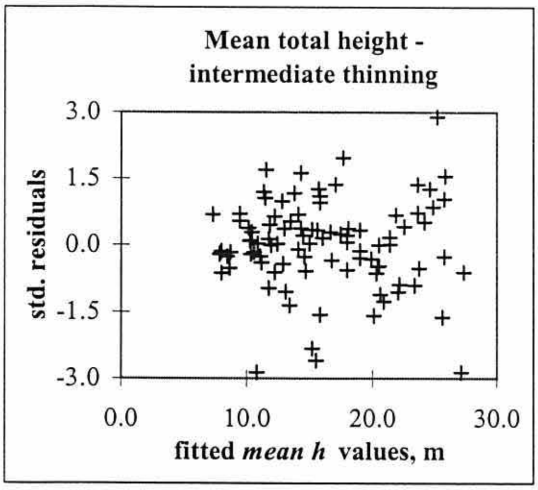
(iv) Model *b* for prediction of the size of thinned trees

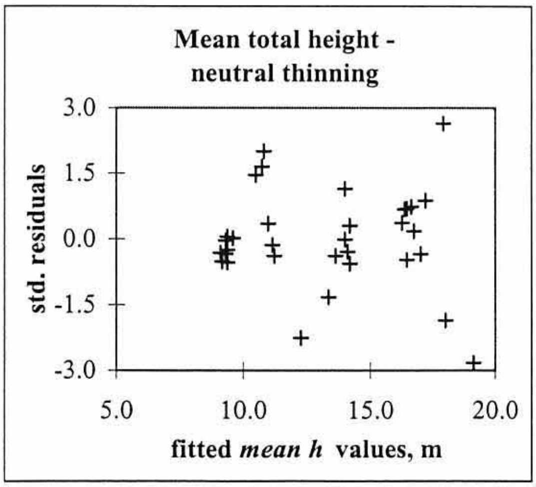












Appendix 3.1: Written programme for the estimation of the parameters of the basal area prediction model constructed by Pienaar and Harrison (1989)

```
"Programme for re-estimating parameters for the basal area
 prediction model developed by Pienaar and Harrison, 1989"

"Written by S.M.C.U.P. Subesinghe "


job 'nonlinear regression'
unit [n=165]
vari [va=1...165] rank

open 'barea.dat'; c=2
read [c=2] a, lnb, lnn, lnh, lnn_a, lnh_a, nt, na, at

"Defines the 7 parameters estiamted by the authors"
scal b[0...7]; va=0.1432, 1.1054, 0.0097, 0.0351, 0.1202, \
       0.2308, 0.0075, 0.1966

"The initial calculations required"
calc ainv = 1/a
calc nt_na = nt/na

"Estimate the fitted values using the initial parameters "
calc guess = b[0] + b[1]*ainv + b[2]*lnn + b[3]*lnh + b[4] \
*lnn_a + b[5]*lnh_a + b[6]*nt_na*(at/a)**b[7]

expr e; value=!e (z = nt_na*(at/a)**b[7])

mode lnb; res=residuals; fitted=fits

rcyc b[7]

"Fits all the paramters together"
fitn [calc=e; selin=y]lnb1, ainv, lnn, \
     lnh, lnn_a, lnh_a, z

print lnb,residuals,fits,guess

stop
```

Appendix 3.2: Programme written for parameter estimation of the basal area projection model built by Pienaar and Harrison (1989)

```
"Programme for re-estimating parameters for the basal area
 projection model developed by Pienaar and Harrison, 1989"

"Written by S.M.C.U.P. Subesinghe"

"Estimates the values for all possible parameter combinations"

OPEN 'barea.dat'; CHAN=2

READ [CHAN=2] lnb1, lnb2, lnn, lnh1, lnh2, na, nt, a1, \
  a2, at

"The essential calculations"
CALC inva1, inva2 = 1/a1, 1/a2
CALC lnh = (lnh2-lnh1)
CALC inva = (inva2-inva1)
CALC lnn_a = ((lnn/a2)-(lnn/a1))
CALC lnh_a = ((lnh2/a2)-(lnh1/a1))
CLAC nt_na, at_a1, at_a2 = nt/na, at/a1, at/a2

"Estiamtes the fitted values without changing the inital
 parameters estiamted by the authors"

"Parameters estimated by the authors for unthinned plantations"
CALC guess1 = lnb1 -25.0905*inva + 0.2255*lnn + 0.9789* \
              lnh +3.0060*lnn_a + 0.8636*lnh_a - 0.1378* \
              (nt_na*((at_a2**2.2995)-(at_a1**2.2995)))

"Parameters estimated by the authors for thinned plantations"
CALC guess2 = lnb1 - 1.1054*inva + 0.0097*lnn + 0.0351* \
              lnh + 0.1202*lnn_a + 0.2308*lnh_a + 0.0013* \
              (nt_na*((at_a2**0.1966)-(at_a1**0.1966)))

EXPR e1; VALUE=!e(z = nt_na*((at_a2**b7)-(at_a1**b7)))
MODE lnb2; RES=residuals; FITTED=fits
RCYC b7
FITN [CALC=e1; CONST=omit; SELIN=yes] lnb1, inva, lnn, \
          lnh, lnn_a, lnh_a, z

PRIN lnb2, residuals, fits, guess1, guess

"Re-parameterization without the parameter b1"
EXPR e2; VALUE=!e(z = nt_na*((at_a2**b7)-(at_a1**b7)))
MODE lnb2; RES=residuals; FITTED=fits
RCYC b7
FITN [CALC=e2; CONST=omit; SELIN=yes; PRIN=summ, esti, \
          fitted] lnb1, lnn, lnh, lnn_a, lnh_a, z

"Re-parameterization without the parameter b2"
EXPR e3; VALUE=!e(z = nt_na*((at_a2**b7)-(at_a1**b7)))
MODE lnb2; RES=residuals; FITTED=fits
RCYC b7
FITN [CALC=e3; CONST=omit; SELIN=yes; PRIN=summ, esti, \
          fitted] lnb1, inva, lnh, lnn_a, lnh_a, z
```

```
"Re-parameterization without the parameter b3"
EXPR e4; VALUE=!e(z = nt_na*((at_a2**b7)-(at_a1**b7)))
MODE lnb2; RES=residuals; FITTED=fits
RCYC b7
FITN [CALC=e4; CONST=omit; SELIN=yes; PRIN=summ, esti, \
          fitted] lnb1, inva, lnn, lnn_a, lnh_a, z

"Re-parameterization without the parameter b4"
MODE lnb2; RES=residuals; FITTED=fits
RCYC b7
FITN [CALC=e4; CONST=omit; SELIN=yes; PRIN=summ, esti, \
          fitted] lnb1, inva, lnn, lnh, lnh_a, z

"Re-parameterion without the parameter b5"
MODE lnb2; RES=residuals; FITTED=fits
RCYC b7
FITN [CALC=e4; CONST=omit; SELIN=yes; PRIN=summ, esti, \
          fitted] lnb1, inva, lnn, lnh, lnn_a, z

"Re-parameterization without the parameter b6"
EXPR e5; VALUE=!e(f2 = (nt_na*((at_a2**b7)- \
                (at_a1**b7))))
MODE lnb2; RES=residuals; FITTED=f2
RCYC b7
FITN [CALC=e5; CONST=omit; SELIN=yes; PRIN=summ,esti, \
          fitted] lnb1, inva, lnn, lnh, lnn_a, lnh_a

"Re-parameterization without the parameters b3 and b4"
EXPR e6; VALUE=!e(z = nt_na*((at_a2**b7)-(at_a1**b7)))
MODE [OFFSET=lnb1] lnb2; RES=residuals; FITTED=fits
RCYC b7
FITN [CALC=e6; CONST=omit; SELIN=yes; PRIN=summ,esti, \
          fittedvalues] lnb1, inva, lnn, lnh_a, z

"Re-parameterization awithout the parameters b3 and b5"
EXPR e7; VALUE=!e(z = nt_na*((at_a2**b7)-(at_a1**b7)))
MODE [OFFSET=lnb1] lnb2; res=residuals; fitted=fits
RCYC b7
FITN [CALC=e7; CONST=omit; SELIN=yes; PRIN=summ,esti, \
          fittedvalues] lnb1, inva, lnn, lnn_a,  z

"Re-parameterization without the parameter b3 and b6"
EXPR e8; VALUE=!e(f3 = nt_na*((at_a2**b7)-(at_a1**b7)))
MODE [OFFSET=lnb1] lnb2; res=residuals; fitted=f3
RCYC b7
FITN [CALC=e8; CONST=omit; SELIN=yes; PRIN=summ,esti, \
          fittedvalues] lnb1, inva, lnn, lnn_a, lnh_a

"Re-parameterization without the parameters b4 and b5"
EXPR e9; VALUE=!e(z = nt_na*((at_a2**b7)-(at_a1**b7)))
MODE [OFFSET=lnb1] lnb2; RES=residuals; FITTED=fits
RCYC b7
FITN [CALC=e9; CONST=omit; SELIN=yes; PRIN=summ,esti, \
          fittedvalues] lnb1, inva, lnn, lnh, z

"Re-parameterization without the parameters b4 and b6"
EXPR e10; VALUE=!e(f4 = (nt_na*((at_a2**b7) - \
          (at_a1**b7))))
MODE [OFFSET=lnb1] lnb2; RES=residuals; FITTED=f2
RCYC b7
FITN [CALC=e10; CONST=omit; SELIN=yes; PRIN=summ,esti, \
          fitted] lnb1, inva, lnn, lnh, lnh_a
```

```
"Re-parameterization without the parameters b5 and b6"
EXPR e11; VALUE=!e(f5 = (nt_na*((at_a2**b7) - \
                                              (at_a1**b7))))
MODE [OFFSET=lnb1] lnb2; RES=residuals; FITTED=f2
RCYC b7
FITN [CALC=e11; CONST=omit; SELIN=yes; PRIN=summ, esti, \
        fitted] lnb1, inva, lnn, lnh, lnn_a

"Re-parameterization without the parameters b3, b4 and b5"
EXPR e12; VALUE=!e(z = nt_na*((at_a2**b7)-(at_a1**b7)))
MODE lnb2; RES=residuals; FITTED=fits
RCYC b7
FITN [CALC=e12; CONST=omit; SELIN=yes; PRIN=summ,esti, \
        fittedvalues] lnb1, inva, lnn, z

"Re-parameterizatio without the parameters b3, b4 and b6"
EXPR e13; VALUE=!e(f6 = nt_na*((at_a2**b7)-(at_a1**b7)))
MODE lnb2; RES=residuals; FITTED=f6
RCYC b7
FITN [CALC=e13; CONST=omit; SELIN=yes; PRIN=summ,esti, \
        fittedvalues] lnb1, inva, lnn, lnh_a

"Re-parmaeterization without the parameter b3, b5 and b6"
EXPR e14; VALUE=!e(f7 = nt_na*((at_a2**b7)-(at_a1**b7)))
MODE lnb2; RES=residuals; FITTED=f7
RCYC b7
FITN [CALC=e14; CONST=omit; SELIN=yes; PRIN=summ,esti, \
        fittedvalues] lnb1, inva, lnn, lnn_a

"Re-parameterization without the parameters b3, b4, b5 and b6"
EXPR e15; VALUE=!e(f8 = nt_na*((at_a2**b7)-(at_a1**b7)))
MODE lnb2; RES=residuals; FITTED=f8
RCYC b7
FITN [CALC=e15; CONST=omit; SELIN=yes; PRIN=summ,esti, \
    fittedvalues] lnb1, inva, lnn

STOP
```

Appendix 3.3: Programme written for estimation of the parameters of the total height prediction model developed by Soares *et al.*(1995)

```
"Estimation of the parameters for the total height prediction
 model developed by Soares et al., 1995"

"Written by S.M.C.U.P. Subasinghe"

OPEN 'ht.dat'; CHANNEL=2
READ [CHANNEL=2] ht,dbh,topht,age,tottree,totba

CALCULATION invage,invdbh = 1/age,1/dbh

"Estimation of all the parameters"
EXPRESSION e1; VALUE=!e(Z1 = (topht**T1*totba**T2*tottree**T3)* \
                        EXP((T4*invage)-(T5*invdbh)))
MODEL ht; res=residuals; fitted=fits
RCYCLE T1,T2,T3,T4,T5
FITNONLINEAR [CALCULATION=e1; CONSTANT=omit; SELINEAR=yes; \
             PRINT=fittedvalues] Z1

"Re-parameterization without the parameter T2"
EXPRESSION e2; VALUE=!e(Z2 = (topht**T1*tottree**T3) \
                       *EXP((T4*invage) \-(T5*invdbh)))
MODEL ht; res=residuals; fitted=fits
RCYCLE T1,T3,T4,T5
FITNONLINEAR [CALCULATION=e2; CONSTANT=omit; SELINEAR=yes] Z2

"Re-parameterization without the parameter T3"
EXPRESSION e3; VALUE=!e(Z3 = (topht**T1*totba**T2) \
                        *EXP((T4*invage)-(T5*invdbh)))
MODEL ht; res=residuals; fitted=fits
RCYCLE T1,T2,T4,T5
FITNONLINEAR [CALCULATION=e3; CONSTANT=omit; SELINEAR=yes] Z3

"Re-parameterization without the parameter T4"
EXPRESSION e4; VALUE=!e(Z4 = (topht**T1*totba**T2*tottree**T3) \
                        *EXP(T5*invdbh))
MODEL ht; res=residuals; fitted=fits
RCYCLE T1,T2,T3,T5
FITNONLINEAR [CALCULATION=e4; CONSTANT=omit; SELINEAR=yes] Z4

"Re-parameterization without the parameters T2,T3"
EXPRESSION e5; VALUE=!e(Z5 = topht**T1*EXP((T4*invage)- \
                        (T5*invdbh)))
MODEL ht; res=residuals; fitted=fits
RCYCLE T1,T4,T5
FITNONLINEAR [CALCULATION=e5; CONSTANT=omit; SELINEAR=yes] Z5

"Re-parameterization without the parameters T3,T4"
EXPRESSION e6; VALUE=!e(Z6 = topht**T1*totba**T2)*EXP(T5*invdbh))
MODEL ht; res=residuals; fitted=fits
RCYCLE T1,T2,T5
FITNONLINEAR [CALCULATION=e6; CONSTANT=omit; SELINEAR=yes] Z6

STOP
```

Appendix 3.4: Programme written for estimation of the parameters of total volume prediction model developed by Soares *et al.* (1995)

```
"Estimation of the parameters of the volume prediction model
  of individual trees developed by Soares et al., 1995."

"Written by S.M.C.U.P.Subasinghe"

"Non significant parameters were ignored and re-parameterized
  at later stages"

OPEN 'vol.dat'; CHANNEL=2

READ [CHANNEL=2] vol,dbh,ht,pidh

CALCULATE invdbh,invht = 1/dbh,1/ht

EXPRESSION e1; VALUE=!e(F1 = pidh*T1*EXP((T2*invht)+(T3*invdbh)))
MODEL vol; res=residuals; fitted=fits

RCYCLE T1,T2,T3
FITNONLINEAR [CALCULATION=e1; CONSTANT=omit]

"Re-parameterization without the parameter T2"
EXPRESSION e2; VALUE=!e(F2 = pidh*T1*EXP(T3*invdbh))
MODEL vol; res=residuals; fitted=fits
RCYCLE T1,T3
FITNONLINEAR [CALCULATION=e2; CONSTANT=omit]

"Re-parameterization without the parameter T3"
EXPRESSION e3; VALUE=!e(F3 = pidh*T1*EXP(T2*invht))
MODEL vol; res=residuals; fitted=fits
RCYCLE T1,T2
FITNONLINEAR [CALCULATION=e3; CONSTANT=omit]

"Re-parameterization without the non linear parameters T2,T3"
EXPRESSION e5; VALUE=!e(F5 = pidh*T1*EXP(invht+invdbh))
MODEL vol; res=residuals; fitted=fits
RCYCLE T1
FITNONLINEAR [CALCULATION=e5; CONSTANT=omit]

STOP
```

Appendix 3.5: Written programme for parameter estimation of total basal area prediction model developed by Soares *et al.* (1995)

```
"Estimation of the parameters of the model developed for
  prediction of the total basal area after thinning by Soares et
  al., 1995"

"Written by S.M.C.U.P. Subasinghe"

OPEN 'ba.dat'; CHANNEL=2

READ [CHANNEL=2] ba1,age1,age2,topht,tottree,ba2

EXPRESSION e1; VALUE=!e(F1 = (ba1**(age1/age2)) \
                 *EXP((1-(age1/age2))*(T1+(T2*topht))))
MODEL ba2; res=residuals; fitted=fits
RCYCLE T1,T2
FITNONLINEAR [CALCULATION=e1; CONSTANT=omit]

"Reparameterization without the parameter T2"
EXPRESSION e2; VALUE=!e(F2 = (ba1**(age1/age2)) \
                 *EXP((1-(age1/age2)) \*(T1+topht)))
MODEL ba2; res=residuals; fitted=fits

RCYCLE T1
FITNONLINEAR [CALCULATION=e2; CONSTANT=omit]

STOP
```

Appendix 3.6: Programme written for parameter estimation of the prediction model of total number of trees after thinning

```
"Estimation of the parameters of the model developed
  by Soares et al. (1995) for prediction of
  number of remaining trees after thinning."

"Written by S.M.C.U.P. Subasinghe"

OPEN 'tree.dat'; CHANNEL=2

READ [CHANNEL=2] tree_at,ba_at,tree_bt,ba_bt
EXPRESSION e1; VALUE=!e(F1 = tree_bt*((1-((1-
(ba_at/ba_bt))**T1))**T2))
MODEL tree_at; FITTED=F1
RCYCLE T1,T2
FITNONLINEAR [CALCULATION=e1; CONSTANT=omit]

STOP
```

Appendix 3.7: Programme written for estimation of parameters of total height prediction model built by West and Mattay (1993)

```
"Estimating of the parameters of total height prediction model
  developed by West and Mattay, 1989"

"Written by S.M.C.U.P. Subasinghe"

OPEN 'ht.dat'; CHANNEL=2

READ [CHANNEL=2] dbh,ht

EXPRESSION e1; VALUE=!e(F1 = 1.3+(dbh/(P+(Q*dbh))))
MODEL ht; FITTED=F1
RCYCLE P,Q
FITNONLINEAR [CALCULATION=e1; CONSTANT=omit]

"Re-parameterization without the parameter Q"
EXPRESSION e2; VALUE=!e(F2 = 1.3+(dbh/(P+dbh)))
MODEL ht; FITTED=F2
RCYCLE P
FITNONLINEAR [CALCULATION=e2; CONSTANT=omit]

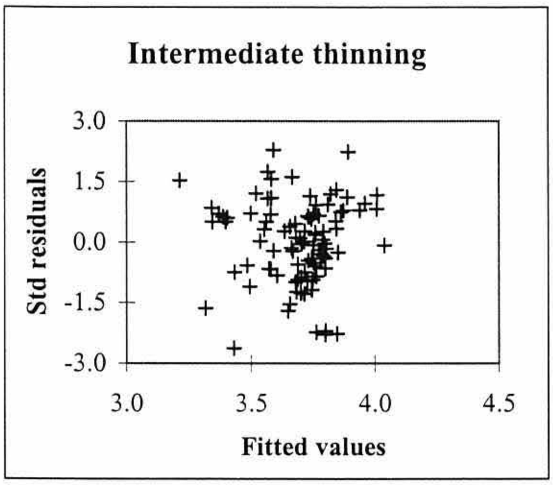
STOP
```

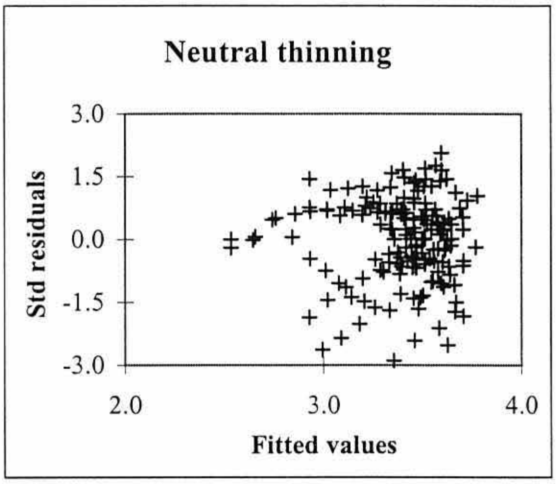
Appendix 3.8: Estimated parameters for total volume prediction model developed by Soares *et al.* (1995)

Age	Intermediate thinning			Neutral thinning		
	R^2	paramet (b_0)	std. error	R^2	paramet (b_0)	std. error
13	0.984	0.4889	0.0046	-	-	-
14	0.966	0.4974	0.0065	-	-	-
16	0.977	0.5150	0.0058	-	-	-
18	0.991	0.4869	0.0031	-	-	-
19	0.992	0.5085	0.0026	0.984	0.5063	0.0013
20	0.978	0.4810	0.0029	-	-	-
21	0.989	0.4854	0.0026	-	-	-
22	0.989	0.4706	0.0027	-	-	-
23	0.991	0.4968	0.0044	-	-	-
24	0.990	0.4917	0.0022	0.989	0.5169	0.0020
25	0.987	0.4919	0.0019	0.986	0.5193	0.0029
26	0.993	0.5003	0.0020	0.997	0.5256	0.0036
27	-	-	-	0.992	0.5218	0.0050
28	0.981	0.4780	0.0035	0.999	0.5355	0.0033
29	0.989	0.5049	0.0025	-	-	-
30	0.998	0.5065	0.0021	-	-	-
31	0.979	0.4907	0.0046	0.985	0.5057	0.0016
32	0.989	0.5111	0.0025	-	-	-
33	0.991	0.4973	0.0032	-	-	-
34	0.981	0.4581	0.0050	-	-	-
36	0.975	0.5162	0.0069	0.984	0.4935	0.0015
37	0.994	0.5294	0.0030	0.986	0.5073	0.0015
38	0.998	0.4983	00025	-	-	-
39	0.976	0.5265	0.0073	-	-	-
40	0.985	0.5061	0.0028	-	-	-
41	0.970	0.5144	0.0068	0.990	0.4896	0.0023
42	0.976	0.5152	0.0033	-	-	-
43	0.962	0.5150	0.0051	-	-	-
44	0.994	0.5211	0.0040	-	-	-
45	0.979	0.4903	0.0027	-	-	-
46	0.991	0.5273	0.0035	-	-	-
47	0.991	0.5095	0.0056	-	-	-
48	0.971	0.5064	0.0045	-	-	-
49	0.989	0.5145	0.0053	-	-	-
50	0.981	0.5016	0.0061	-	-	-
51	0.979	0.5298	0.0042	-	-	-
52	0.952	0.5011	0.0046	-	-	-
53	0.984	0.5093	0.0036	-	-	-
54	0.964	0.5079	0.0063	-	-	-
55	0.988	0.4696	0.0045	-	-	-
56	0.973	0.5167	0.0039	-	-	-
58	0.955	0.5105	0.0095	-	-	-
60	0.908	0.4914	0.0068	-	-	-
62	0.984	0.5053	0.0037	-	-	-
65	0.964	0.4986	0.0055	-	-	-
67	0.935	0.4894	0.0062	-	-	-
68	0.950	0.4742	0.0083	-	-	-

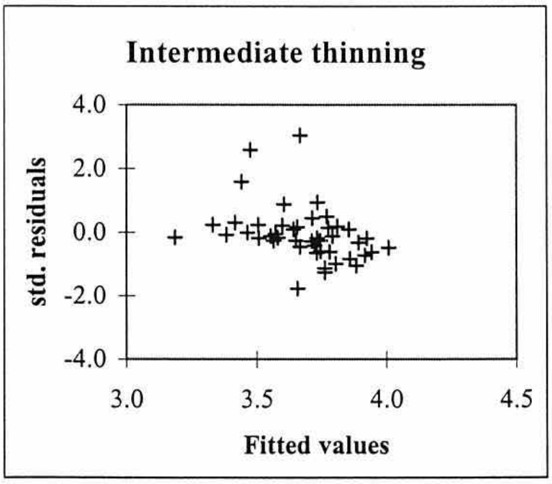
Appendix 3.9: Standardised residuals of the selected models

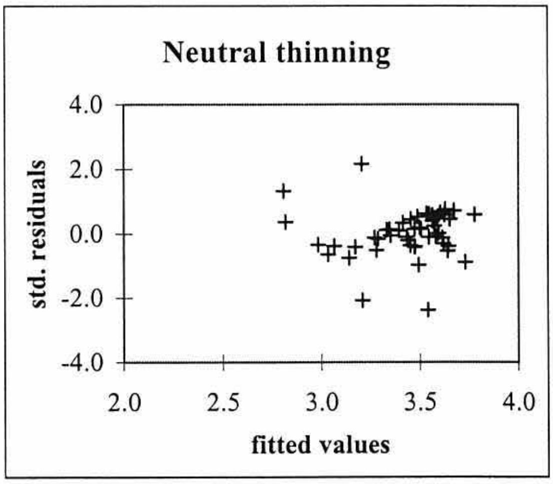
(i) Standard residual distributions of the selected basal area prediction model *b* after re-calibrating the model constructed by Pienaar and Harrison (1989)



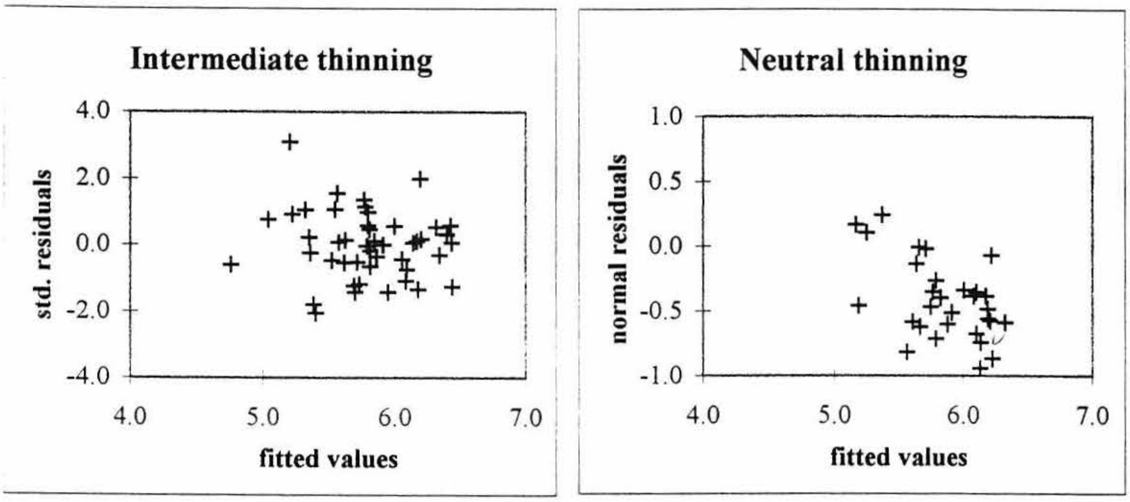


(ii) Standard residual distributions of the selected basal area projection model *b* after re-calibrating the model constructed by Pienaar and Harrison (1989)

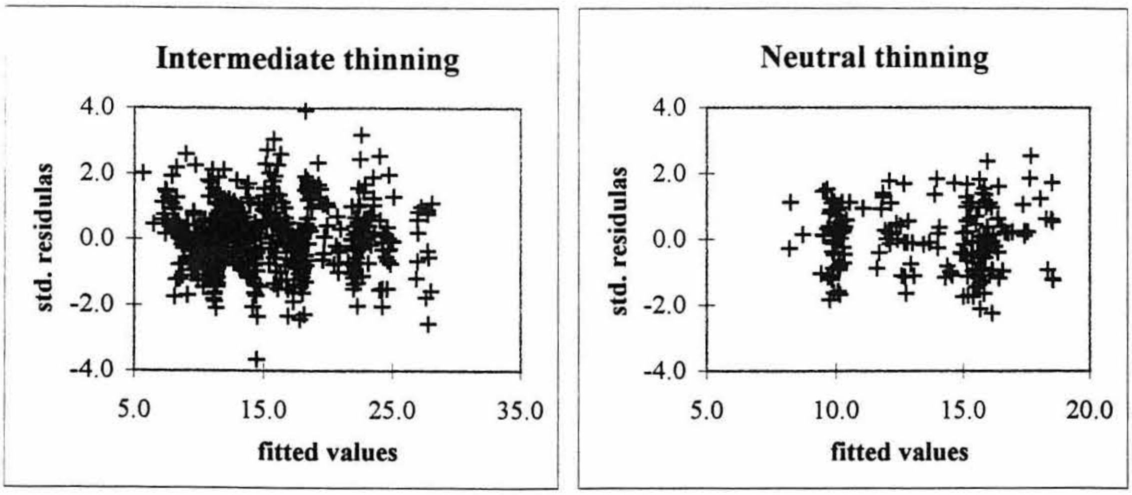




(iii) Residual distributions of the total volume projection model *b* after re-calibrating the model constructed by Pienaar and Harrison (1989)



(iv) Standard residual distributions of the total height prediction model *b* after re-calibrating the initial model developed by Soares *et al*. (1995)

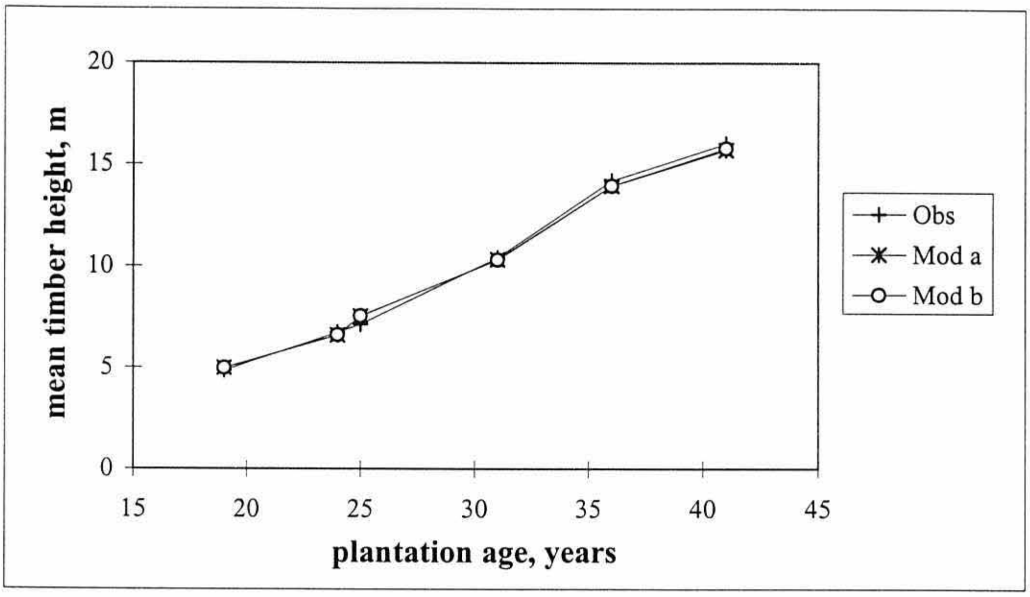


Appendix 4.1: Comparison of the model predictions with the observed values for the neutral thinningtype

(i) Diameter at breast height

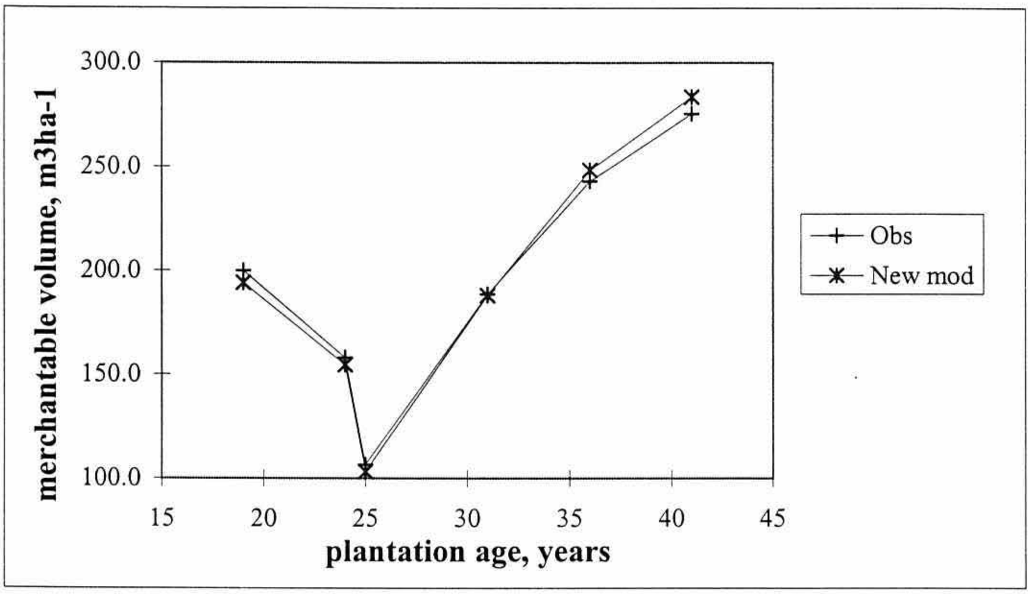
(ii) Total height

(iii) Timber height



(iv) Total volume

(v) **Merchantable volume**



(vi) **Total basal area**

Appendix 5.1: A yield table constructed from the new models

Species: **Corsican pine**

Initial spacing: **1.4m**

Yield Class: **12**

		MAINCROP after Thinning						Yield from THINNINGS						CUMULATIVE PRODUCTION		MAI
Age yrs	Top ht	Trees /ha	Mean dbh	Mean ht	B A /ha	Mean vol	Vol /ha	Trees /ha	Mean dbh	Mean ht	B A /ha	Mean vol	Vol /ha	B A /ha	Vol /ha	Vol /ha
18	7.8	3878	10	7	33	0.02	94	0	0	0	0	0	0	33	94	5.2
23	10.3	2126	13	9	29	0.06	125	1453	11	9	14	0.03	42	43	167	7.3
38	12.5	1421	16	11	29	0.12	164	705	13	10	9	0.06	42	52	248	8.6
33	14.6	1053	18	13	28	0.18	193	368	15	12	7	0.11	42	58	319	9.7
38	16.5	820	20	15	27	0.26	213	233	17	14	5	0.18	42	62	381	10.1
43	18.2	670	23	18	29	0.29	265	150	20	17	5	0.28	42	69	475	11.0
48	19.7	560	25	19	29	0.50	280	110	22	18	4	0.38	42	73	542	11.3
53	21.1	486	27	20	29	0.62	302	74	25	19	4	0.50	37	77	601	11.4
58	22.3	433	29	22	30	0.77	333	53	26	21	3	0.59	31	81	663	11.5
63	23.4	394	31	23	31	0.91	360	39	28	22	2	0.71	27	84	723	11.5
68	24.3	365	33	24	32	1.06	387	29	30	23	2	0.85	25	87	775	11.4
73	25.1	342	35	25	34	0.23	420	23	32	25	2	1.00	23	91	831	11.4
78	25.8	322	37	26	35	1.37	443	20	33	26	2	1.10	22	94	876	11.2