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External optical feedback effects in electrically pumped semiconductor nano-laser arrays

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Abstract: Detailed simulations have been undertaken of the dynamical response of linear and triangular arrays of nano-lasers to external optical feedback from a common external mirror. Careful attention is given to forms of dynamics that arise in such devices when experimentally accessible parameters such as the optical feedback strength, laser bias current, and external cavity length are changed – including for the latter on wavelength scales. In addition, the role played by the strength of the coupling between the nano-lasers is indicated. A salient feature of nano-lasers is the possibility of enhanced spontaneous emission via the Purcell effect and its impact on obtainable dynamics is illustrated. In general, the elements of the arrays display a combination of stable, periodic, quasi-periodic, and chaotic behaviour. Prospects for significant generalizations of the analysis undertaken here are briefly addressed.

1. Introduction

External optical feedback effects on semiconductor lasers have been a topic of interest almost from the beginning of this technology [1]. With a view to their applications in optical fibre communications, much initial interest was stimulated by the deleterious effects on semiconductor laser behaviour when subject to unwanted optical feedback, *e.g.*, from optical fibre facets. On the other hand it was demonstrated that optical feedback could stabilize semiconductor lasers [2]. Theoretical exploration of this behaviour included the development of the Lang-Kobayashi (L-K) model [3] which has proved to be of remarkable utility over subsequent decades (including in the present paper). A classification of the regimes of feedback effects was provided in early work [4]. Much effort was subsequently given to studying the effects of deliberate optical feedback on the semiconductor lasers [5]. Similarly, semiconductor laser arrays have a very long pedigree being specifically designed to provide high optical output powers [6]. However, it was recognized that such devices were susceptible to dynamical instabilities which could compromise their practical utility [7]. Subsequently a large body of work has been undertaken on semiconductor laser arrays [8].

As the designs of semiconductor lasers have evolved over the last half century it has proved worthwhile to consider the impact of optical feedback on novel semiconductor lasers. In recent years attention has been given to the development of optically and electrically pumped semiconductor nano-lasers [9–14]. Attention has been given to the dynamics of mutually-coupled nano-lasers [15] and more recently such structures have received interest from the viewpoint of time-delay suppression for chaotic optical communications [16]. The effects of optical feedback on stand-alone semiconductor nano-lasers has been previously reported [17]. In the present work we extend those studies by considering for the first time the effects of optical feedback on linear and triangular three-element arrays of electrically-pumped nano-lasers. This contribution complements our recent work which has detailed the dynamical behaviour of

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unpertubed nano-laser arrays [18], effects arising due to direct current modulation of such arrays [19] and their response to external optical injection [20]. The novel physical feature in such arrays is the Purcell enhancement of spontaneous emission [21,22] which is well-established a general feature of wave-length scale electromagnetic cavities and which, ipso facto, plays a salient role in the results reported here.

Following a description of the model in Section 2, Section 3 is devoted to representative results illustrating the dynamical behavour in three-element electrically-pumped nano-laser arrays with external optical feedback. Conclusions drawn from the work and indications of further developments are summarized in Section 4.

2. Model

The dynamical model of semiconductor nano-laser arrays with external optical feedback is modified from the traditional L-K coupled rate equations [3,16,23] to accommodate the Purcell enhancement of spontaneous emission. For arrays consisting of M identical nano-lasers coupling to each other and each laser with an identical optical feedback, the rate equations are:

$$\begin{aligned}\frac{dS_j(t)}{dt} = {} & \frac{\Gamma F\beta N_j(t)}{\tau_n} + \Gamma g_n[N_j(t) - N_0]S_j(t) - \frac{S_j(t)}{\tau_p} \\ & - \sum_{\substack{m=1 \\ m \neq j}}^{M} 2k_{jm}S_m(t)\sin[\phi_m(t) - \phi_j(t)] \\ & + 2\kappa_j\sqrt{S_j(t)S_j(t-\tau_j)}\cos[\theta_j(t)]\end{aligned} \tag{1}$$

$$\begin{aligned}\frac{d\phi_j(t)}{dt} = {} & \frac{\alpha}{2}\left\{\Gamma g_n[N_j(t) - N_0] - \frac{1}{\tau_p}\right\} \\ & + \sum_{\substack{m=1 \\ m \neq j}}^{M} \left\{\Delta\omega_{jm} + k_{jm}\frac{S_m(t)}{S_j(t)}\cos[\phi_m(t) - \phi_j(t)]\right\} \\ & - \kappa_j\frac{S_j(t-\tau_j)}{S_j(t)}\sin[\theta_j(t)]\end{aligned} \tag{2}$$

$$\frac{dN_j(t)}{dt} = \frac{I_j}{eV_a} - \frac{N_j(t)}{\tau_n}[F\beta + (1-\beta)] - g_n[N_j(t) - N_0]S_j(t) \tag{3}$$

$$\theta_j(t) = \Delta\phi + \phi_j(t) - \phi_j(t - \tau_j) \tag{4}$$

where the subscripts 'j' and 'm' represent j_{th} and m_{th} laser respectively. t is the time. $S(t)$ is the photon density, $\phi(t)$ is the optical phase and $N(t)$ is the carrier density, $\theta(t)$ is the phase change.

In lasers with external optical feedback, there are two important parameters of importance: feedback strength κ and external cavity length L_{ext}. In principle, using an array of M nano-lasers, one may consider that each laser is subject to optical feedback from individual external mirrors of differing reflectivities and located at differing distances from the lasers. Erring on the side of experimental caution, in the present work it is considered that the array is subject to feedback from a common external mirror and hence only one reflectivity is considered. For simplicity it is further assumed that the nano-lasers in the array are identical. Relaxing either or both of these constraints opens the opportunity for further simulations.

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As illustrated in Fig. 1, simulations are effected here for a linear and triangular array of 3 nano-lasers where nearest-neighbour coupling between the nano-lasers is assumed. The strength of that coupling is determined by the detailed waveguide structure of the nano-lasers and the distance between them. In order to obtain generic results, the coupling strength is treated here as a parameter whose variation will be found to have significant impact on the dynamical behaviours obtained. Such arrays may be accommodated within photonic integrated circuits (PICs) and then consideration will need to be given to the formation of the feedback mirrors. Emphasizing that such fabrication is far from imminent, one may speculate on means for forming such mirrors depending upon the location of the array within a PIC. For an array in the interior of such a PIC one may consider the use of etched grooves whilst in the case that the array is at the edge of the PIC then e.g. a butt-coupled optical fibre can provide access to an external mirror. These two array configurations have been explored in previous work where, in some circumstances the behaviour of linear and triangular arrays has been found to be similar [18]. It may be anticipated that such similarity of behaviour may be encountered when both configurations are subject to similar operating conditions. This conjecture is explored in the simulations reported below.

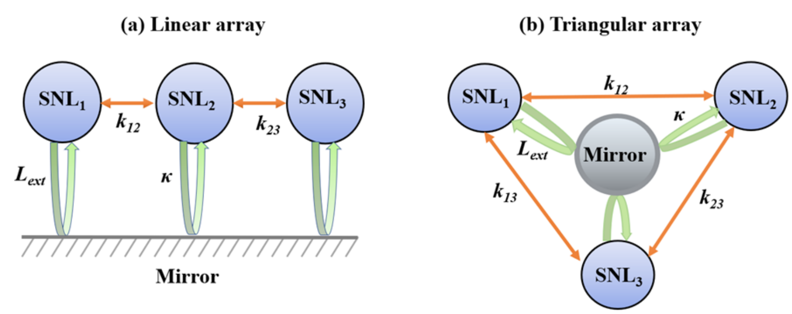

Fig. 1. A schematic diagram of nano-laser arrays with external optical feedback. (a) Linear array, (b) Triangular array.

As indicated, the two main controllable parameters are the feedback strength κ, and the external cavity length L_{ext} or the corresponding external cavity round trip delay time, τ. $\kappa = \eta(1\text{-}r_2)(r_1/r_2)c/(2nL_{in})$, where η is the fraction of reflected field which couples back into the lasing mode. $\eta = (f_{ext}\ r_2)^{1/2}/\ r_1$, where f_{ext} is the feedback fraction. $r_1 = 0.95$ and $r_2 = 0.85$ are the power reflectivities of the external mirror and the front laser facet respectively, c is the speed of light, $n = 3.4$ is the refractive index and $L_{in} = 1.39$ µm is the semiconductor nano-laser (SNL) internal cavity length. Γ is the confinement factor, τ_n is the carrier life time, g_n is the differential gain, N_0 is the transparency carrier density, k is the coupling rate between the two lasers, α is the linewidth enhancement factor, $\Delta\omega$ is the frequency detuning between the two lasers, I is the injection current, e is the elementary charge, V_a is the volume of the active region. Note that $k_{jm} = k_{mj}$ and the external cavity roundtrip delay τ in (1), (2), and (4) is related to L_{ext} via $\tau = 2\ L_{ext}/c$. Note that external cavity length can also be expressed in a way that $L_{ext} = L_0 + \Delta L$, where L_0 is the initial external cavity length, and ΔL is a small variation of external cavity length within the range of [0, $\lambda_0/2$], where λ_0 is the wavelength of the solitary SNL. Feedback phase $\Delta\phi$ is related to ΔL via $\Delta\phi = 4\pi\Delta L/\lambda_0$. The influence of feedback phase is of particular importance due to its laser-wavelength-scale sensitivity. For instance, a half-wavelength change of external cavity length will lead to a phase change of 2π. Within such a change of phase, it has been seen in conventional semiconductor lasers that transitions between stable and unstable states can easily occur [24–28]. However, in practice, it is challenging to maintain a fixed feedback phase even with use of precise phase control elements [26]. For semiconductor nano-lasers which may be operated in photonic integrated circuits, it can be anticipated that the feedback phase will

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be difficult to control. Therefore, from a practical point of view, it is important to investigate the impact of variations feedback phase. Equations (1)–(3) have been solved numerically using a fourth order Runge-Kutta integration method. In the simulations, a temporal resolution of $\Delta t = 0.1$ ps is selected and the duration of the time series is set to be 1 µs. The dynamics of the nano-lasers is analysed using the device parameters given in Table 1 which are taken mainly from [11].

Table 1. Physical Meaning and Value of symbols in (1)–(4)

Symbol	Physical Meaning	Value
F	Purcell factor	5, 20
I	injection current	1.56~2.75 mA
κ	feedback strength	0~106.4 ns^{-1}
τ	external cavity roundtrip delay	6.67~13.3 ps
Γ	confinement factor	0.65
β	spontaneous emission coupling factor	0.05
τ_n	carrier life time	1.00×10^{-9} s
g_n	differential gain	1.65×10^{-12} m^3/s
N_0	carrier density at transparency	1.10×10^{24} m^{-3}
τ_p	photon life time	0.36×10^{-12} s
α	line-width enhancement	5
e	Factor elementary charge	1.60×10^{-19} C
V_a	volume of the active region	3.96×10^{-19} m^3

3. Results

First, analysis of the dynamics of the nano-lasers is performed for $F = 5$, $L_{ext} = 2000$µm, $\Delta\phi = \pi/4$ and $I = 2I_{th}$, where $I_{\mathrm{th}} = 1.1$ mA is the threshold current. Note that the values of feedback phases are also fixed as π/4 for nano-lasers 1 and 3. Figure 2 shows the time series of nano-laser 2 in the linear array with change of feedback coupling fraction when $k_{12} = k_{23} = 5 \times 10^8$. Figure 2(a) shows the time series when the nano-laser is stable with η is 8.16×10^{-4}. Figure 2(b-d) shows the time series when the nano-laser is periodic with η is 3.67×10^{-3},1.34×10^{-2},6.53×10^{-3} respectively. Figure 2(e) shows the time series when the nano-laser is quasi-periodic with η is 1.38×10^{-2}. Figure 2(f) is the time series when the nano-laser is chaotic with η is 1.67×10^{-2}. Note that the time series are normalized to a fixed photon density of 1×10^{22} m^{-3}.

A convenient means for summarizing such dynamical behaviour is by using so-called bifurcation diagrams wherein the nature of the dynamics can be indicated without displaying the detailed time series. Figure 3 provides three such bifurcation diagrams showing the change in the nature of the dynamics with increasing optical feedback strength for the cases of three different external cavity lengths for the nano-laser 2 in either linear or triangular array. These bifurcation diagrams are obtained by recording the local extremum of the time series of the photon density at every different point of η. When the SNL is stable (S), only one extremum, can be found in the temporal waveform of photon density. When the SNL is unstable, two or more extrema are located, and the number of the extrema can be used to define different types of dynamics. For example, a few extrema indicate periodic oscillations (P), clusters of extrema indicate quasi-periodic (QP) oscillations whereas many extrema indicate chaos (C). In Fig. 3(a) the laser is shown to exhibit several forms of periodic behaviour. For the cases of longer external cavities periodic behaviours are interspersed with regions of quasi-periodic and chaotic dynamics.

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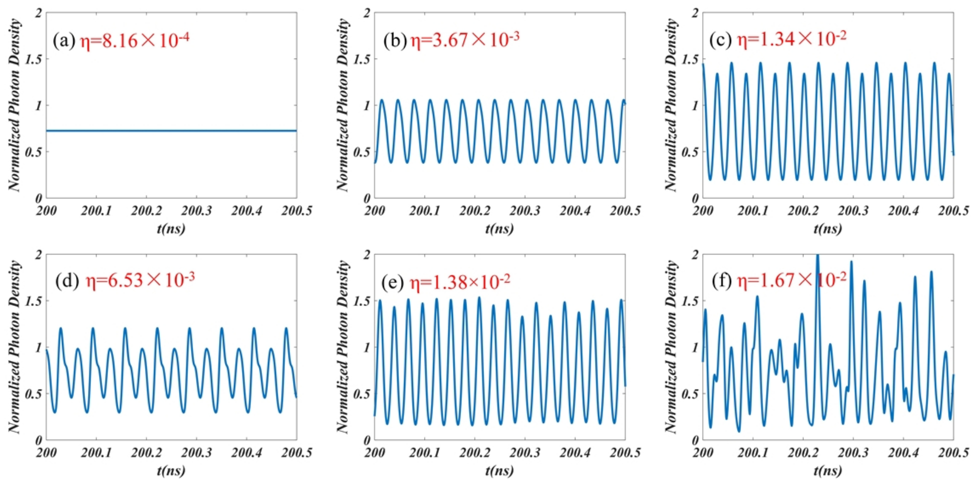

Fig. 2. Time series of nano-laser 2 in the linear array. (a) Stable state, (b-d) periodic states, (e) quasi-periodic state, and (f) chaotic state.

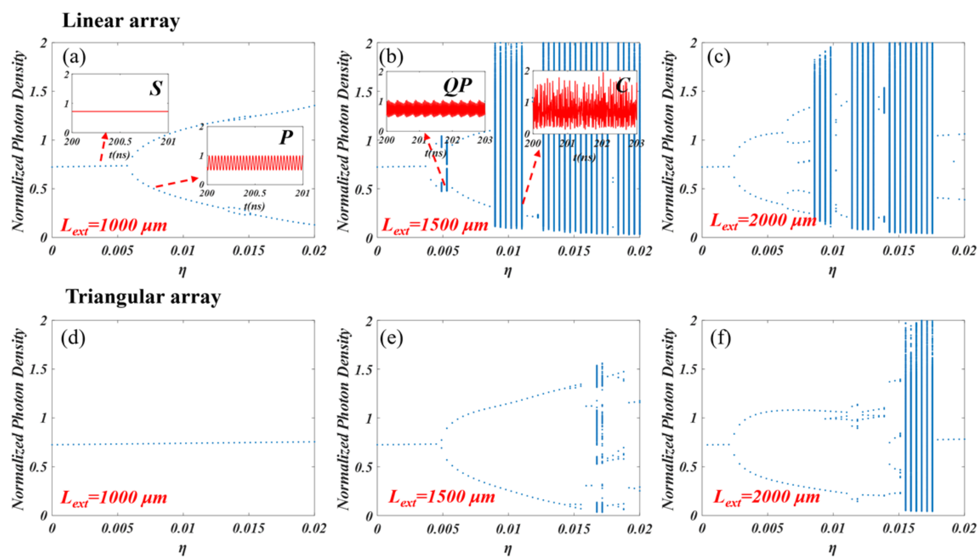

Fig. 3. The bifurcation diagram of linear (when $k_{12} = k_{23} = 5 \times 10^8$) and triangular (when $k_{12} = k_{13} = k_{23} = 5 \times 10^8$) nano-laser 3-element arrays for different external cavity lengths L_{ext} when $F = 5$, $I = 2I_{\text{th}}$, and $\Delta\phi = \pi/4$.

Having identified the species of dynamics which arise in the cases of different nominal external cavity lengths, attention is now given to the sensitivity of the array response to wavelength scale changes in the external cavity length. It has been indicated that a salient feature of nano-lasers is the occurrence of enhanced spontaneous emission via the Purcell effect. The two-dimensional plots in Fig. 4 succinctly illustrate the combined effects of changing the feedback phase and external feedback strengths for different Purcell enhancement factors, F for the linear array. Considering the case of $F = 5$ it is seen that, with the exception of relatively low feedback strengths, variation of the feedback phase effects qualitative changes in the dynamics with varying admixtures of stable, periodic, quasi-periodic and chaotic dynamics arising. Swathes

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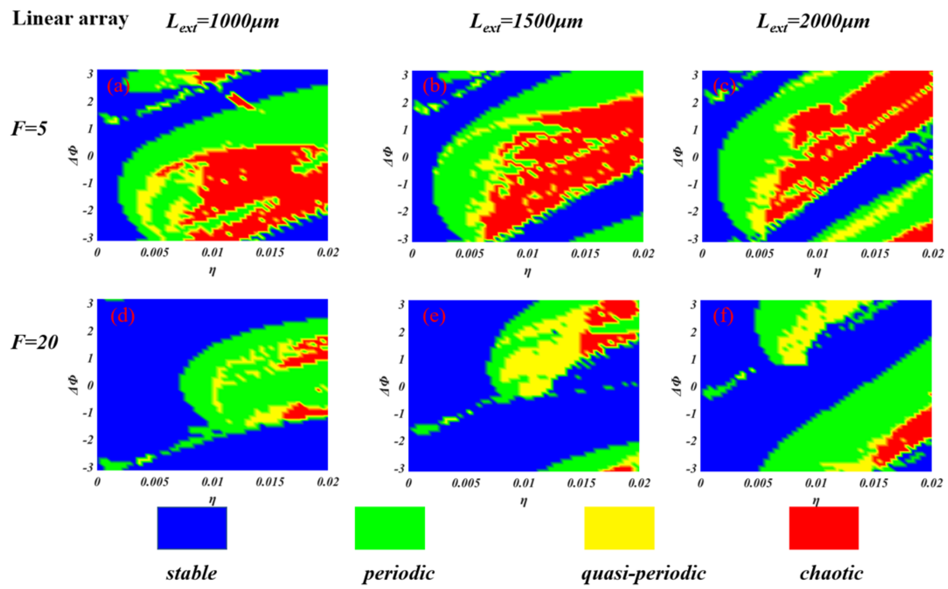

Fig. 4. The two-dimensional dynamics diagram of nano-laser 2 in the linear array when $k_{12} = k_{23} = 5 \times 10^8$ for different values of the Purcell factor and the external cavity length L_{ext}.

of chaotic dynamics are particularly prevalent for feedback strengths in excess of about 0.075. The regions of chaos become more extensive with longer nominal external cavity lengths. In addition, for identifiable ranges of feedback phase variations, the regions of stable and indeed periodic behaviour are enhanced with longer external cavity lengths. Turning attention to the case of Purcell factor, $F = 20$, it is apparent that the enhanced spontaneous emission in this case dampens the response [17] so that the predominant forms of dynamics are stable and periodic behaviour. Islands of quasi-periodic and chaotic dynamics are found but in relatively small bands of the phase-change versus feedback strength parameter space. Such relative stability will be of great benefit for many applications of such arrays.

Figure 5 displays the effect of wavelength scale variations in the external cavity for the triangular array for two values of the Purcell factor. For the lower value ($F = 5$) there is a similar range of dynamical behaviours as obtained in the linear array. However, regions of quasi-periodic and chaotic behaviour are rarer and smaller and thus the predominant features are stable and periodic dynamics. The envelopes of the regions of periodic dynamics for the cases of external cavity lengths of 1500 μm and 2000μm are similar to those for linear arrays but the infills of more complex dynamics are very different. The area of chaotic behaviour is larger for the shortest external cavity considered but in that case stable behaviour is the principal feature of the dynamics. Such stable behaviour is particularly pronounced when attention is given to the case of the higher value of the Purcell factor ($F = 20$). Small residues of chaotic dynamics are evident as well as the occasional appearance of quasi-periodic dynamics but the dynamics is mainly characterized by stability inter-leaved with some periodic dynamics. In essence the difference arises due to the fact that in the triangular array all lasers have two nearest neighbour lasers to which they are coupled. In the linear array, only the central laser is coupled to two lasers.

Apart from the strength of the external feedback and the length of the external cavity, a basic operating parameter which is available for easy experimental variation is the laser bias current. The results presented above were for a bias current of $I = 2I_{\text{th}}$. In Fig. 6 and 7 bias currents of 1.5 and 2.5 the laser threshold current are considered for both linear and triangular arrays respectively.

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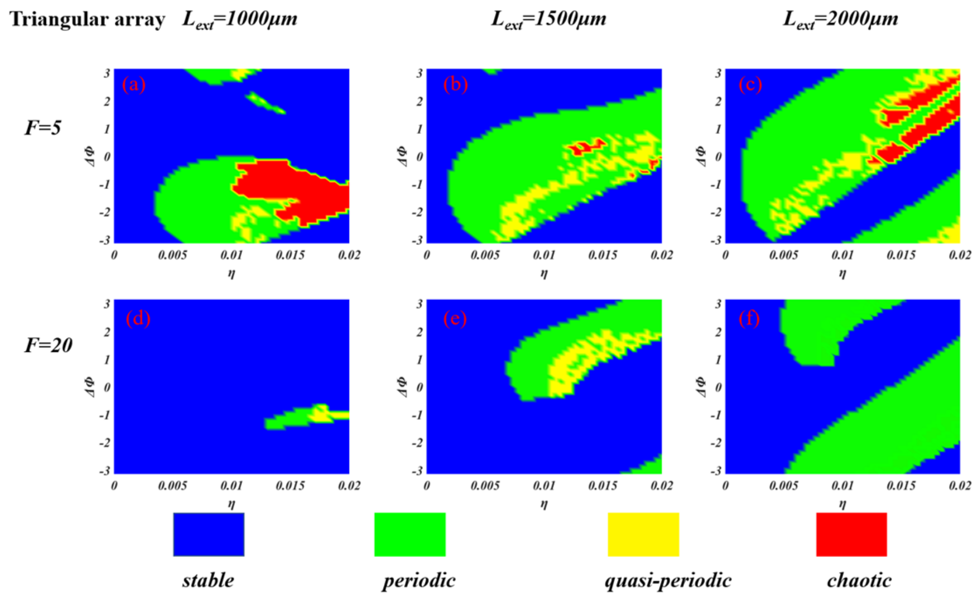

Fig. 5. The two-dimensional dynamics diagram of nano-laser 2 in the triangular array when $k_{12} = k_{13} = k_{23} = 5 \times 10^8$ for different values of the Purcell factor and the external cavity length L_{ext}.

Similarities in behaviour between linear and triangular arrays are again confirmed. The main effect of increasing the bias current is to enlarge the region of chaotic behaviour and shrink the domains of quasi-periodic dynamics. Areas of stable and periodic behaviour are maintained if not enhanced.

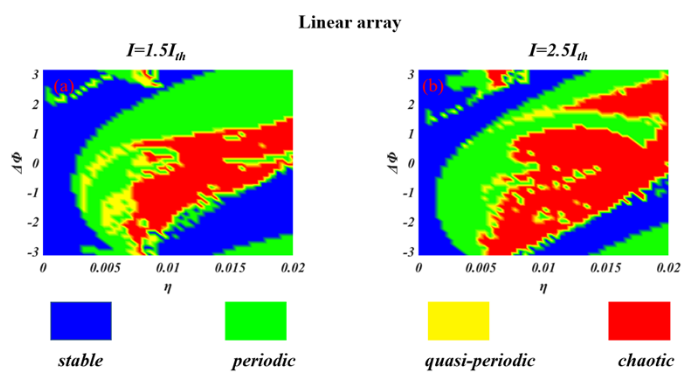

Fig. 6. The two-dimensional dynamics diagram of nano-laser 2 in the linear array when $F = 5$, $L_{\text{ext}} = 1500\,\mu\text{m}$, $k_{12} = k_{23} = 5 \times 10^8$ and (a) $I = 1.5\ I_{\text{th}}$, (b) $I = 2.5\ I_{\text{th}}$.

A key design parameter for the nano-laser array is the strength of the inter-element coupling. It is thus of importance to delineate how this impacts the collective dynamics of the array when subject to external optical feedback. The two dimensional diagrams in Fig. 8 categorise the response of each element of the linear array when the coupling between lasers 2 and 3 are changed for different values of feedback phase and external feedback strength. It is underlined that these varieties of behaviours cannot be explored in a given 3-element array where such strengths are

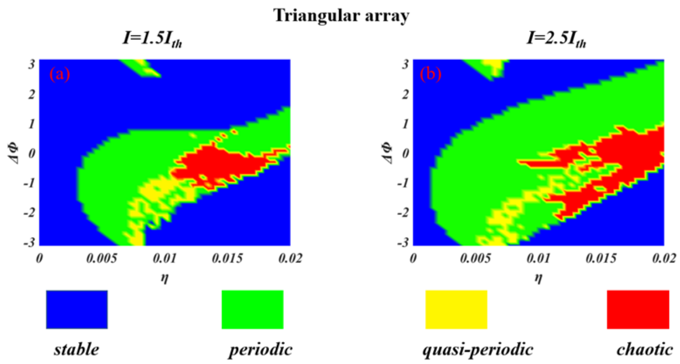

Fig. 7. The two-dimensional dynamics diagram of nano-laser 2 in the triangular array when $F = 5$, $L_{\text{ext}} = 1500\,\mu\text{m}$, $k_{12} = k_{13} = k_{23} = 5 \times 10^8$ and (a) $I = 1.5\,I_{\text{th}}$, (b) $I = 2.5\,I_{\text{th}}$.

fixed. Rather a comparison is made of different arrays appropriately designed to provide the indicated coupling strengths.

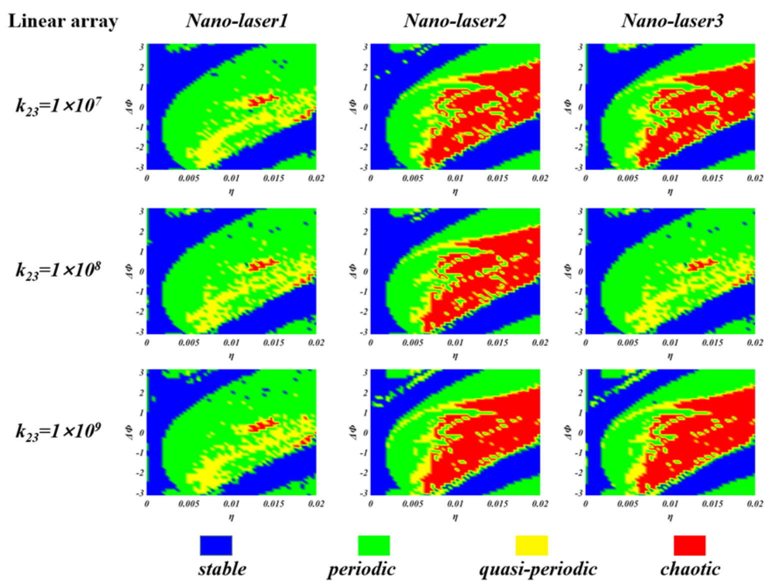

Fig. 8. The two-dimensional dynamics diagrams of nano-lasers 1-3 in the linear array when $k_{12} = 1 \times 10^8$ for different values of k_{23}.

Changing the coupling strengths removes the symmetry which characterized the preceding results and leads to the array elements potentially exhibiting different behaviours. For both the linear array (Fig. 8) and the triangular array (Fig. 9), unsurprisingly the behaviour of nano-laser1 is largely unaffected as the coupling between nano-lasers 2 and 3 is increased by two orders of magnitude. The results here apply to the case of the 1500 μm external cavity and with the Purcell factor, $F = 5$. The predominant feature of the dynamical behaviour of nano-laser 1 is periodic behaviour complemented by stable behaviour and very small regions of quasi-periodic or chaotic dynamics. Nano-lasers 2 and 3 in the linear array (Fig. 8) generally have significant areas of chaotic behaviour with stable behaviour being mainly present at low feedback levels and in bands at higher feedback strengths. For the intermediate value of coupling strength nano-laser 3 in the

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linear array displays rather little chaotic behaviour and more extensive periodic dynamics. That characterization applies again in the case of the triangular array whose dynamics are captured in Fig. 9. There, for the intermediate value of the coupling strength, all three nano-lasers exhibit the same general dynamical features. For higher and lower values of the coupling strength nano-lasers 2 and 3 exhibit prominent domains of chaotic dynamics together with bands of stability and periodic behaviour.

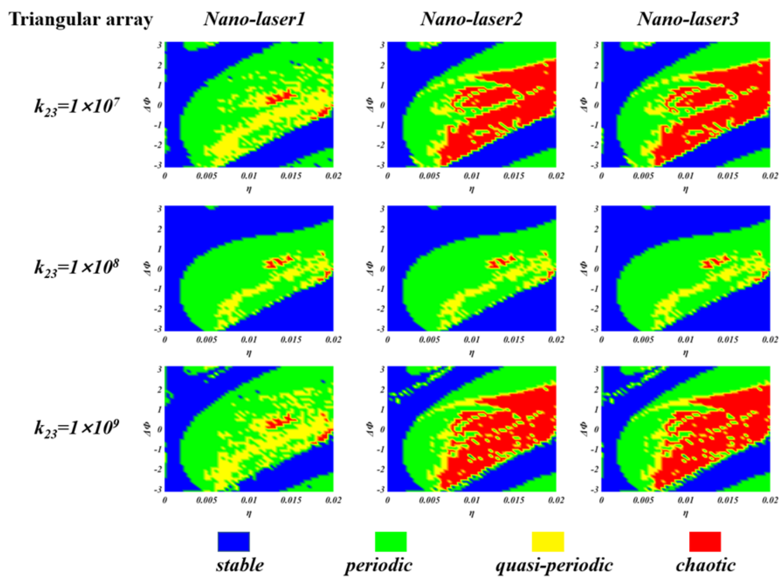

Fig. 9. The two-dimensional dynamics diagrams of nano-lasers 1-3 in a triangular array when $k_{12} = k_{13} = 1 \times 10^8$ for different values of k_{23}.

4. Conclusions

The response of 3-element linear and triangular arrays of nano-lasers to external optical feedback from a common mirror has been explored in some detail. Attention has been given to varying the optical feedback strength, laser bias current, inter-element coupling strength and the external cavity length - including on wavelength scales. A salient feature of nano-lasers is the possibility of enhanced spontaneous emission via the Purcell effect and its impact has been shown here. In combination these parameters have been shown to excite several varieties of dynamics in individual elements of the nano-laser arrays. In most applications it would be expected that stable operation would be the requisite behaviour but also the delivery of periodic outputs may be of interest. The results reported here would inform the operating conditions to obtain such behaviours and indeed chaotic output if that were desired in some applications. It may be anticipated following [25,29] that multistability may be found in the current system by systematically investigating the influence of the initial conditions and the sweep directions of the parameters.

It is recalled that all the simulations reported above are for the case of identical lasers. Despite this restriction it is apparent that considerable variations in the dynamical response of the elements of the arrays may arise. It is to be expected that consideration of non-identical nano-lasers will amplify the variety of dynamics which may be excited in such arrays when subject to external optical feedback. Exploration of such options will form part of our future work. It is intriguing to speculate on the possibilities which may arise if it were possible to effect individual feedback into the array elements and having the options of varying both external cavity length and feedback strengths. A further means for generalizing this work will be to treat arrays with a larger number of elements. There are a number of possible means for efficiently investigating the phenomena which may arise in larger arrays and these also will feature in forthcoming work.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

References

1. R. Broom, E. Mohn, C. Risch, *et al.*, "Microwave self-modulation of a diode laser coupled to an external cavity," IEEE J. Quantum Electron. **6**(6), 328–334 (1970).
2. N. Chinone, K. Aiki, and R. Ito, "Stabilization of semiconductor laser outputs by a mirror close to a laser facet," Appl. Phys. Lett. **33**(12), 990–992 (1978).
3. R. Lang and K. Kobayashi, "External optical feedback effects on semiconductor injection laser properties," IEEE J. Quantum Electron. **16**(3), 347–355 (1980).
4. R. Tkach and A. Chraplyvy, "Regimes of feedback effects in 1.5-μm distributed feedback lasers," J. Lightwave Technol. **4**(11), 1655–1661 (1986).
5. D. M. Kane and K. A. Shore, *Unlocking Dynamical Diversity: Optical Feedback Effects on Semiconductor Lasers*, John Wiley & Sons Ltd, Chichester, U.K. (2005).
6. D. R. Scifres, W. Streifer, and R. D. Burnham, "High-power coupled-multiple-stripe phase-locked injection laser," Appl. Phys. Lett. **34**(4), 259–261 (1979).
7. S. S. Wang and H. G. Winful, "Supermode analysis of phase-locked semiconductor laser arrays," Appl. Phys. Lett. **52**(21), 1774–1776 (1988).
8. D. Botez and D. R. Scifres, eds., *Diode Laser Arrays*, (Cambridge University Press, 1994).
9. M. T. Hill, Y.-S. Oei, B. Smalbrugge, *et al.*, "Lasing in Metallic-Coated Nanocavities," Nat. Photonics **1**(10), 589–594 (2007).
10. M. P. Nezhad, A. Simic, O. Bondarenko, *et al.*, "Room-Temperature Subwavelength Metallo-Dielectric Lasers," Nat. Photonics **4**(6), 395–399 (2010).
11. K. Ding, M. T. Hill, Z. C. Liu, *et al.*, "Record Performance of Electrical Injection Sub-Wavelength Metallic-Cavity Semiconductor Lasers at Room Temperature," Opt. Express **21**(4), 4728–4733 (2013).
12. K. H. Li, X. Liu, Q. Wang, *et al.*, "Ultralow-Threshold Electrically Injected AlGaN Nanowire Ultraviolet Lasers on Si Operating at Low Temperature," Nat. Nanotechnol. **10**(2), 140–144 (2015).
13. Y. Fan and K. A. Shore, "Design of Room Temperature Electrically Pumped Visible Semiconductor Nanolasers," IEEE J. Quantum Electron. **54**(5), 1–7 (2018).
14. K. Ren, C. Li, Z. Fang, *et al.*, "Recent Developments of Electrically Pumped Nano-lasers," Laser Photonics Rev. **17**(5), 2200758 (2023).
15. H. Han and K. A. Shore, "Dynamics and Stability of Mutually Coupled Nano-Lasers," IEEE J. Quantum Electron. **52**(11), 1–6 (2016).
16. X. Zhang, G. Guo, X. Liu, *et al.*, "Dynamics and Concealment of Time-Delay Signature in Mutually Coupled Nano-Laser Chaotic Systems," Photonics **10**(11), 1196 (2023).
17. Z. A. Sattar and K. A. Shore, "External optical feedback effect in semiconductor nano-lasers," IEEE J. Sel. Top. Quantum Electron. **21**(6), 500–505 (2015).
18. Y. Fan, K. A. Shore, and X. Shao, "Dynamics of electrically-pumped semiconductor nano-laser arrays," Photonics **10**(11), 1249 (2023).
19. Y. Fan, S. An, K. A. Shore, *et al.*, "Tailoring the direct current modulation response of electrically-pumped semiconductor nano-laser arrays," Photonics **10**(12), 1317 (2023).
20. Y. Fan, T. Shi, J. Zhang, *et al.*, "Optical injection effects in electrically pumped semiconductor nano-laser arrays," Opt. Express **32**(11), 19361 (2024).
21. E. M. Purcell, "Spontaneous emission probabilities at radio frequencies," Phys. Rev. **69**(1-2), 37–38 (1946).
22. Q. Gu, B. Slutsky, F. Vallini, *et al.*, "Purcell effect in sub-wavelength semiconductor lasers," Opt. Express **21**(13), 15603–15617 (2013).
23. T. Suhr, N. Gregersen, K. Yvind, *et al.*, "Modulation response of nanoLEDs and nanolasers exploiting Purcell enhanced spontaneous emission," Opt. Express **18**(11), 11230–11241 (2010).
24. M. Wolfrum and D. Turaev, "Instabilities of lasers with moderately delayed optical feedback," Opt. Commun. **212**(1-3), 127–138 (2002).
25. S. Wieczorek, B. Krauskopf, T. Simpson, *et al.*, "The dynamical complexity of optically injected semiconductor lasers," Phys. Rep. **416**(1-2), 1–128 (2005).
26. D. Lenstra, "Relaxation oscillation dynamics in semiconductor diode lasers with optical feedback," IEEE Photonics Technol. Lett. **25**(6), 591–593 (2013).
27. D. D'Agostino, H. P. M. M. Ambrosius, M. K. Smit, *et al.*, "Integrated laser with optical feedback shows suppressed relaxation-oscillation dynamics," IEEE Photonics Technol. Lett. **27**(21), 2292–2295 (2015).

Optics EXPRESS

28. D. Zhao, S. Andreou, W. Yao, *et al.*, "Monolithically integrated multiwavelength laser with optical feedback: damped relaxation oscillation dynamics and narrowed linewidth," IEEE Photonics J. **10**(6), 1–8 (2018).
29. N. Li, H. Susanto, B. R. Cemlyn, *et al.*, "Stability and bifurcation analysis of spin-polarized vertical-cavity surface-emitting lasers," Phys. Rev. A **96**(1), 013840 (2017).